Defining Fundamental Constants of Nature: The New SI

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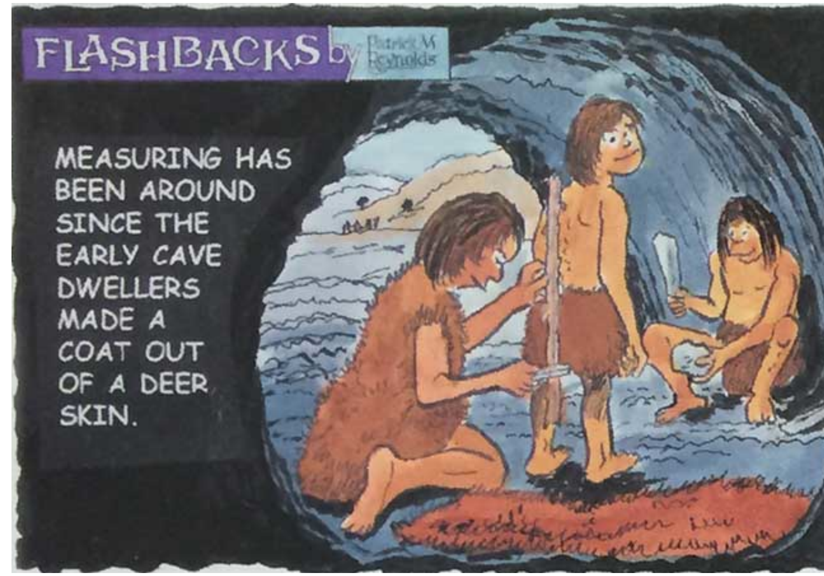
Outline

- **Brief history of the SI**
- **The new SI**
- **Communication about New SI**

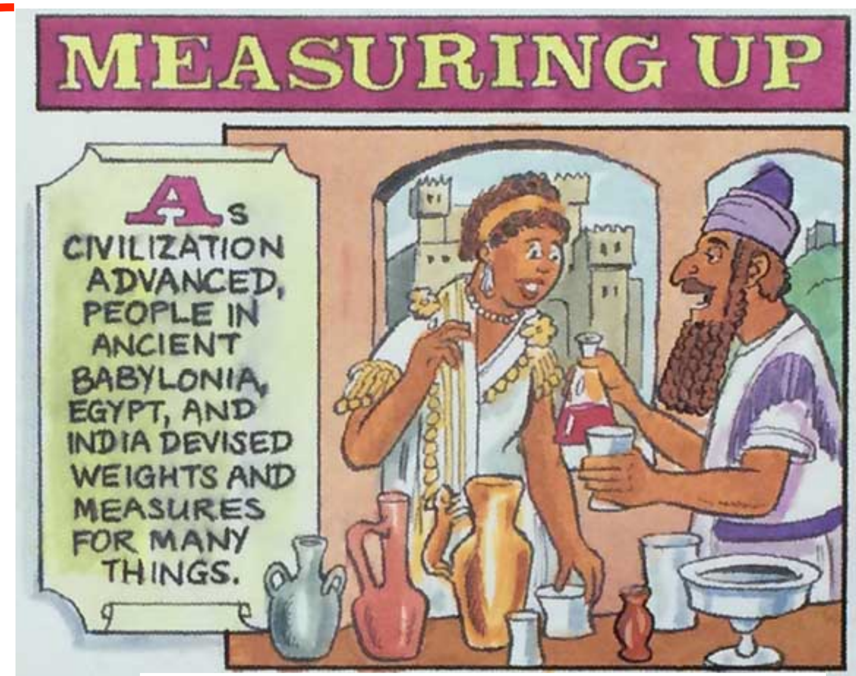
Outline

- **Brief history of the SI**
  - J. H. Williams, "Defining and Measuring Nature: The Make of All Things," Morgan & Claypool San Rafael, CA (2014)

