Twenty-first Century Lattice Gauge Theory:

Results from the QCD Lagrangian

Andreas S. Kronfeld
Fermilab
Fermilab Colloquium
8 August 2012

as part of the

arXiv:1203.1204

Aim of this talk

- Provide a survey of results about QCD, obtained using numerical lattice gauge theory, that are both
  - quantitatively impressive;
  - qualitatively noteworthy.
- Some quoted results have replaced ignorance, guesses, and beliefs with scientific knowledge.
- Others aid the interpretation of experiments or observations in particle physics, nuclear physics, and astrophysics.

Quantum Chromodynamics—QCD

e.g., H. Fritzsch, M. Gell-Mann, H. Leutwyler, *PLB* **47** (1973) 365

- Modern theory of the strong force: quarks+gluons → hadrons → nuclei.

- A gauge theory, mathematically similar to quantum electrodynamics:

$$\mathcal{L}_{\rm QED} = -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} - \sum_{\text{charged } l} \bar{\psi}_l(\not{D}_l + m_l)\psi_l$$

$$\not{D}_l = \gamma^\mu\left(\partial_\mu + q_l e_0 A_\mu\right)$$

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2g_0^2}F^a_{\mu\nu}F^{\mu\nu a} - \sum_{\text{colored } f} \bar{\psi}^i_f(\not{D} + m_f)_{ij}\psi^i_f$$

$$\not{D}_{ij} = \gamma^\mu\left(\partial_\mu\delta_{ij} + A^a_\mu t^a_{ij}\right)$$

- Now the gauged quantum number is not electric charge, but color.

- SU(3) gauge symmetry: gauge boson “gluon” carries color.

- Laws of Nature.

Color *vs*. colour

- With SU(3), states with equal amounts of colors red, green, and blue (or equal amounts of cyan, magenta, and yellow) are neutral.

- In vision, light (ink) with equal amounts of colours red, green, and blue (equal amounts of cyan, magenta, and yellow) are white (black) or gray.

- Should you spell QCD's color like physicists Greenberg, Gell-Mann, Nambu, …, or like francophile administrators and secretaries at CERN?

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\begin{aligned}\mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \\ &- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \\ &+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}]\end{aligned}$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
  - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$;
  - bafflingly implausible cancellation called the strong CP problem.

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\mathcal{L}_{\rm QCD} = \frac{1}{g_0^2}\,{\rm tr}[F_{\mu\nu}F^{\mu\nu}] \qquad r_1 \text{ or } m_\Omega \text{ or } \Upsilon(2S\text{-}1S), \ldots.$$

$$- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \qquad m_\pi, m_K, m_{J/\psi}, m_\Upsilon, \ldots.$$

$$+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,{\rm tr}[F_{\mu\nu}F_{\rho\sigma}] \qquad \theta = 0.$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
  - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$;
  - bafflingly implausible cancellation called the strong CP problem.

Quantum Chromodynamics

- The most perfect theory—asymptotic freedom.
- Triumph of reductionism: quark model ⊕ parton model ⊕ color = QCD.
- Multi-scale problem: m_{u}, m_{s}, m_π, m_K, Λ_{QCD}, m_{c}, m_{b}, m_{t}; Q^2; a^{-1}; L^{-1}.
- Rich in symmetry: C, P, T; chiral symmetry, heavy-quark symmetry.
- Rich in emergent phenomena: hadron masses, chiral symmetry breaking, phase transitions, atomic nuclei ...
  - ... requiring nonperturbative methods (lattice gauge theory) and a full exploitation of symmetries, asymptotic freedom, *etc*.

Asymptotic Freedom

Politzer, *PRL* **30** (1973) 1346;
Gross, Wilczek, *PRL* **30** (1973) 1343

- At short-distances, the force in QCD looks similar to QED:

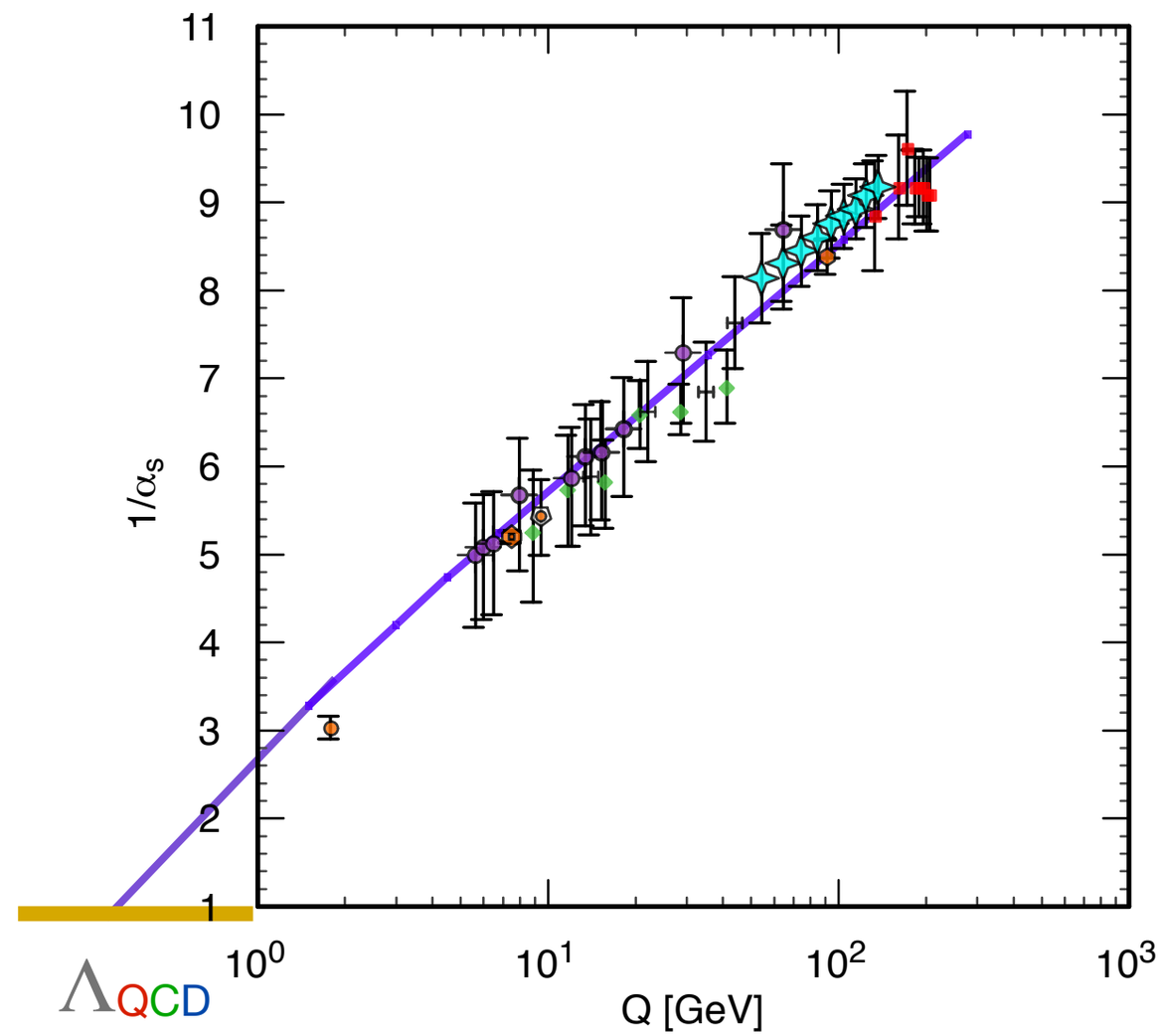
$$F(r) = -\frac{4}{3}\frac{\alpha_s(1/r)}{r^2}$$

where the 4/3 is a color factor.

- The key difference is that virtual gluons reduce the effective α_s at short distances.

- Verified in experiment.

- Relates α_s to a physical scale, Λ_{QCD}.



ASK & Quigg, arXiv:1002.5032

Lattice Gauge Theory

K. Wilson, *PRD* **10** (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].

- Gauge symmetry on a spacetime lattice:

  - mathematically rigorous definition of QCD functional integrals;

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

  - enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.

- Lowest-order strong coupling expansion demonstrates confinement.

Numerical Lattice QCD

- Nowadays “lattice QCD” usually implies a numerical technique, in which the functional integral is integrated numerically on a computer.

- A big computer.

- Some compromises:

  - finite human lifetime ⇒ Wick rotate to Euclidean time: $x^4 = ix^0$;

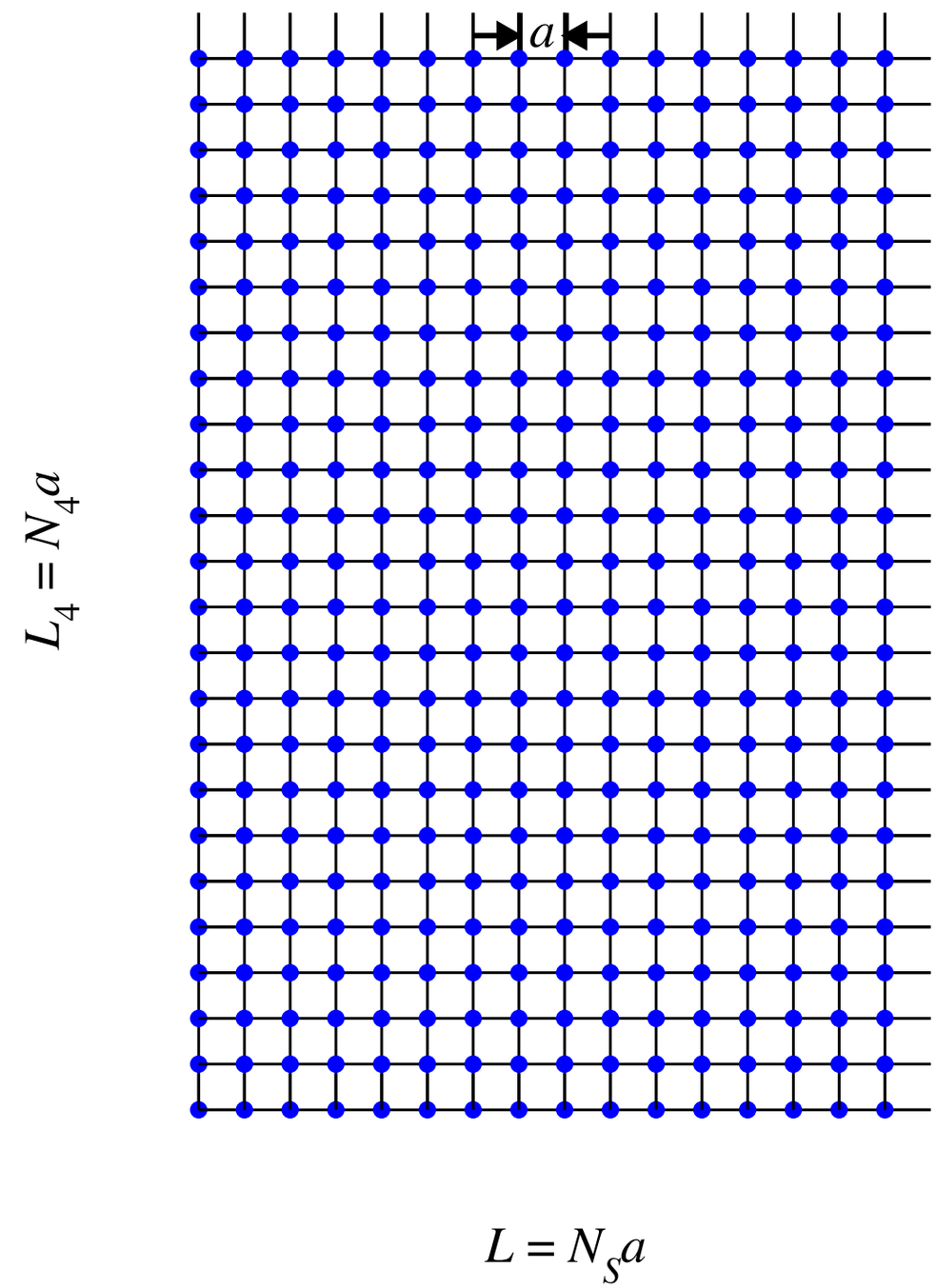
  - finite memory ⇒ finite space volume & finite time extent;

  - finite CPU power ⇒ light quarks often heavier than up and down.

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);
- Infinite lattice: countably many; used to define QFT;
- Finite lattice: finite dimension ~ 10^8, so compute integrals numerically.

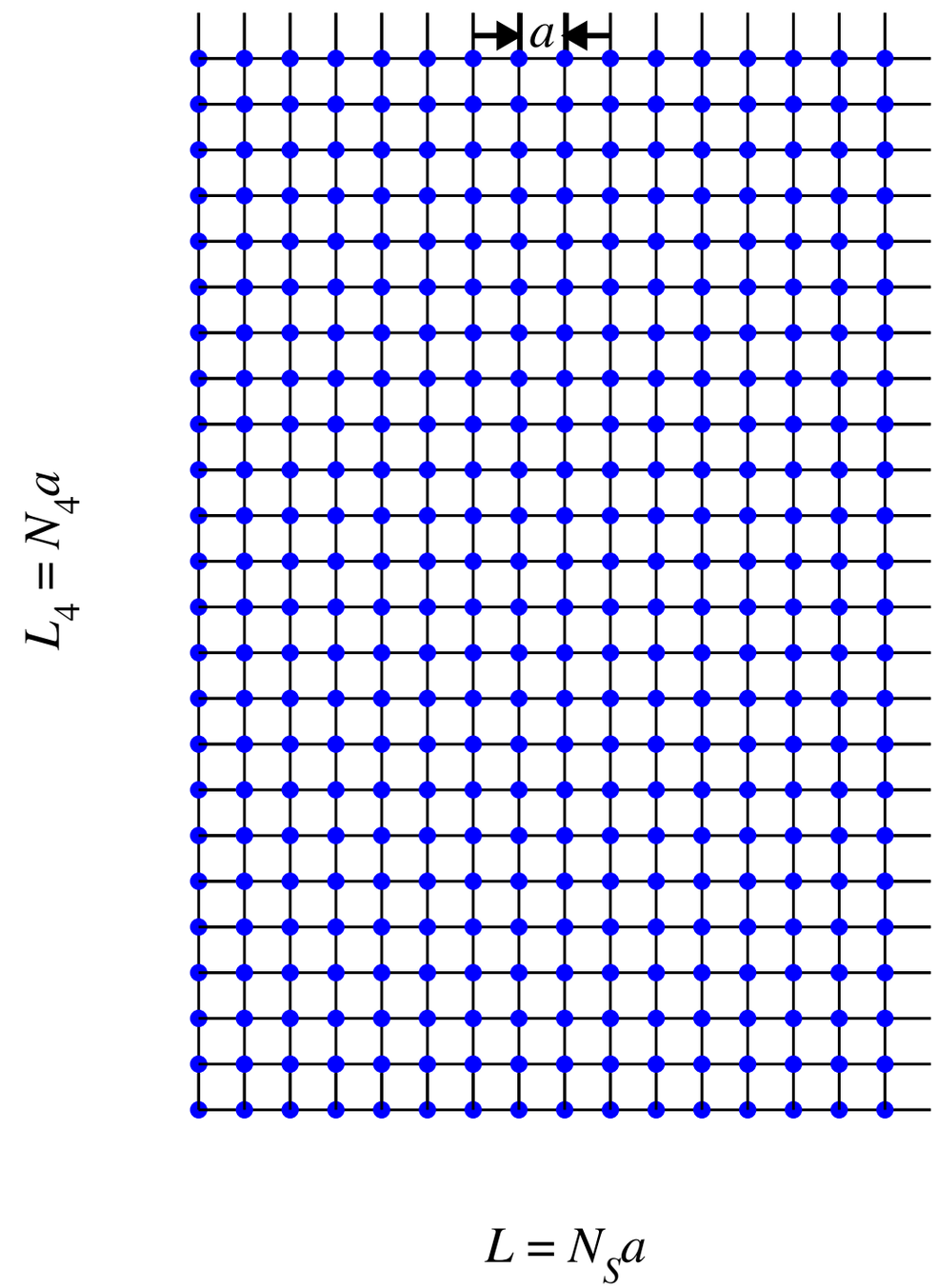


Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

hand

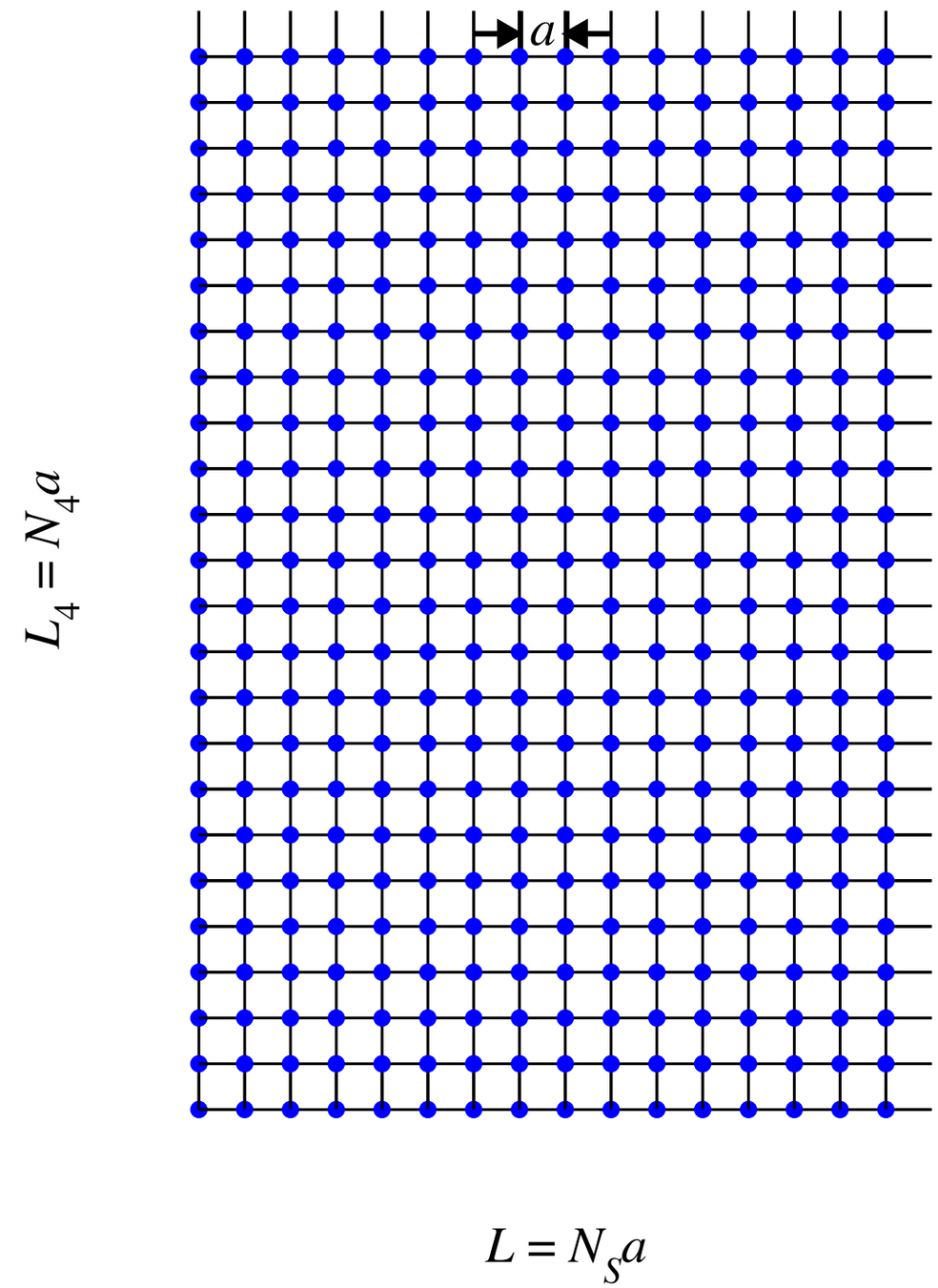
- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);
- Infinite lattice: countably many; used to define QFT;
- Finite lattice: finite dimension ~ 10^8, so compute integrals numerically.



Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \underbrace{\mathcal{D}U}_{\text{MC}} \underbrace{\mathcal{D}\psi \mathcal{D}\bar{\psi}}_{\text{hand}} \exp\left(-S\right) [\bullet]$$

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);
- Infinite lattice: countably many; used to define QFT;
- Finite lattice: finite dimension ~ 10^8, so compute integrals numerically.



Some Jargon

- QCD observables (quark integrals by hand):

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D} + m_f) \exp\left(-S_{\text{gauge}}\right) [\bullet']$$

- *Quenched* means replace $\det$ with 1. (Obsolete.)

- *Un*quenched means not to do that.

- *Partially* quenched (usually) doesn't mean "n_f too small" but $m_{\text{val}} \neq m_{\text{sea}}$, or even $\not{D}_{\text{val}} \neq \not{D}_{\text{sea}}$ ("mixed action").

Some algorithmic issues

e.g., ASK, hep-lat/0205021

- lattice $N_S^3 \times N_4$, spacing a
- memory $\propto N_S^3 N_4 = L_S^3 L_4 / a^4$
- $\tau_g \propto a^{-(4+z)}$, $z = 1$ or 2.
- $\tau_q \propto (m_q a)^{-p}$, $p = 1$ or 2.
- Imaginary time:
  - static quantities
- size $L_S = N_S a$, $L_4 = N_4 a$;
- dimension of spacetime = 4
- critical slowing down
- especially dire with sea quarks
- thermodynamics: $T = 1/N_4 a$

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) [\bullet]$$
$$= \mathrm{Tr}\{\bullet\, e^{-\hat{H}/T}\} / \mathrm{Tr}\{e^{-\hat{H}/T}\}$$

Sea Quarks

- Staggered quarks, with rooted determinant, $\mathrm{O}(a^2)$.
- Wilson quarks, $\mathrm{O}(a)$:
  - tree or nonperturbatively $\mathrm{O}(a)$ improved $\Rightarrow \mathrm{O}(a^2)$;
  - twisted mass term—auto $\mathrm{O}(a)$ improvement $\Rightarrow \mathrm{O}(a^2)$.
- Ginsparg-Wilson (domain wall or overlap), $\mathrm{O}(a^2)$:
  - $\not{D}\gamma_5 + \gamma_5\not{D} = 2a\not{D}^2$ implemented w/ $\mathrm{sign}(\not{D}_\mathrm{W})$.

Sea Quarks

- Staggered quarks, with rooted determinant, $\mathrm{O}(a^2)$.
- Wilson quarks, $\mathrm{O}(a)$:
  - tree or nonperturbatively $\mathrm{O}(a)$ improved $\Rightarrow \mathrm{O}(a^2)$;
  - twisted mass term—auto $\mathrm{O}(a)$ improvement $\Rightarrow \mathrm{O}(a^2)$.
- Ginsparg-Wilson (domain wall or overlap), $\mathrm{O}(a^2)$:
  - $\not{D}\gamma_5 + \gamma_5\not{D} = 2a\not{D}^2$ implemented w/ $\mathrm{sign}(\not{D}_\mathrm{W})$.

fast

clean

- Many numerical simulations with sea quarks are called (perhaps misleadingly) “unquenched” or “full QCD.”

  - $n_f = 2$: with same mass, omitting strange sea;

  - $n_f = 3$: may (or may not) imply 3 of same mass;

  - $n_f = 2+1$: strange sea + 2 as light as possible for up and down;

  - $n_f = 2+1+1$: add charmed sea to $2+1$.

- “Full QCD” can also mean $m_{\rm val} = m_{\rm sea}$, or $\not{D}_{\rm val} = \not{D}_{\rm sea}$.

- Many numerical simulations with sea quarks are called (perhaps misleadingly) "unquenched" or "full QCD."

  - $n_f = 2$: with same mass, omitting strange sea;

  - $n_f = 3$: may (or may not) imply 3 of same mass;

  - $n_f = 2+1$: strange sea + 2 as light as possible for up and down;

  - $n_f = 2+1+1$: add charmed sea to $2+1$.

- "Full QCD" can also mean $m_{\rm val} = m_{\rm sea}$, or $\not{D}_{\rm val} = \not{D}_{\rm sea}$.

- Many numerical simulations with sea quarks are called (perhaps misleadingly) “unquenched” or “full QCD.”

  - $n_f = 2$: with same mass, omitting strange sea;

  - $n_f = 3$: may (or may not) imply 3 of same mass;

  - $n_f = 2{+}1$: strange sea + 2 as light as possible for up and down;

  - $n_f = 2{+}1{+}1$: add charmed sea to $2{+}1$.

- “Full QCD” can also mean $m_{\rm val} = m_{\rm sea}$, or $\not{D}_{\rm val} = \not{D}_{\rm sea}$.

Correlators Yield Masses & Matrix Elements

- Two-point functions for masses $\pi(t) = \bar{\psi}_u \gamma_5 S \psi_d$:

$$\langle \pi(t) \pi^\dagger(0) \rangle = \sum_n |\langle 0 | \hat{\pi} | \pi_n \rangle|^2 \exp(-m_{\pi_n} t)$$

- Two-point functions for decay constants:

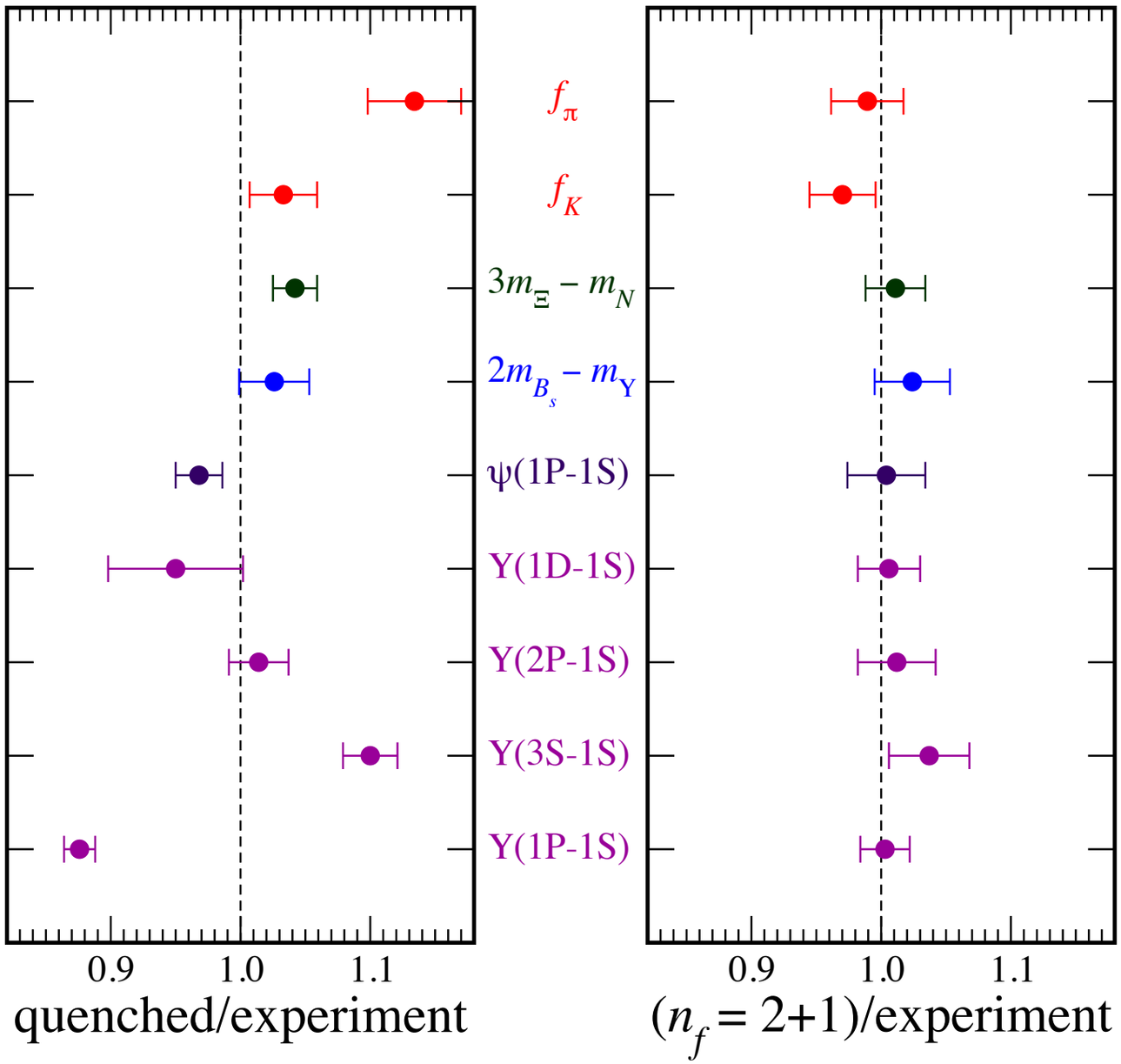
$$\langle J(t) \pi^\dagger(0) \rangle = \sum_n \langle 0 | \hat{J} | \pi_n \rangle \langle \pi_n | \hat{\pi}^\dagger | 0 \rangle \exp(-m_{\pi_n} t)$$

- Three-point functions for form factors, mixing:

$$\langle \pi(t) J(u) B^\dagger(0) \rangle = \sum_{mn} \langle 0 | \hat{\pi} | \pi_m \rangle \langle \pi_n | \hat{J} | B_m \rangle \langle B_m | \hat{B}^\dagger | 0 \rangle$$
$$\times \exp[-m_{\pi_n}(t-u) - m_{B_m} u]$$

2+1 Sea Quarks!

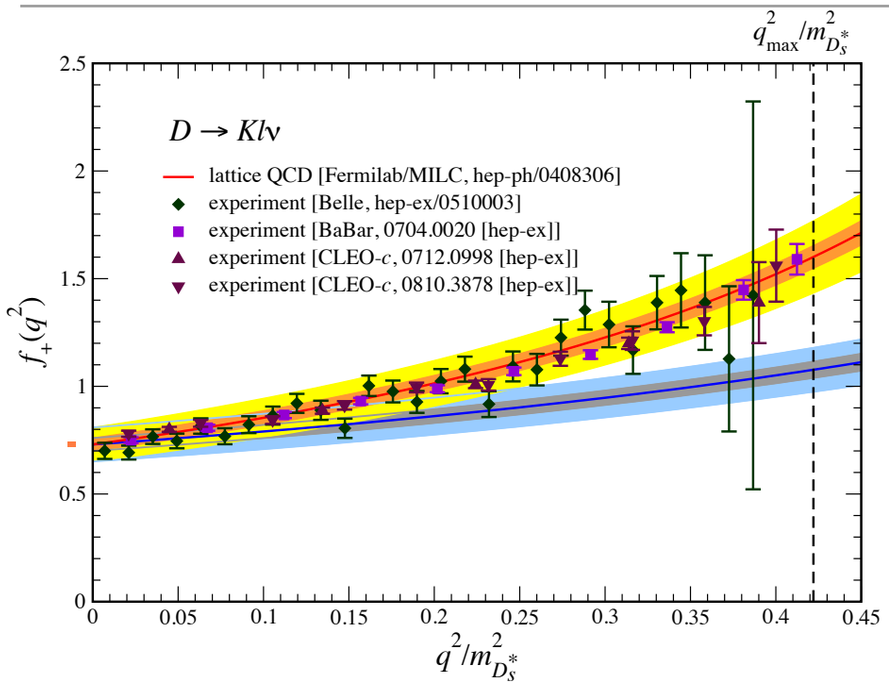
HPQCD, MILC, Fermilab Lattice, hep-lat/0304004



- $a = 0.12$ & 0.09 fm;
- $O(a^2)$ improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, Υ(2S-1S) input.

Predictions

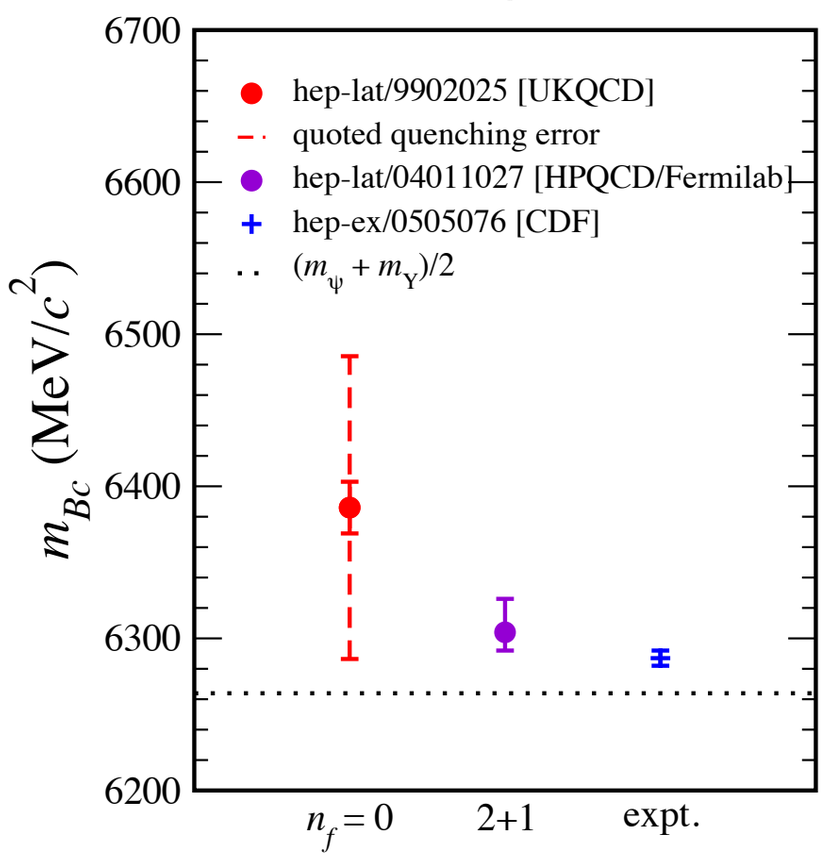
Fermilab Lattice, MILC, HPQCD,
hep-ph/0408306, hep-lat/0411027, hep-lat/0506030

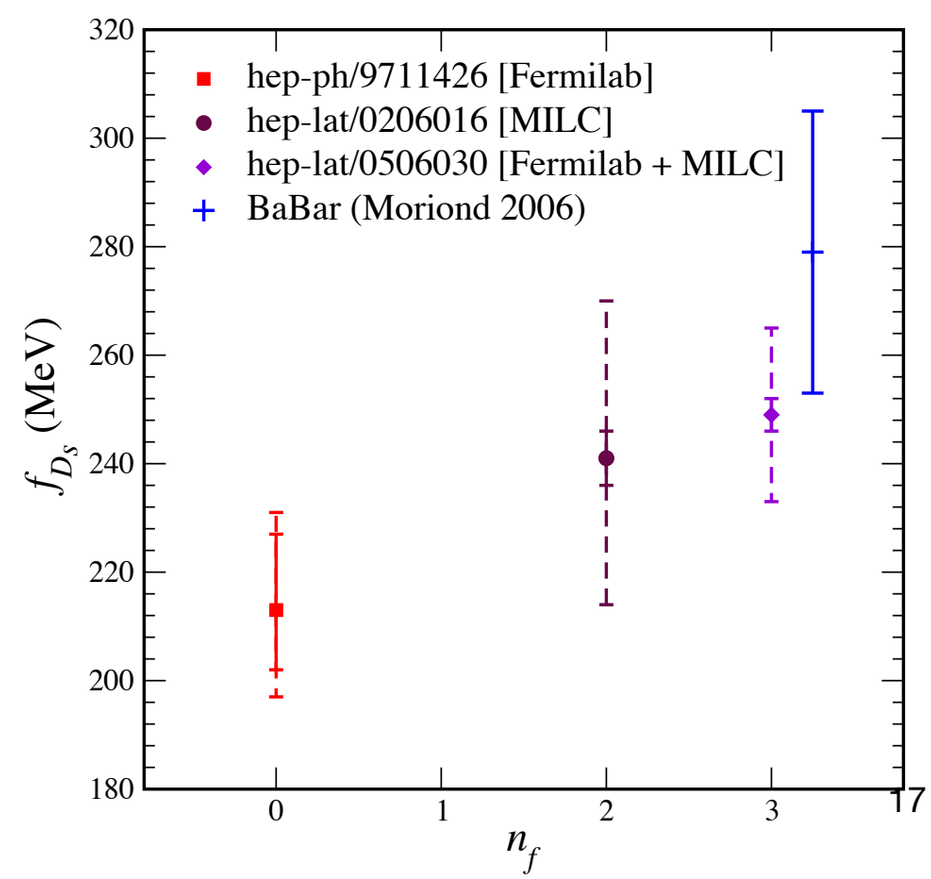


- Semileptonic form factor for $D \to Kl\nu$
- Mass of B_c meson
- Charmed-meson decay constants

2004

2005





Outline

- Introduction
- Chiral Symmetry Breaking
- Hadron Spectrum
- QCD Parameters
- Flavor Physics
- Thermodynamics
- Summary & Challenges

Chiral Symmetry Breaking

Chiral Symmetry

Y. Nambu, *PRL* **4** (1960) 380

- The hadron spectrum has a striking feature:
  - $m_\pi = 135\ \mathrm{MeV}$ but $m_\varrho = 770\ \mathrm{MeV}$, $m_\mathrm{p} = 938\ \mathrm{MeV}$, *etc*.
- Nambu applied lessons from superconductivity, noting (4 years before quarks) that the pion's small mass could obtain from a *spontaneously* broken *axial* symmetry (moderated with a small amount of explicit breaking).
- QCD explained the origin: if up and down quark masses are neglected, the Lagrangian has an $SU_L(2)\times SU_R(2)$ chiral symmetry, which provides candidate axial symmetry.
- BTW, pions break $SU_L(2)\times U_Y(1)$: without terascale EWSB, $W^\pm$ and Z would eat the pions and have masses around 100 MeV; in nature, ~0.1% mixing.

