What Can Lattice Gauge Theory Do for YOU ?

Fermilab Colloquium

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Statements expressing Goals of Lattice Gauge Theory

- provide reliable theoretical tools for studies of strongly coupled Quantum Field Theories and apply them wherever they are needed to deepen understanding of Particle Physics

- investigate a wide range of nonperturbative phenomena from first principles

- make accurate comparisons between theory and experiment possible

- help test the Standard Model of Particle Physics and the search for Physics Beyond the SM

In this talk I wish to provide brief overviews concerning ;

- basics of Lattice Gauge Theory (LGT)

- some examples where LGT has had impact

  — precision flavor physics

  B, B_s, D, D_s, K decays

  Testing the SM, searches for New Physics

  — SM parameters

  CKM matrix elements, quark masses, α_s

  — Other (mainly in the future)

  $g-2$, hadronic matrix elements for ν-physics, dark matter searches

- LGT as a tool to understand Quantum Field Theories

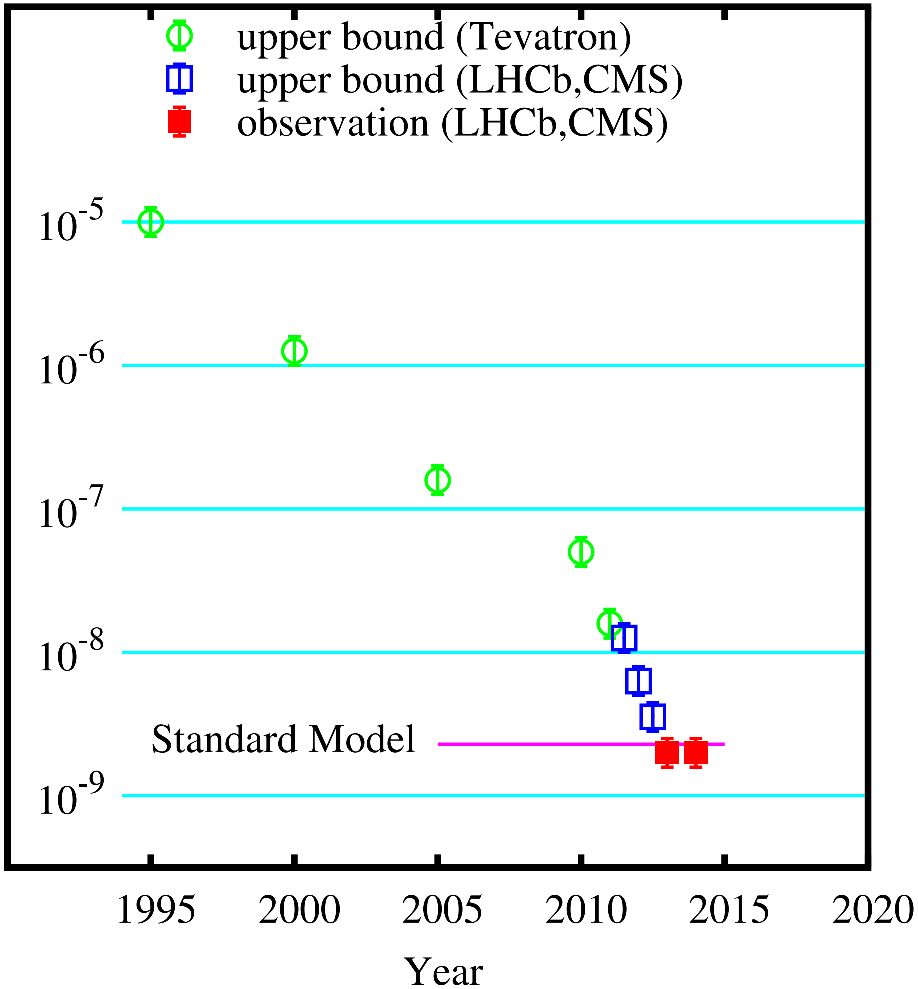
Example $\boxed{B_q \to \mu^+\mu^-}$ **(q = s,d)**

Highly supressed in the SM and hence sensitive to New Physics, in principle.

History:

$B_s \to \mu^+\mu^-$

Branching Fraction



$\boxed{B_q \to \mu^+\mu^-}$ (cont'd)

Long history of searching for this FCNC process at the Tevatron, LHCb and CMS. Highly supressed in the SM and hence sensitive to New Physics. Upper bounds established at the Tevatron kept on coming down until the branching fraction essentially hit the Standard Model (SM) prediction. Finally actual observation by LHCb and CMS.

Experiment (LHCb and CMS; F.Archilli CKM2014)

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$$

SM Prediction (Bobeth et al. PRL 112:101801 (2014))

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)|_{SM} = (3.65 \pm 0.23) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)|_{SM} = (1.06 \pm 0.09) \times 10^{-10}$$

Ingredients in the SM Prediction

$$\mathcal{B}(B_q \to \mu^+\mu^-) = \frac{|N|^2 M_{B_q}^3 f_{B_q}^2}{8\pi\Gamma_H^q} f(2m_l/M_{B_q})|C_A(\mu_b)|^2 \ + \ \mathcal{O}(\alpha_{em})$$

f_{B_q} : B_q **meson decay constant.**

$$\langle 0|A^\mu|B_q(p)\rangle = \langle 0|\bar{b}\gamma^\mu\gamma_5 q|B_q(p)\rangle = ip^\mu f_{B_q}$$

$C_A(\mu_b)$: **Wilson coefficient.**

N = function of G_F, M_W and CKM matrix elements ($|V_{cb}|$, $|V_{tb}^* V_{td}|$)

f_{B_q} and $|V_{cb}|$ from nonperturbative QCD calculations

C_A **recently updated to include NLO EW and NNLO QCD corrections. "Non-parametric" errors down to 1.5%**

$\Rightarrow$ **dominant errors from CKM and** f_{B_q}

Main Features of Lattice Gauge Theory

The Standard Model is built on basic principles of

Quantum Mechanics and Special Relativity

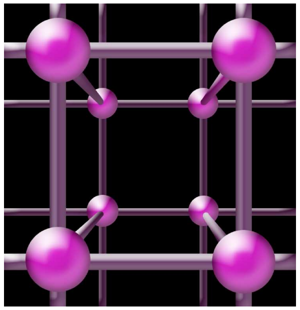
$\Longrightarrow$ **Quantum Field Theory** (QFT)

Conventional QFT's describe systems with ∞ degrees of freedom and are notoriously complicated to deal with. Even within perturbation theory UV divergencies must be regularised and the theory renormalized. Extracting reliable nonperturbative (QCD) information is even harder, although crucial for many SM predictions.

Lattice Gauge Theory provides a gauge invariantly regularized version of the original QFT that lends itself to both perturbative and nonperturbative treatment (K.Wilson 1974).

Main Features of Lattice Gauge Theory (cont'd)

Continuous space-time $\rightarrow$
a discrete grid of lattice sites.



Fields $\Psi(x)$, $A_\mu(x)$ specified only at lattice sites or links

The lattice spacing **'a'** provides an UV regulator. Such a cutoff in coordinate space compatible with local gauge invariance (a different symmetry transformation at each site).

Physics extracted by calculating vacuum expectation values of a wide variety of appropriate operators.

$$\langle ...... \rangle \propto \int \mathcal{D}A_\mu \{.....\} e^{-S}$$

On the lattice such path integrals can be calculated non-perturbatively.

$\boxed{\text{Main Features of Lattice Gauge Theory}}$ (cont'd)

On the lattice, path integrals (i.e. functional integrals) become

<u>ordinary multidimensional integrals</u>

(Riemann integrals for bosons, Grassmann integrals for fermions)

Path integral over fermionic fields can be done explicitely (action bilinear in fermion fields)

Remaining bosonic integrals $\int \Pi_l dA_\mu(l)$ done numerically via Monte Carlo methods.