Science of measurement - metrology

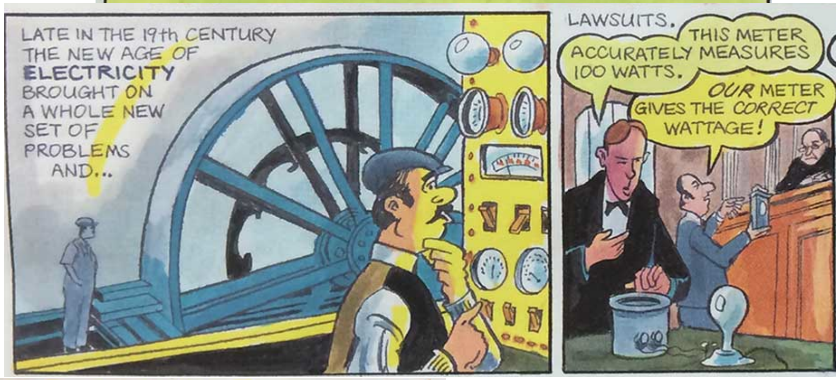


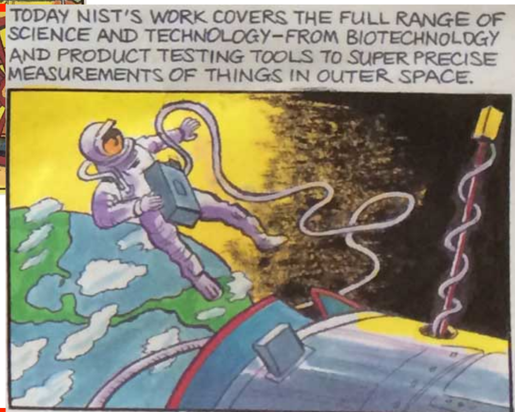
Universal measure for trade and commerce



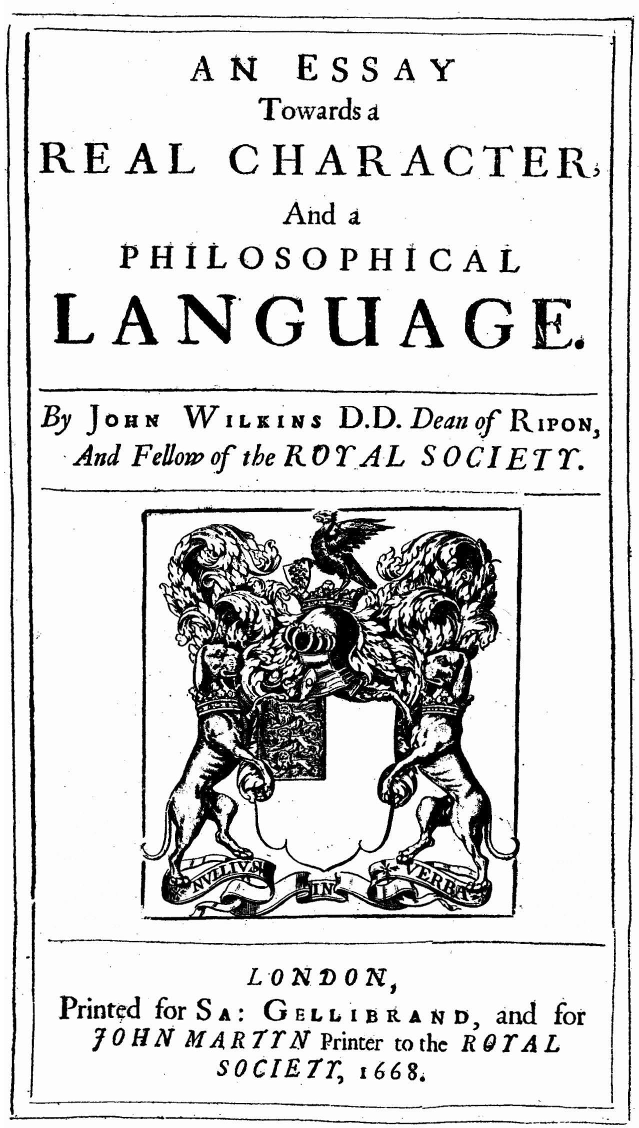
"Ceratonia siliqua MHNT.BOT.2011.3.89" by Roger Culos

International standardization





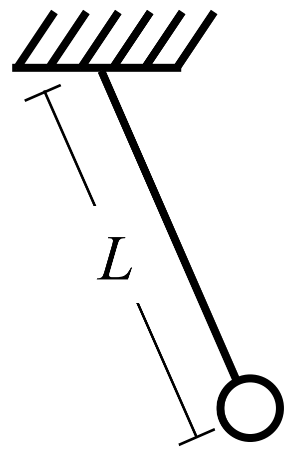
Universal Measure – John Wilkins 1668


AN ESSAY
Towards a
REAL CHARACTER,
And a
PHILOSOPHICAL
LANGUAGE.

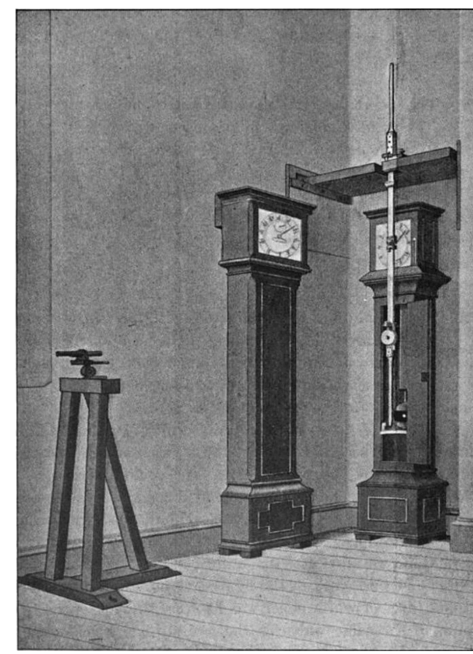
By JOHN WILKINS D.D. Dean of RIPON,
And Fellow of the ROYAL SOCIETY.