- In the 20th century, we were already confident that QCD was a good theory of the strong interactions, based on, *e.g.*, its explanation of the SLAC deep-inelastic scattering experiments.

- Because QCD was (considered) right, and since Nambu's picture was (considered) right, it was believed that QCD must drive spontaneous chiral symmetry breaking.

- But does it?

- Goldstone formula: $m_\pi^2 \langle \bar{\psi}\psi \rangle = 0$, if $\langle \bar{\psi}\psi \rangle \neq 0$, then $m_\pi = 0$.

- What is it?

Chiral Condensate

e.g., H. Fukaya *et al*. [JLQCD], *PRD* **83** (2011) 074501

$m_{\mathrm{u}},\ m_{\mathrm{d}} \to 0,\ \ m_{\mathrm{s}}$ physical

$$\langle\bar{\psi}\psi\rangle^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = \left[234 \pm 4_{\mathrm{stat}} \pm 17_{\mathrm{syst}}\ \mathrm{MeV}\right]^3$$

- At the hadronic level, the spontaneous breaking of chiral symmetry allows the nucleon mass to be nonzero, even when $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$.

- In nature, m_{u} & m_{d} are small, so the physical picture of chiral symmetry is:
  - dominantly spontaneously broken (Nambu's mechanism);
  - small corrections from explicit breaking (chiral perturbation theory).

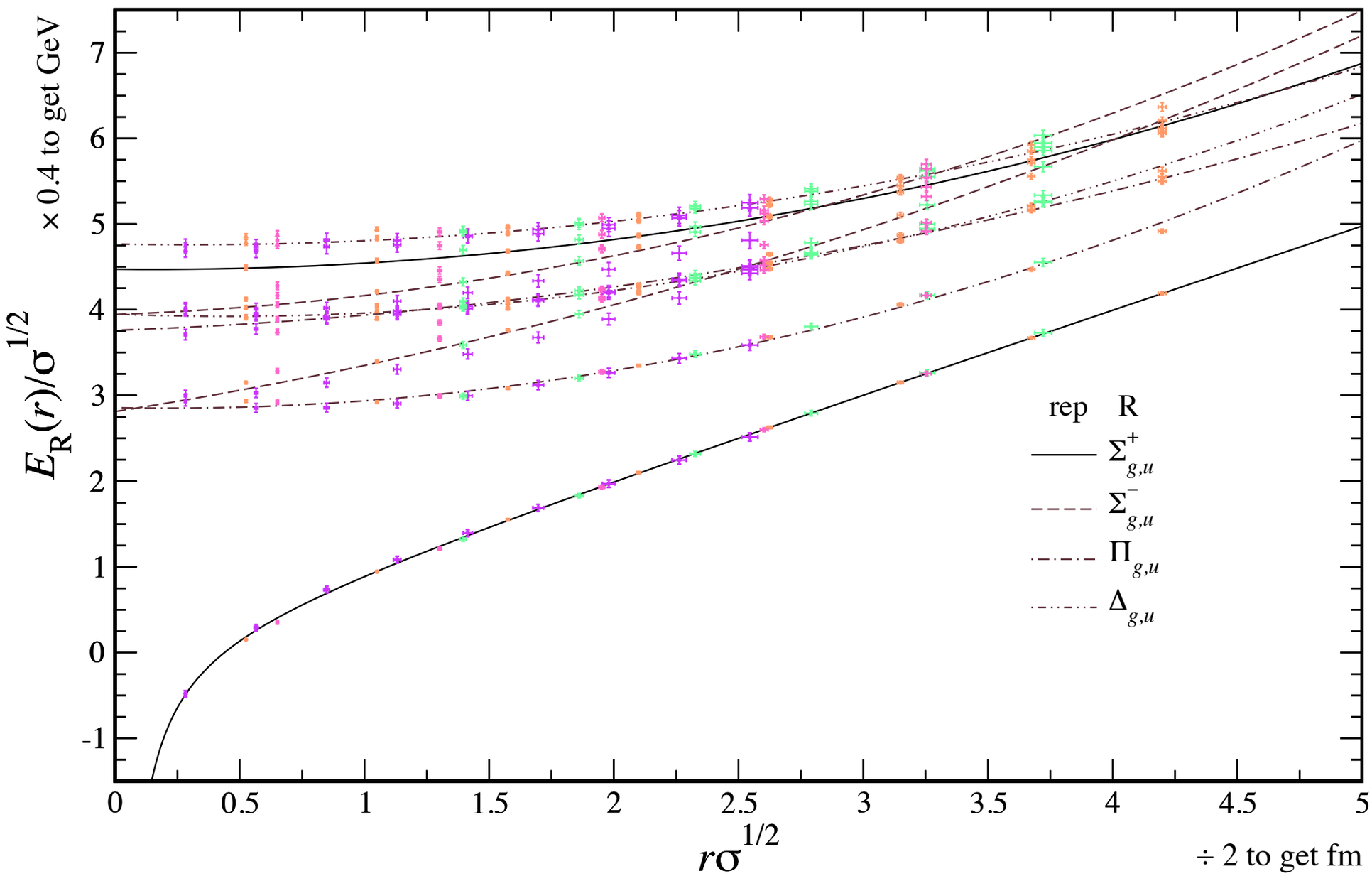
Hadron Spectrum

Why Compute Hadron Masses?

- Show that the QCD Lagrangian generates hadron masses.
- Understand more deeply the only known mechanism for generating masses.
- Study first the chromodynamic energy stored between static sources:
  - lowest level is the potential energy: at short distances, it is Coulombic, but at large distances, it soon turns linear, as a string would;
  - excitations are interesting too: at short distances, level-ordering and level-spacing is QED-like; at hadronic distances (~1–2 fm), the level-ordering becomes string-like; at very large distances (> 2 fm), the spacing too.

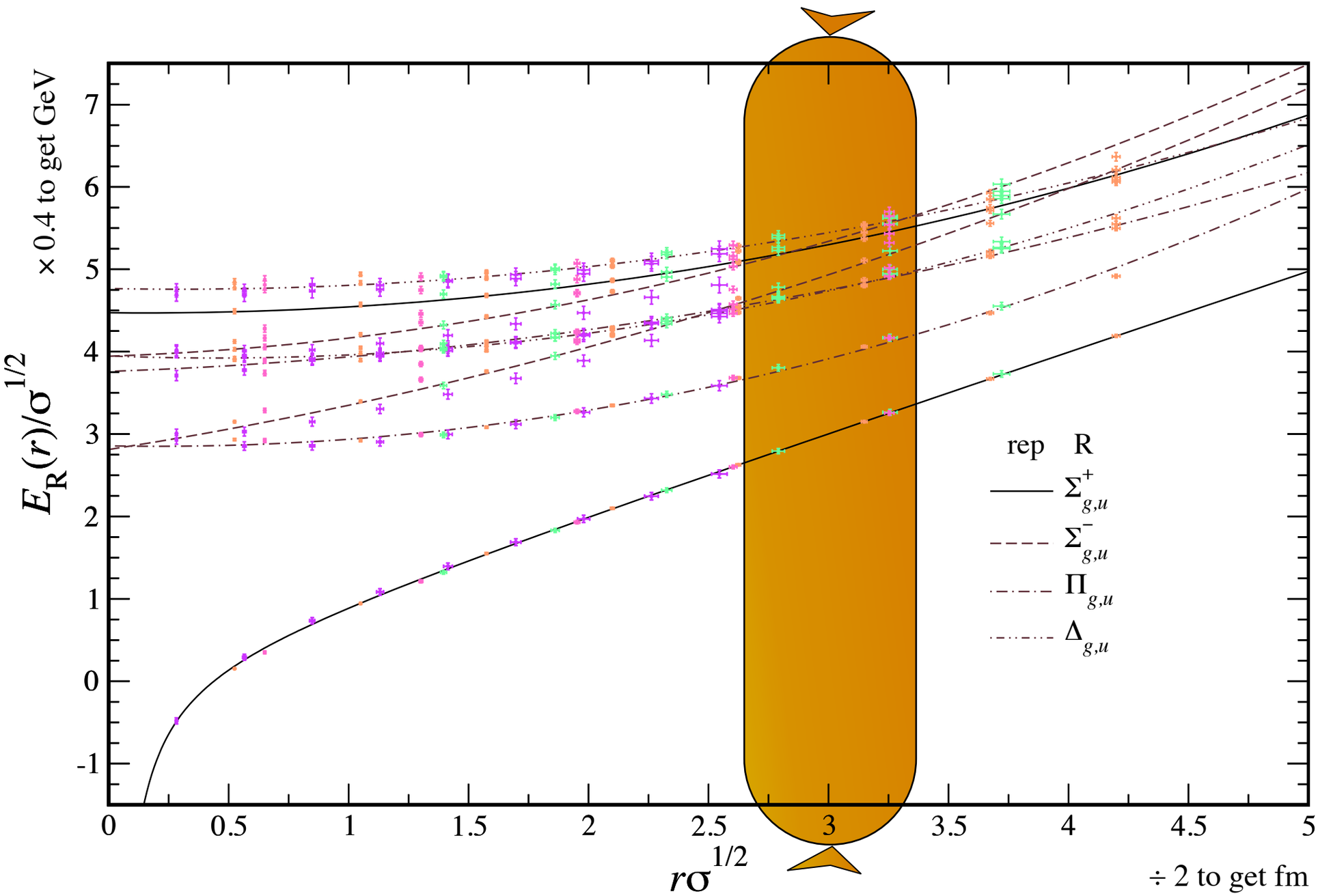
The QCD String

K. Juge, J. Kuti, & C. Morningstar, *PRL* **90** (2003) 161601



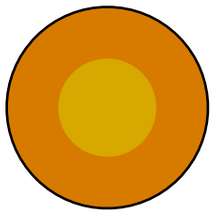
The QCD ~~String~~ Sausage

K. Juge, J. Kuti, & C. Morningstar, *PRL* **90** (2003) 161601



Hadron Masses: Qualitative & Quantitative

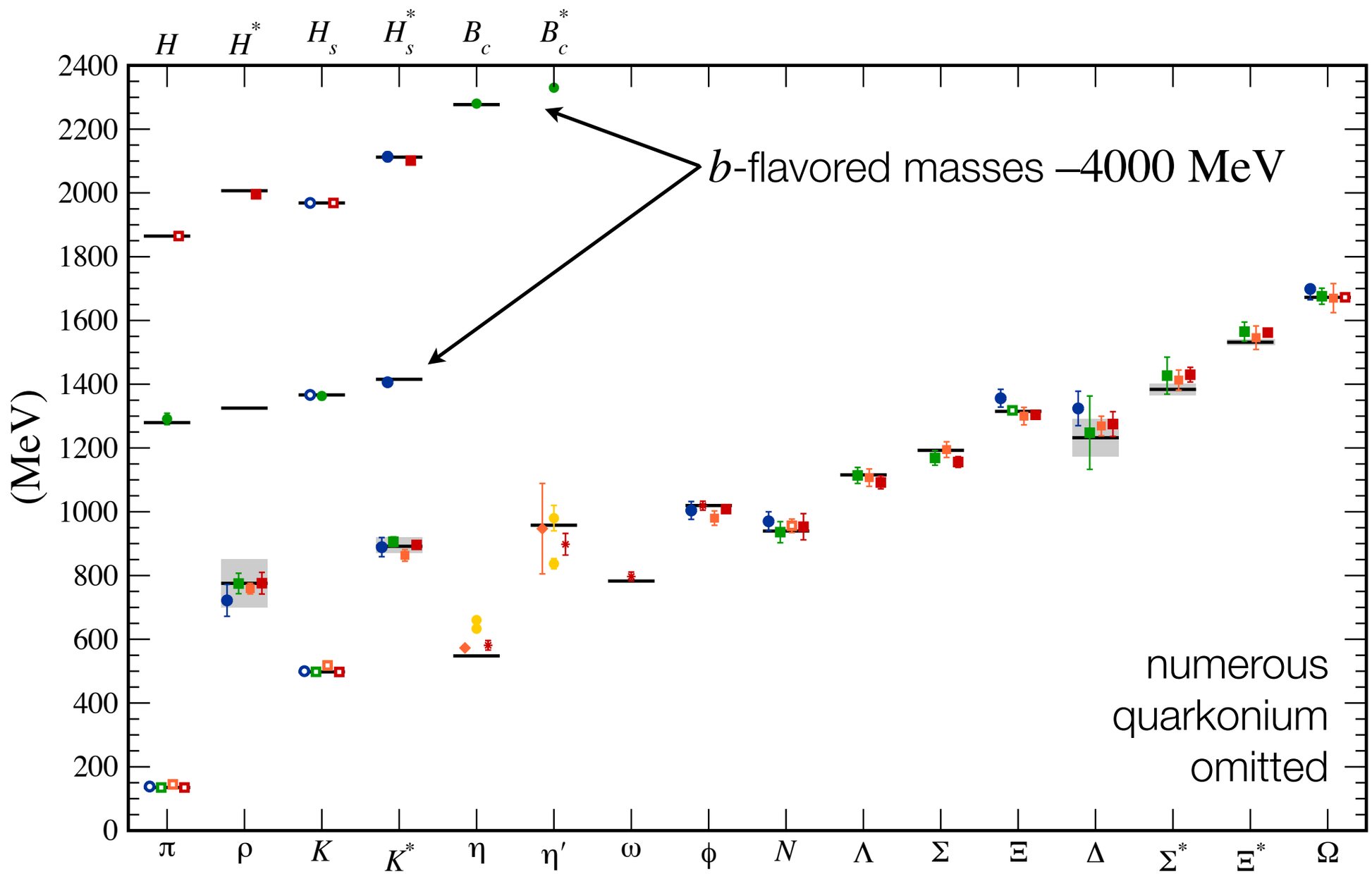
- In QCD, the energy in the sausage generates the mass of all hadrons.
- Glueballs (hadrons without quarks) consist only of this stuff.



  - $M(0^{++}) \approx 1700$ MeV [$f_J(1710)$]; $M(0^{-+})$ & $M(2^{++}) \sim 800$–900 MeV higher.
- The mass of hadrons with light quarks (like protons and neutrons) comes from this kind of energy, plus the kinetic energy of (relativistic) quarks confined.
- This is the source of mass for atomic nuclei and, thus, everyday objects …
  - … including you.