$\langle ...... \rangle \propto \int \mathcal{D}A_\mu \{.....\} e^{-S} \Longrightarrow \int \Pi_l dA_\mu(l)\{.......\}e^{-S}$

So, the lattice allows us to obtain vacuum expectation values. By choosing appropriate operators one can extract: spectroscopy, decay constants, form factors, mixing parameters + more

Back to $B_s \to \mu^+\mu^-$ Example

We were interested in f_{B_s} and $|V_{cb}|$.

f_{B_s}

Want $\langle 0|A_0|B_s\rangle = M_{B_s}\, f_{B_s}$

Start from $\langle ....\rangle \Longrightarrow \langle A_0\, \Phi^\dagger_{B_s}\rangle$
$\Phi^\dagger_{B_s}$ **= interpolating operator with quantum numbers of the B_s meson. It creates the groundstate B_s meson plus excitations. e.g.** $\Phi^\dagger_{B_s} = \overline{\Psi}_b\gamma_5\Psi_s$.

Then,

$\langle A_0(t)\Phi^\dagger_{B_s}(0)\rangle \sim \sum_k a_k e^{-E_k t}$ **with** $a_k \propto \langle 0|A_0|E_k\rangle\, \langle E_k|\Phi^\dagger_{B_s}|0\rangle$

Fit and extract $\langle 0|A_0|E_0\rangle \equiv \langle 0|A_0|B_s\rangle$

$\underline{|V_{cb}|}$ **Study $B \to D^{(*)} l\nu$ decays**

Want $\langle D|V_\mu|B\rangle \Rightarrow f_+(q^2)$ **and** $f_0(q^2)$

$\frac{d\Gamma(B\to Dl\nu)}{dq^2} \propto |V_{cb}|^2\, f_+(q^2)^2$

$\langle ....\rangle \Longrightarrow \langle \Phi_D(T)\; V_\mu(t)\; \Phi^\dagger_B(0)\rangle \sim \sum_j \sum_k A_{jk} e^{-(T-t)E_D^{(j)}}\; e^{-tE_B^{(k)}}$

$A_{jk} \sim \langle 0|\Phi_D|E_D^{(j)}\rangle\; \langle E_D^{(j)}|V_\mu|E_B^{(k)}\rangle\; \langle E_B^{(k)}|\Phi^\dagger_B|0\rangle$

So,

$\boxed{\textbf{Lattice Gauge theorists make a living by calculating VEV's}}$

$\langle ......\rangle \propto \int \mathcal{D}A_\mu\{.....\}e^{-S} \Longrightarrow \int \Pi_l dA_\mu(l)\{.......\}e^{-S}$

As mentioned already, this gives us

Spectrum, decays constants, form factors, mixing parameters, quarks masses, strong coupling, ...

Some Numbers

Typical lattice sizes

$N_s^3 \times N_t$: $20^3 \times 48$, $28^3 \times 96$, $48^3 \times 144$, $64^3 \times 192$, ...
($10^6 \sim 10^8$ number of sites)

Typical lattice spacings

a : 0.15fm - 0.045fm
(a^{-1} : 1.5GeV - 4.4GeV)

$L = N_s a$: 3.fm - 5.fm

Typical pion masses

Now simulations at physical pion masses are starting
Still many results with $m_\pi \sim$ 250MeV - 500MeV.

Must control finite "a", finite volume and heavier than physical pion masses in our simulations.

Fixing Parameters in the QCD Action

The QCD action, both in the continuum and on the lattice, includes several parameters that must be fixed by experiment before any predictions can be made. These are the **bare quark masses** and the **scale** (or coupling).

$$
\begin{aligned}
\Upsilon(2S-1S)\ \text{splitting}\ \ \text{or}\ \ f_\pi &\longrightarrow a^{-1} \\
\text{pion} &\longrightarrow m_{u,d} \\
\text{kaon} &\longrightarrow m_s \\
\eta_c\ \ \text{or}\ \ D_s\ \text{meson} &\longrightarrow m_c \\
\Upsilon\ \ \text{or}\ \ B_s &\longrightarrow m_b
\end{aligned}
$$

After this, no adjustable parameters left.

Estimating Errors

A trustworthy lattice result should come with a full Error Budget that lists errors coming from:

— **finite lattice spacing, i.e. $a \to 0$ extrapolation**

— **larger than physical pion masses (note: more and more simulations done now with physical pions)**

— **finite volume effects**

— **operator matching**

— **bare mass tunings**

— **scale setting**

— **e.&m. effects etc.**

Many Choices for Lattice Actions

Several transcriptions of continuum QCD onto the lattice exist and are being employed by different lattice collaborations. Choices for how to discretize

$\frac{1}{4}\left(F^2_{\mu\nu}\right)^2$ (gluons)
$\gamma^\mu D_\mu + m_l$ (light quarks)
$\gamma^\mu D_\mu + M_H$ (heavy quarks)

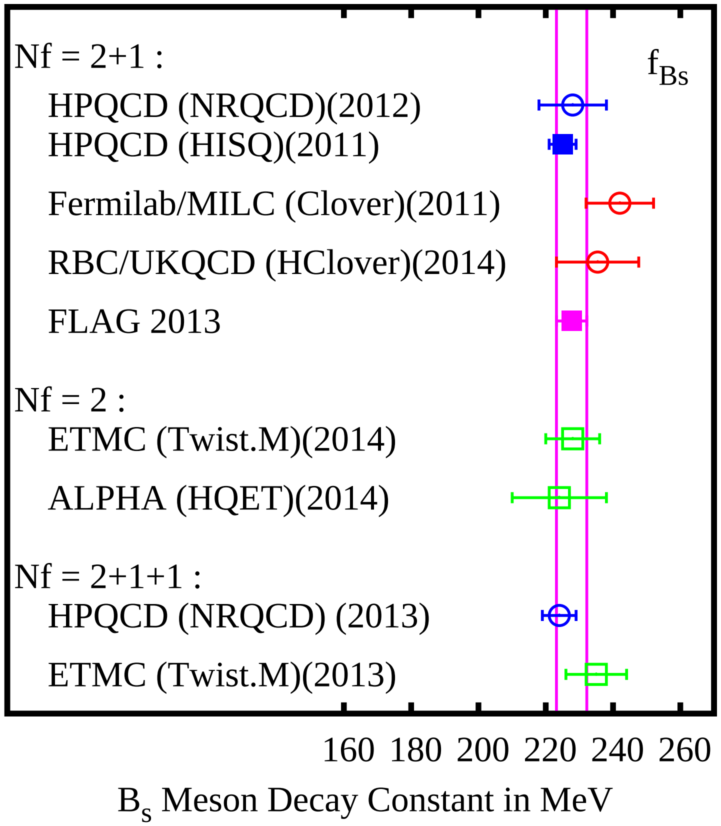
Important to cross check lattice results based on different actions

