Whither† SUSY?

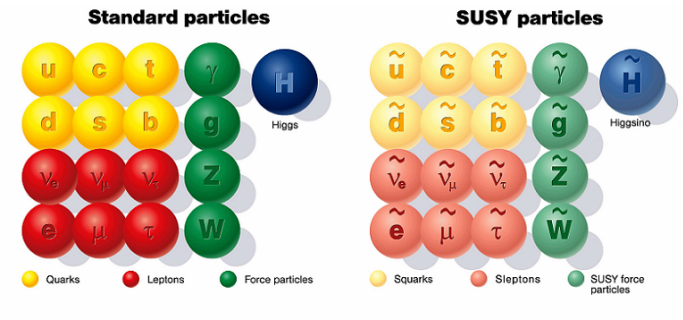
G. Ross, Fermilab, August 2013

†
whither *Archaic or poetic*
adv
1. to what place?
2. to what end or purpose?
conj
to whatever place, purpose, etc.
[Old English *hwider, hwæder;* related to Gothic *hvadrē;* modern English form influenced by HITHER]

Low energy

SUSY – to what end or purpose?

^



Low energy

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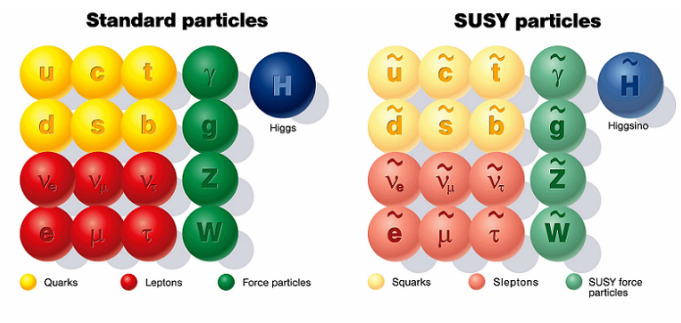
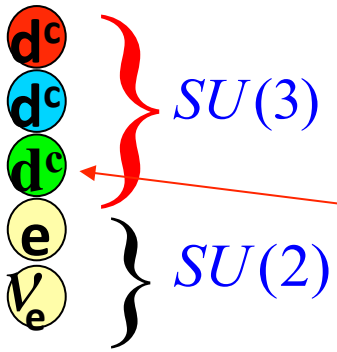


Unification:

$$e.g.\quad SO(10) \supset \underset{g_5}{SU(5)} \supset \underset{g_3}{SU(3)} \otimes \underset{g_2}{SU(2)} \otimes \underset{g_1}{U(1)}$$

Georgi Glashow 1974

$(\bar{5})_L$:

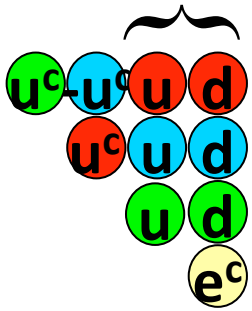


$$3Q_? + Q_{e^-} = 0$$

$$Q_{d^c} = 1/3$$

LH states SU(2) doublets

$(10)_L$:

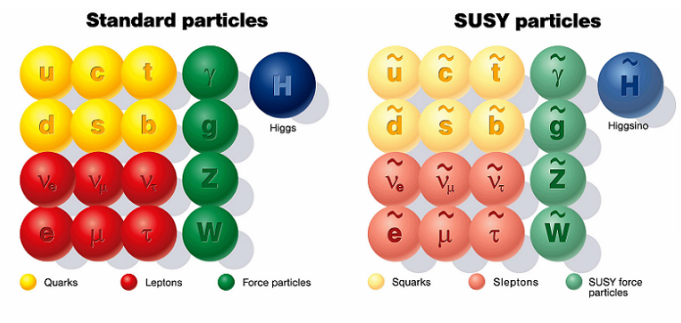


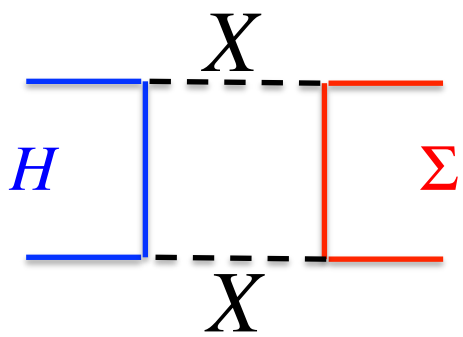
$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

$$\nu_{e,L}{}^c \equiv \nu_{e,R}$$

Low energy

SUSY - to what end or purpose?





SUSY Unification: the hierarchy problem

$$e.g.\quad \underset{}{SO(10)} \supset \underset{g_5}{SU(5)} \supset \underset{g_3}{SU(3)} \otimes \underset{g_2}{SU(2)} \otimes \underset{g_1}{U(1)}$$

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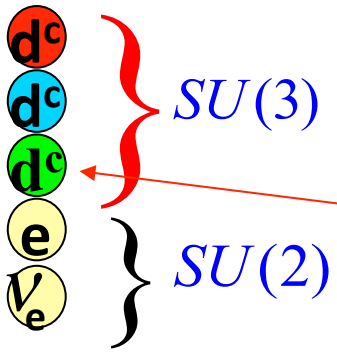

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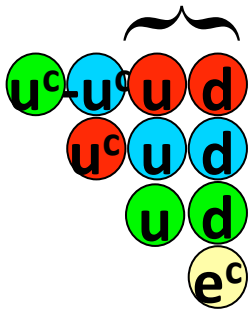

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$$\nu_{e,L}{}^c \equiv \nu_{e,R}$$

Low energy

SUSY – to what end or purpose?

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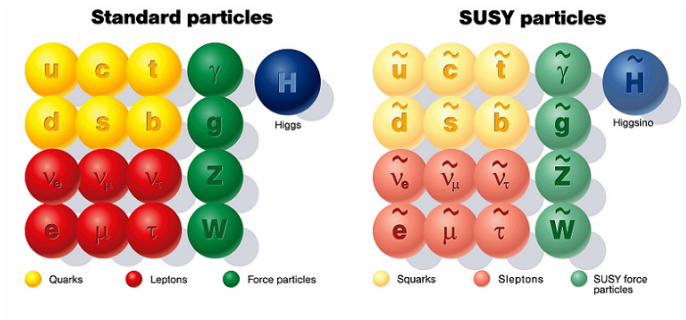
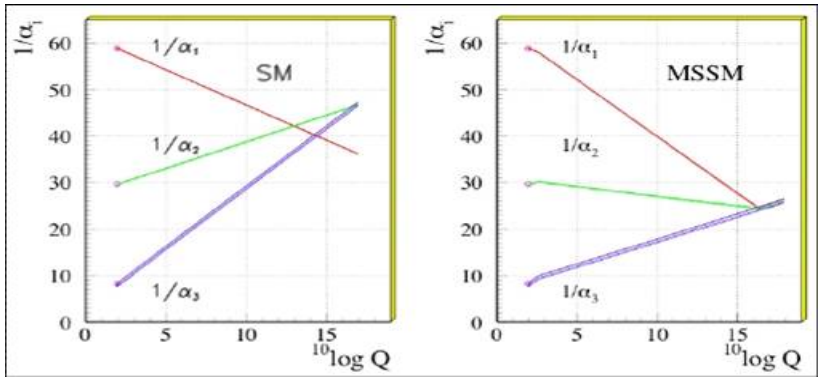


Unification:



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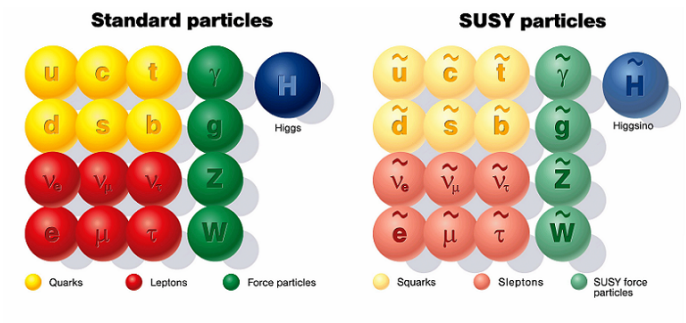
Low energy

SUSY – to what end or purpose?



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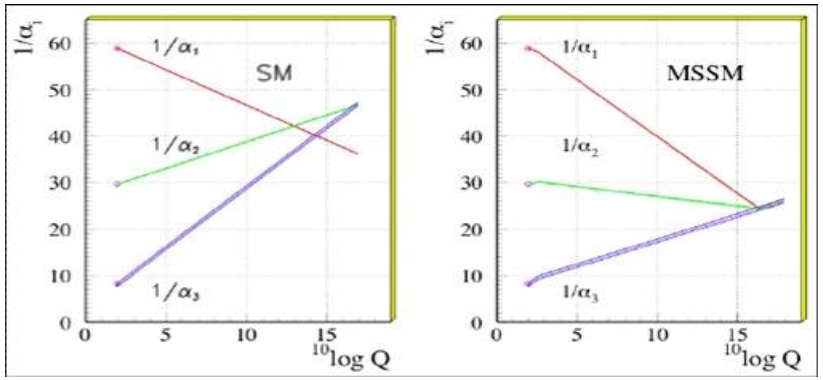


The (SUSY) Standard Model as an EFT:

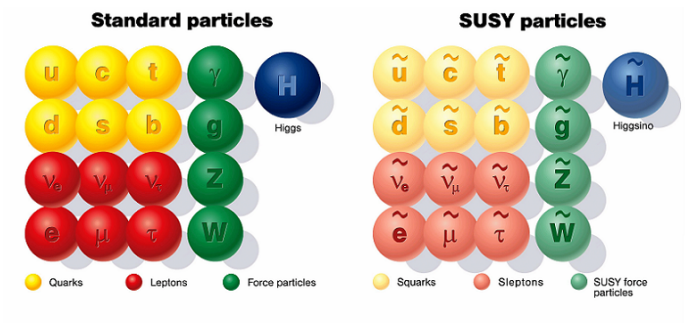
A_μ ✔, Ψ ✔ H ✔ ?

$$M_{Higgs}, M_{W,Z} \ll M_{Planck}, M_{GUT}, ..$$ ✔

Low energy

SUSY – to what end or purpose?

Λ



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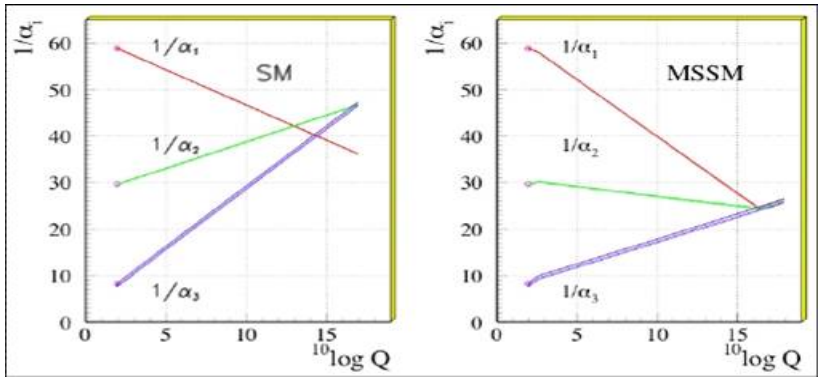

✔

The (SUSY) Standard Model as an EFT:

A_μ ✔, Ψ ✔ H ✔ ?

$$M_{Higgs}, M_{W,Z} \ll M_{Planck}, M_{GUT}, ..$$ ✔

$$\delta m_{H_u}^2 \simeq -\frac{3y_t^2}{4\pi^2}\left(m_{stop}^2 + \frac{g_s^2}{3\pi^2} m_{gluino}^2 \log\left(\frac{\Lambda}{m_{gluino}}\right)\right)\log\left(\frac{\Lambda}{m_{stop}}\right)$$?