LONDON,
Printed for SA: GELLIBRAND, and for
JOHN MARTYN Printer to the ROYAL
SOCIETY, 1668.

- **Decimal system**
- **Time as a universal measure (the second)**
- **Length derived through a seconds pendulum**

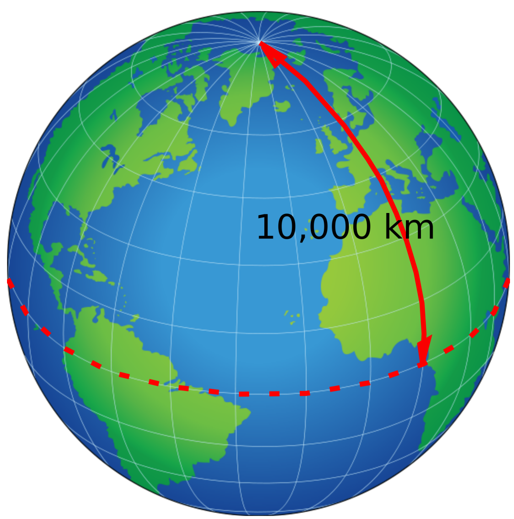


$$T = 2\pi\sqrt{\frac{L}{g}}$$

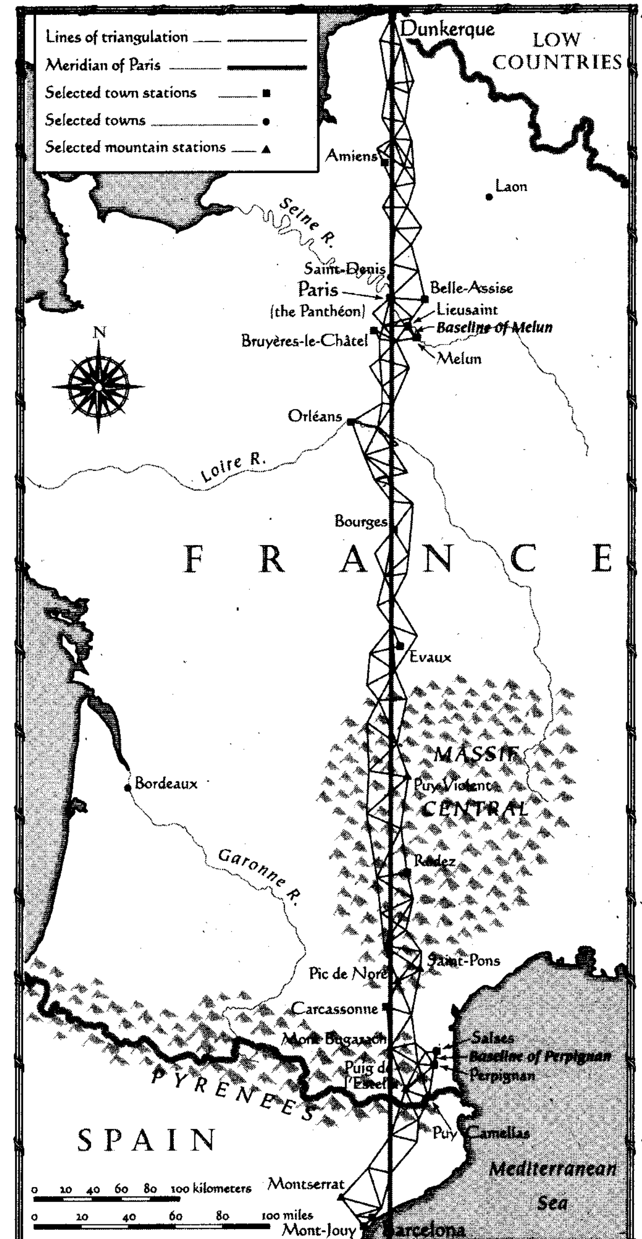


Universal measure – Gabriel Mouton 1670

- **Decimal system**
- **Length as universal measure (earth)**
- **Jean-Baptiste Delambre and Pierre François-André Méchain**
  - Survey of 1792-1798
  - K. Adler, “The Measure of All Things,” Simon & Schuster, New York (2002).