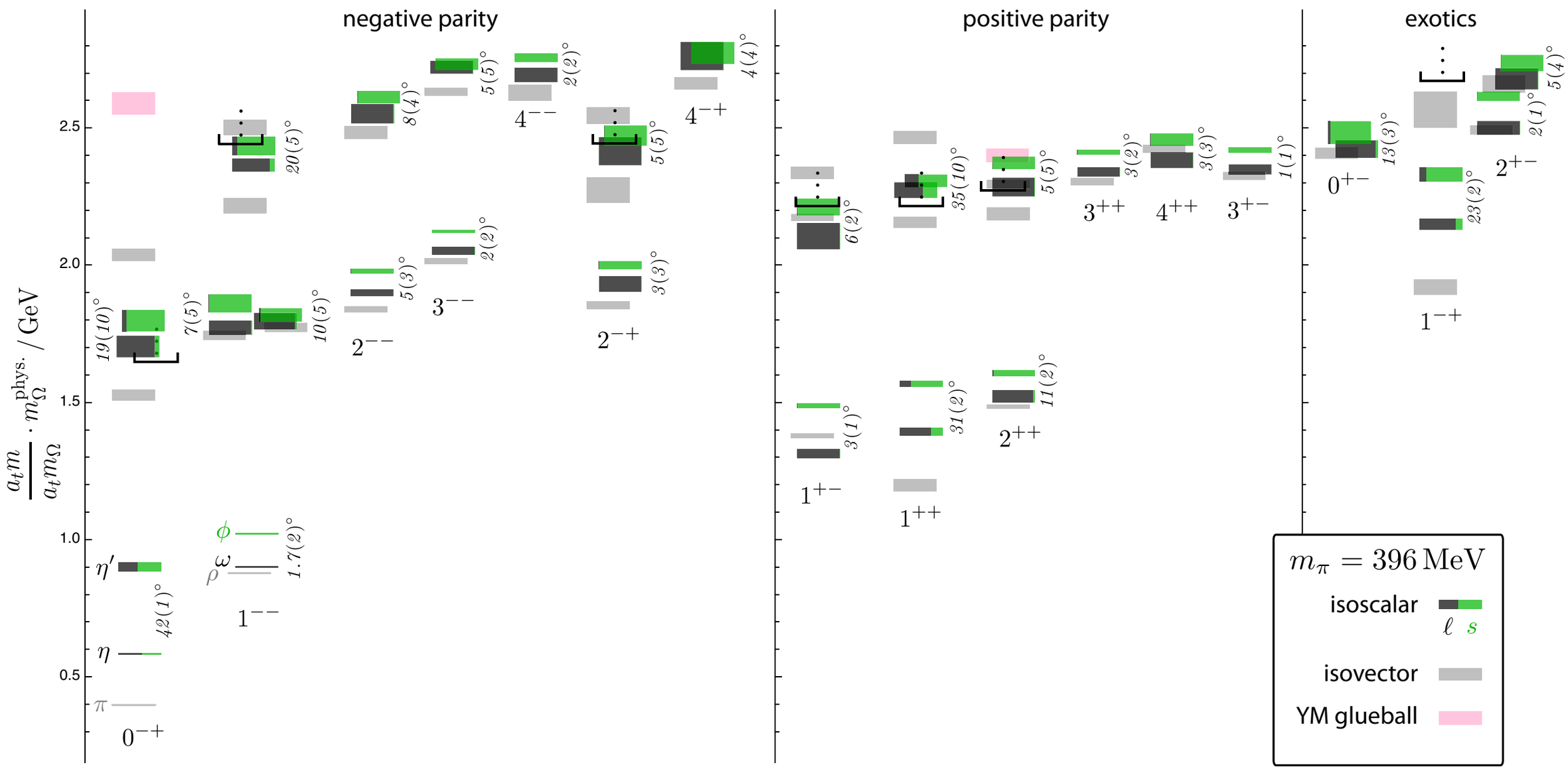
QCD Hadron Spectrum

$\pi \ldots \Omega$: BMW, MILC, PACS-CS, QCDSF;
η-η': RBC, UKQCD, Hadron Spectrum (ω);
D, B: Fermilab, HPQCD, Mohler&Woloshyn



Excited States

e.g., Hadron Spectrum Collaboration, *PRD* **83** (2011) 111502



- Future applications to glueball spectra, hybrids, excited baryons, and mixing.

Now, quark masses are MeV not GeV (see below), therefore

$$m_N = E_/c^2$$

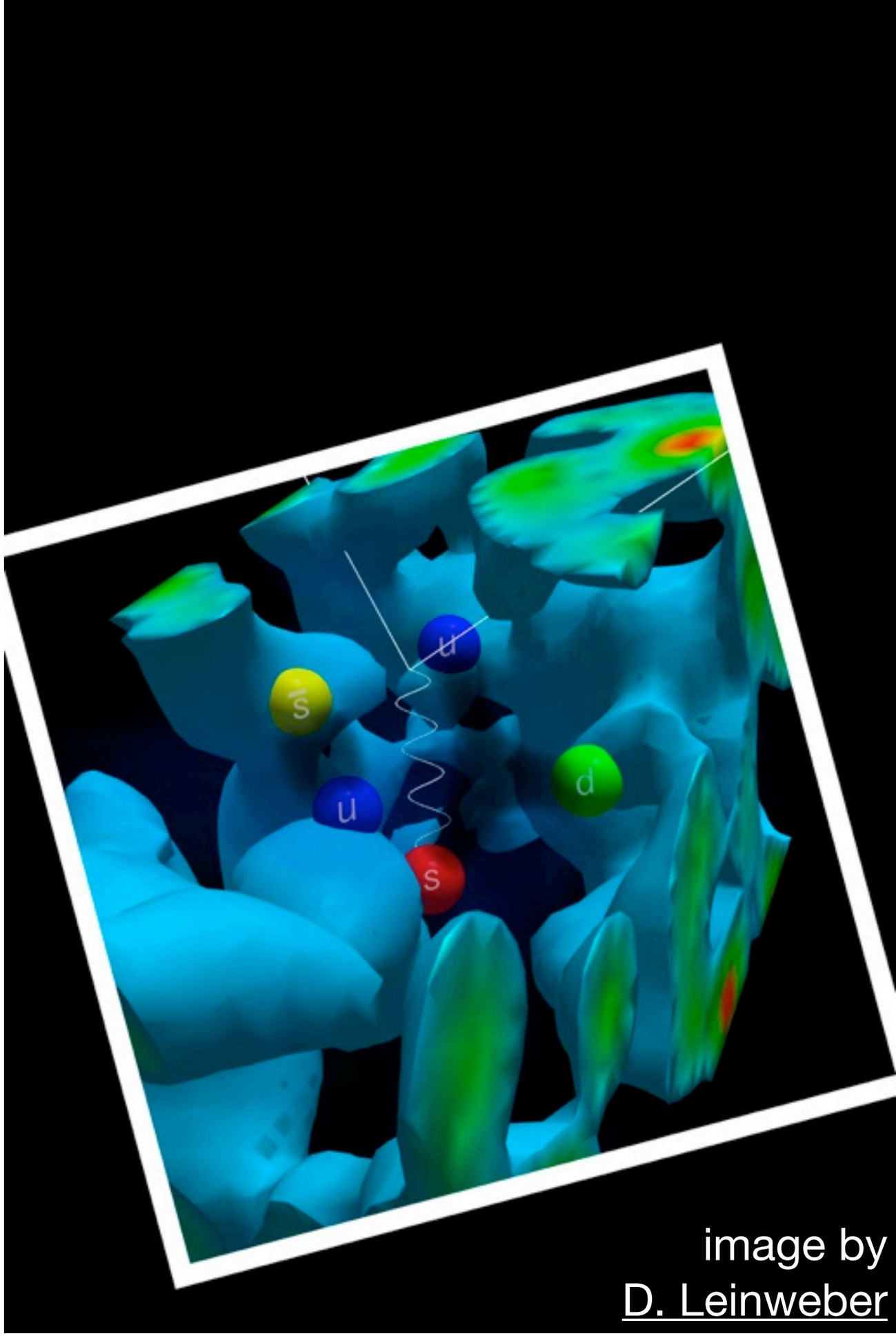
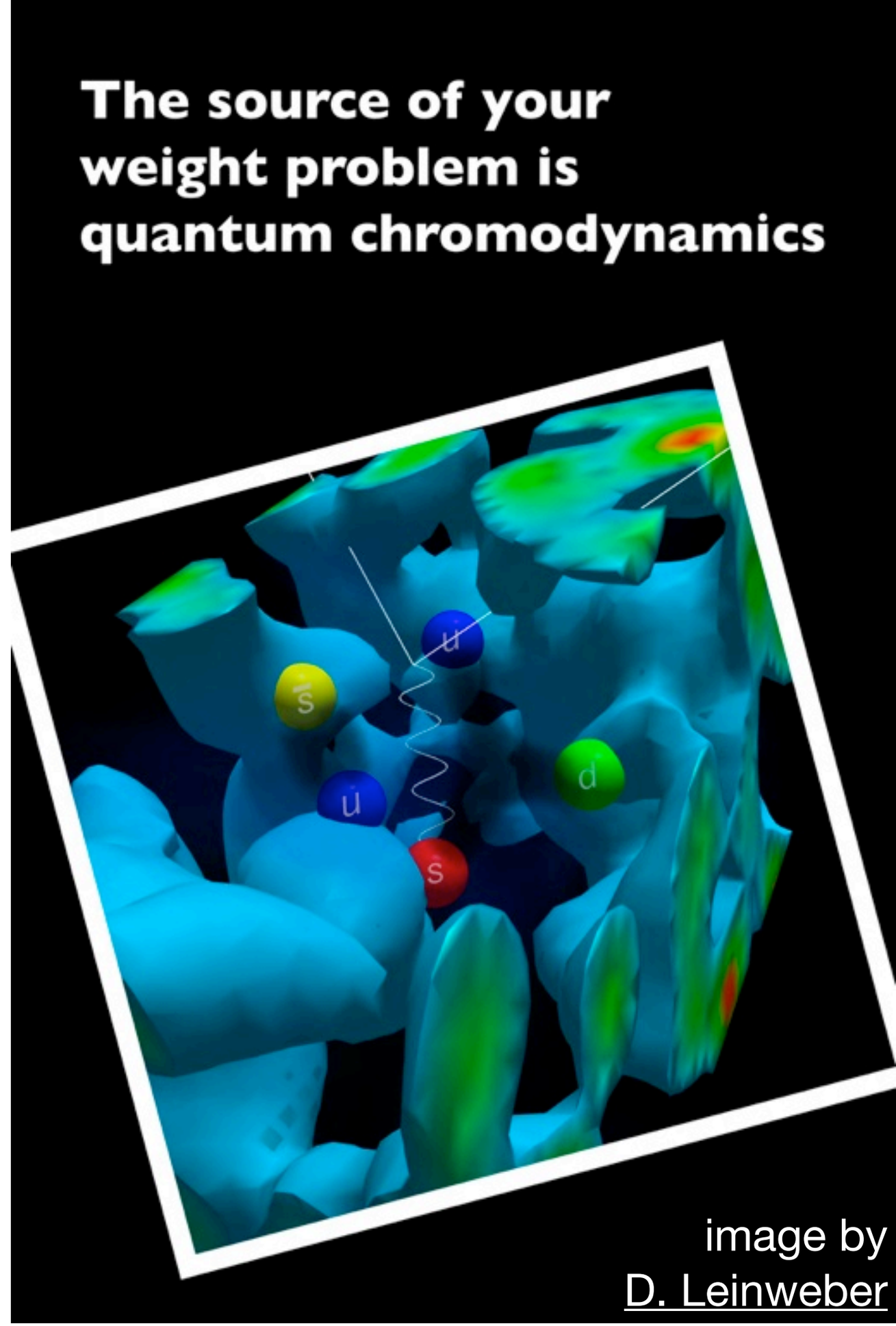


image by
D. Leinweber

Now, quark masses are MeV not GeV (see below), therefore

$$m_N = E_/c^2$$



QCD Parameters α_s and Quark Masses:
Derived from Hadronic Properties

Standard Model: 19 Parameters

- Gauge couplings: α_s, α_{QED}, $\alpha_W = (m_W/v)^2/\pi$;
- Lepton masses: m_e, m_μ, m_τ;
- Quark masses: $m_u e^{i\theta}$, m_d, m_s, m_c, m_b, m_t;
- CKM: $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $\exp(i\delta_{KM})$;
- EWSB: $v = 246$ GeV, $\lambda = (m_H/v)^2/2$.
- Need lattice QCD, lattice Yukawa.

Standard Model: 19 Parameters or 28

- Gauge couplings: α_s, α_{QED}, $\alpha_W = (m_W/v)^2/\pi$;
- Lepton masses: m_e, m_μ, m_τ; $m_{\nu 1}$, $m_{\nu 2}$, $m_{\nu 3}$;
- Quark masses: $m_u e^{i\theta}$, m_d, m_s, m_c, m_b, m_t;
- CKM: $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $\exp(i\delta_{KM})$; θ_{12}, θ_{23}, θ_{13}, δ_{PMNS}, ϕ_1, ϕ_2;
- EWSB: $v = 246$ GeV, $\lambda = (m_H/v)^2/2$.
- Need lattice QCD, lattice Yukawa.

Standard Model: 19 Parameters or 28

- Gauge couplings: α_s, α_{QED}, $\alpha_W = (m_W/v)^2/\pi$;
- Lepton masses: m_e, m_μ, m_τ; $m_{\nu 1}$, $m_{\nu 2}$, $m_{\nu 3}$;
- Quark masses: $m_u e^{i\theta}$, m_d, m_s, m_c, m_b, m_t; "Instability" → "renormalization."
- CKM: $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $\exp(i\delta_{KM})$; θ_{12}, θ_{23}, θ_{13}, δ_{PMNS}, ϕ_1, ϕ_2;
- EWSB: $v = 246$ GeV, $\lambda = (m_H/v)^2/2$. "Infinite λ" → "triviality."
- Need lattice QCD, lattice Yukawa.

Light Quark Masses

- The nonzero pion (kaon) mass is very sensitive to the light (strange) masses.

- Chiral perturbation theory predicts ratios of masses, but not the overall scale.

Lattice QCD	MILC	RBC	BMW	HPQCD
$\overline{m}_u(2\text{ MeV})$	1.9 ± 0.2	2.24 ± 0.35	2.15 ± 0.11	2.01 ± 0.14
$\overline{m}_u(2\text{ MeV})$	4.6 ± 0.3	4.65 ± 0.35	4.79 ± 0.14	4.79 ± 0.16
$\overline{m}_s(2\text{ MeV})$	88 ± 5	97.6 ± 6.2	95.5 ± 1.9	92.4 ± 1.5

- These are small—up & down masses are ~4 & ~9 times electron mass.

- The up mass is far from 0: the strong CP problem is indeed a problem.

Strong *CP* Problem

- Quark masses arise from Yukawa couplings, $\boldsymbol{m} = v\boldsymbol{y}/\sqrt{2}$, and from low-energy QCD instantons (tunneling between classical vacua):
  - observable *CP* violation $\propto \vartheta = \theta_{\mathrm{QCD}} - \arg\det \boldsymbol{y} < 10^{-11}$.
- If $\boldsymbol{y}$ has a zero mode, then its phase can be anything and, thus, chosen so that $\vartheta = 0$; no *CP* violation arises.
- Though m_u is small, lattice QCD calculations show no evidence for an instanton effect big enough to allow a zero mode in $\boldsymbol{y}$.
- A non-Standard symmetry (Peccei-Quinn) is then the least implausible explanation for the cancellation. Consequence: particles called axions.

Heavy Quark Masses

- The charmonium correlator also yields impressive precision on the charm mass [Bochkarev & de Forcrand, hep-lat/9505025]; analogous to determination from e^+e^- by Chetyrkin *et al.*:

  - lattice + PT: $m_c(m_c) = 1.273(6)\ \mathrm{GeV}$ [arXiv:1004.4285];

  - e^+e^- + PT: $m_c(m_c) = 1.279(13)\ \mathrm{GeV}$ [arXiv:0907.2110].

- Similarly, $m_b(m_b) = 4.164(23)\ \mathrm{GeV}$ [HPQCD, arXiv:1004.4285],
  cf. $m_b(m_b) = 4.163(16)\ \mathrm{GeV}$ from e^+e^- + PT [arXiv:0907.2110].

- Also, $m_c/m_s = 11.85(16)$ [HPQCD, arXiv:0910.3102], which led to the right-most column in the table of light-quark masses.

Strong Coupling α_s

- In lattice gauge theory, the bare coupling—$g_0^2(1/a)$— is an input. Aim is to relate this to $\alpha_s(m_Z) = g_s^2(m_Z)/4\pi$. Alas, conversion in PT does not converge.

- Two main strategies:

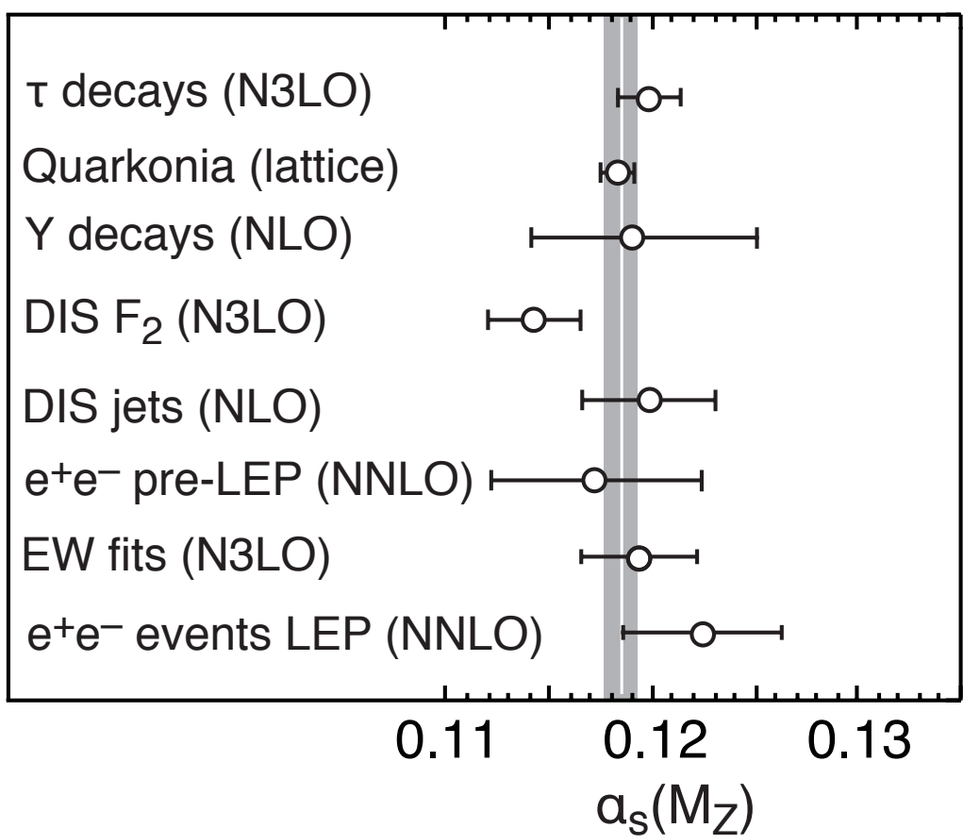
  - compute a short-distance lattice quantity (*e.g.*, small Wilson loop, Creutz ratio of small Wilson loops, ...); compare MC with (lattice) PT $\rightarrow \alpha_s(1/a)$;

  - compute short-distance "$1/Q$" continuum quantity (*e.g.*, J-J correlator, Schrödinger functional, ghost vertex); compare MC with PT $\rightarrow \alpha_s(Q)$.