Little hierarchy problem

SUSY – to what place?

breaking

Little hierarchy problem $\Rightarrow$ definite SUSY structure

^

MSSM: 105 +(19) Parameters

$$M_Z^2 = \sum_{\tilde{q},\tilde{l}} a_i \tilde{m}_i^2 + \sum_{\tilde{g},\tilde{W},\tilde{B}} b_i \tilde{M}_i^2 + \ldots$$

$$m_{\tilde{q}} > 0.6 - 1 TeV \quad \Rightarrow \quad \Delta > a \frac{\tilde{m}^2}{M_Z^2} \sim 100$$

(Unless light stop $m_{\tilde{t},LHC} > 250\ GeV$)

$\Longrightarrow$ Correlations between SUSY breaking parameters and/or additional low-scale states

SUSY – to what place?

Little hierarchy problem $\Rightarrow$ definite SUSY breaking structure

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$\Rightarrow$ Correlations between SUSY breaking parameters and/or additional low-scale states

Fine Tuning measure:

$$\Delta(a_i) = \left| \frac{a_i}{M_Z} \frac{\partial M_Z}{\partial a_i} \right|,$$

$$\Delta_m = Max_{a_i} \Delta(a_i), \quad \Delta_q = \left(\sum \Delta_{\gamma_i}^2 \right)^{1/2}$$

Ellis, Enquist, Nanopoulos, Zwirner

Barbieri, Giudice

Fine tuning from a likelihood fit:

"Nuisance" variable

$$L(\text{data} \mid \gamma_i) \propto \int d\text{v}\,\delta\left(m_Z - m_Z^0\right)\delta\left(\text{v-}\left(-\frac{m^2}{\lambda}\right)^{1/2}\right) L(\text{data} \mid \gamma_i;\text{v})$$

$$= \frac{1}{\Delta_q}\delta\left(n_q(\ln\gamma_i - \ln\gamma_i^S)\right) L(\text{data} \mid \gamma_i;\text{v}_0)$$

Fine tuning not optional!

Ghilencea, GGR
Casas et al

Probabilistic interpretation:

$$\chi^2_{new} = \chi^2_{old} + 2\ln\Delta_q$$

$$\Delta_q \ll 100$$

- The CMSSM

$$\gamma_i = \mu_0, m_0, m_{1/2}, A_0, B_0$$

assume correlation between SUSY breaking parameters

The CMSSM

$\mu_0, m_0, m_{1/2}, A_0, B_0$

$$
\begin{aligned}
V &= m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (m_3^2\,H_1\cdot H_2 + h.c.) \\
&+ \frac{1}{2}\lambda_1\,|H_1|^4 + \frac{1}{2}\lambda_2\,|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2 + \lambda_4\,|H_1\cdot H_2|^2 \\
&+ \left[\frac{1}{2}\lambda_5\,(H_1\cdot H_2)^2 + \lambda_6\,|H_1|^2\,(H_1\cdot H_2) + \lambda_7\,|H_2|^2\,(H_1\cdot H_2) + h.c.\right]
\end{aligned}
$$

Minimisation conditions:

$$v^2 = -m^2/\lambda, \qquad 2\lambda\frac{\partial m^2}{\partial\beta} = m^2\frac{\partial\lambda}{\partial\beta}$$

$$
\begin{aligned}
m^2 &= m_1^2\cos^2\beta + m_2^2\sin^2\beta - m_3^2\sin 2\beta \\
\lambda &= \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\,(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta)
\end{aligned}
$$

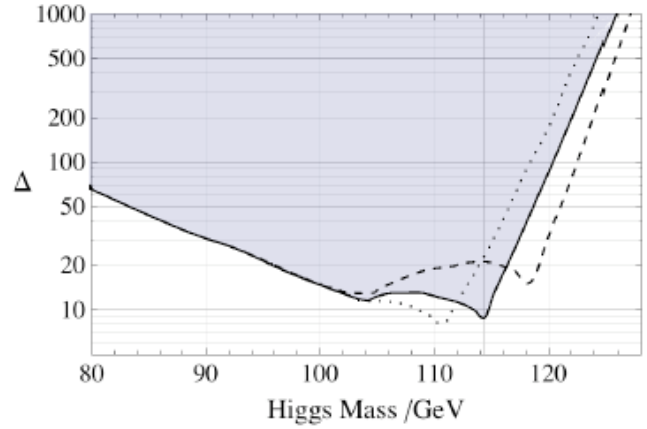
$$\Delta \equiv \max\left|\Delta_p\right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p}$$

Couplings and masses evaluated to two loop (leading log) order

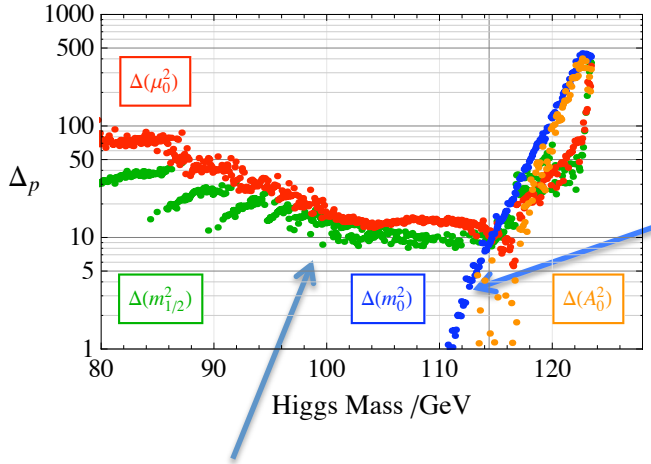
...enhanced sensitivity due to small tree-level $\lambda = \frac{1}{8}\left(g_1^2 + g_2^2\right)\cos^2 2\beta$

Cassel, Ghilencea, GGR
c.f. earlier work : Dimopoulos, Giudice
Chankowski, Ellis, Olechowski, Pokorski

The CMSSM - before LHC



$$\Delta_i,\ i = \mu_0, m_0, m_{1/2}, A_0, B_0$$



Limit of focus point

λ increase with m_H

$$v^2 = -\frac{m^2}{\lambda}$$

Constraints

SUSY particle masses

$$3.20 < 10^4\ \mathrm{Br}(b \to s\gamma) < 3.84$$

$$\mathrm{Br}(b \to \mu\mu) < 1.8 \times 10^{-8}$$

$$\delta a_\mu < 292 \times 10^{-11}$$

$$-0.0007 < \delta\rho < 0.0012$$

Radiative EW breaking

Relic density unrestricted

Guuge coupling unification

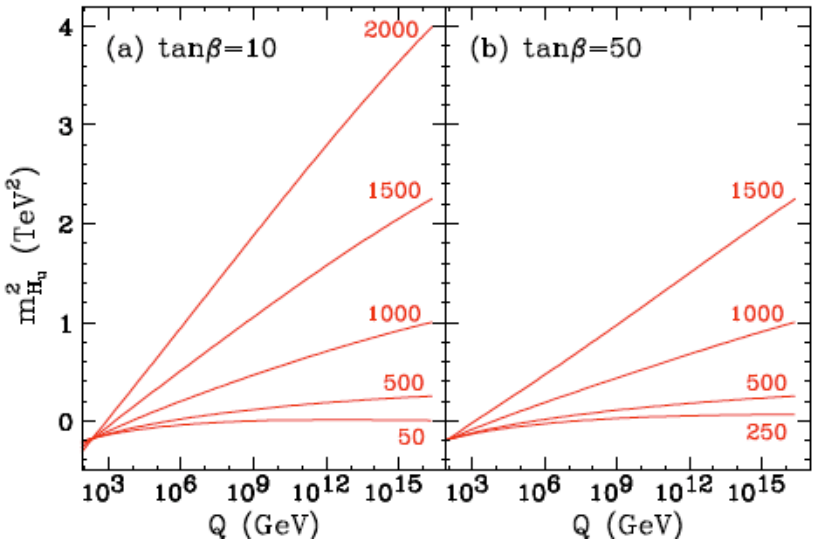
Focus Point

$$2|y_t|^2(m_{H_u}^2+m_{Q_3}^2+m_{\bar{u}_3}^2)+2|a_t|^2$$

$$16\pi^2\frac{d}{dt}m_{H_u}^2=3X_t-6g_2^2|M_2|^2-\frac{6}{5}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m_{Q_3}^2=X_t+X_b-\frac{32}{3}g_3^2|M_3|^2-6g_2^2|M_2|^2-\frac{2}{15}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m_{\bar{u}_3}^2=2X_t-\frac{32}{3}g_3^2|M_3|^2-\frac{32}{15}g_1^2|M_1|^2$$



$$m_{H_u}^2(Q^2)=m_{H_u}^2(M_P^2)+\frac{1}{2}\left(m_{H_u}^2(M_P^2)+m_{Q_3}^2(M_P^2)+m_{\bar{u}_3}^2(M_P^2)\right)\left[\left(\frac{Q^2}{M_P^2}\right)^{\frac{3y_t^2}{4\pi^2}}-1\right]$$

m_0^2 $\quad$ $3m_0^2$ $\quad$ $\simeq -\frac{2}{3},\ Q^2\simeq M_Z^2$

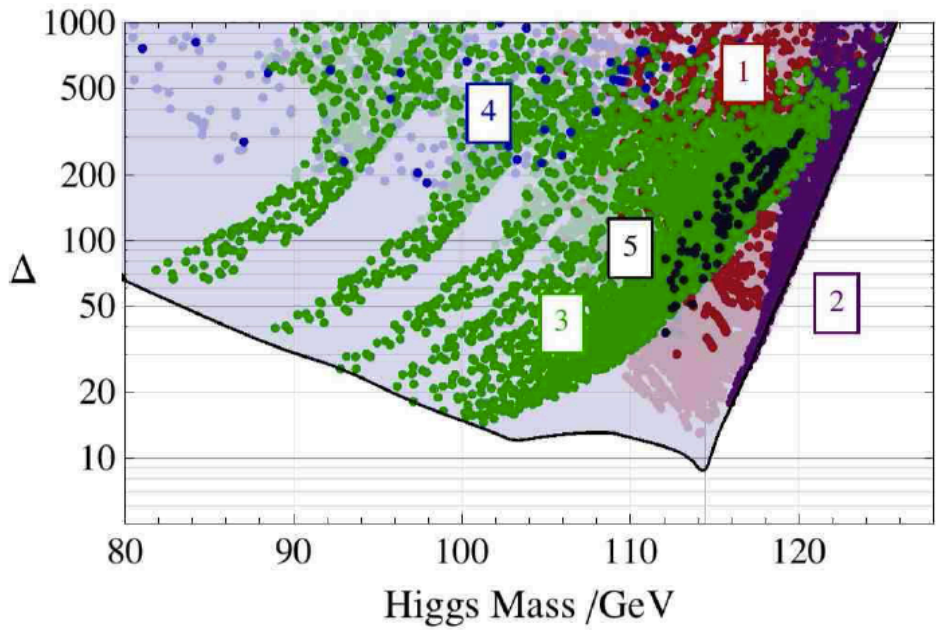
"Focus point": $m_{H_u}^2(0)=m_{Q_3}^2(0)=m_{\bar{u}_3}^2(0)\equiv m^2$