wikipedia.org

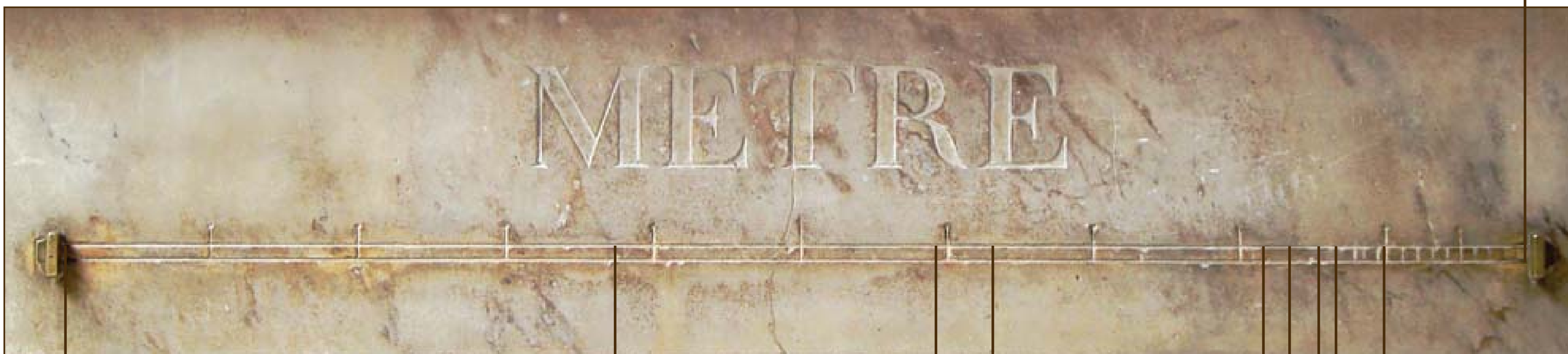


Birth of the metric system

- **20 June 1799, two platinum standards representing the meter and the kilogram deposited in the Archives de la République, Paris**

(Musée des Arts et Métiers, Paris)

Fast forward evolution of the SI



1668
John Wilkins's essay is published.

1799
The metric system is born. The Archives de la République in Paris receives two platinum artifact standards representing the meter and kilogram.

1875
Seventeen member nations sign the Meter Convention. Work begins on constructing new international prototypes for the meter and kilogram.

1889
The first General Conference on Weights and Measures (CGPM) approves a system of measures with the base units meter, kilogram, and second.

1954
The ampere, kelvin, and candela are officially adopted as base units by the 10th CGPM.

1960
The 11th CGPM adopts the name International System of Units (SI) with the base units meter, kilogram, second, ampere, kelvin, and candela. The meter is redefined as the wavelength of radiation from a specific excitation in krypton-86.

1967
The second is redefined in terms of the hyperfine splitting frequency of the cesium-133 atom.

1971
The mole becomes a new base unit of the SI, and the list of base units grows to seven.

1983
A new definition of the meter links it to the speed of light in vacuum.

2018
The new SI will specify the exact values of seven fundamental constants, shown in table 2. All SI units will be based on those defining constants.

Present SI – units defined

Base Quantity	*Base Unit*	*Definition*
time	second	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.
length	meter	The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.
mass	kilogram	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram
electric current	ampere	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length
thermodynamic temperature	kelvin	The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water
amount of substance	mole	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12; the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles
luminous intensity	candela	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian

Outline

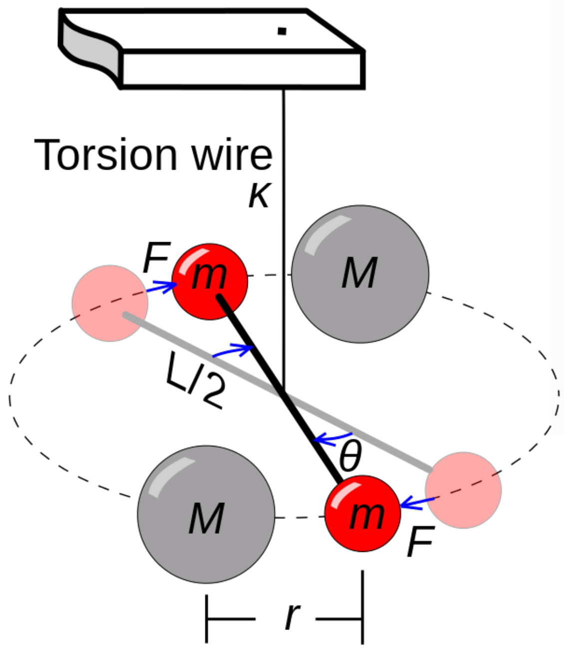
- Brief history of the SI
- **The new SI**
- Communication about New SI