Results for α_s (all $n_f = 2+1$):

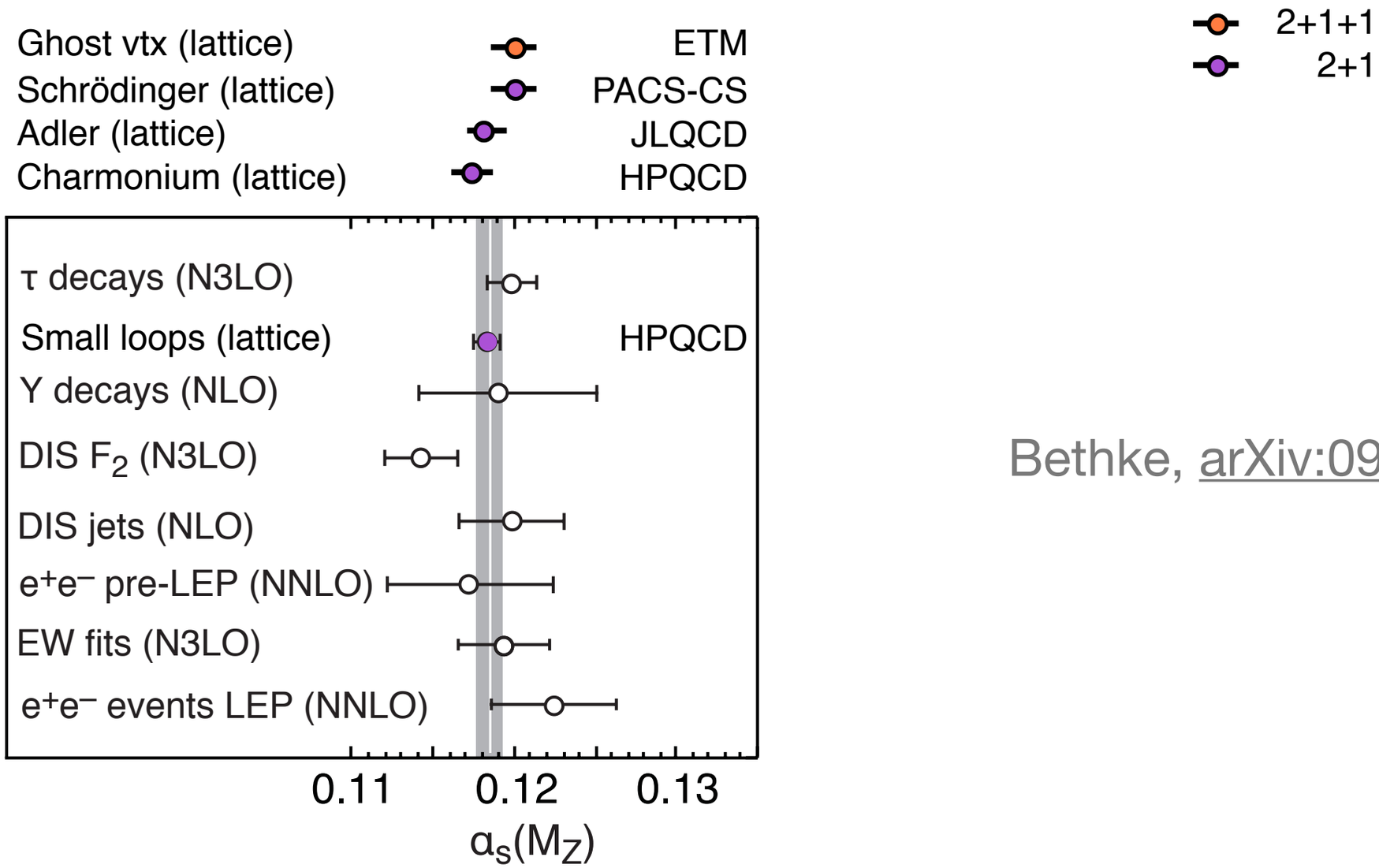
- Schrödinger [PACS-CS, arXiv:0906.3906] | Adler [JLQCD, arXiv:1002.0371]:
  - $\alpha_s(m_Z) = 0.1205(8)(5)(+0/-17)$ | $\alpha_s(m_Z) = 0.1181(3)(+13/-4)$;
- Wilson Creutz, *etc.* [HPQCD, arXiv:0807.1687 | Maltman, arXiv:0807.2020]:
  - $\alpha_s(m_Z) = 0.1183(8)$ | $0.1192(11)$;
- Charmonium correlator [HPQCD + Karlsruhe, arXiv:0805.2999]:
  - $\alpha_s(m_Z) = 0.1174(12)$ | $0.1184(6)$ [update in arXiv:1004.4285];
- Bethke's world average (without lattice | with HPQCD WL) [arXiv:0908.1135]:
  - $\alpha_s(m_Z) = 0.1186(11)$ | $0.1184(7)$.

QCD of hadrons = QCD of partons



Bethke, arXiv:0908.1135

QCD of hadrons = QCD of partons



Bethke, arXiv:0908.1135

Flavor Physics

Weak Interactions

- At energies probed by the Tevatron and the LHC, left- and right-handed quarks are different (weak eigenbasis):

$$Q_L = \left(\begin{pmatrix} u \\ d' \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L \begin{pmatrix} t \\ b' \end{pmatrix}_L \right), \qquad Y_{Q_L} = -\frac{1}{6}$$

$$U_R = \begin{pmatrix} u_R & c_R & t_R \end{pmatrix}, \qquad Y_{U_R} = +\frac{2}{3}$$

$$U_R = \begin{pmatrix} d_R & s_R & b_R \end{pmatrix}, \qquad Y_{D_R} = -\frac{1}{3}$$

9 fields: 3 doublets and 6 singlets under $SU_L(2) \times U_Y(1)$.

CKM Matrix

Cabibbo, *PRL* **10** (1963) 531;
Kobayashi, Maskawa, *Prog. Theor. Phys.* **49** (1973) 652

- Mass couples left to right: has to break $SU_L(2)$ and $U_Y(1)$ (spontanesously).
- Weak and mass eigenbases related by unitary transformation, $D_L = V_{\rm CKM} D'_L$.
- Global symmetries reduce parameter count of CKM matrix to 4:
  - $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $\delta_{\rm KM} = \arg V^*_{ub}$—as fundamental as electron mass.
- Unitarity relations, *e.g.*, $V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{tb} = 0$: triangles in the complex plane.
- Probed by many measurements + corresponding QCD.

CKM and Lattice QCD

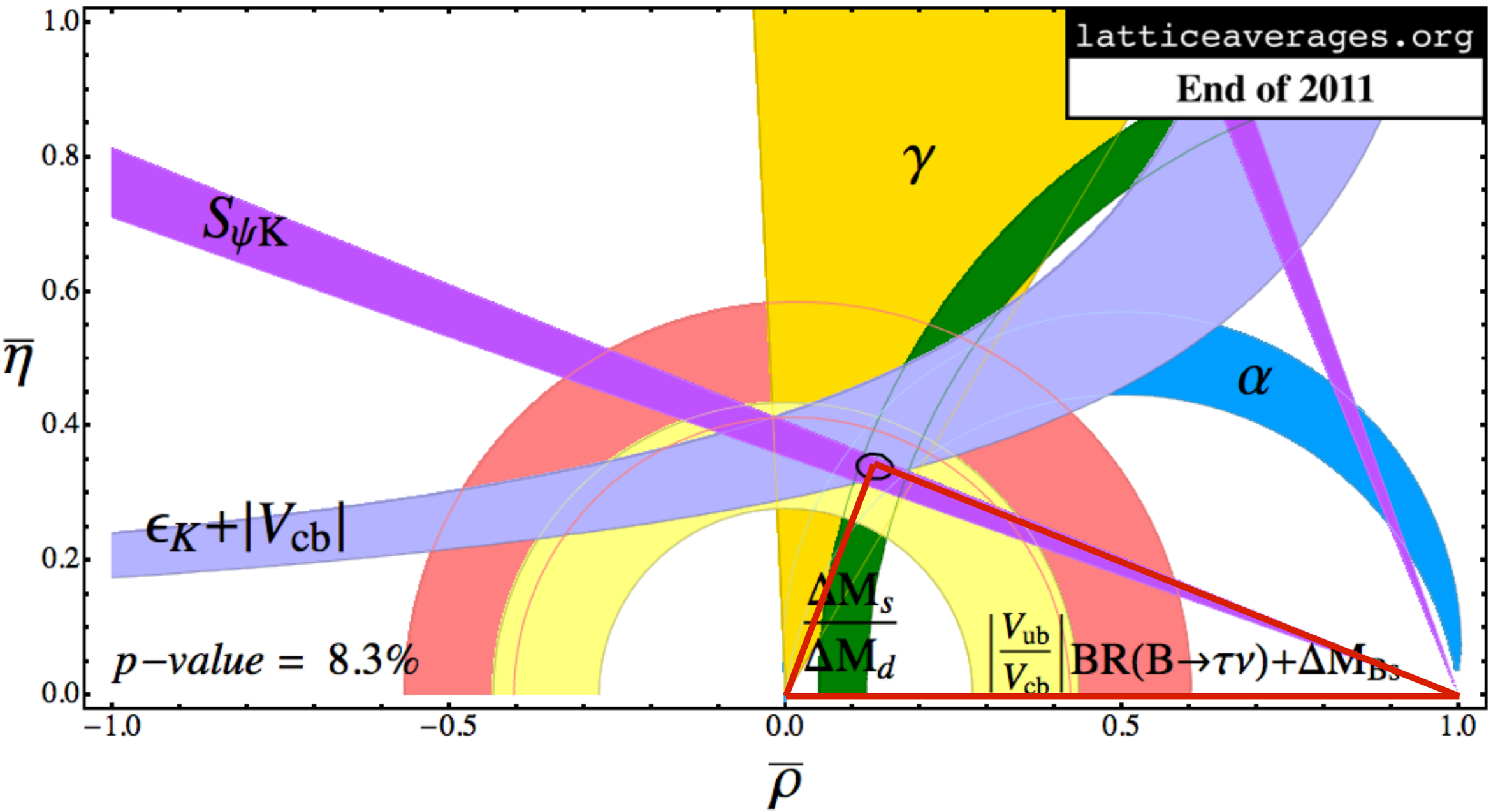
- Gold-plated quantities available to (over)determine CKM matrix:

$$V = \begin{pmatrix} \begin{array}{c} |V_{ud}| \\ \pi \to \ell\nu \\ n \to pe^-\bar{\nu} \end{array} & \begin{array}{c} |V_{us}| \\ K \to \ell\nu \\ K \to \pi\ell\nu \end{array} & \begin{array}{c} |V_{ub}| \\ B \to \tau\nu \\ B \to \pi\ell\nu \end{array} & \begin{array}{c} \arg V_{ub}^* \\ \langle K^0|\bar{K}^0\rangle \\ \\ \end{array} \\ \begin{array}{c} |V_{cd}| \\ D \to \ell\nu \\ D \to \pi\ell\nu \end{array} & \begin{array}{c} |V_{cs}| \\ D_s \to \ell\nu \\ D \to K\ell\nu \end{array} & \begin{array}{c} |V_{cb}| \\ B \to D\ell\nu \\ B \to D^*\ell\nu \end{array} & \\ \begin{array}{c} |V_{td}| \\ \langle B_d|\bar{B}_d\rangle \end{array} & \begin{array}{c} |V_{ts}| \\ \langle B_s|\bar{B}_s\rangle \end{array} & \begin{array}{c} |V_{tb}| \\ (\text{no } t\bar{q} \text{ hadrons}) \end{array} & \end{pmatrix}$$

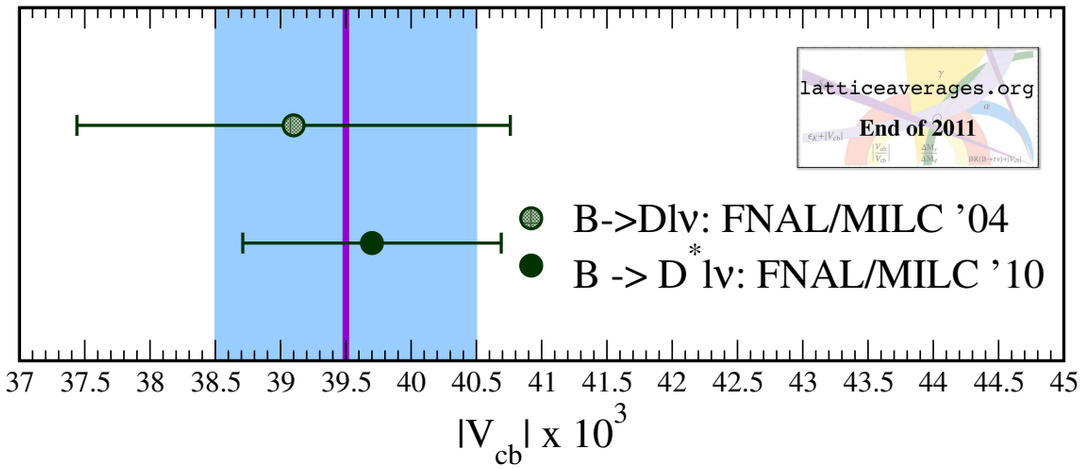
- loops vs. trees

Unitarity Triangle

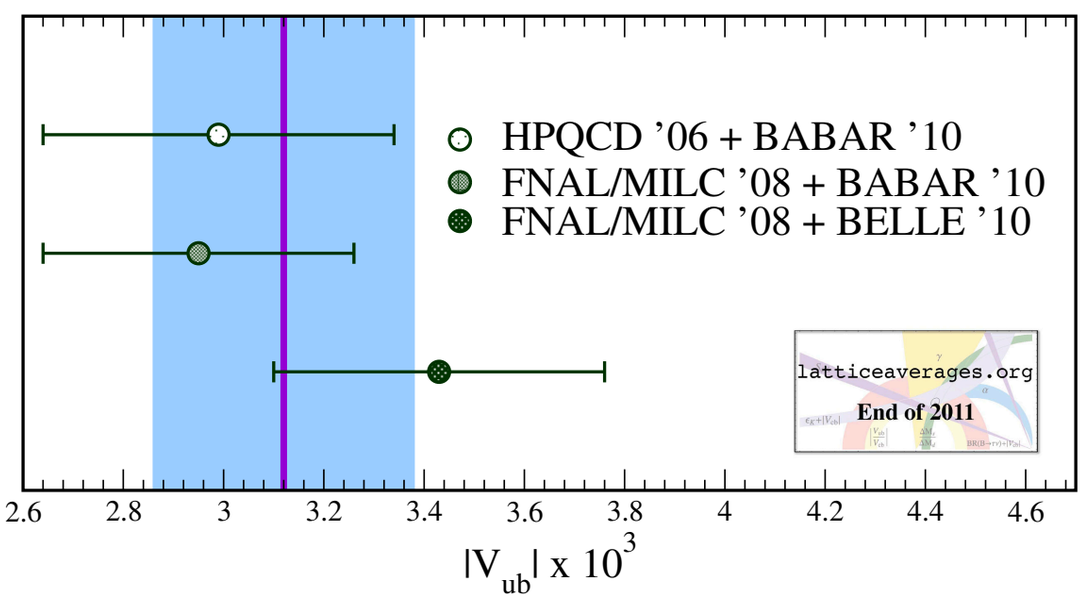
c.f., Laiho, Lunghi, Van de Water, arXiv:0910.2928