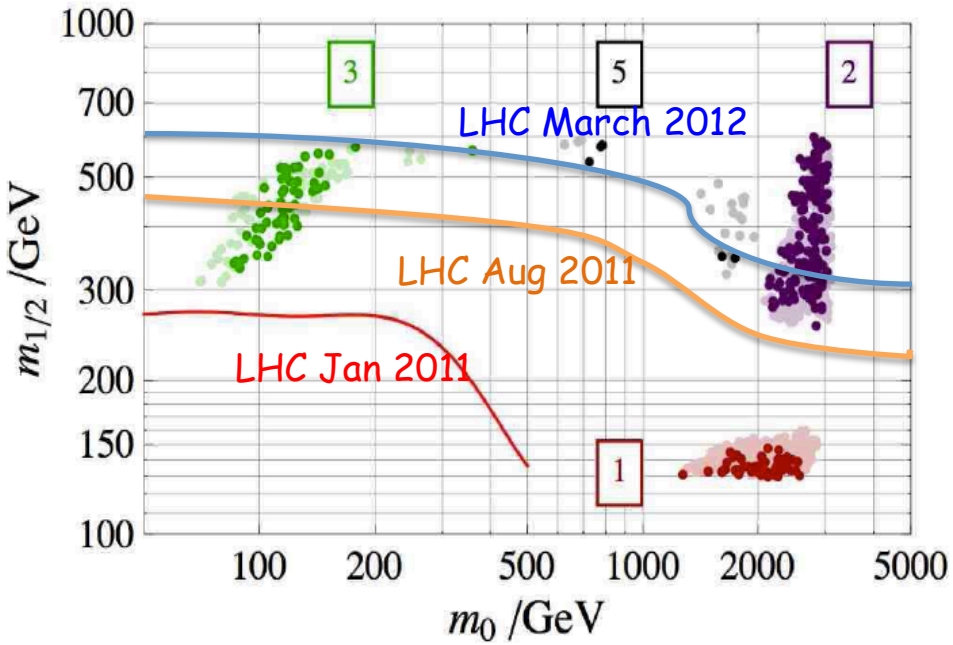
$$m_{H_u}^2(t_0)=a_0\,m^2+..,\ a_0\le 0.1$$

Natural choice

$i.e.\ m_{Q_3}^3, m_{\bar{u}_3}^2 \gg M_Z^2$ possible

Feng, Matchev, Moroi
Chan, Chattopadhay, Nath
Barbieri, Giudice
Feng, Sanford





Relic density restricted

1 h^0 resonant annihilation

2 $\tilde{h}$ t-channel exchange

3 $\tilde{\tau}$ co-annihilation

4 $\tilde{t}$ co-annihilation

• 5 A^0 / H^0 resonant annihilation

Within 3σ WMAP:

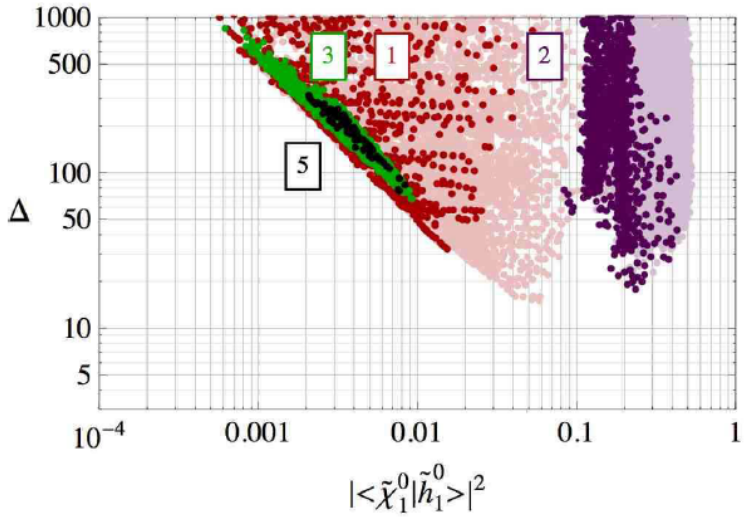
$$\Delta_{Min} = 15, \quad m_h = 114.7 \pm 2GeV$$

$< 3\sigma$ WMAP:

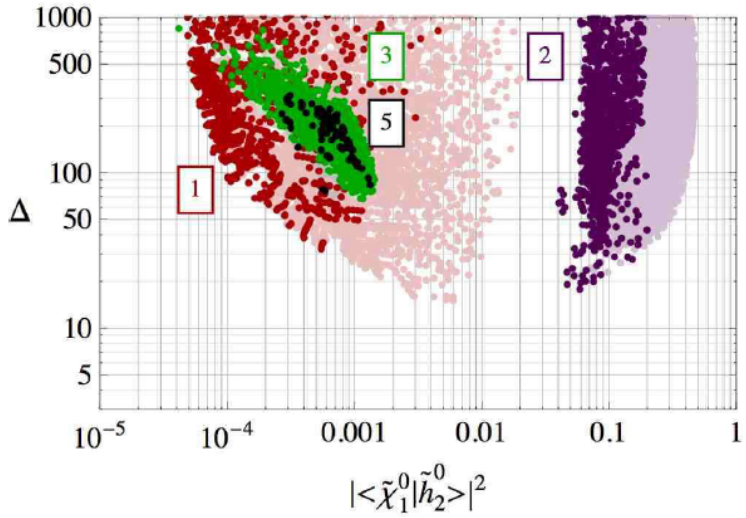
$$\Delta_{Min} = 18, \quad m_h = 115.9 \pm 2GeV$$

Cassel, Ghilencea, GGR

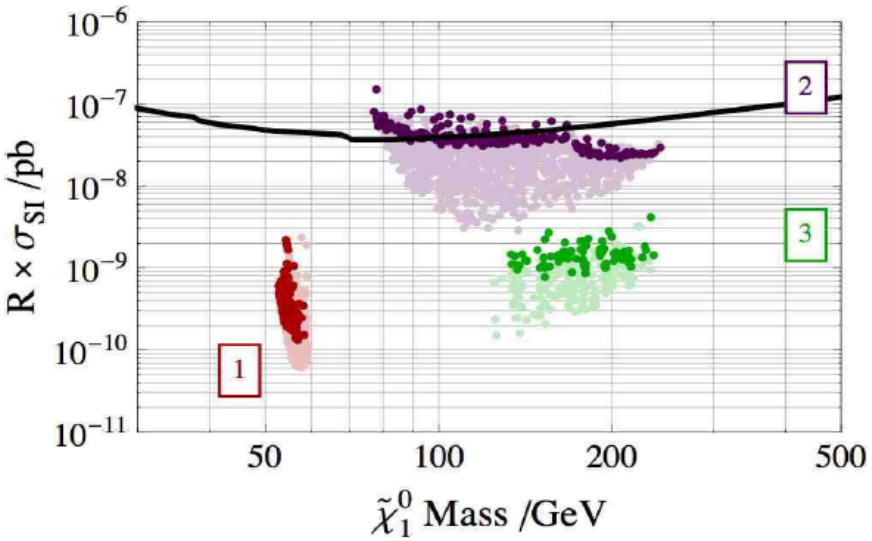
DM - Scaled spin independent cross section for LSP-proton scattering:



(a) LSP $\tilde{h}_1^0$ component

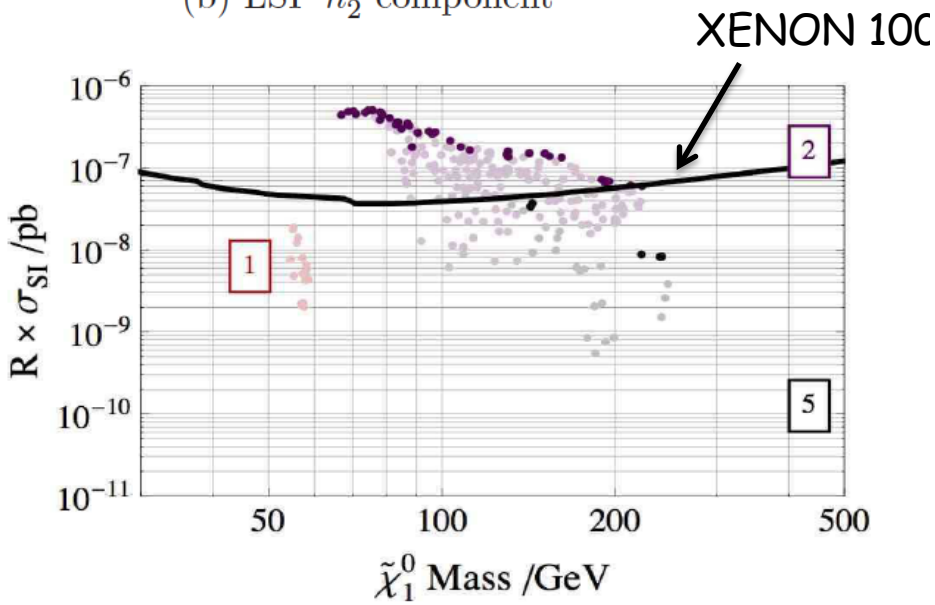


(b) LSP $\tilde{h}_2^0$ component



(a) $\tan\beta \leq 45$

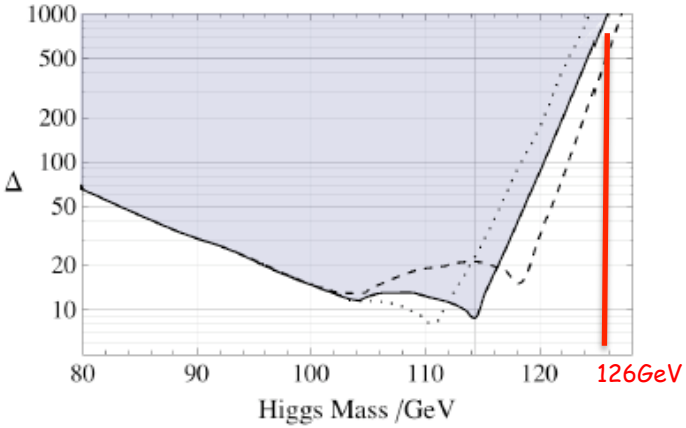
$\Delta < 100$



(b) $50 \leq \tan\beta \leq 55$

$\Delta < 100$

The CMSSM – after Higgs discovery



$$M_S^2 = m_{q_3} m_{U_3}$$

$$M_{h^0}^2 = M_Z^2 \cos^2 2\beta + \frac{3M_t^2 h_t^2}{4\pi^2}\left(\ln\left(\frac{M_S^2}{M_t^2}\right) + \delta_t\right) + \cdots \qquad \simeq 126 GeV$$

$$\boxed{\Delta_{Min} > 350\,, \quad m_h = 125.6 \pm 3 GeV}$$

Reduced fine tuning (c.f. CMSSM)

- New focus points?