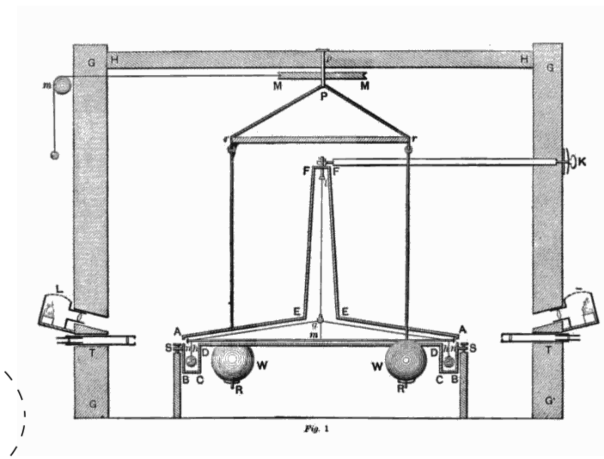
What are fundamental constants?

Newton’s Principia, 1686

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

Cavendish, 1798





$$G = 6.74 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$$

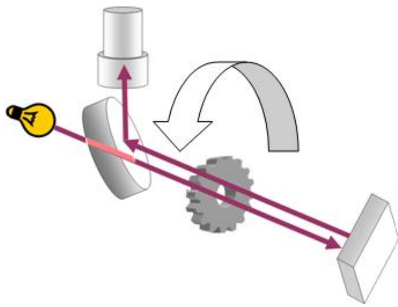
wikipedia.org

Other Fundamental Constants

Constant	Basic Law	Equation
G	Newtonian Universal Gravitation	$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$
k_1	Coulomb's	$\vec{F} = k_1\frac{q_1q_2}{r^2}\hat{r}$
k_2	Biot-Savart	$\vec{B} = k_2\int\frac{\vec{I}\times\hat{r}dl}{r^2}$
c	Maxwell's General Relativity	$c^2 = \frac{k_1}{k_2}; E = mc^2$
h	Planck's Relation	$E = h\nu$
R_∞	Rydberg-Ritz Formula	$\frac{1}{\lambda} = R_\infty\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$
α	Bohr Theory of Hydrogen	$E_n = -\frac{1}{2}\left(\frac{k_1e^2}{\hbar c}\right)^2\frac{m_ec^2}{n^2} \equiv -\frac{1}{2}\frac{\alpha^2m_ec^2}{n^2}$

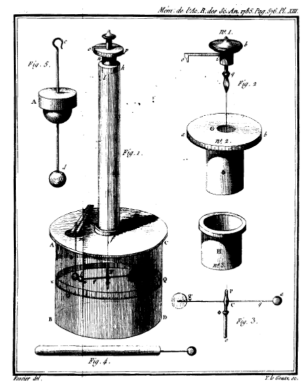
Comparing Theory from Measured Values of Fundamental Constants