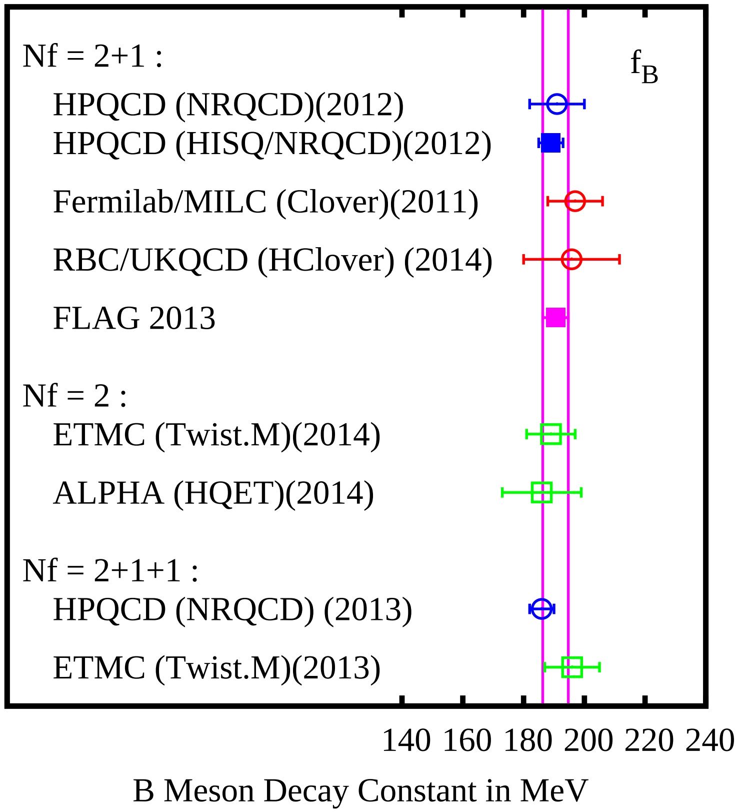
Collaborations

I will be presenting results mainly from

- Fermilab Lattice and MILC Collab.
- European Twisted Mass (ETM) Collab.
- Alpha Collab.
- PACS-CS Collab., JLQCD
- UK-Riken-Brookhaven-Columbia (UKQCD/RBC) Collab.
- High Precision QCD (HPQCD) Collab.

Results for B and B_s Decay Constants





For the Standard Model prediction for $\mathcal{B}(B_q \to \mu^+\mu^-)$ Bobeth et al. used the FLAG averages

$$f_{B_d} = 190.5(4.2)\,\text{MeV}, \qquad f_{B_s} = 227.7(4.5)\,\text{MeV}$$

Results for $|V_{cb}|$

The most accurate exclusive determination of $|V_{cb}|$ comes from studies of $B \to D^*, l\nu$ decays at zero recoil. The relevant form factor, $\mathcal{F}(1)$, was updated by the Fermilab/MILC collaboration this year [arXiv:1403.0635] with significantly reduced errors.

$$|V_{cb}|_{excl.} = (39.04 \pm 0.49_{expt} \pm 0.53_{QCD} \pm 0.19_{QED}) \times 10^{-3}$$

The corresponding inclusive determination [Gambino and Schwanda;arXiv:1307.4551] stands at

$$|V_{cb}|_{incl.} = (42.42 \pm 0.86) \times 10^{-3}$$

i.e. $\sim 3\sigma$ tension between exclusive and inclusive

Bobeth et al. use inclusive $|V_{cb}|^2$ value and quote ~4.5% CKM errors.

Indirect approach to $\mathcal{B}(B \to \mu^+\mu^-)$

A.Buras : hep-ph/0303060 $\frac{\mathcal{B}(B_q \to \mu^+\mu^-)}{\Delta M_q} \propto \frac{\tau(B_q)}{\hat{B}_{B_q}}$

$\hat{B}_{B_q}$ = **" Bag Parameter"** $\langle \overline{B_q}|(V-A)(V-A)|B_q\rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 B_{B_q}$

Use HPQCD's lattice results (0902.1815) $\hat{B}_{B_s} = 1.33(6)$, $\hat{B}_{B_d} = 1.26(11)$

Buras 1303.3820

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)|_{indir.} = (3.71 \pm 0.17) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)|_{indir.} = (1.03 \pm 0.09) \times 10^{-10}$$

Error now dominated by that of bag parameters B_{B_q}.

Compare with direct method (Bobeth et al.)

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)|_{SM} = (3.65 \pm 0.23) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)|_{SM} = (1.06 \pm 0.09) \times 10^{-10}$$

Other Heavy Meson Rare Decays

$B \to K l^+ l^-$

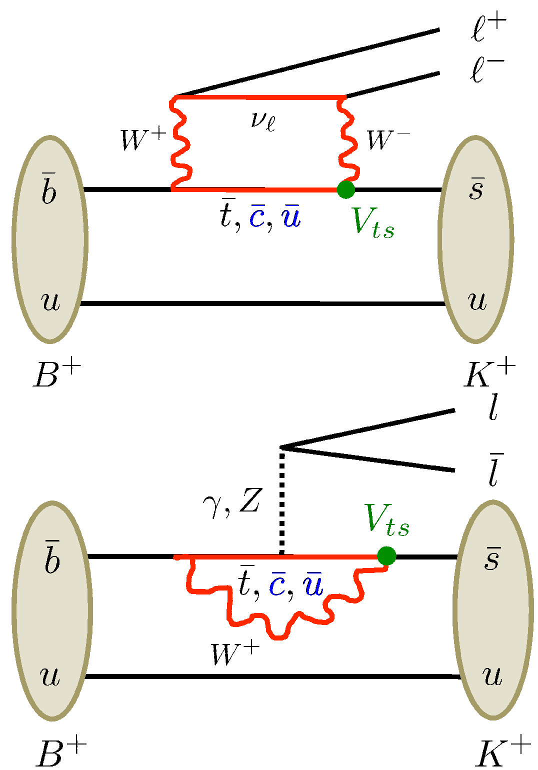
$B \to \pi l^+ l^-$

form factors f_+, f_0, f_T, angular distributions, constraints on Wilson Coeff.

$B \to K^* l^+ l^-$

$B_s \to \Phi l^+ l^-$

need many more form factors, ang. distr.

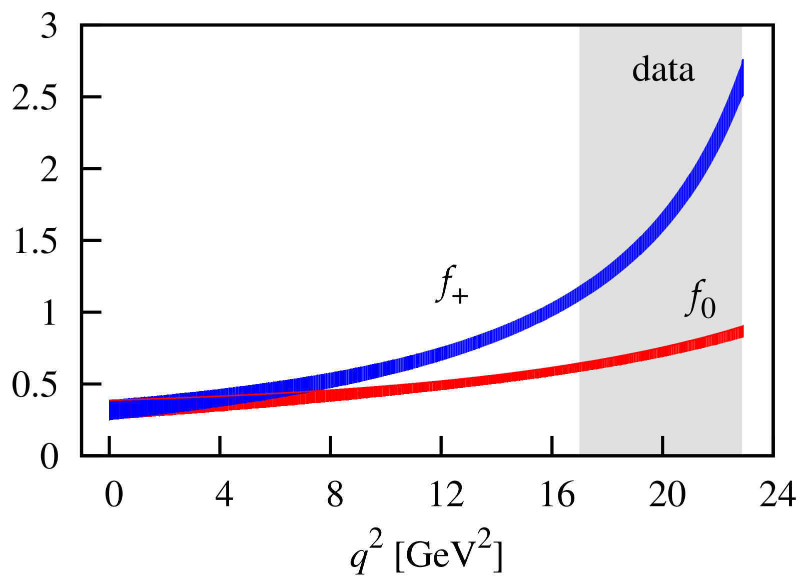


These are all FCNC processes which occur via loops and are highly suppressed in the SM. Sensitive to New Physics.