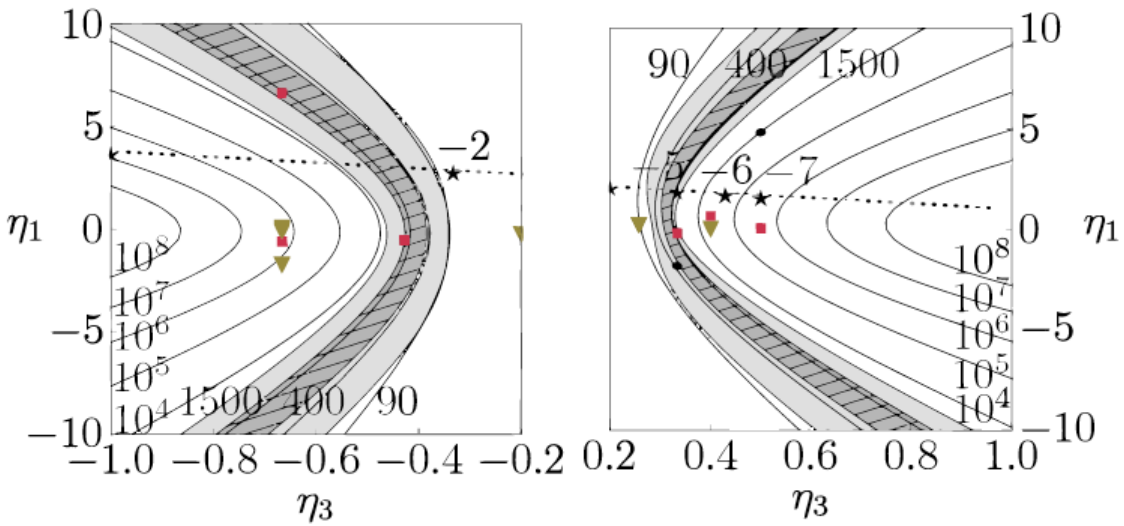
  Gauginos: $M_{\tilde{g},\widetilde{W},\tilde{B}}$ Non-universal gaugino correlations

- New degrees of freedom

I. Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}



$$M_3 : M_2 : M_1 = \eta_3 : 1 : \eta_1$$

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Natural ratios? e.g.:

GUT: $SU(5): \quad \Phi^N \subset (24 \times 24)_{symm} = 1 + 24 + 75 + 200; \quad SO(10): \quad (45 \times 45)_{symm} = 1 + 54 + 210 + 770$

	$\eta_3 : 1 : \eta_1$	$2.7\eta_3 : 1 : 0.5\eta_1$
Representation	$M_3 : M_2 : M_1$ at M_{GUT}	$M_3 : M_2 : M_1$ at M_{EWSB}
1	1:1:1	6:2:1
24	2:(-3):(-1)	12:(-6):(-1)
75	1:3:(-5)	6:6:(-5)
200	1:2:10	6:4:10

String: $(3+\delta_{GS}):(-1+\delta_{GS}):\left(-\frac{33}{5}+\delta_{GS}\right)$ (OII, also mixed moduli anomaly)

I. Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

$$\Delta_{Min}^{CMSSM} = 60\ (500), \quad m_h = 125.6 \pm 3 GeV$$

LHC8 SUSY bounds ✔

DM relic abundance ✔

DM searches ✗

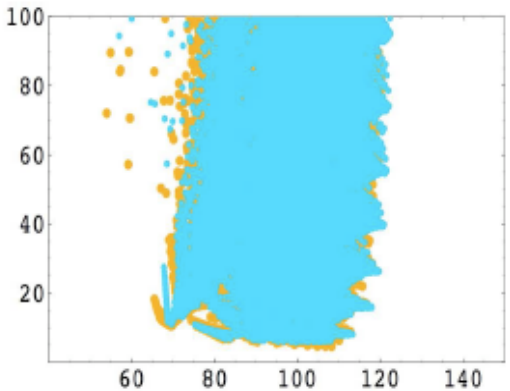
II. Reduced fine tuning : New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0 \theta\theta$$

Dimension 5

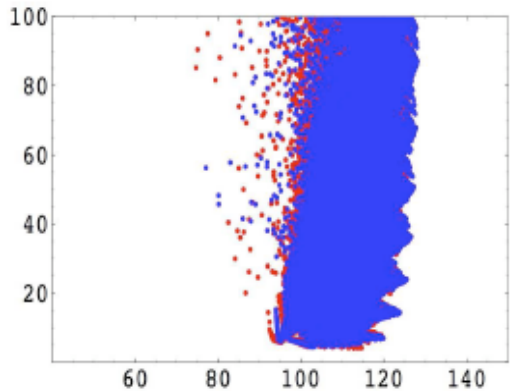
$$\delta V = \varsigma_1 \left(|h_u|^2 + |h_d|^2\right) h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \; \varsigma_2 = \frac{c_0 m_0}{M_*}$$

Δ



m_h

MSSM



m_h

+ dim 5 operators

Cassel, Ghilencea, GGR
Casas, Espinosa, Hidalgo
Dine, Seiberg, Thomas
Batra, Delgardo, Tait
Kaplan,

Even for M_*=65 μ_0 a significant shift of m_h for constant Δ

…effect mainly comes from ς_1 term … origin?

II. Reduced fine tuning : New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0\theta\theta$$

Dimension 5

$$\delta V = \varsigma_1\left(|h_u|^2 + |h_d|^2\right)h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \ \varsigma_2 = \frac{c_0 m_0}{M_*}$$

Singlet extensions

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

NMSSM

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$

GNMSSM

$$\mu_S >> m_{3/2} : \quad W_{eff}^{GNMSSM} = (H_u H_d)^2 / \mu_s + \mu H_u H_d$$

$$\delta V = \frac{\mu}{\mu_S}\left(|H_u|^2 + |H_d|^2\right) H_u H_d$$

✔

II. Reduced fine tuning : New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0 \theta\theta$$ Dimension 5

$$\delta V = \varsigma_1 \left(|h_u|^2 + |h_d|^2\right) h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \ \varsigma_2 = \frac{c_0 m_0}{M_*}$$

Singlet extensions

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$ NMSSM

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$ GNMSSM

$$\mu_S >> m_{3/2} : \quad W_{eff}^{GNMSSM} = (H_u H_d)^2 / \mu_s + \mu H_u H_d$$

$$\delta V = \frac{\mu}{\mu_S}\left(|H_u|^2 + |H_d|^2\right) H_u H_d$$ ✔

but are μ, μ_s naturally small?

SUSY extensions of the Standard Model

$$W = h^E L H_d \overline{E} + h^D Q H_d \overline{D} + h^U Q H_u \overline{U} + \mu H_d H_u$$
$$+ \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_u + \lambda'' \overline{U}\,\overline{D}\,\overline{D}$$
$$+ \frac{1}{M}\left(Q Q Q L + Q Q Q H_d + Q \overline{U}\,\overline{E} H_d + L L H_u H_u \right)$$

R-parity: Z_2

$H_u, H_d \quad +1$

$L, \overline{E}, Q, \overline{D}, \overline{U}, \quad \theta \quad -1$

SUSY states odd

Weinberg, Sakai

SUSY extensions of the Standard Model

$$W = h^E L H_d \overline{E} + h^D Q H_d \overline{D} + h^U Q H_u \overline{U} + \mu H_d H_u + \mu_s H_d H_u$$
$$+ \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_u + \lambda'' \overline{U}\,\overline{D}\,\overline{D}$$
$$+ \frac{1}{M}\left(Q Q Q L + Q Q Q H_d + Q \overline{U}\,\overline{E} H_d + L L H_u H_u \right)$$

R-parity: Z_2

Z_N^R R-symmetry

N=4,6,8,12,24

Discrete gauge symmetry
-anomaly free

Ibanez, GGR

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_S
4	1	1	0	0	2
8	1	5	0	4	6

R-symmetry ensures singlets light

$SU(5), SO(10)$
compatible

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange

SUSY extensions of the Standard Model

$$W = h^E L H_d \overline{E} + h^D Q H_d \overline{D} + h^U Q H_u \overline{U} + \mu H_d H_u + \mu_s H_d H_u$$

$$+ \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_u + \lambda'' \overline{U}\,\overline{D}\,\overline{D}$$

$$+ \frac{1}{M}\left(Q Q Q L + Q Q Q H_d + Q \overline{U}\,\overline{E} H_d + L L H_u H_u \right)$$

R-parity: Z_2

Z_N^R R-symmetry N=4,6,8,12,24

SUSY breaking:

Domain walls safe

$\langle W \rangle, \langle \lambda\lambda \rangle$ R=2, non=perturbative breaking

$Z_{4R} \to Z_2^R \quad R-parity$

LSP stable

$\mu, \mu_s \sim m_{3/2}, \quad O(\frac{m_{3/2}}{M^2} QQQL)$

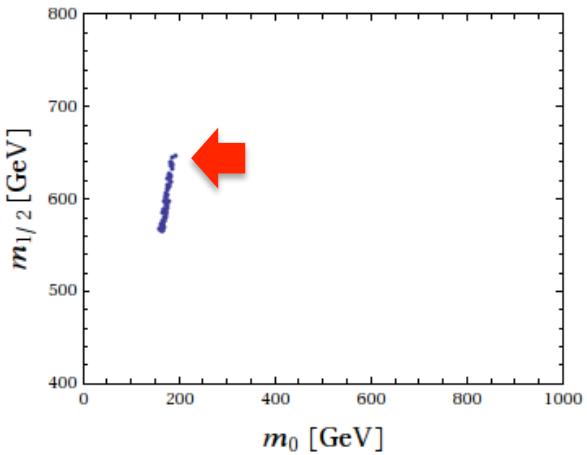
Fine tuning in the CGNMSSM $(\lambda \leq 0.7^{\dagger})$