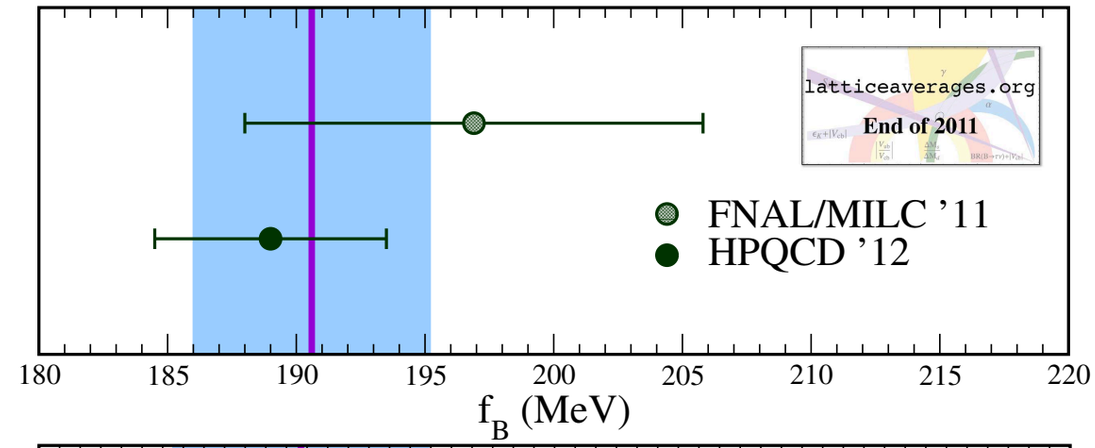
B-Meson Averages from Lattice QCD

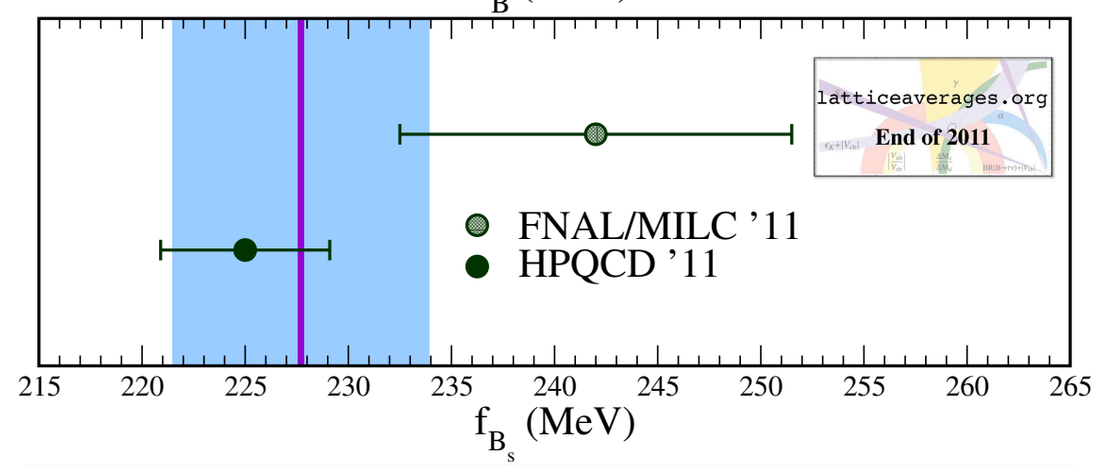


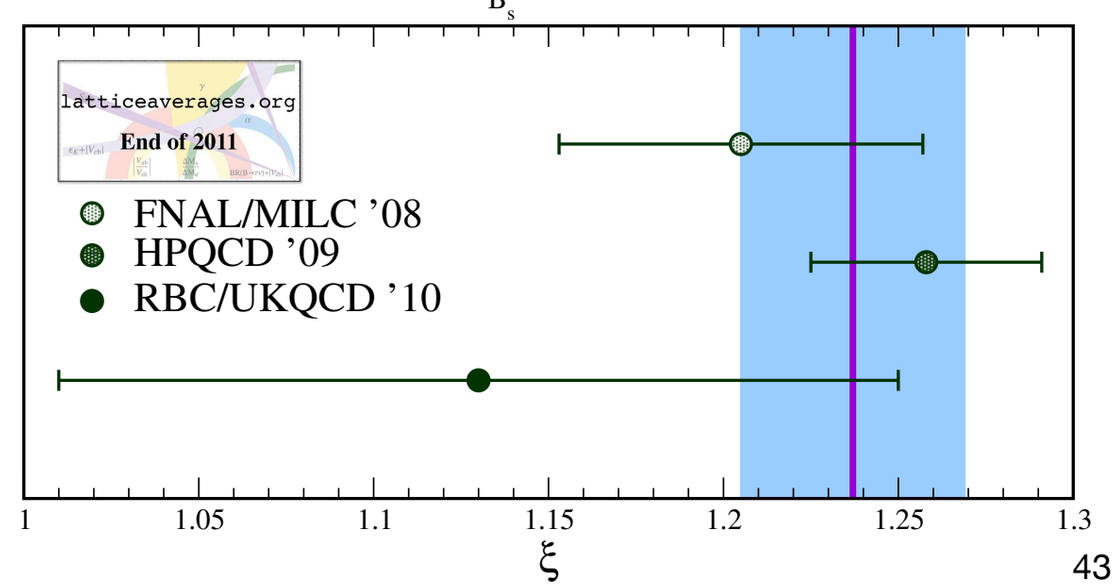
inclusive $|V_{cb}|$ is ~2σ higher



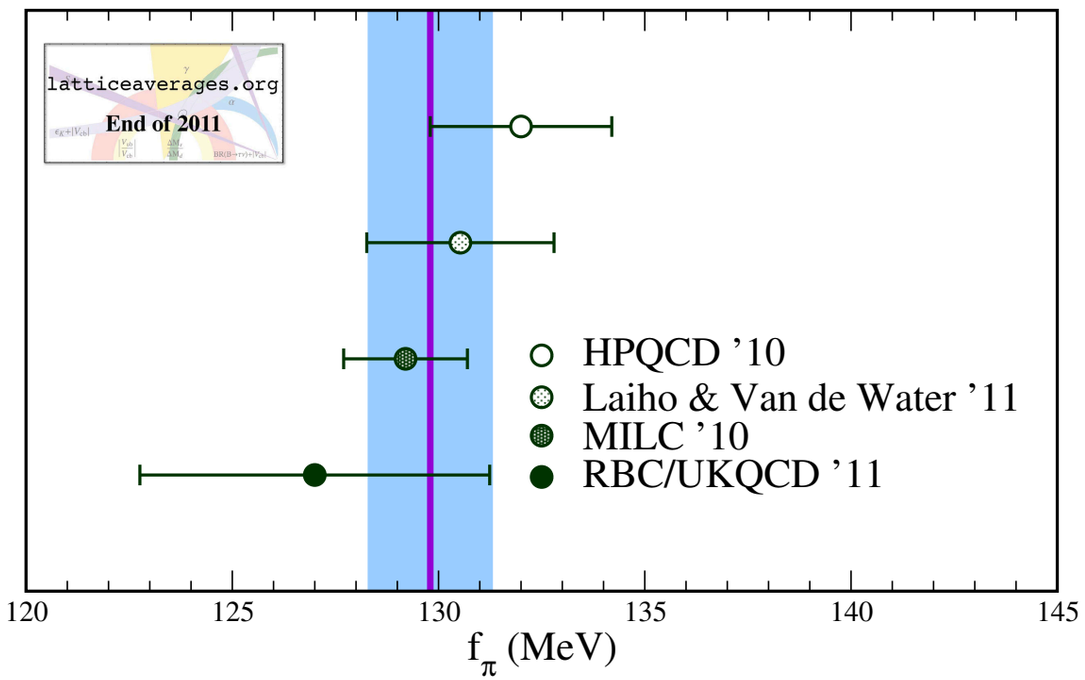
inclusive $|V_{ub}|$ is ~1σ higher

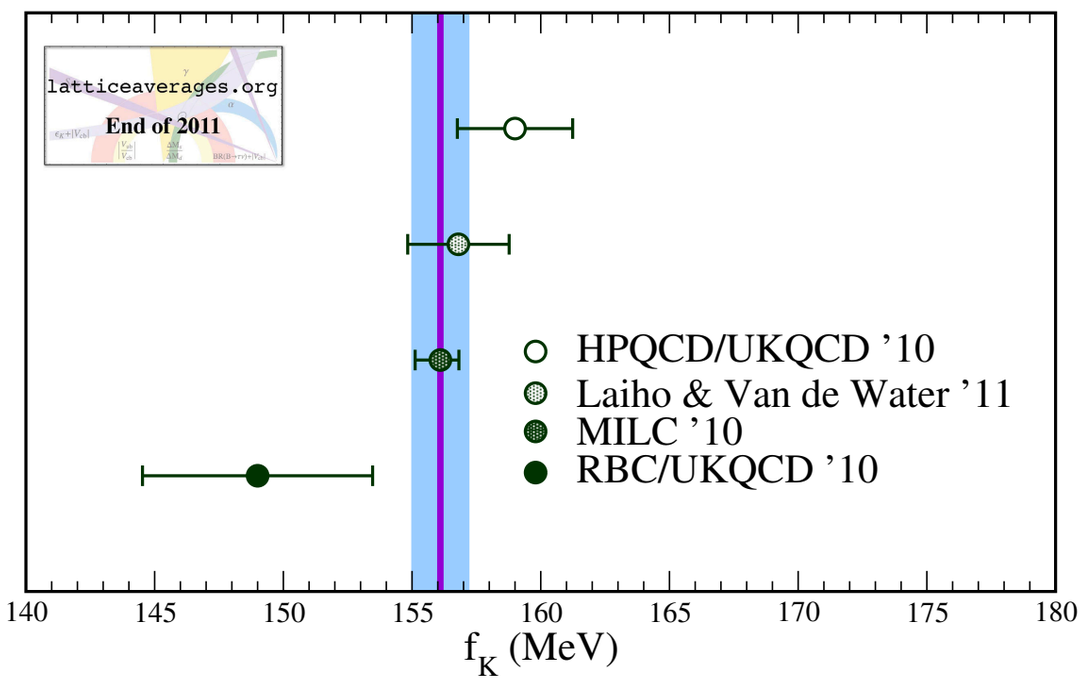




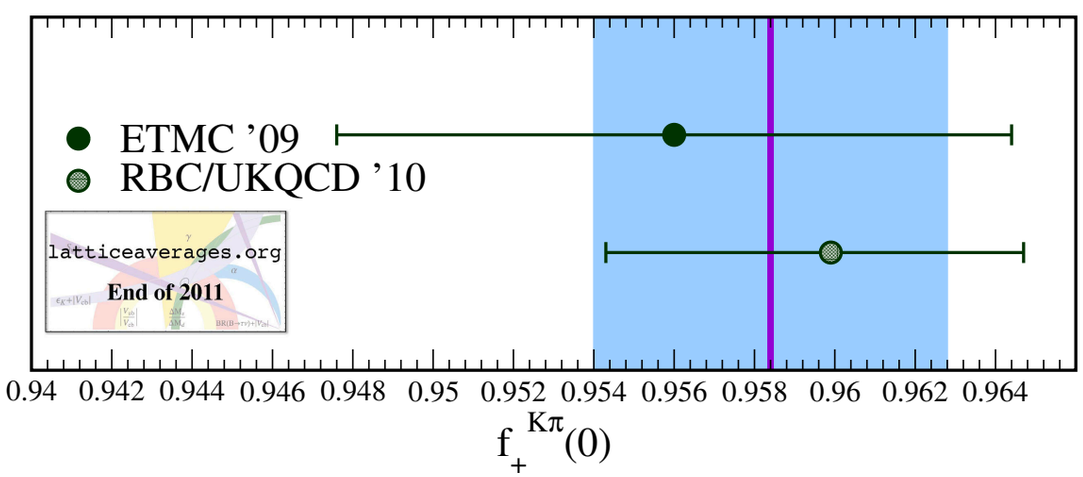


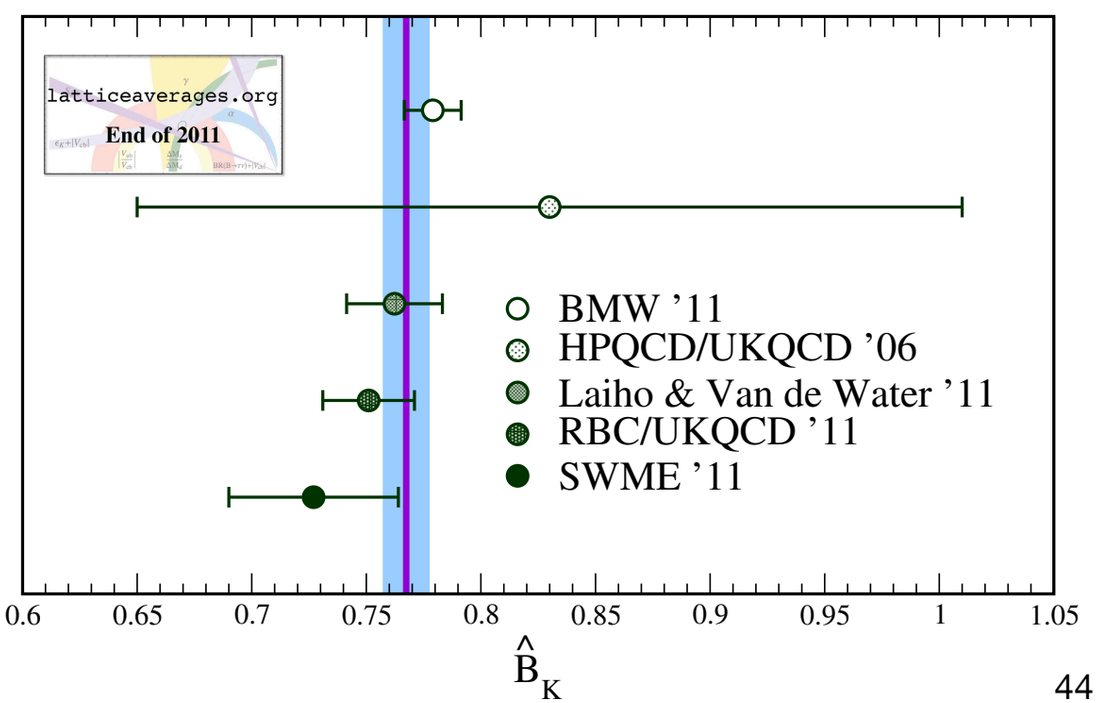
π- & *K*-Meson Averages from Lattice QCD





plots from latticeaverages.org



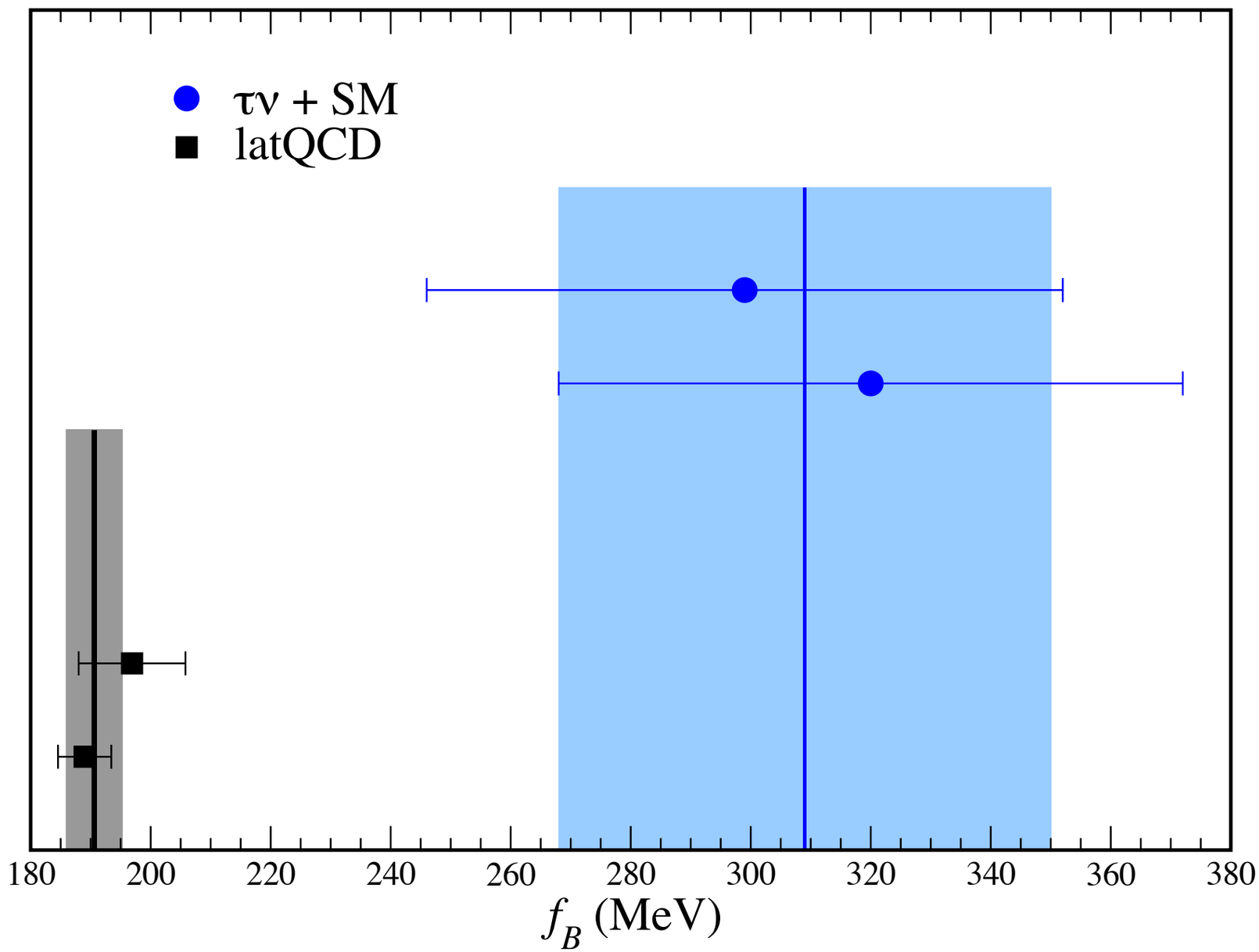


Lessons

- Lattice QCD plays a crucial role for neutral-meson mixing (K, B, B_s).
- Lattice QCD plays a key role in $|V_{us}|$, $|V_{cs}|$, $|V_{ub}/V_{cb}|$, $|V_{cb}|$.
- Suite of experiments, pQCD, and lQCD shows that CKM flavor violation and KM CP violation predominates.
- Still room for new physics: tension at 2–3σ level:
  - confidence level of global fit improves more, if NP in kaon mixing [LLV];
  - ε_K band uses corrections of Nierste, Ligeti, ASK [hep-ph/0201071].

Tension in $B \to \tau\nu$?