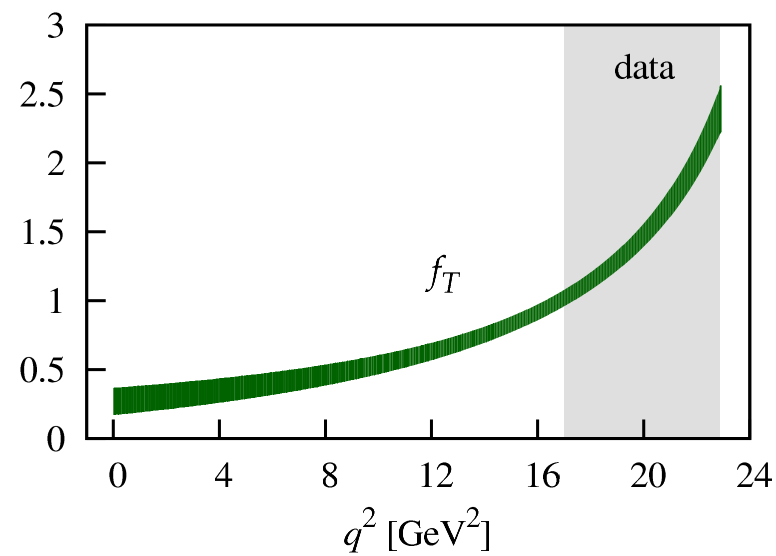
$B \to K l^+ l^-$ Form Factors (from $\langle K|V_\mu|B\rangle$, $\langle K|t_{\mu\nu}|B\rangle$)

(HPQCD: C.Bouchard et al. [arXiv:1306.0434, 1306.2384])

$f_+(q^2)$ **and** $f_0(q^2)$



$f_T(q^2)$



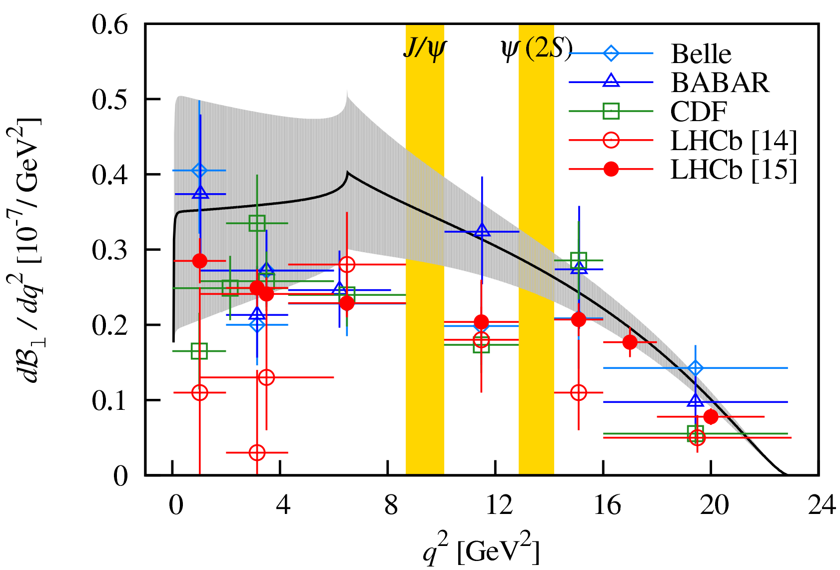
Extrapolations to small q^2 using the BCL (Bourrely-Caprini-Lellouch) “z-expansion”.

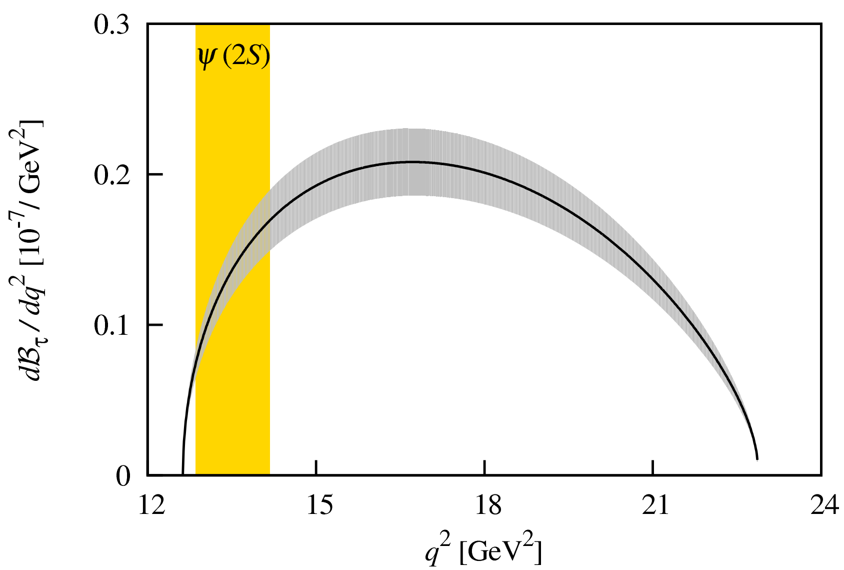
Differential Branching Fractions: Comparisons with Experiment

In SM $\frac{d\Gamma_l}{dq^2} = 2a_l + \frac{2}{3}c_l, \qquad l = e, \mu, \tau.$

a_l, c_l **: functions of form factors, Wilson coeff. masses etc.**

$B \to K l^+ l^-$ $\qquad\qquad$ $B \to K \tau^+ \tau^-$



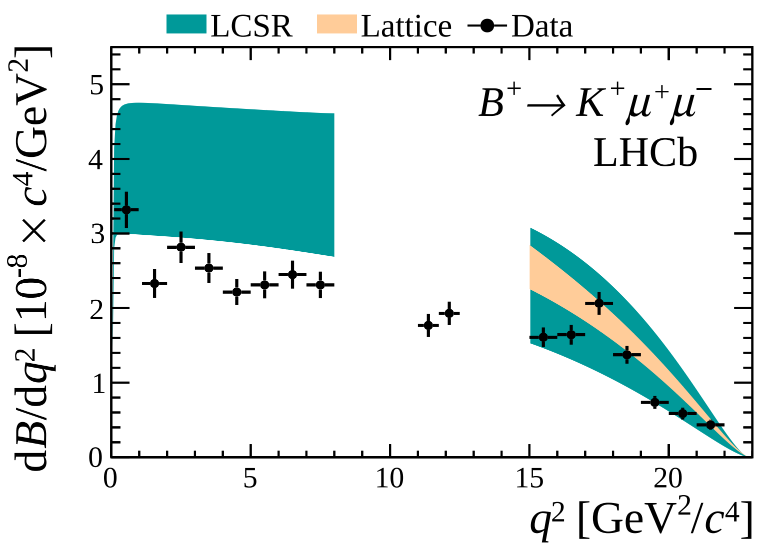


Note: $c\bar{c}$ **effects/resonances treated in very naive way**

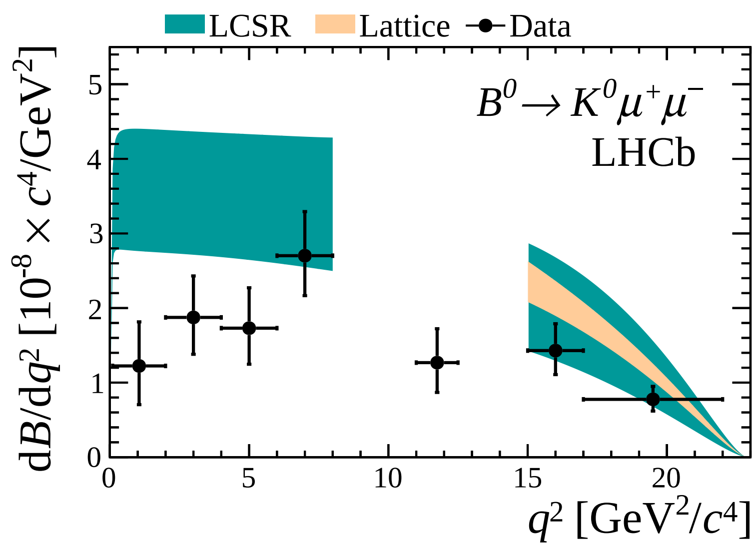
Lattice and LCSR Form Factors + Experiment