$$\Delta_{Min} = 60\,(500), \quad m_h = 125.6 \pm 3GeV$$

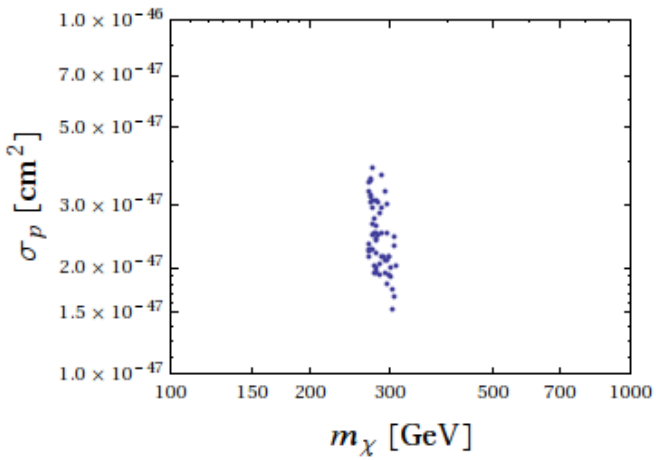
LHC8 SUSY bounds ✗

DM relic abundance ✔

DM searches ✔



Stau co-annihilation



DM searches insensitive

GGR, Schmidt-Hoberg , Staub

† c.f. Hall, Pinner, Ruderman

Fine tuning in the (C)GNMSSM $(\lambda \leq 0.7^{\dagger})$

Non-unversal gaugino masses

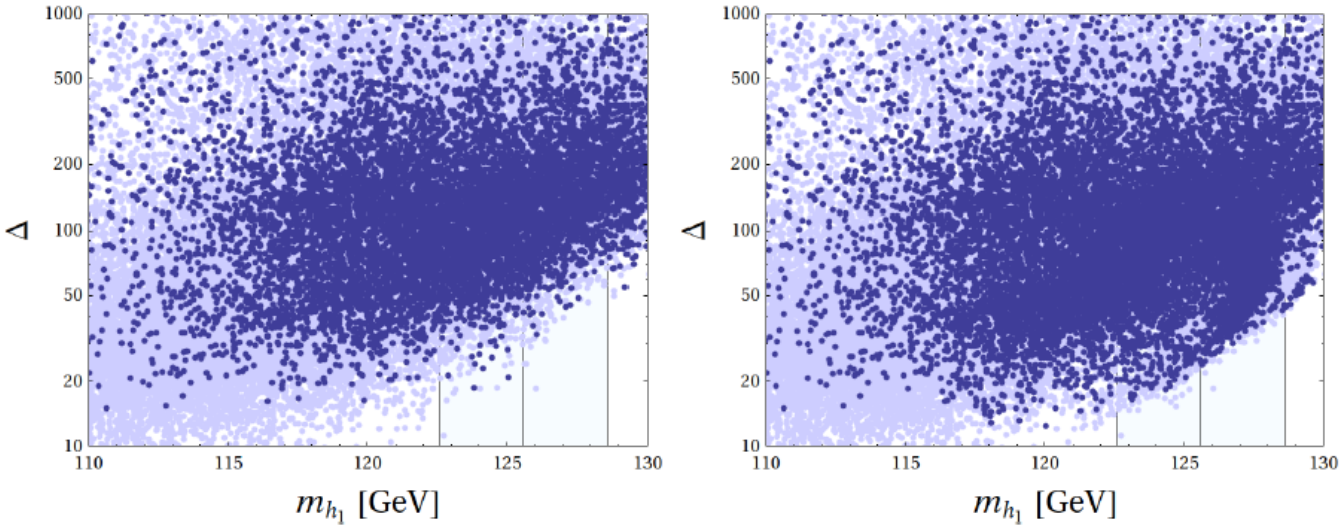
$$\Delta_{Min} = 20\,, \quad m_h = 125.6 \pm 3 GeV$$

LHC8 SUSY bounds ✔

DM relic abundance ✔

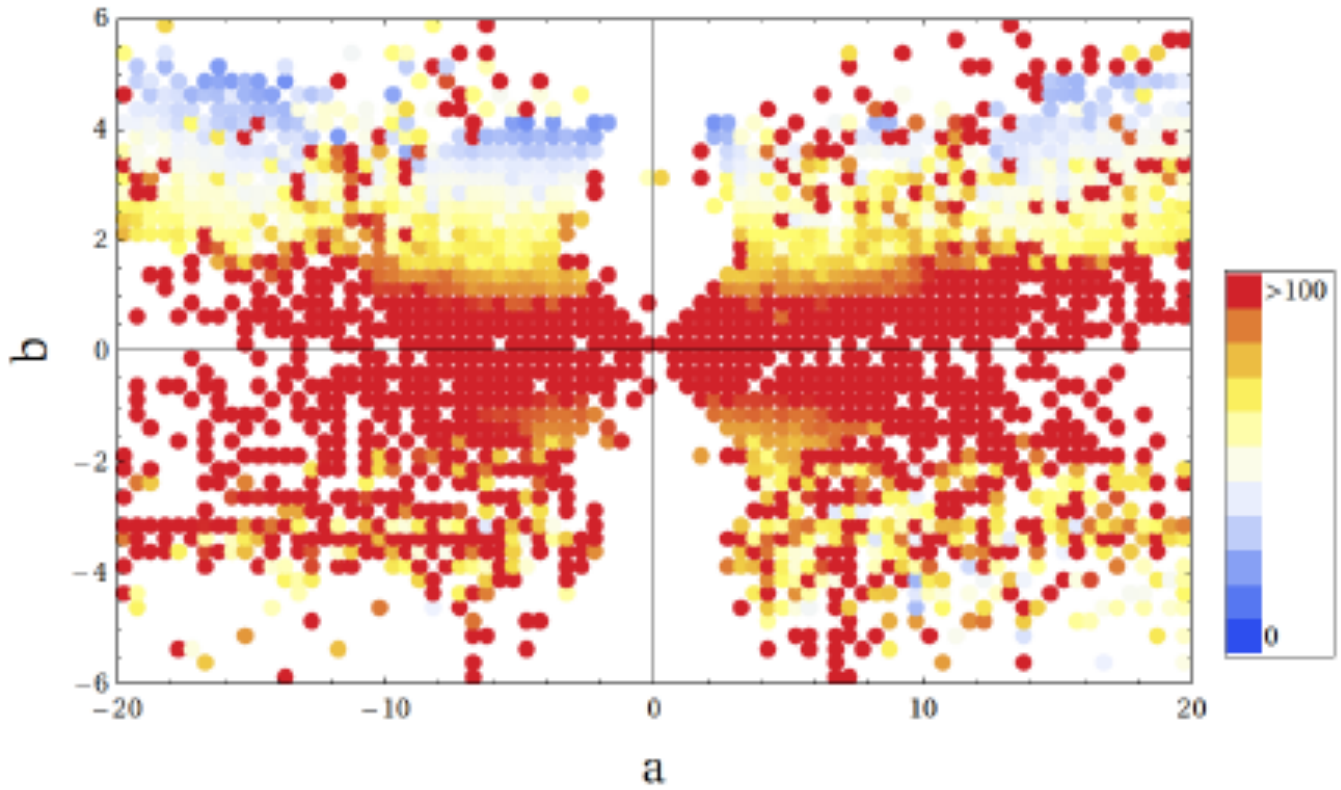
DM searches ✔

Δ



GGR, Kaminska, Schmidt-Hoberg

Fine tuning v/s gaugino mass ratios



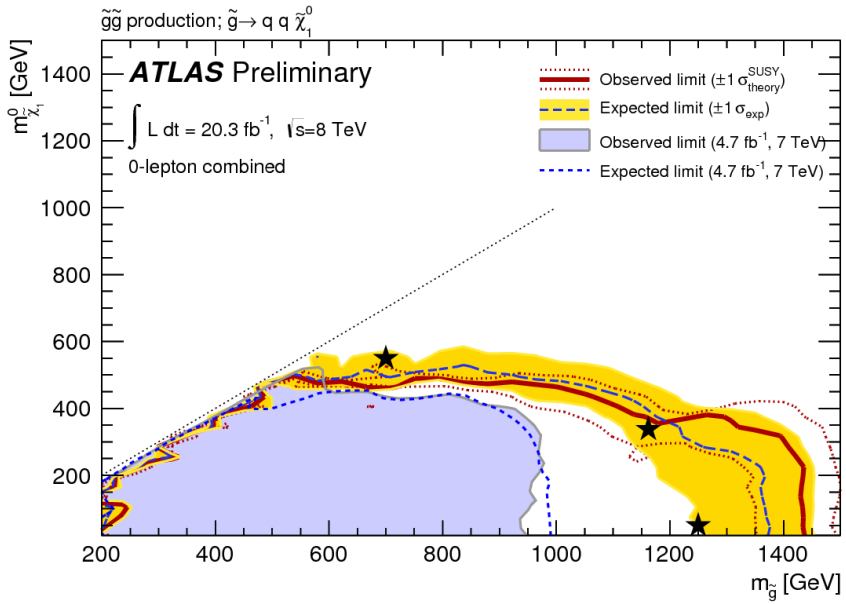
F.T.