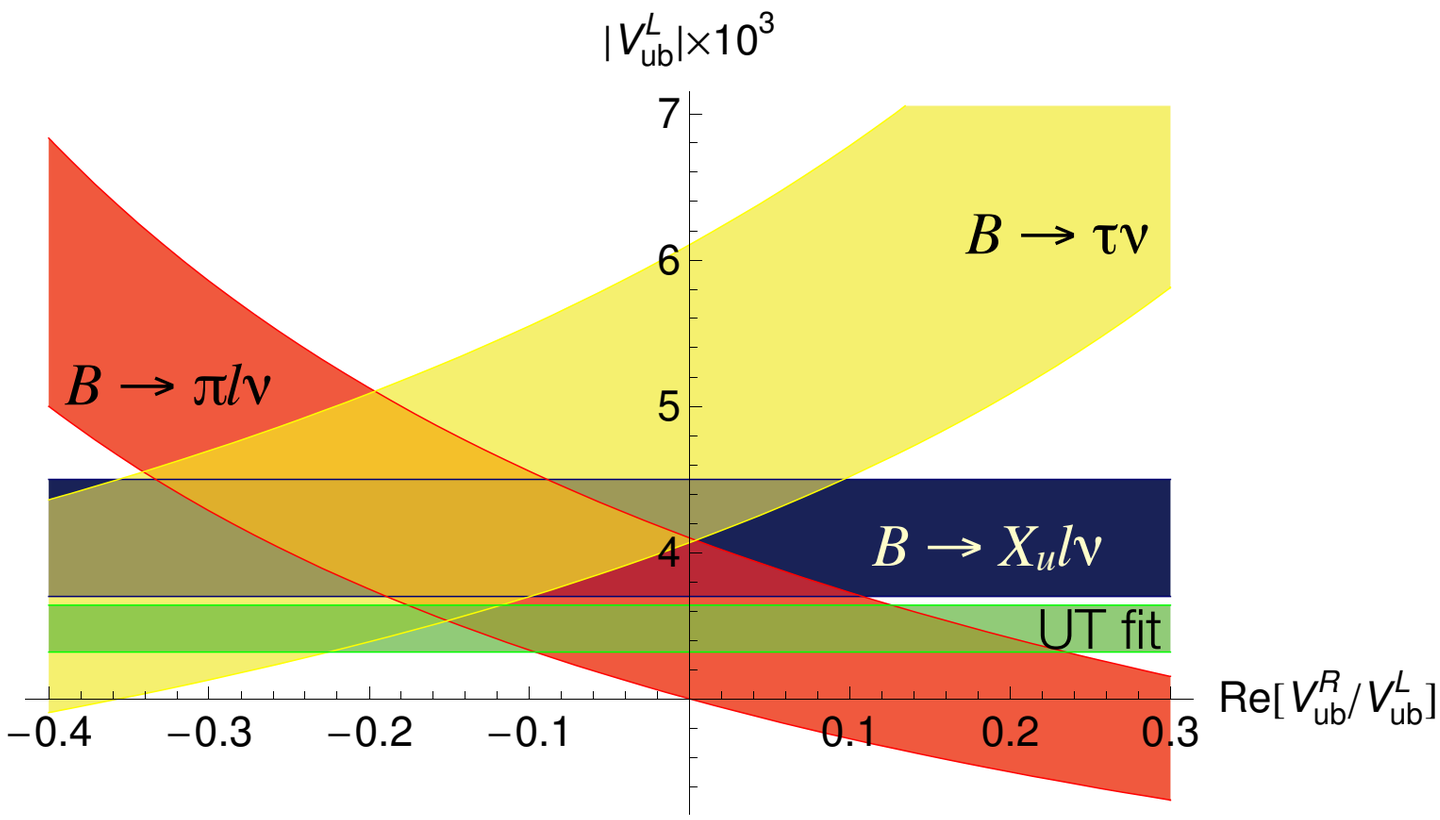
$\mathrm{BR} \propto |f_B V_{ub}|^2$



- New physics, *e.g.*, charged Higgs of MSSM?
- w/ $A_{MSSM} \sim -1.1 A_{SM}$?

Right-Handed Currents?

Crivellin, arXiv:0907.2461

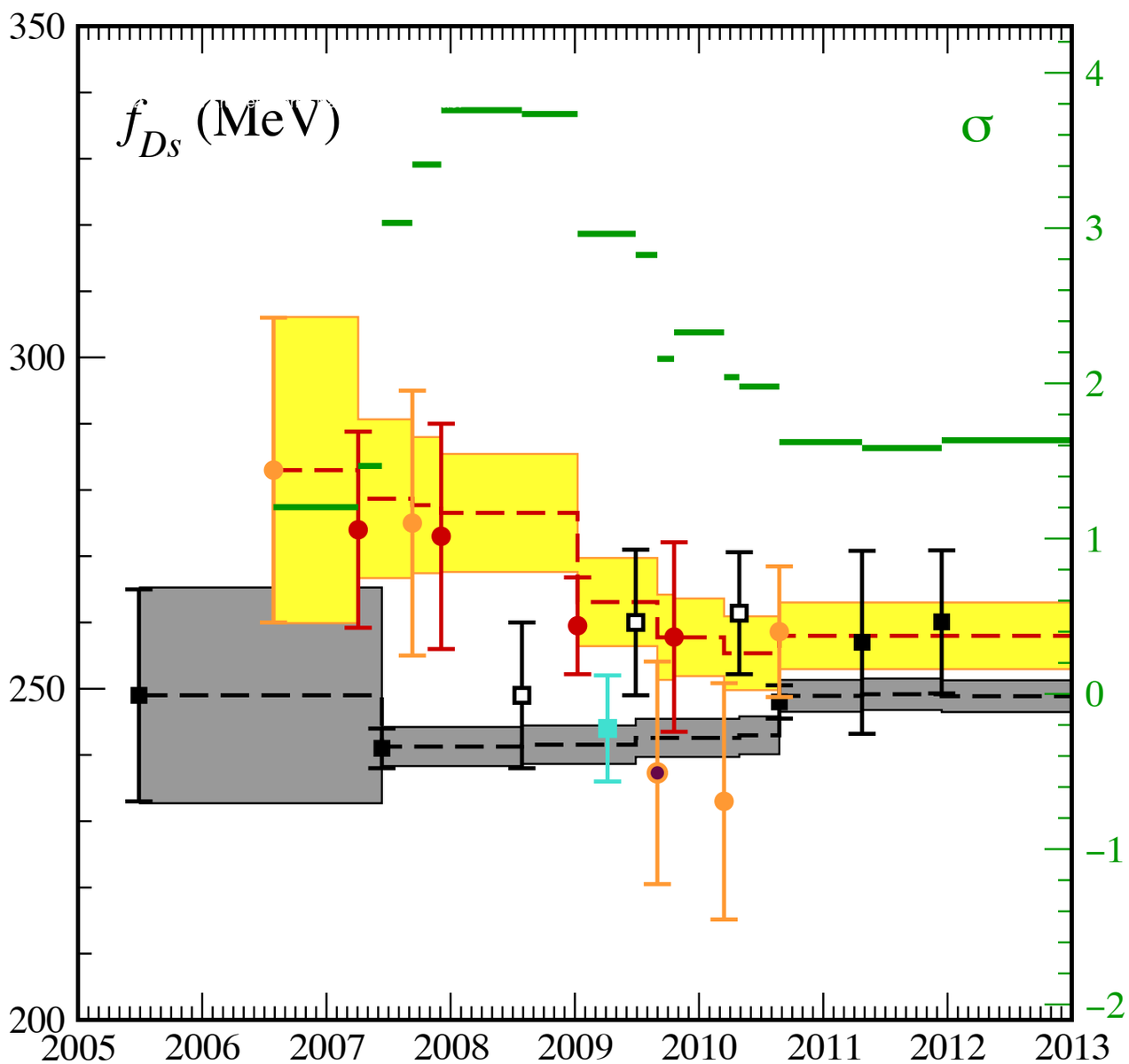


- Realistic models?

- Exclusive 3.12(26):
  - $|V_{ub}| = |V^L_{ub} + V^R_{ub}|$
- Inclusive 4.34(40):
  - $|V_{ub}| \approx |V^L_{ub}|$
- Leptonic 4.95(55):
  - $|V_{ub}| = |V^L_{ub} - V^R_{ub}|$

Tension in $D_s \to l\nu$?

update from ASK, arXiv:0912.0543

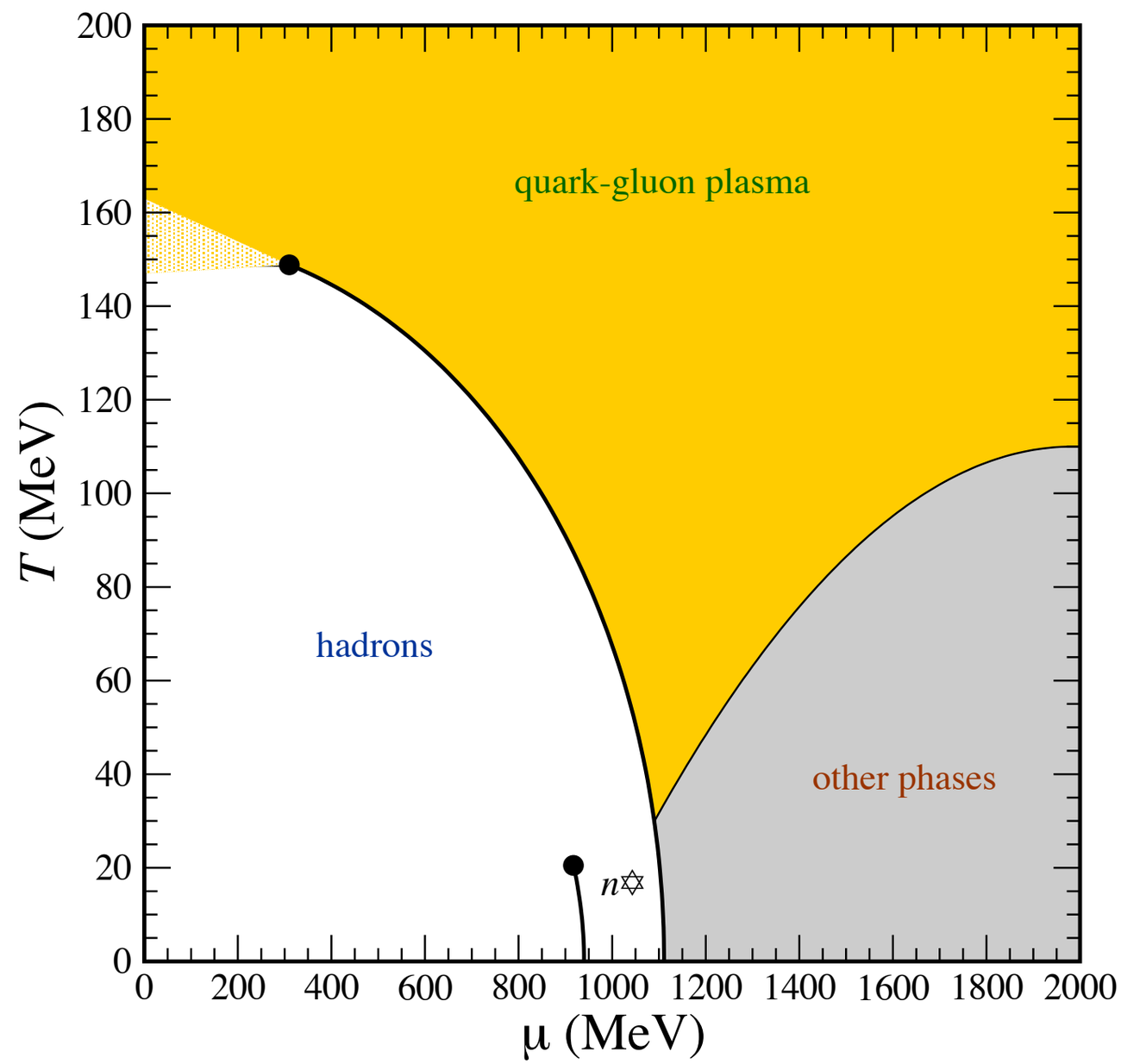


- Gray: running lQCD avg.
- Yellow: running expt avg.
- Orange: BaBar, Belle.
- Red: CLEO.
- Green (right y axis): running deviation in σ.
- σ is mostly exptl stats.

Thermodynamics

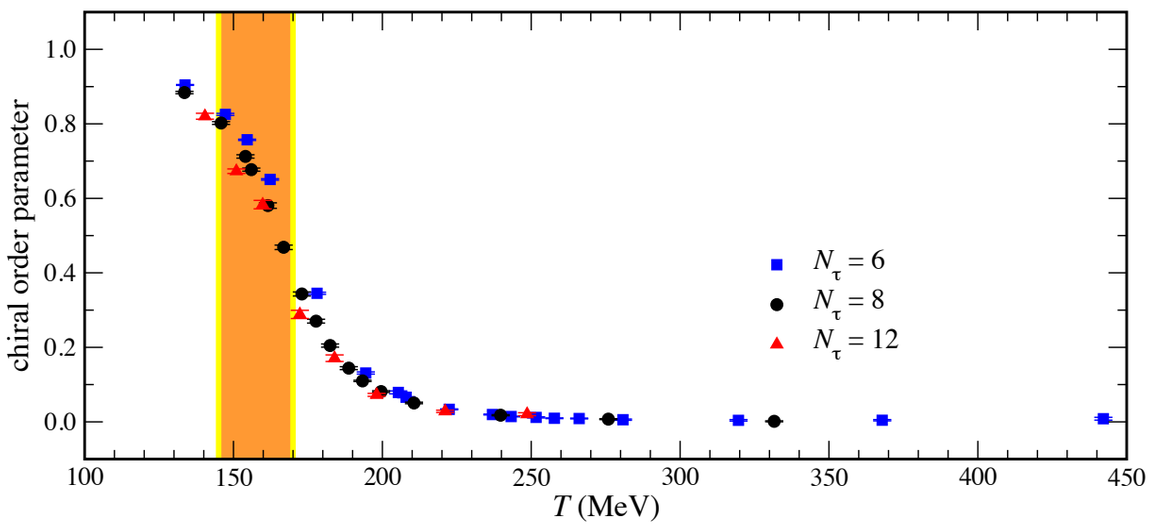
QCD Phase Diagram

- 20th century conventional wisdom: first order PT at $\mu = 0$.
- Why? Chiral symmetry says so.
- Consequence: bubbles of hadrons formed as the universe cooled.

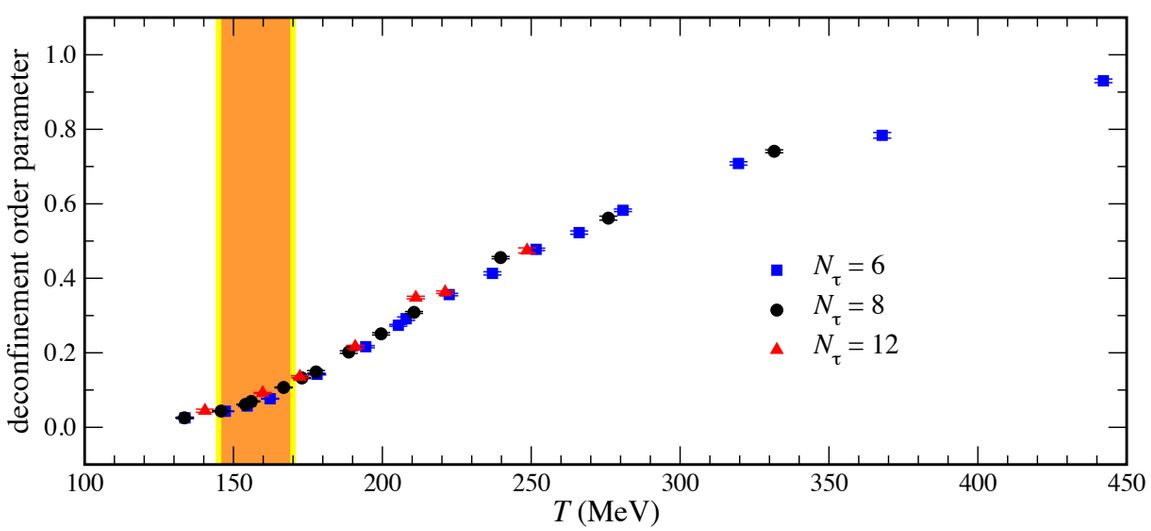


QCD Phase Transition ($\mu = 0$)

Y. Aoki *et al*. *Nature* **443** (2006) 675; A. Bazavov *et al.*, *PRD* **85** (2012) 054503



10^{12} K



- Temperature $T = 1/N_\tau a$.
- Smooth crossover to phase, in which the grand canonical average becomes:
  - chirally symmetric;
  - deconfined.
- Same transition temperature: also for other observables, *e.g.*, susceptibilities peak.

Quarks and gluons vs. hadrons

- The thermal average is

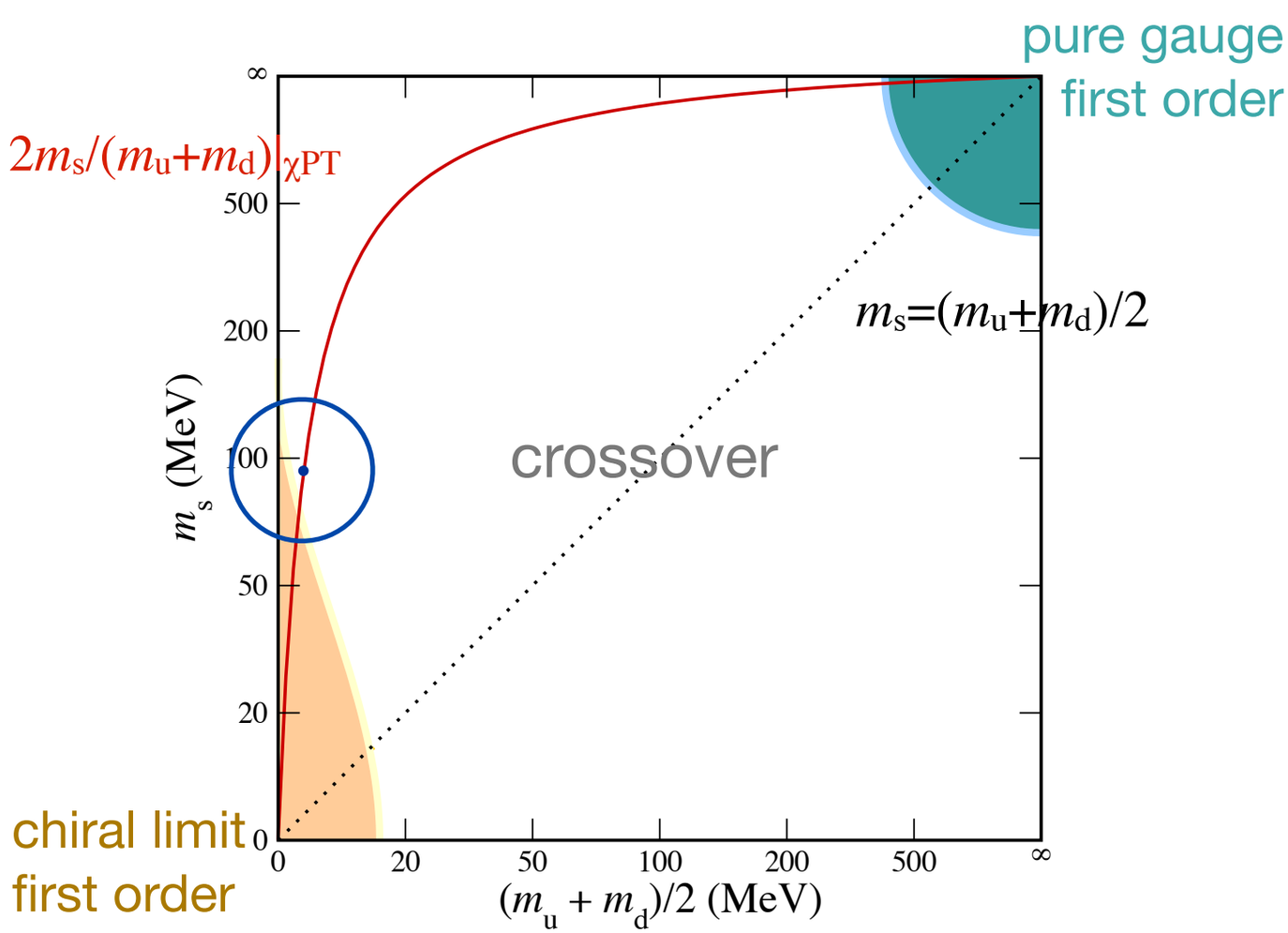
$$\langle \bullet \rangle = \frac{\mathrm{Tr}\left[\bullet\, e^{-\hat{H}/T}\right]}{\mathrm{Tr}\, e^{-\hat{H}/T}}$$

  which is as “inclusive” as possible.