(R.Aaij et al. (LHCb) arXiv:1403.8044 [hep-exp])

$B^+ \to K^+\mu^+\mu^-$

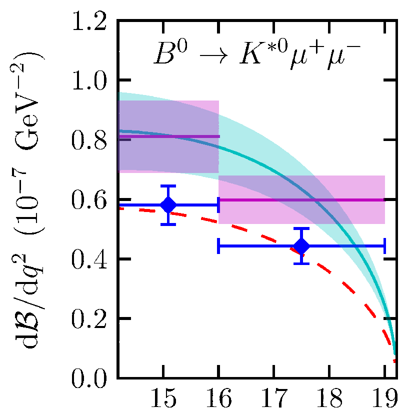


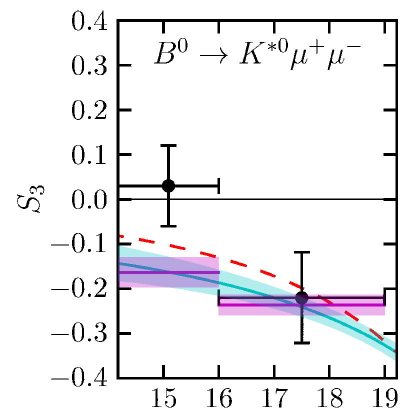
$B^0 \to K^0\mu^+\mu^-$

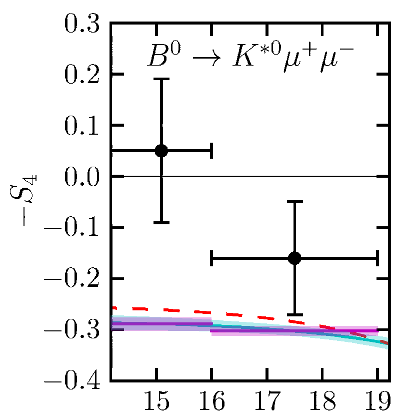


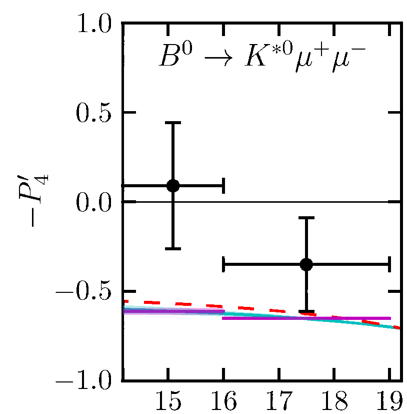
$B \to K^* l^+ l^-$: Comparisons between Experiment and Theory

(HPQCD: R.R.Horgan et al. [arXiv: 1310.3722, 1310.3887])



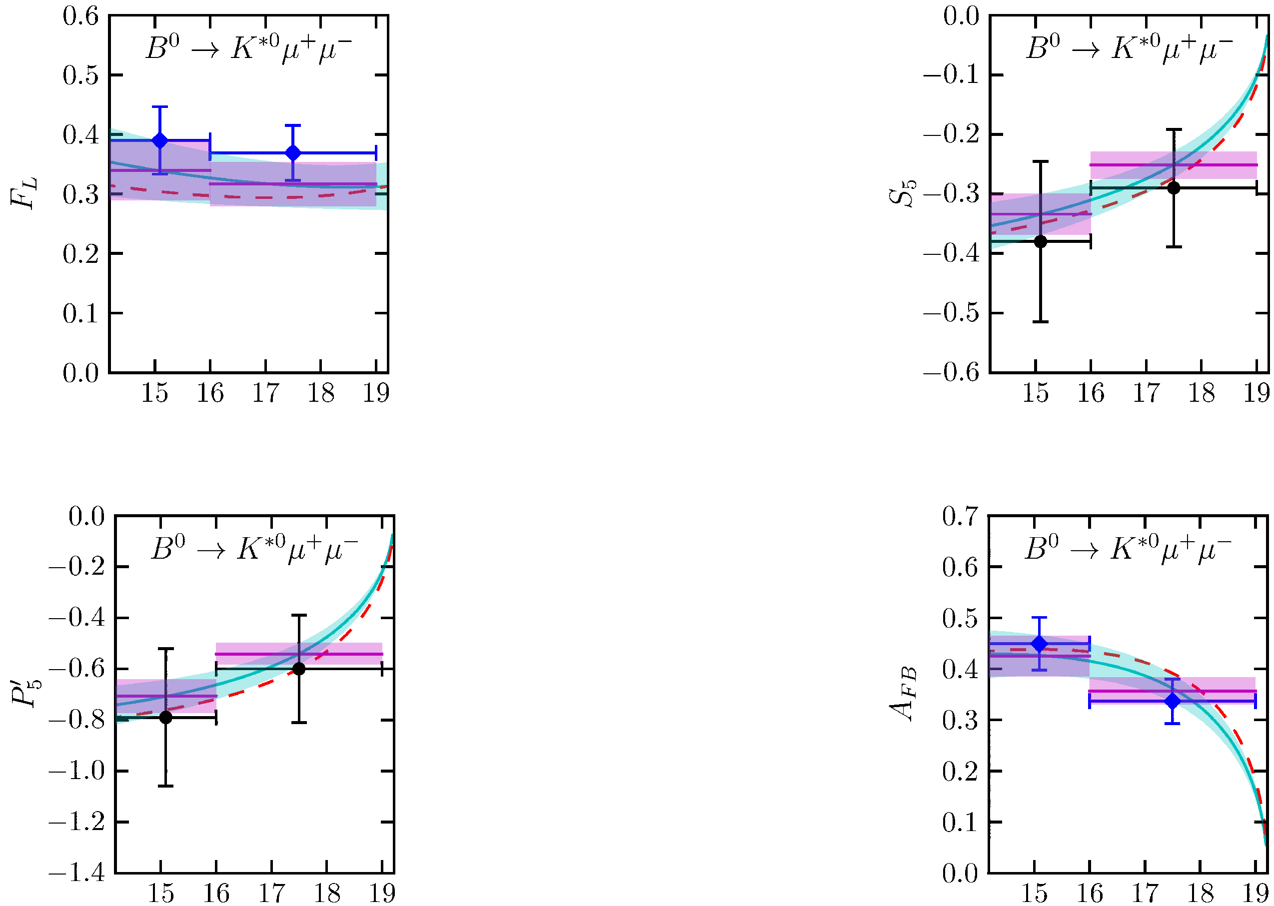






– – – $C_9^{NP} = -1.1$, $C_9' = 1.1$ **Bands = SM ($C_9^{NP} = C_9' = 0$)**

$B \to K^* l^+ l^-$: Comparisons (cont'd)



– – – $C_9^{NP} = -1.1$, $C'_9 = 1.1$ **Bands = SM** ($C_9^{NP} = C'_9 = 0$)

Implications for $B \to K^{(*)}\nu\overline{\nu}$ Studies at Belle II

The same form factors used in $B \to K^{(*)}l^+l^-$ decays also needed for $B \to K^{(*)}\nu\overline{\nu}$ branching fractions ($\langle K|V_\mu|B\rangle$).

The theory behind these rare decays is particularly clean (no internal photon complications, factorization holds).

SM Prediction A.Buras, J.Girrbach-Noe et al. : arXiv:1409.4557

$$\overline{\mathcal{B}}(B^+ \to K^+\nu\overline{\nu})|_{SM} = (4.2 \pm 0.4) \times 10^{-6}$$

$$\overline{\mathcal{B}}(B^0 \to K^{*0}\nu\overline{\nu})|_{SM} = (9.9 \pm 0.8) \times 10^{-6}$$

These predictions use Form factors from the lattice at large q^2 and from light-cone sum rules at small q^2.
Theory errors under good control and can be improved .