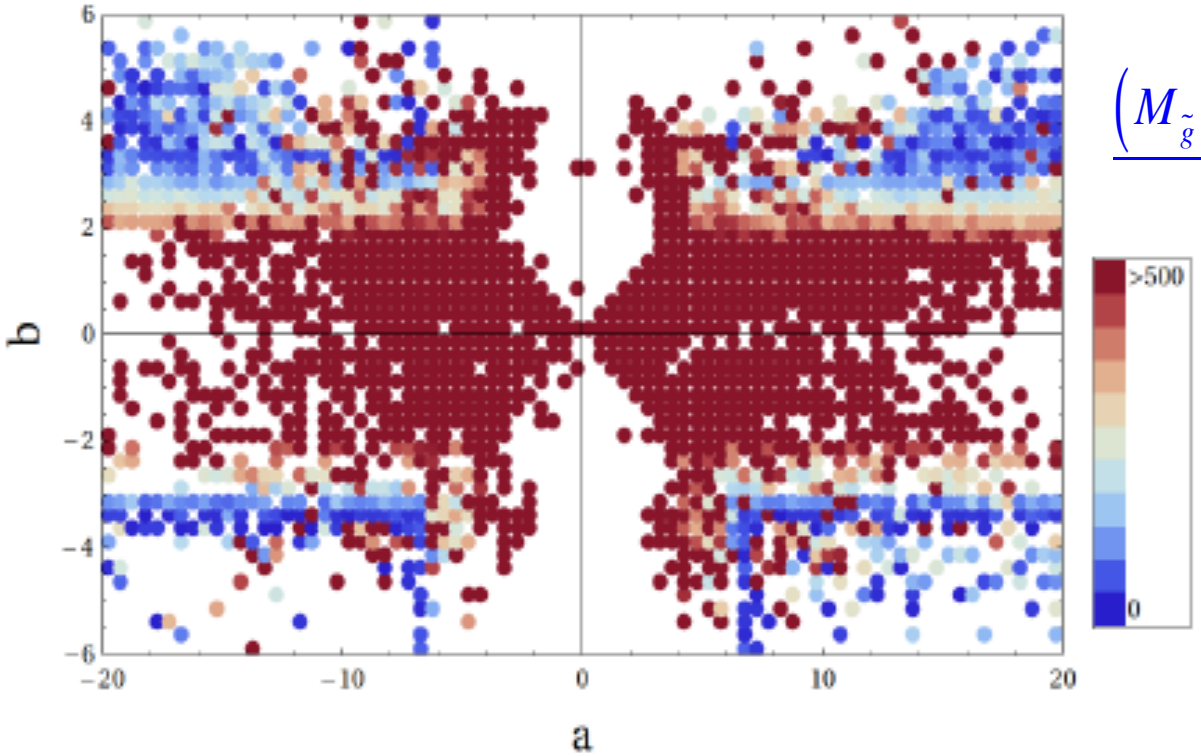
> 100

0

$$M_3 = m_{1/2}, M_2 = b.m_{1/2}, M_1 = a.m_{1/2}$$

Compressed spectrum





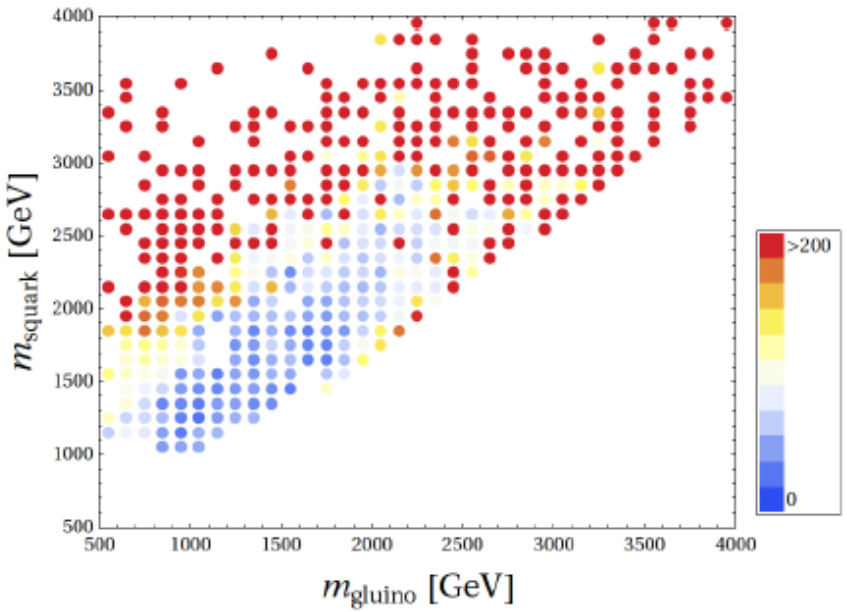
$$\frac{\left(M_{\tilde{g}} - M_{LSP}^{neutralino}\right)}{GeV}$$

> 500

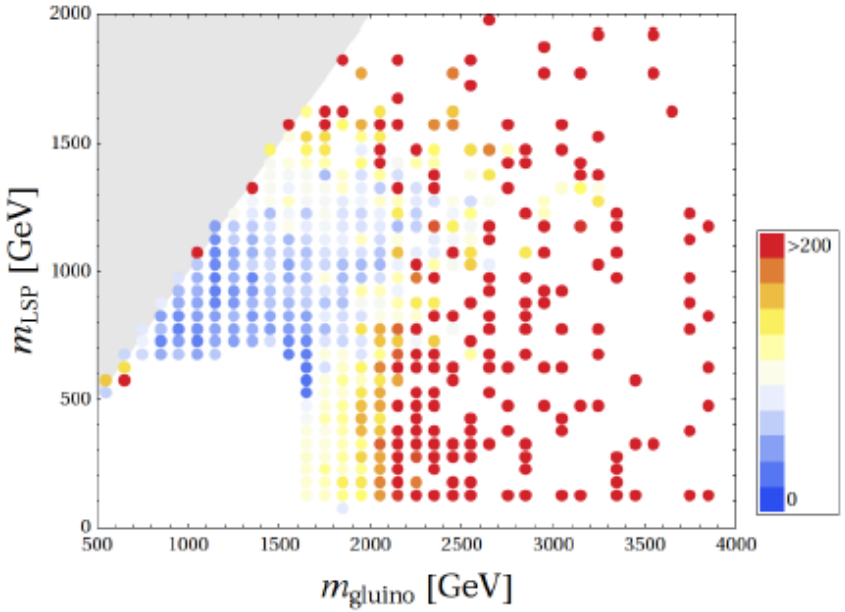
0

Masses v/s fine tuning

m_{squark}



m_{LSP}

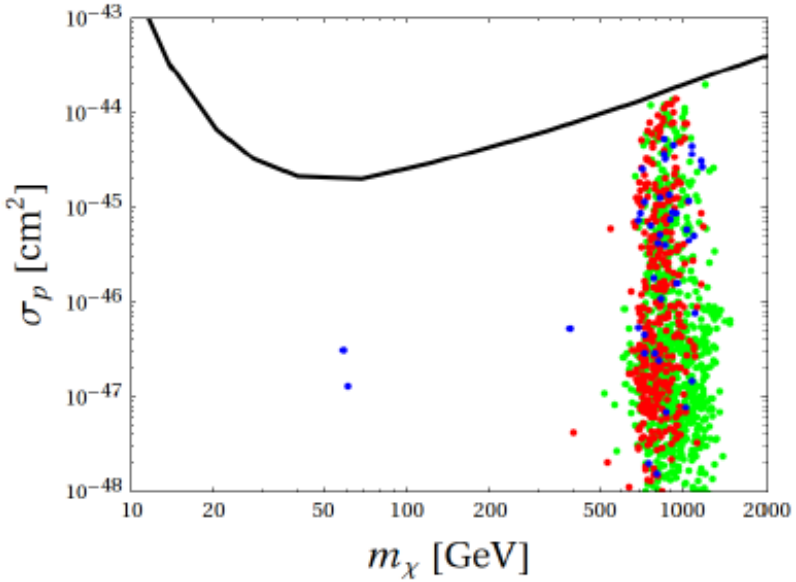


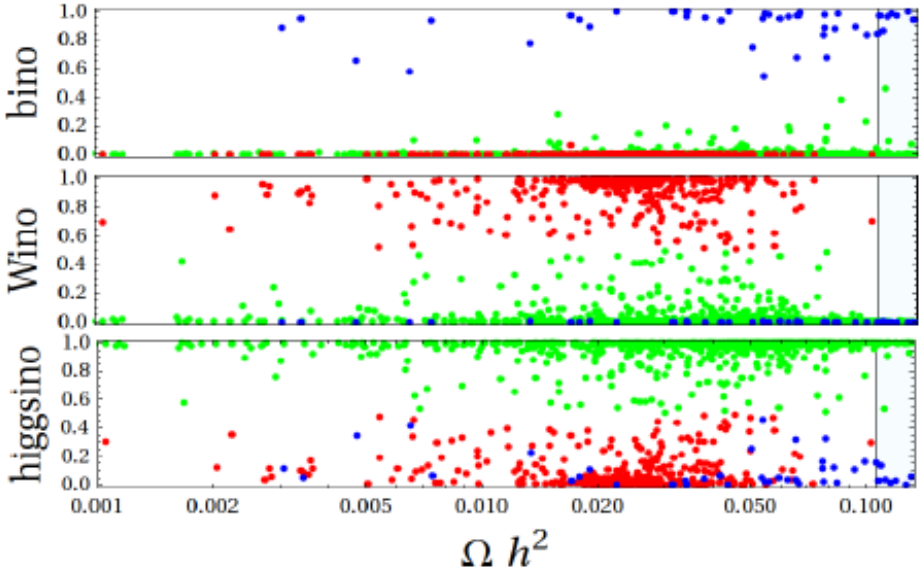
> 500

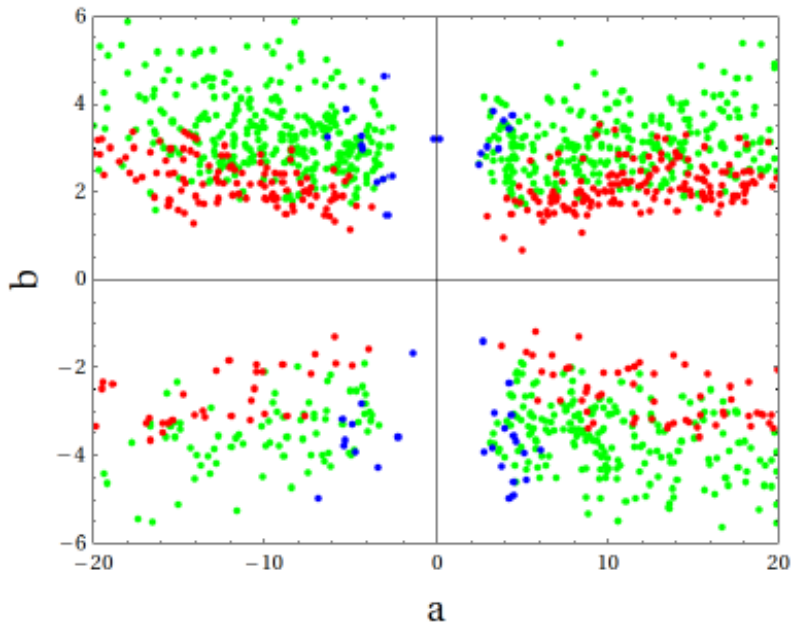
0

M_{gluino}

Dark matter







Summary

- GUTs $\Longrightarrow$ SUSY-GUTS (hierarchy problem)
- Low fine tuning not optional
- Fine tuning sensitive to SUSY spectrum

  ...scalar and gaugino focus points
- $\Delta^{CMSSM} > 350$ ✗ $\Delta^{(C)MSSM} > 60$ ✗

  $\Delta^{CGMSSM} > 60$ ✗ $\Delta^{(C)GNMMS} > 20$ ✔

$$c.f. \quad \Delta^{CMSSM}_{Low\ scale} = (10-30), \quad m_{\tilde{t}} = (1-5)TeV$$