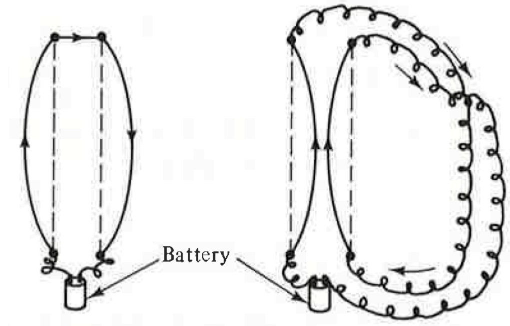
Time of Flight



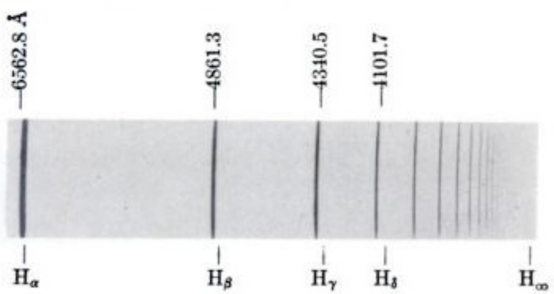
$$c^2 = \frac{k_1}{k_2}$$

Electromagnetic Forces



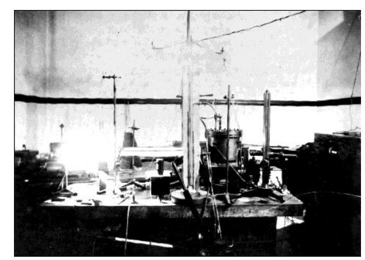


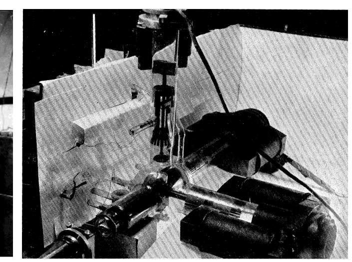
Hydrogen Spectroscopy

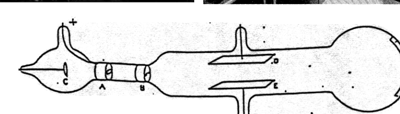


$$R_\infty = \frac{1}{2}\frac{\alpha^2 m_e c}{h}$$

Measurements of m_e, e, and h

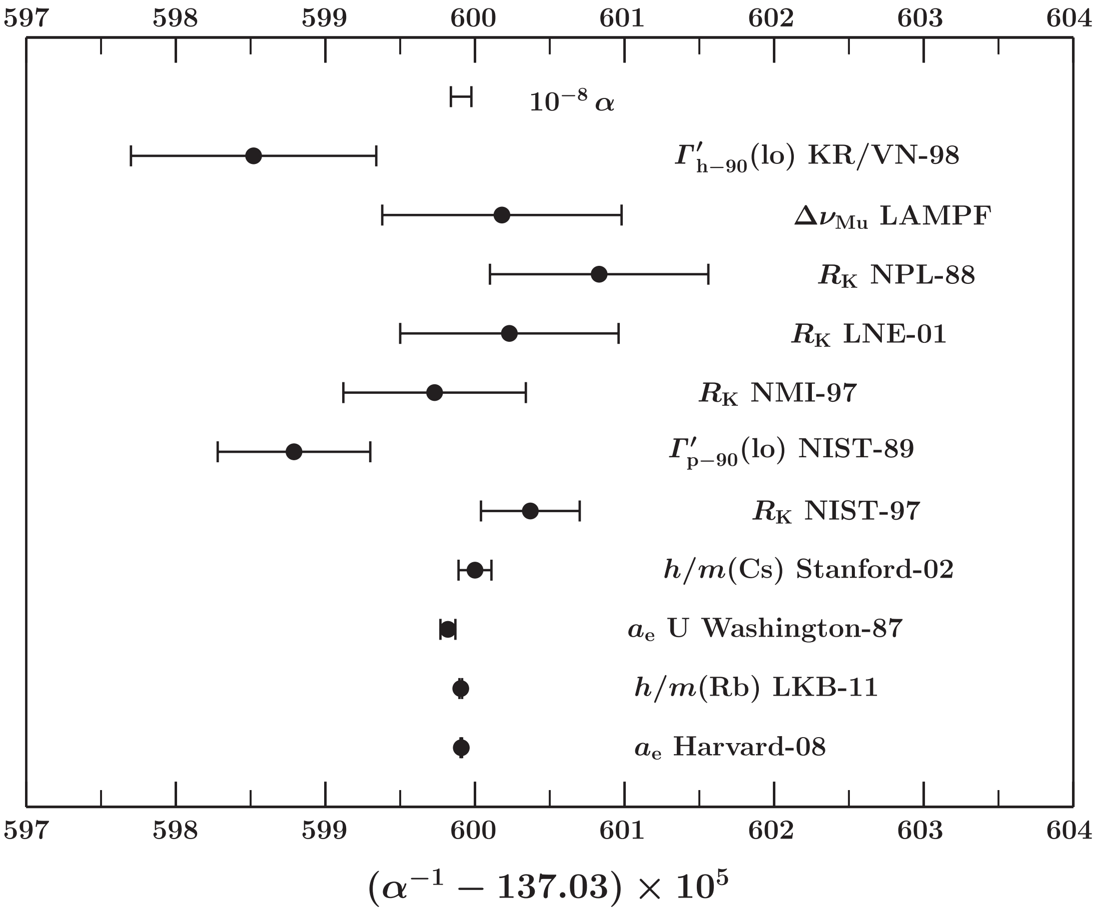






Better Determination of the fundamental constants

Input data related to the 2010 CODATA TGFC determination of the fine structure constant



$$\gamma_x = \frac{|\mu_x|}{i\hbar}$$

$$\Delta\nu_{\mathrm{Mu}} = \frac{16}{3} c R_\infty \alpha^2 \frac{m_{\mathrm{e}}}{m_\mu}\left(1+\frac{m_{\mathrm{e}}}{m_\mu}\right)^{-3} F\left(\alpha, m_{\mathrm{e}}/m_\mu\right)$$

$$R_{\mathrm{K}} = \frac{\mu_0 c}{2\alpha}$$

$$R_\infty = \frac{1}{2}\frac{\alpha^2 m_{\mathrm{e}} c}{h}$$

$$a_{\mathrm{e}} = \sum_{n=1}^{\infty} C_{\mathrm{e}}^{(2n)}\left(\frac{\alpha}{\pi}\right)^n + a_{\mathrm{e}}(\mathrm{had}) + a_{\mathrm{e}}(\mathrm{weak})$$

How to make a system of units

- Six quantities (force, mass, acceleration, velocity, length, time) and three equations:

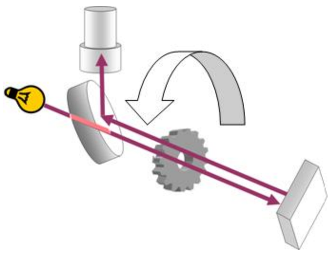
$$F = ma = m\,dv/dt = m\,d^2x/dt^2$$

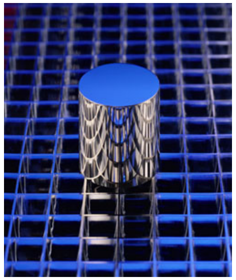
- Choose independent subset of base quantities – either force or mass and any two of the remaining five:

$$x, m, t$$

- Choose specific reference quantity for each base quantity – define a unit:

$x = 1$ meter $\quad m = 1$ kilogram $\quad t = 1$ second





How to make a system of units

- **Choose specific reference quantity for each base quantity – define a fundamental constant:**

$$E = hv = mc^2 = eV = kT$$

- ***h*, *c*, e, and *k* are invariants of nature – a true universal measure**

- **Available to everybody, anytime, anywhere**

Paradigm Shift of SI

Present SI ("explicit units")

- **7 base units with definitions**
  - s, kg, m, A, K, mo, cd
- **All other units derived**

New SI ("explicit constants")

- **7 fundamental constants defined as exact**
  - $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$, c, h, e, k, N_{A}, K_{cd}
- **All units realized through the "defining constants"**

Present SI – units defined

Base Quantity	***Base Unit***	***Definition***
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luminous intensity	candela	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian

New SI – constants defined

Base Quantity	*Defining Constant*	*Definition*
frequency	$\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$	the unperturbed ground state hyperfine splitting frequency of the cesium 133 atom $\Delta\nu\,(^{133}\text{Cs})_{\text{hfs}}$ is exactly 9 192 631 770 hertz.
velocity	c	the speed of light in vacuum c is exactly 299 792 458 meter per second.
action	h	the Planck constant h is exactly 6.626X × 10^{-34} joule second
electric charge	e	the elementary charge e is exactly 1.602X × 10^{-19} coulomb
heat capacity	k	the Boltzmann constant k is exactly 1.380X × 10^{-23} joule per kelvin
amount of substance	N_A	the Avogadro constant N_A is exactly 6.022X × 10^{23} reciprocal mole
luminous flux/ power	K_{cd}	the luminous efficacy K_{cd} of monochromatic radiation of frequency 540 × 10^{12} hertz is exactly 683 lumen per watt

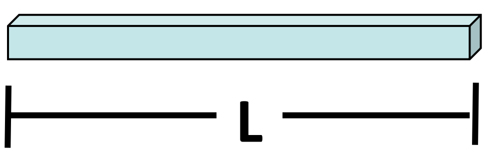
Paradigm shift of SI

- **Measurement of a dimensional Quantity, Q**

Value of Quantity	=	**Numerical value**	×	**unit**
Q	=	**{Q}**	×	**[Q]**

Present SI: $[Q] = m^{p_1}\ kg^{p_2}\ s^{p_3}\ A^{p_4}\ K^{p_5}\ mol^{p_6}\ cd^{p_7}$

New SI: $[Q] = [c_1]^{s_1}\ [c_2]^{s_2}\ [c_3]^{s_3}\ [c_4]^{s_4}\ [c_5]^{s_5}\ [c_6]^{s_6}\ [c_7]^{s_7}$



L	=	**1**	×	**m**
L	=	**30.663 . .**	×	$[c]\ [\nu(Cs)]^{-1}$

P. J. Mohr, "Defining units in the quantum based SI," Metrologia **45**, 129 (2008)

Some Fundamental Constants in the SI

- Newtonian constant of gravitation:

  $G = 6.673\,84(80) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ [1.2×10^{-4}]

- Avogadro constant:

  $N_\mathrm{A} = 6.022\,141\,29(27) \times 10^{23}\ \mathrm{mol^{-1}}$ [4.4×10^{-8}]

- Electron mass:

  $m_\mathrm{e} = 9.109\,382\,91(40) \times 10^{-31}\ \mathrm{kg}$ [4.4×10^{-8}]

- Planck constant:

  $h = 6.626\,069\,57(29) \times 10^{-34}\ \mathrm{J\,s}$ [4.4×10^{-8}]

- Fine-structure constant:

  $\alpha = 1/137.035\,999\,074(44)$ [3.2×10^{-10}]

- Rydberg constant:

  $R_\infty = 10\,973\,731.568\,539(55)\ \mathrm{m^{-1}}$ [5.0×10^{-12}]