What Did We Learn from Rare Decay Examples ?

- precision experiments and accurate SM predictions needed to test SM and search for New Physics

- SM predictions involve both perturbative and nonperturbative (QCD) inputs

- LGT is doing its part in this endeavor

- More accuracy needed (f_{B_q},B_{B_q}, FF, $|V_{cb}|$....)

- Using form factors from the lattice, one can start playing around by allowing Wilson Coefficents to take on non-SM values.

Many Other Channels in Flavor Physics

Similar story repeats itself again and again.
Lattice needed for nonperturbative inputs etc. etc.

$B \to \pi l\nu$, $B_s \to K l\nu$, $B \to \tau\nu_\tau$ .....

e.g. $|V_{ub}|_{excl}$ **from Lattice + Belle** $= 3.47(22) \times 10^{-3}$ **(FLAG 2013)**

D **and** D_s **decays** **(**$|V_{cd}|$, $|V_{cs}|$, f_D, f_{D_s}, **FF ....)**

e.g. $|V_{cs}|_{semilep} = 0.963(14)(5)$ **(HPQCD 2013)**

Kaon physics **(**$|V_{us}|$, f_K/f_π, **FF)**

e.g. $|V_{us}|_{semilep} = 0.22290(74)(52)$ **(FNAL/MILC 2014)**

Main focus has been consistency checks of the CKM matrix using both leptonic and semileptonic decays.

Cabibbo-Kobayashi-Maskawa (CKM) Physics

Transformation between the mass eigenstates and the weak (charged current) basis.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$\hat{V}$ includes 4 out of the 19 fundamental parameters of the Standard Model.
If $\hat{V}$ has complex entries $\Longrightarrow$ source of CP violation in the SM.

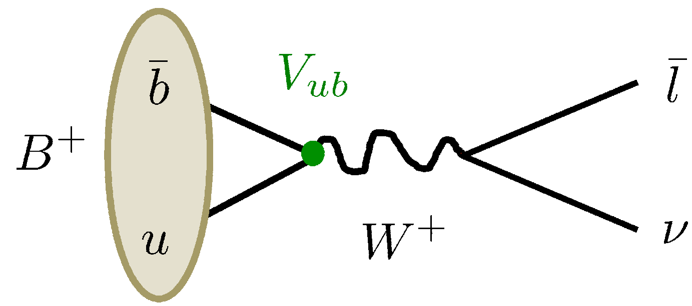
Verification of this mechanism for CP violation in the Standard Model
$\Longrightarrow$ 2008 Nobel Prize in Physics for Kobayashi & Maskawa.

Heavy Meson Leptonic Decays

$B \to \tau\nu_\tau \qquad \propto |V_{ub}|^2 f_B^2$

$D \to \tau\nu_\tau, \mu\nu_\mu \qquad \propto |V_{cd}|^2 f_D^2$

$D_s \to \mu\nu_\mu \qquad \propto |V_{cs}|^2 f_{D_s}^2$



Heavy Meson Semileptonic Decays

$B \to \pi l\nu, \quad B_s \to K l\nu,$

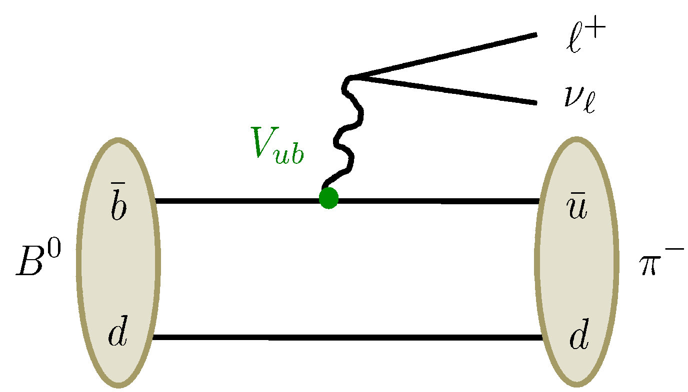
$B \to D l\nu, \quad B_s \to D_s l\nu$

$\Longrightarrow |V_{ub}|^2 \left[\text{or } |V_{cb}|^2\right] \left(f_+(q^2)\right)^2$

$D \to \pi l\nu, \quad D \to K l\nu$

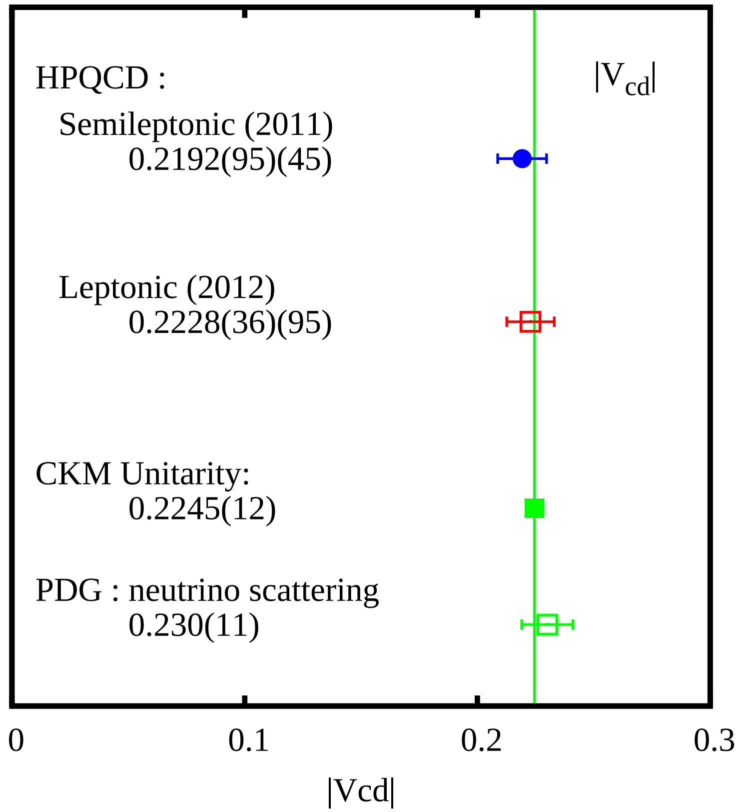
$\Longrightarrow |V_{cd}|^2 \left[\text{or } |V_{cs}|^2\right] \left(f_+(q^2)\right)^2$

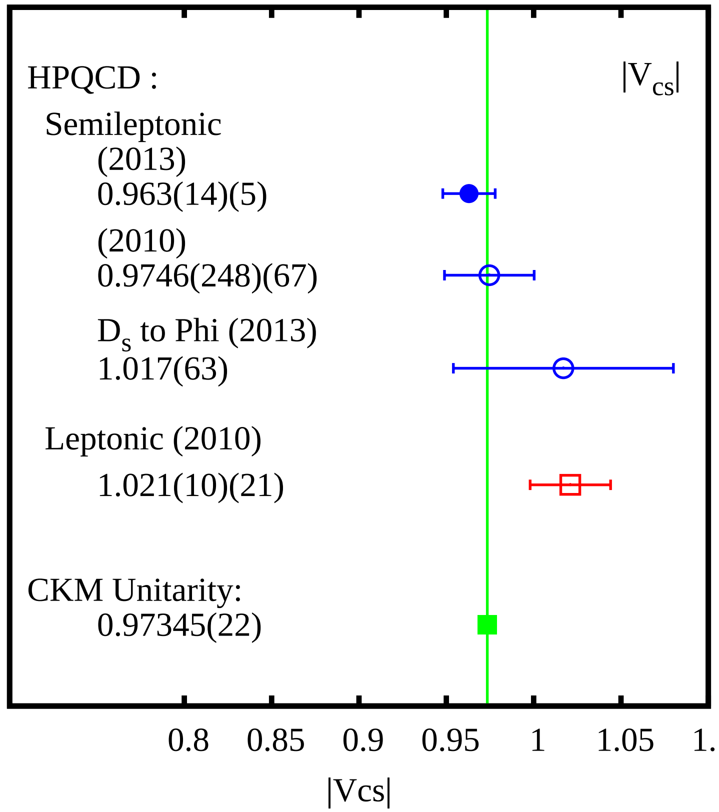
$D_s \to \Phi l\nu \qquad \Longrightarrow |V_{cs}|^2$