Barger et al

Summary

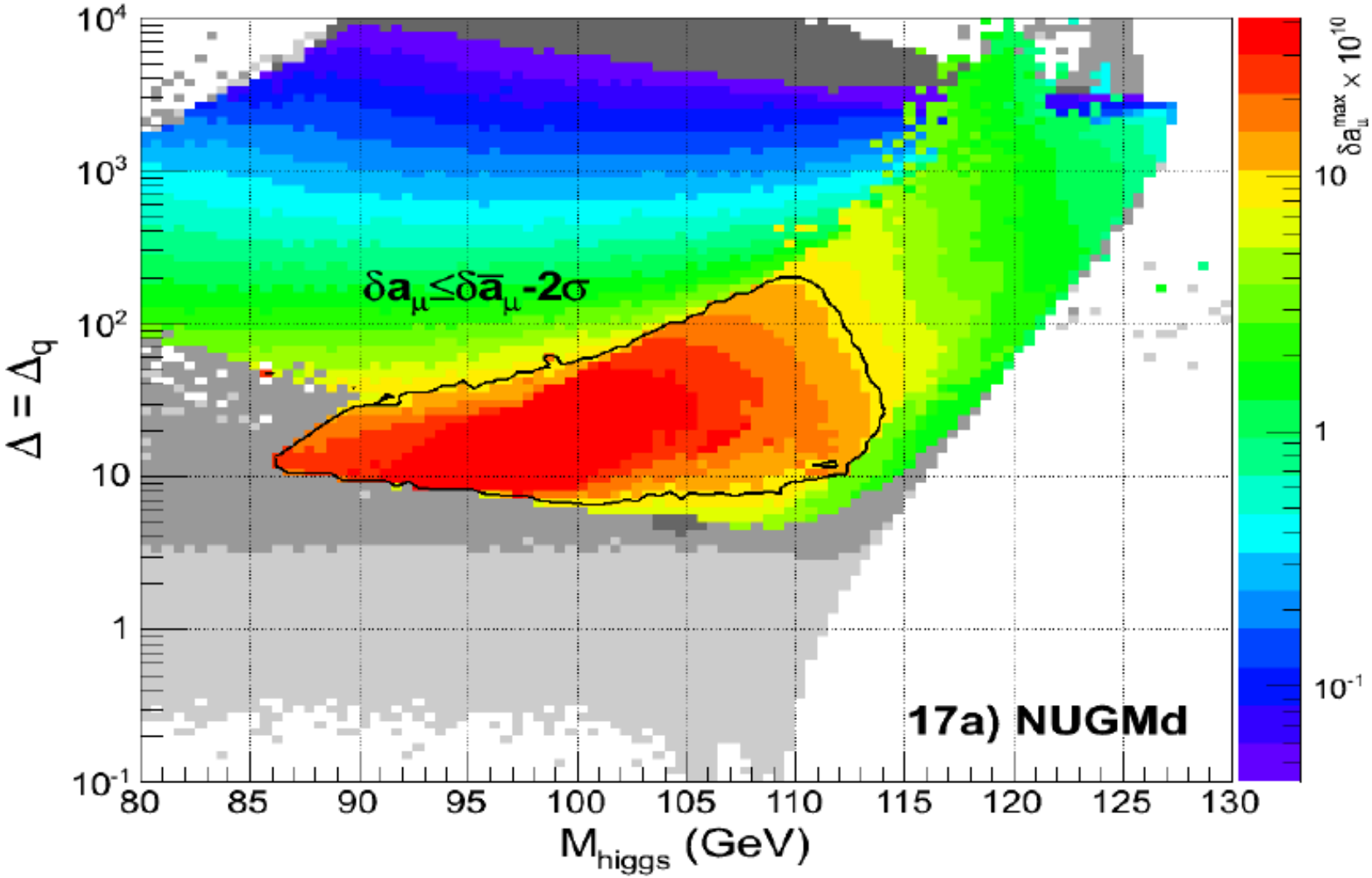
- GUTs $\Longrightarrow$ SUSY-GUTS (hierarchy problem)
- Low fine tuning not optional
- Fine tuning sensitive to SUSY spectrum

  ...scalar and gaugino focus points
- $\Delta^{CMSSM} > 350$ $\Delta^{(C)MSSM} > 60$

  $\Delta^{CGMSSM} > 60$ $\Delta^{(C)GNMMS} > 20$
- Well motivated SUSY models remain to be tested

  LHC14?

  Compressed spectra, TeV squarks and gluinos



2-loop fine tuning in 75 case

Ghilencea, Lee, Park

Muon g-2

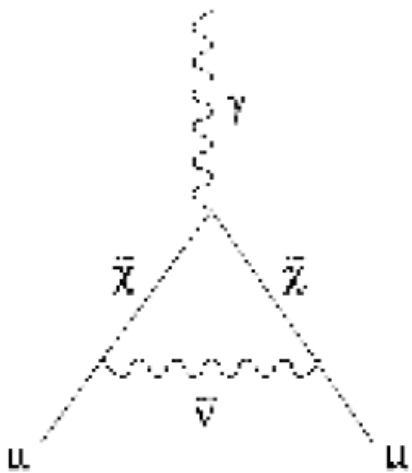
a_μ is a plausible location for a new physics signal!!

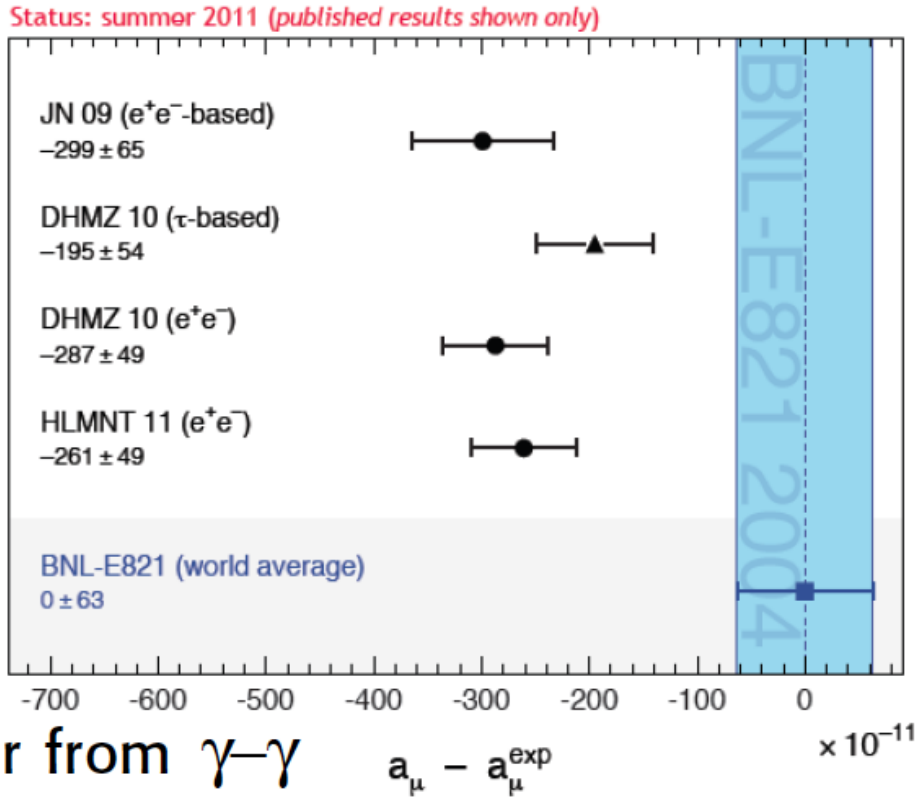
eg could be light SUSY (now tension with LHC)

$$a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 8.0) \times 10^{-10}$$

- ➡ 3.6 "standard deviations" (e^+e^-)
- ➡ 2.4 "standard deviations" (τ)

$$\delta a_\mu = 13 \cdot 10^{-10} \left(\frac{100 GeV}{M_{SUSY}}\right)^2 tg\beta$$





⊕ Error dominated by th error from γ–γ

Benchmark points

(preliminary)

	BP1	BP2	BP3	BP4	BP5
m_0 [GeV]	746	163	957	573	752
$m_{1/2}$ [GeV]	476	568	557	482	472
$\tan\beta$	2.7	2.9	2.8	3.4	2.8
A_0 [GeV]	1433	1666	782	27	-198
λ	1.43	1.47	1.58	1.34	1.12
κ	-0.1	0.09	-0.005	1.52	1.03
A_λ [GeV]	A_0	A_0	A_0	400	192
A_κ [GeV]	A_0	A_0	A_0	-323	-326
v_S [GeV]	-841	-190	-929	390	281
μ_S [GeV]	-5931	-5354	-5799	131	-37
$m_{h_d}^2$ [GeV^2]	m_0^2	m_0^2	m_0^2	$9.1 \cdot 10^5$	$5.4 \cdot 10^5$
$m_{h_u}^2$ [GeV^2]	m_0^2	m_0^2	m_0^2	$2.3 \cdot 10^6$	$2.4 \cdot 10^6$
m_s^2 [GeV^2]	m_0^2	m_0^2	m_0^2	$2.8 \cdot 10^6$	$1.7 \cdot 10^6$
μ [GeV]	-750	-1136	-934	-33	10
$b\mu$ [GeV^2]	$-2.4 \cdot 10^6$	$-1.2 \cdot 10^6$	$-2.3 \cdot 10^6$	147	26
b_s [GeV^2]	$-1.9 \cdot 10^7$	$-5.4 \cdot 10^6$	$-1.4 \cdot 10^7$	326	144
ξ_s [GeV^3]	$2.2 \cdot 10^9$	$1.5 \cdot 10^9$	$3.0 \cdot 10^9$	22	-8
m_{squark} [GeV]	1256-1293	1207-1263	1507-1548	1211-1248	1280-1315
$m_{\tilde{g}}$ [GeV]	1219	1389	1416	1242	1235
m_{h_1} [GeV]	124	123.5	125	93.5	78
m_{h_2} [GeV]	1002	856	1257	125	124
h_1 singletfraction	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-4})$	0.8	0.85
$\text{Br}(h \to \gamma\gamma)$	$2.29 \cdot 10^{-3}$	$2.28 \cdot 10^{-3}$	$2.2 \cdot 10^{-3}$	$2.5 \cdot 10^{-3}$	$2.66 \cdot 10^{-3}$
$\text{Br}(b \to s\gamma)$	$3.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$
Δa_μ	$-7.8 \cdot 10^{-11}$	$-2.5 \cdot 10^{-10}$	$-5.4 \cdot 10^{-11}$	$1.7 \cdot 10^{-10}$	$8 \cdot 10^{-11}$
$\delta\rho$	$6.2 \cdot 10^{-5}$	$6.6 \cdot 10^{-5}$	$7.5 \cdot 10^{-5}$	$1.9 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$
$m_{\tilde{\chi}_1^0}$ [GeV]	229	270	168	99	70
$\tilde{\chi}_1^0$ singlinofraction	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$	0.1	0.2
Ωh^2	7.5	0.10	7.4	0.017	0.11
$\sigma_p [cm^2]$	$2.8 \cdot 10^{-47}$	$2.2 \cdot 10^{-47}$	$6 \cdot 10^{-47}$	$1.2 \cdot 10^{-44}$	$1.3 \cdot 10^{-45}$
Δ (Fine-tuning)	34.9	51.0	51.8	44.9	52.7

Table 1: Benchmark scenarios for the GNMSSM for the universal (BP1-BP3) and the general (BP4-BP5) case. m_{squark} shows the range of squark masses of the first two generations. For the last two points the second lightest Higgs is mostly MSSM-like.