- Parton-hadron duality in scattering & decay seems to work once $E > 2\ \mathrm{GeV}$; at such T exchange trace over hadronic states for trace over partons.

- Does not contradict “chiral symmetry restoration” or “deconfinement” at lower temperatures: average includes a state and its chiral partner & and includes states with lots of hadrons.

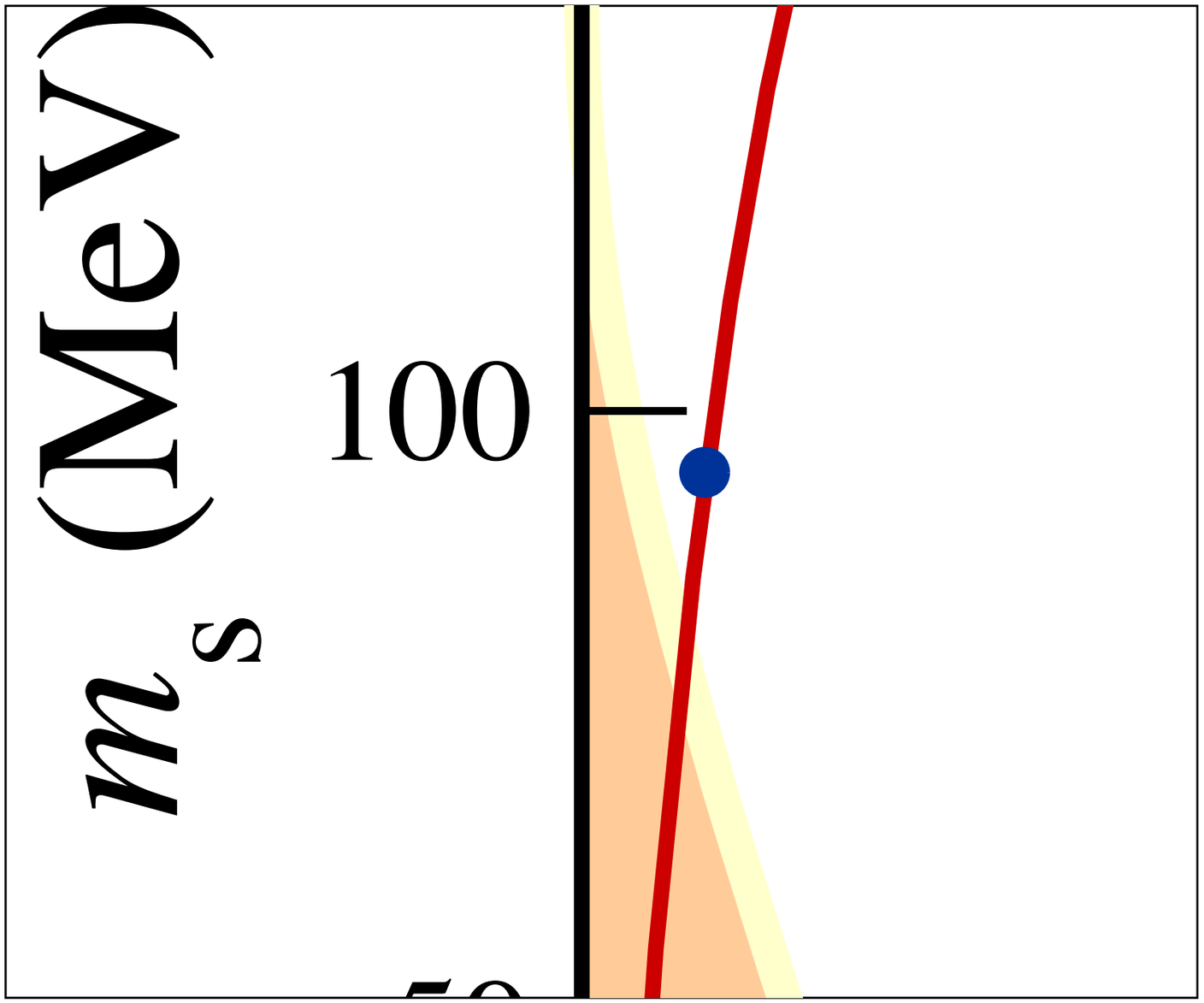
Quark Masses are Key



after de Forcrand & Philipsen
arXiv:0808.1096

- Explicit χSB softens the transition.
- Quark masses are small, but ...
- ... if even smaller, the $\mu = 0$ transition would be second order, or even first order.
- Implications for the early universe.

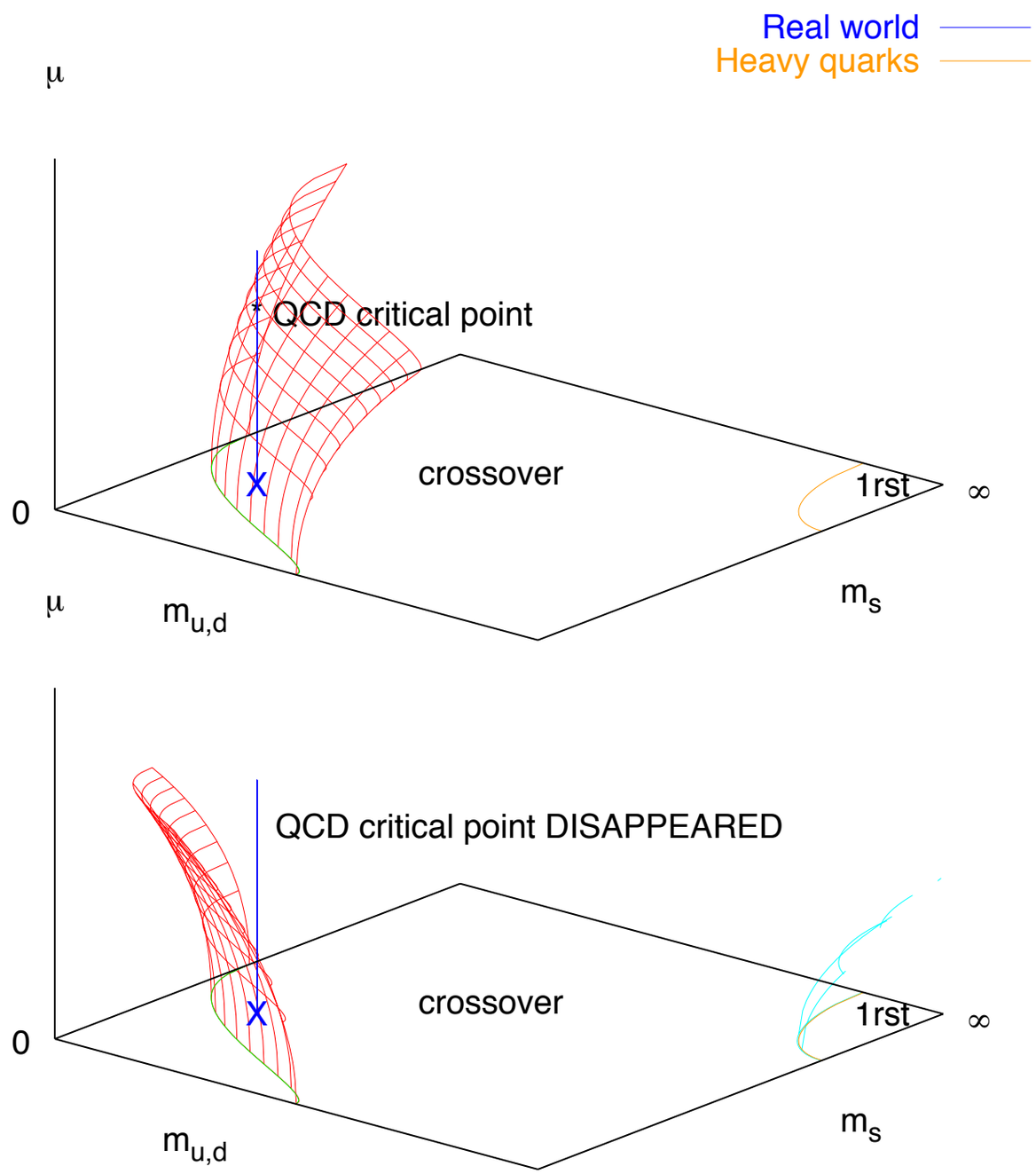
Quark Masses are Key



after de Forcrand & Philipsen
arXiv:0808.1096

- Explicit χSB softens the transition.
- Quark masses are small, but ...
- ... if even smaller, the $\mu = 0$ transition would be second order, or even first order.
- Implications for the early universe.

Equation of State ($\mu \neq 0$)



- Studies limited to $\mu \approx 0$.
- Curvature has been controversial.
- Models and qualitative arguments suggest top picture, whence a critical point for some $\mu \neq 0$.
- Several groups find the opposite:
  - a cutoff effect?

Summary and Challenges

Summary: from "QCD *should* work this way" to "QCD *does* work this way"

- Quantitative
  - Precise α_s, m_c, m_b:
  - Precise m_s, m_d, m_u:
  - Hadron masses:
  - CKM V_{cb}, V_{ub}, $V_{td}/V_{ts} \oplus B_K, f_B$:
  - Chiral condensate $\langle \bar{q}q \rangle$:
  - Smooth crossover:
- Qualitative
  - $\text{QCD}_{\text{hadrons}} = \text{QCD}_{\text{partons}}$
  - strong CP is a problem
  - Your mass = E_0/c^2;
  - $\text{Nobel}_{\text{KM}} \oplus$ BSM hints;
  - QCD breaks chiral symmetry;
  - Cooling universe.

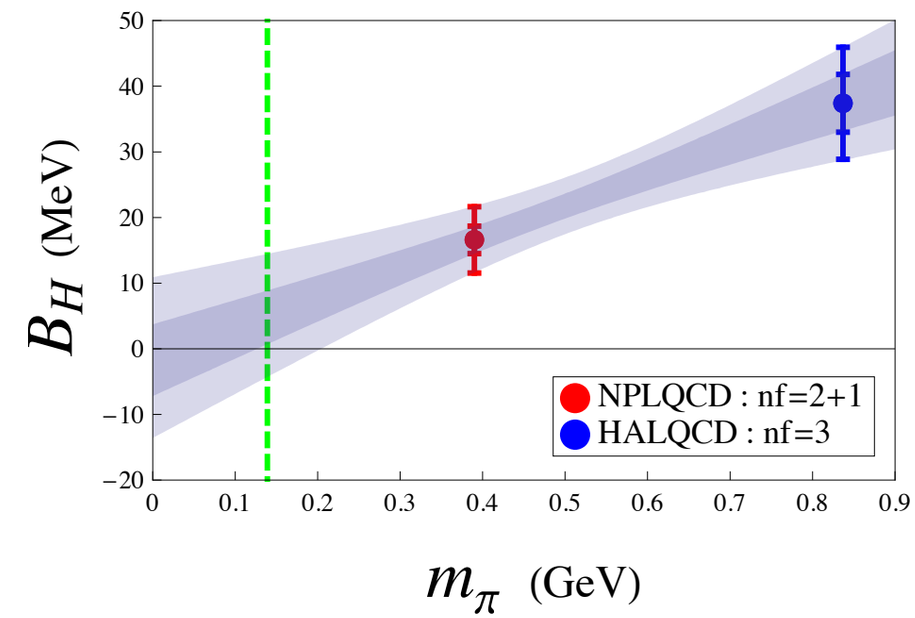
Challenges

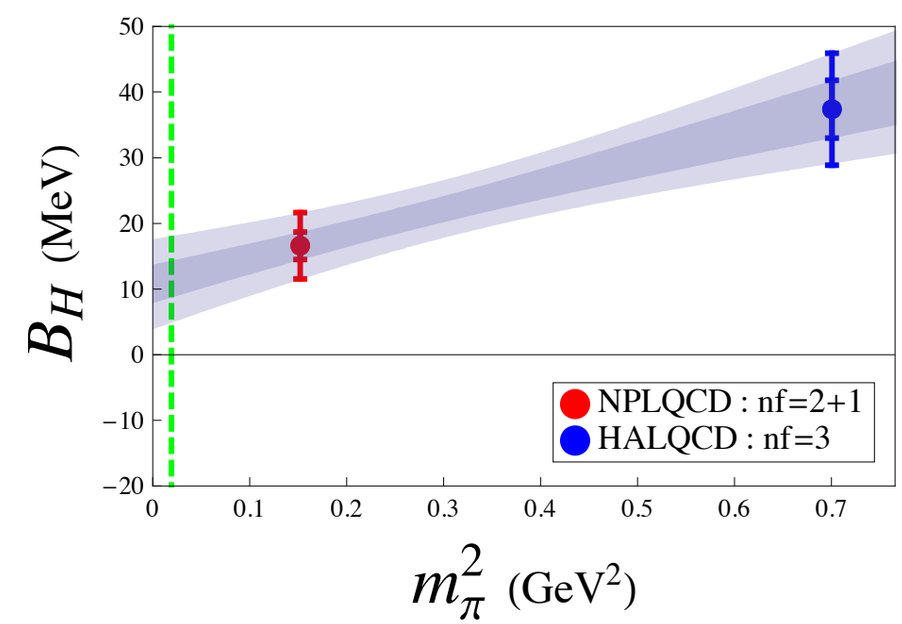
- Particle physics:
  - 1% precision for flavor physics;
  - strangeness content of the nucleon, needed for WIMP cross sections;
  - reliable moments for the nucleon's gluon distribution;
  - non-QCD gauge theories of electroweak symmetry breaking.
- Nuclear physics:
  - larger chemical potential;
  - excited states;
  - multi-hadron states, mixing, and, soon enough, nuclei—

Atomic Nuclei from QCD

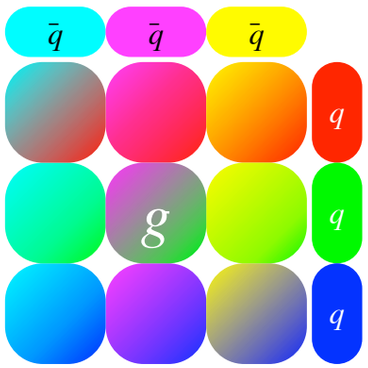
HALQCD, *PRL* **106** (2011) 162002;
NPLQCD, *PRL* **106** (2011) 162001, *MPLA* **26** (2011) 2587

- The simplest nucleus is the deuteron, $d = pn$.
- Barely bound: fine tuning of QCD parameters.
- Do other dibaryons exist? Conjectured $H = \Lambda\Lambda$.
- Recent lattice QCD calculations, with unphysical quark masses, suggest a bound state, but not yet definitive.

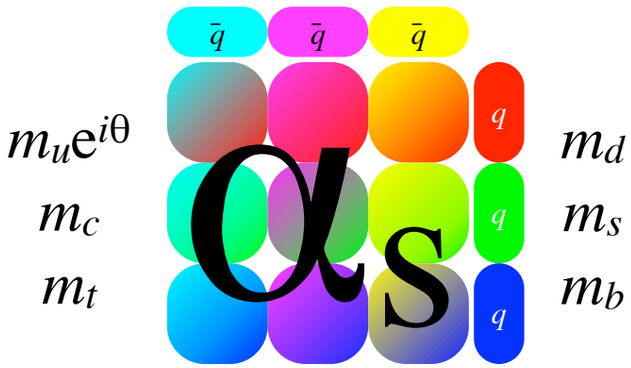




Emergent Phenomena in QCD



Emergent Phenomena in QCD



Emergent in

Phenomena QCD

nuclear physics

nuclear forces

neutron stars

hadron-hadron interactions

phase shifts

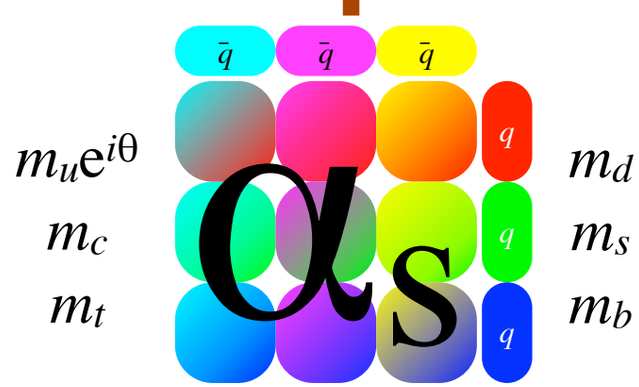
excited states

hadrons

weak decays

phase transitions in early universe

confinement & χSB



hadron structure: pdfs, $\chi N \to \chi N$

Thank you!