Many consistency checks possible

Summary of $|V_{cs}|$ and $|V_{cd}|$ Results

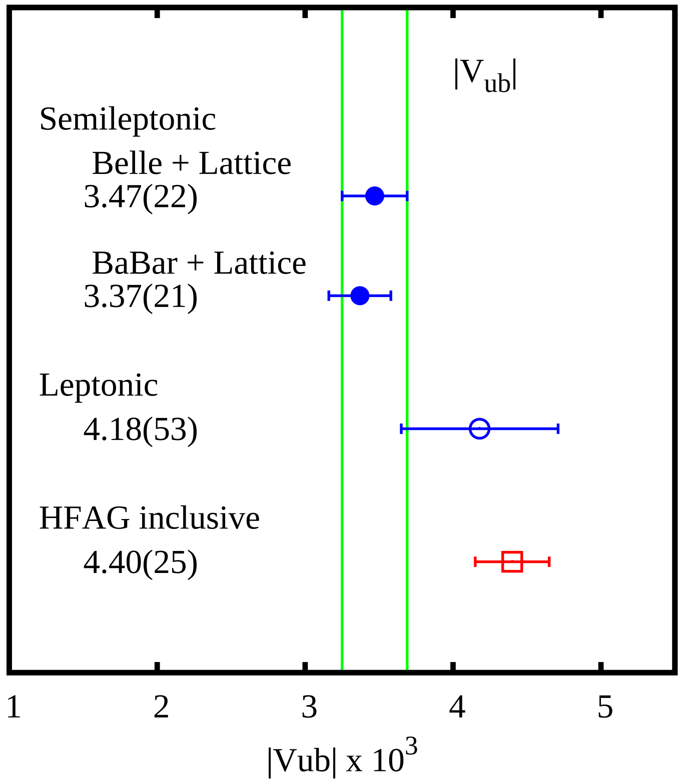




Wonderful consistency for $|V_{cd}|$.

$\sim 2\sigma$ **tension for $|V_{cs}|$ between Leptonic and CKM Unitarity. Experimental (theory) errors dominate Leptonic (semileptonic) determinations.**

Summary: $|V_{ub}|$ from Semileptonic and Leptonic B Decays

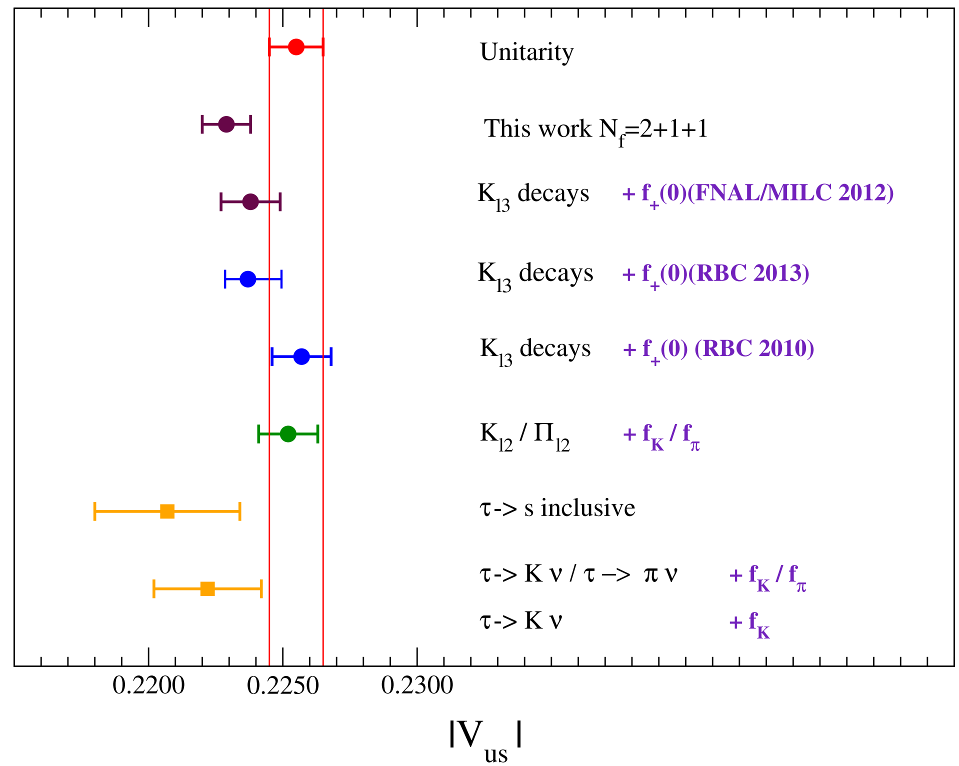


Again tension between exclusive and inclusive semileptonic determinations.

Improvements in lattice results for $B \to \pi l \nu$ forthcoming soon. Need reduction in experimental errors for $\mathcal{B}(B \to \tau \nu_\tau)$ (Belle II).

Summary: $|V_{us}|$ from Semileptonic and Leptonic Kaon Decays

Plot by Elvira Gamiz (March 2014)



A.Bazavov et al. (FNAL/MILC); PRL 112 (2014) 112001:

$$\Delta_{CKM}| = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00115(40)_{Vus}(43)_{Vud}$$

Current Tensions ($2 \sim 3\,\sigma$) in Flavor Physics

- **Inclusive versus Exclusive $|V_{cb}|$**

- **Inclusive versus Exclusive $|V_{ub}|$**

- **Leptonic versus Semileptonic $|V_{cs}|$**

- $R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}l\nu)}$ **($l = \mu, e$, BaBar sees excess)**

- $B \to K^{(*)}l^+l^-$ **angular variables**

- $R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)}$ **(LHCb finds $R_K < 1.$)**

All these need to be scrutinized.

$\boxed{\text{Determination of Quark Masses and } \alpha_s}$

Sofar

$\{$ experimental measurement $\}$ = $\{$ theory expression [pert. & non-pert. inputs] $\}$

$\Rightarrow$ checks of SM predictions, CKM etc.