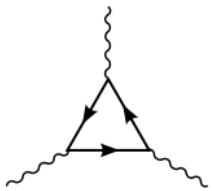
A unique solution : Z_4^R discrete R symmetry

MSSM spectrum
No perturbative μ term
Commutes with SO(10)
Anomaly cancellation

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_N
4	1	1	0	0	2

Green Schwarz term

$$A_{G-G-\mathbb{Z}_N} = \rho \mod \eta$$



$$A_{\mathrm{SU}(3)-\mathrm{SU}(3)-\mathbb{Z}_N} = \frac{1}{2}\sum_i \left[3\cdot q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 3R$$

$$A_{\mathrm{SU}(2)-\mathrm{SU}(2)-\mathbb{Z}_N} = \frac{1}{2}\sum_i \left[3\cdot q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 2R + \frac{1}{2}\left(q_H + q_{\bar{H}} - 2R\right)$$

$$A_{\mathrm{U}(1)_Y\text{-}\mathrm{U}(1)_Y\text{-}\mathbb{Z}_N^R} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\overline{\mathbf{5}}}^g\right) + \frac{3}{5}\left[\frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 11\right] \qquad (R=1)$$

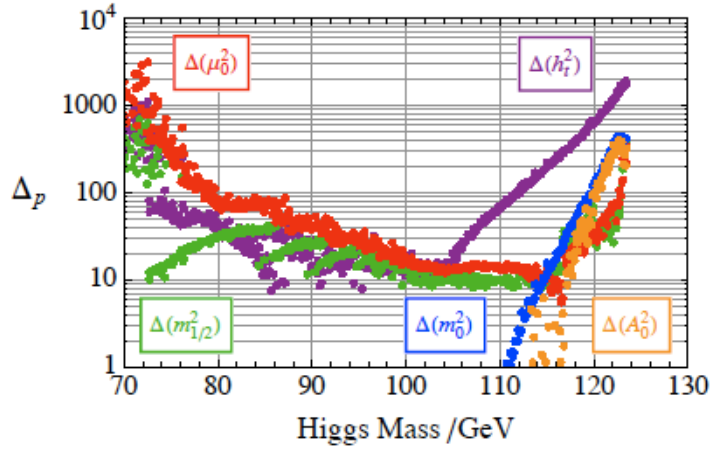
$$\Rightarrow N = 3,4,6,8,12,24$$

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange

Effect of focus point limited by h_t :

Fine tuning for measured parameter :

$$\Delta'(h_t) \simeq \left| \frac{\Delta h_t}{M_Z} \frac{\partial M_Z}{\partial h_t} \right|$$



$\Delta(p)^{-1}$ is probability p lies in range $p, p+\delta p$

For measured parameter, probability p lies in range,

compatible with measured value is: $P \sim \frac{\delta p}{\sigma_p} = \Delta(p)^{-1} \frac{p}{\sigma_p}$

Ciafolini, Strumia
Romanino, Strumia

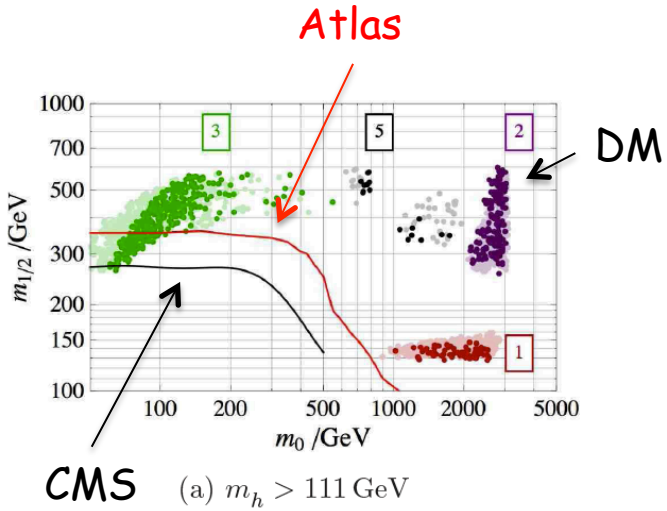
$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3}{(4\pi)^2} \frac{m_t^4}{v^2} \left[\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{X_t^2}{12 m_{\tilde{t}}^2} \right) \right]$$

$$X_t = A_t - \mu \cot \beta$$

$$|X_t| = X_t^{\max} = \sqrt{6}\, m_{\tilde{t}}$$

LHC – Regions of low fine tuning $\Delta < 100$:

	SUG0	SUG1	SUG2	SUG3	SUG5
m_0	1455	1508	2270	113	725
$m_{1/2}$	160	135	329	383	535
A_0	238	1492	30	-220	1138
$\tan\beta$	22.5	22.5	35	15	50
μ	191	433	187	529	581
$m_{\tilde{g}}$	482	414	900	898	1252
$m_{\tilde{u}_L}$	1469	1509	2331	826	1315
$m_{\tilde{t}_1}$	876	831	1423	602	1000
$m_{\tilde{\chi}_1^+}$	106	104	168	293	416
$m_{\tilde{\chi}_2^0}$	108	104	181	293	416
$m_{\tilde{\chi}_1^0}$	60	53	123	155	222
Δ	9	50	45	68	84
$\Omega_{\tilde{\chi}_1^0}h^2$	0.41	0.13	0.10	0.13	0.10
$\mathrm{BR}(b \to s\gamma) \times 10^4$	3.4	3.7	3.4	3.2	3.2
$\mathrm{BR}(B_s \to \mu^+\mu^-) \times 10^9$	3.0	2.9	2.9	3.4	1.7
$\delta a_\mu \times 10^{10}$	4.5	3.2	3.2	22.5	16.6
$\sigma^{\mathrm{SI}}_{\chi p}$ (pb) $\times 10^{10}$	108	5	432	24	101
$\sigma^{(LO)}$(7 TeV) (pb)	8	12	0.9	0.4	0.02
$\sigma^{(LO)}$(14 TeV) (pb)	40	75	3	5	0.4



- (General) Gauge mediation in the MSSM

$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_G$$

$$m^2_{\tilde{f}}(M_{mess}) = 2\sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda_S^2$$

No focus point

$$k_i = (\frac{5}{3},1,1)$$

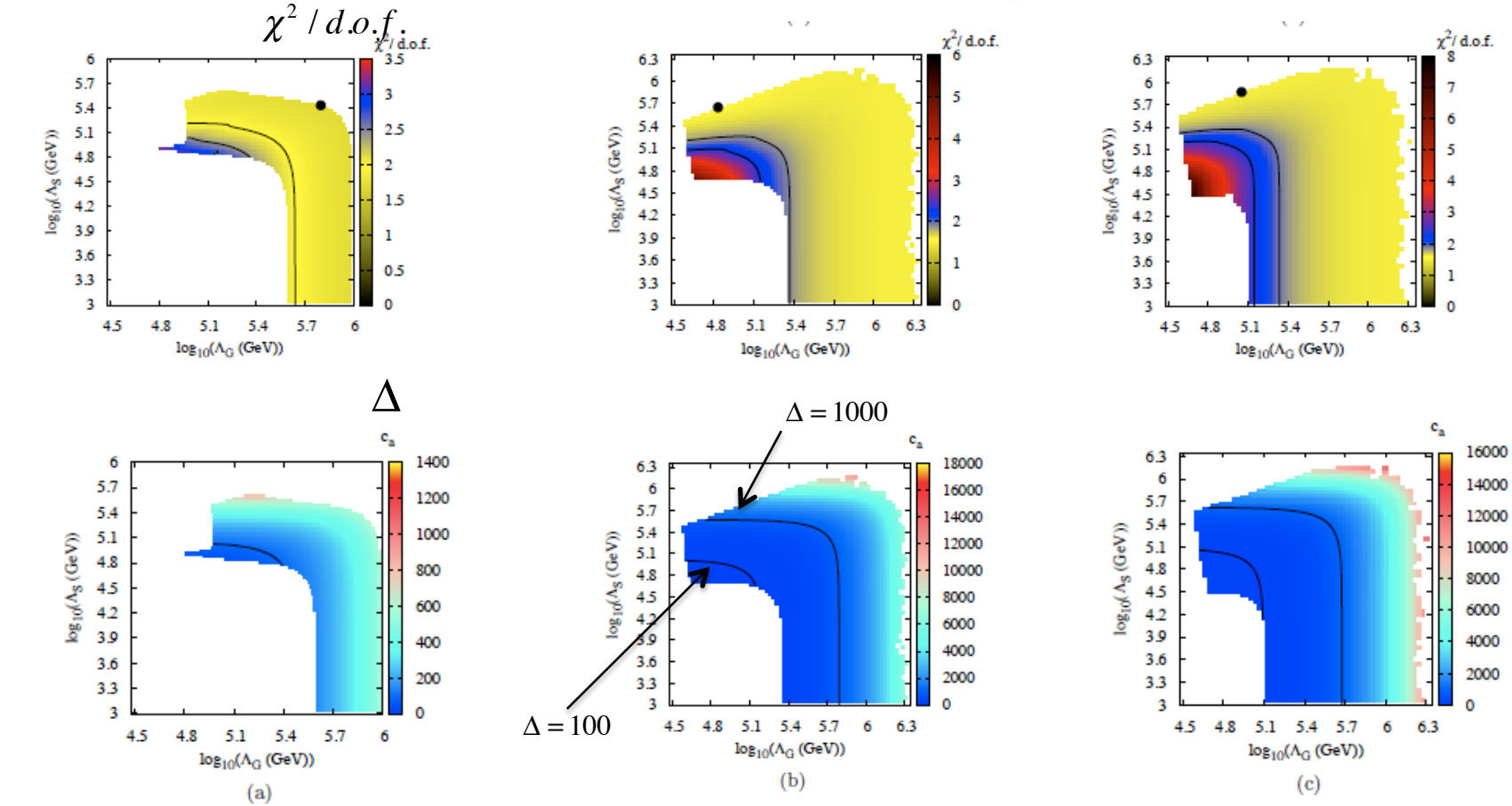
$$k_i \alpha_i \left(M_{GUT}\right) = 1, \quad i = 1,2,3$$

(Ordinary gauge mediation $\Lambda_G = \Lambda_S$)

Meade, Seiberg, Shih

Fine tuning in General Gauge Mediation

$B \to X_s\gamma, B \to \tau\mu, B \to \mu^+\mu^-, B \to D\tau\mu,$
$D_s \to \mu\nu, D_s \to \tau\nu, K \to \mu\nu / \pi \to \mu\nu, \Delta_{0-}$



$M_{Messenger}$ $10^6 GeV$ $10^{10} GeV$ $10^{14} GeV$

$\Delta > 100$ no focus point

Abel, Dolan, Jaeckel, Khoze
(Giusti, Romanino, Strumia)

Phenomenology

- Gaugino mass ratios

$$\frac{M_i(Q)}{M_{1/2}} = \eta_i \frac{\alpha_i(Q)}{\alpha_i(M_X)} \quad \Rightarrow \quad \begin{aligned} \frac{M_1(Q)}{M_2(Q)} &\approx 0.5\eta_1 \\ M_2(Q) &\approx 0.8 M_{1/2} \\ \frac{M_3(Q)}{M_2(Q)} &\approx 2.7\eta_3 \end{aligned}$$

…. gauginos can be very heavy

- Light neutralino and 2 charginos nearly degenerate

$$m_{\chi_2^0} - m_{\chi_1^0} = M_Z^2 \left(\frac{s_W^2}{M_1} + \frac{c_W^2}{M_2} \right) + \mathcal{O}(\frac{M_Z^3}{M_2^2})$$

$$m_{\chi_1^\pm} - m_{\chi_1^0} = \frac{1}{2} M_Z^2 \left(\frac{s_W^2}{M_1} + \frac{c_W^2}{M_2} \right) + \frac{1}{2} M_Z^2 \left(\frac{s_W^2}{M_1} - \frac{c_W^2}{M_2} \right) \epsilon \sin 2\beta + \mathcal{O}(\frac{M_Z^3}{M_2^2})$$

$+$ for $|M_1| < \mu$, Bino or Higgsino LSP candidate