Useful physics also from

$$\boxed{\{ \text{ simulation data } \} = \{ \text{ theory expression } \}}$$

Examples :

LOGs or Ratios of $\{$Wilson Loops $\}$ = $\sum_n c_n \alpha_s^n$
$(\langle e^{i \oint A \cdot dl} \rangle)$

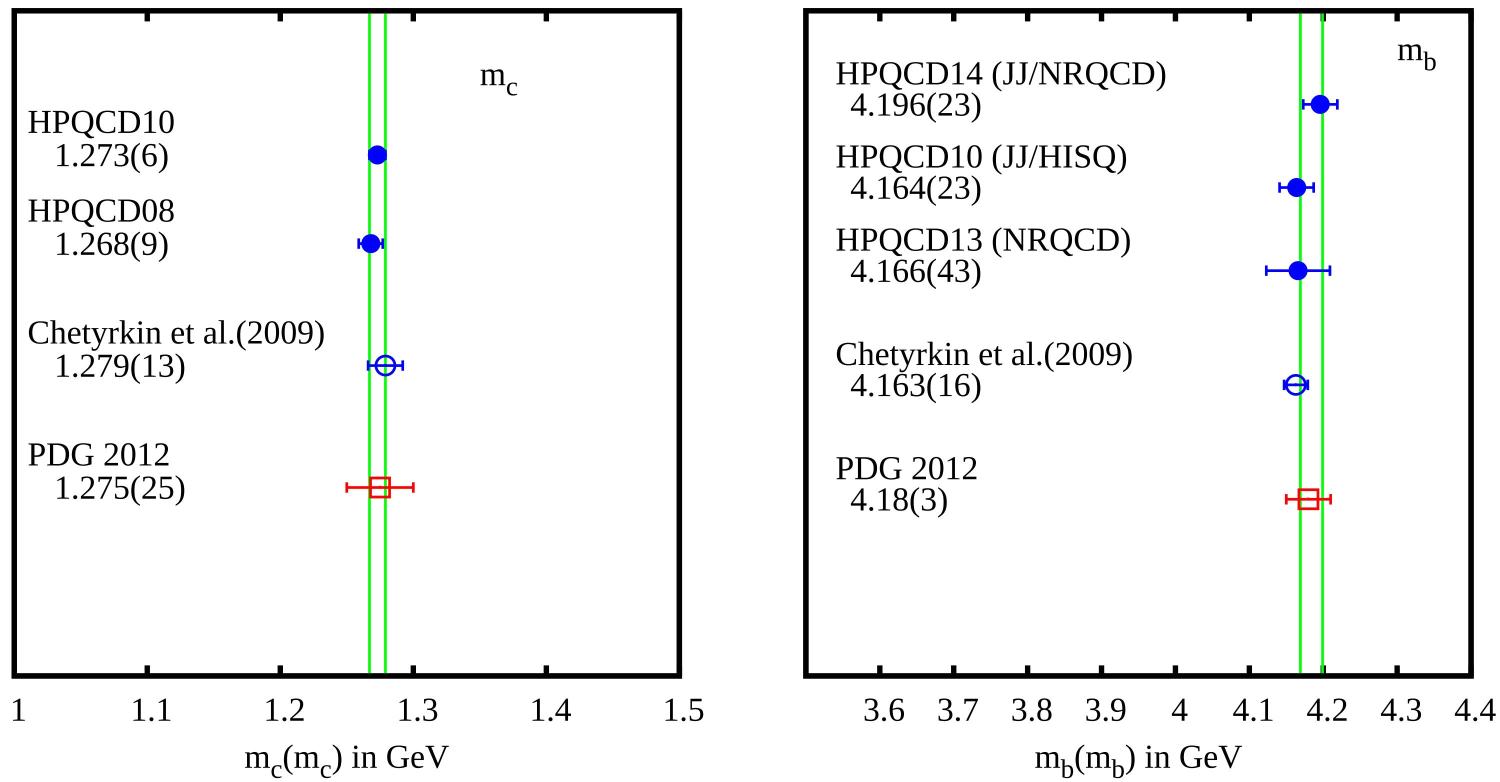
Moments of $\{$ J-J correlators $\}$ = $\{$ funct. of α_s and M_Q $\}$

LHS : can be obtained very accurately

RHS : known to very high orders in cont./lattice Pert. Th.

$$\Longrightarrow \quad \boxed{\alpha_s \text{ and } M_Q}$$

Results for Charm and Bottom Masses

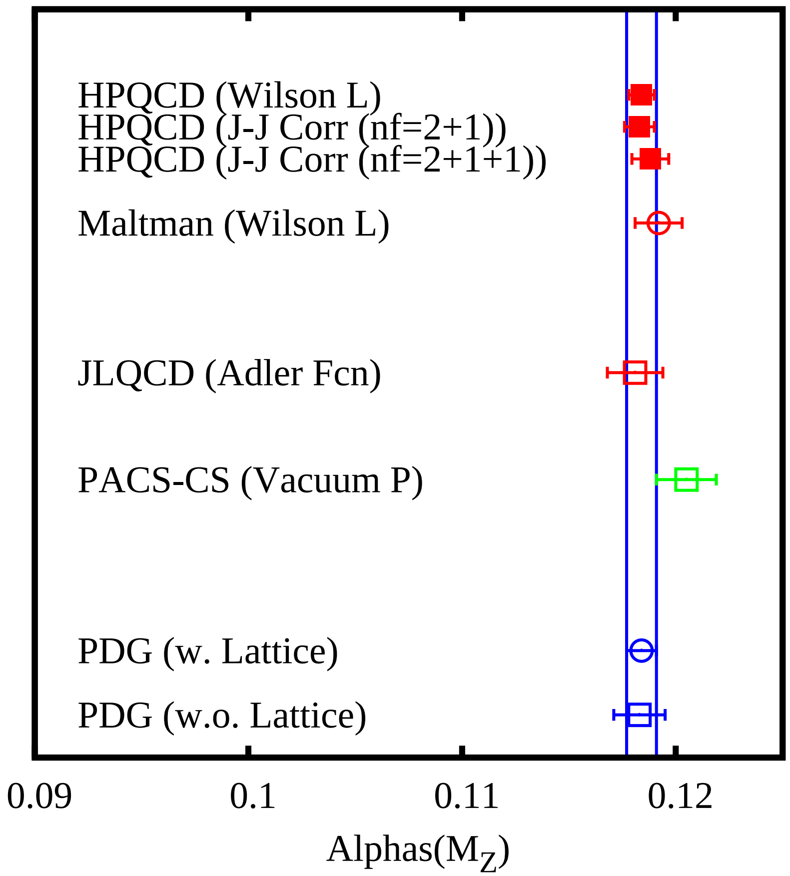


Charm $\overline{MS}$ Mass

Bottom $\overline{MS}$ Mass

(Weighted lattice average 4.184(15) GeV)

Results for $\alpha_s^{\overline{MS}}(M_Z)$



Non-lattice results from : DIS, τ decays, Z pole fits, e^+e^- event shapes and jets, ...

Summary and Outlook

- Precision experimental measurements searching for new physics are useful only when combined with commensurately accurate SM predictions

- SM predictions require both perturbative and non-perturbative inputs. Lattice Gauge Theory is playing crucial role here.

- This talk touched on some examples including rare decays ($B_q \to \mu^+\mu^-$, $B \to Kl^+l^-$, $B \to K\nu\overline{\nu}$), $B_{(s)}$, $D_{(s)}$ and Kaon leptonic and semileptonic decays and extraction of CKM matrix elements and determinations of α_s and M_Q.

- Several $2 \sim 3\ \sigma$ tensions exist now in Flavor Physics. We can look forward to significant improvements in experimental and theory errors in coming years.

$\boxed{\text{Summary and Outlook}}$ (cont'd)

- **Accurate heavy quark masses and α_s will become crucial in future precision Higgs Physics (P.Lepage, P.Mackenzie, M.Peskin arXiv:1404.0319)**

- **The flexibility of LGT can be exploited to**
  calculate quantities not easily accessibe to experiments
  recall:
  $\{$ **measured quantity** $\} = \{$ **funct. of** α_s **and** M_Q $\}$
  Are there other $\{$ **measured quantity** $\}$**'s that may prove useful ?**

- **LGT can explore Quantum Field Theories other than those entering the SM.**
  Change # of flavors, gauge groups etc.
  This is currently an active area of research in LGT, an area that needs guidance and feedback from the larger HEP community.

What Can Lattice Gauge Theory Do for YOU ?

$\boxed{\text{YOU}}$

means

particle experimentalists
phenomenologists
BSM model builders
dark matter scientists
nuclear structure scientists
etc. etc. .....

$\boxed{?}$

The lattice community welcomes feedback.
Please let us know what you need