Present SI

Quantity	=	{Quantity}	[Quantity]
$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	=	9 192 631 770	Hz
c	=	299 792 458	m/s
IPK	=	1	kg
μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{N/A^2}$
T_{TPW}	=	273.16	K
$M(^{12}\mathrm{C})$	=	0.012	kg/mol
K_{cd}	=	683	lm/W

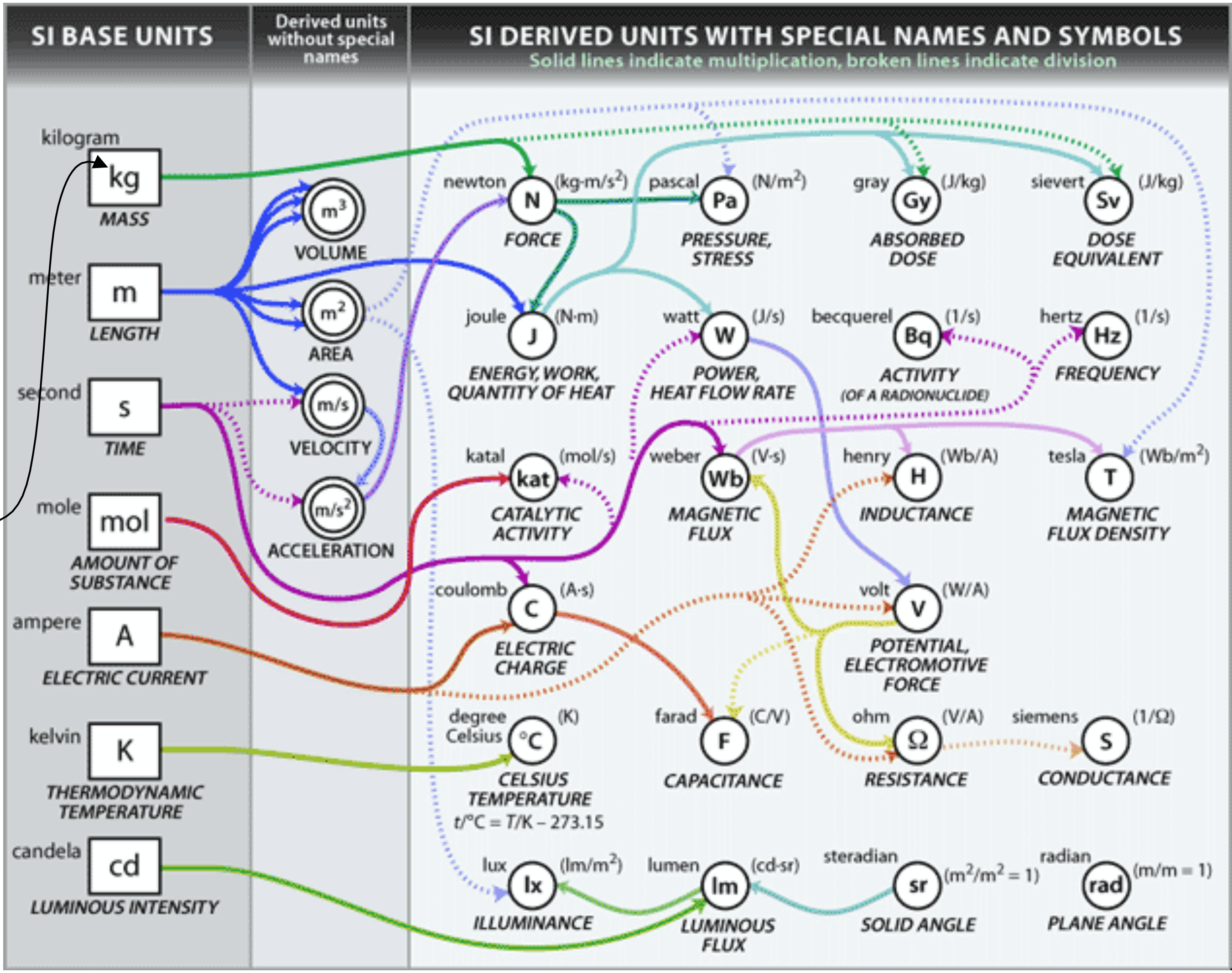
New SI

Quantity	=	{Quantity}	[Quantity]
$\Delta\nu(^{133}Cs)_{hfs}$	=	9 192 631 770	Hz
c	=	299 792 458	m/s
h	=	$6.626\,.\,.\ \times 10^{-34}$	J s
e	=	$1.602\,.\,.\ \times 10^{-19}$	C
k	=	$1.380\,.\,.\ \times 10^{-23}$	J/K
N_A	=	$6.022\,.\,.\ \times 10^{-23}$	1/mol
K_{cd}	=	683	lm/W

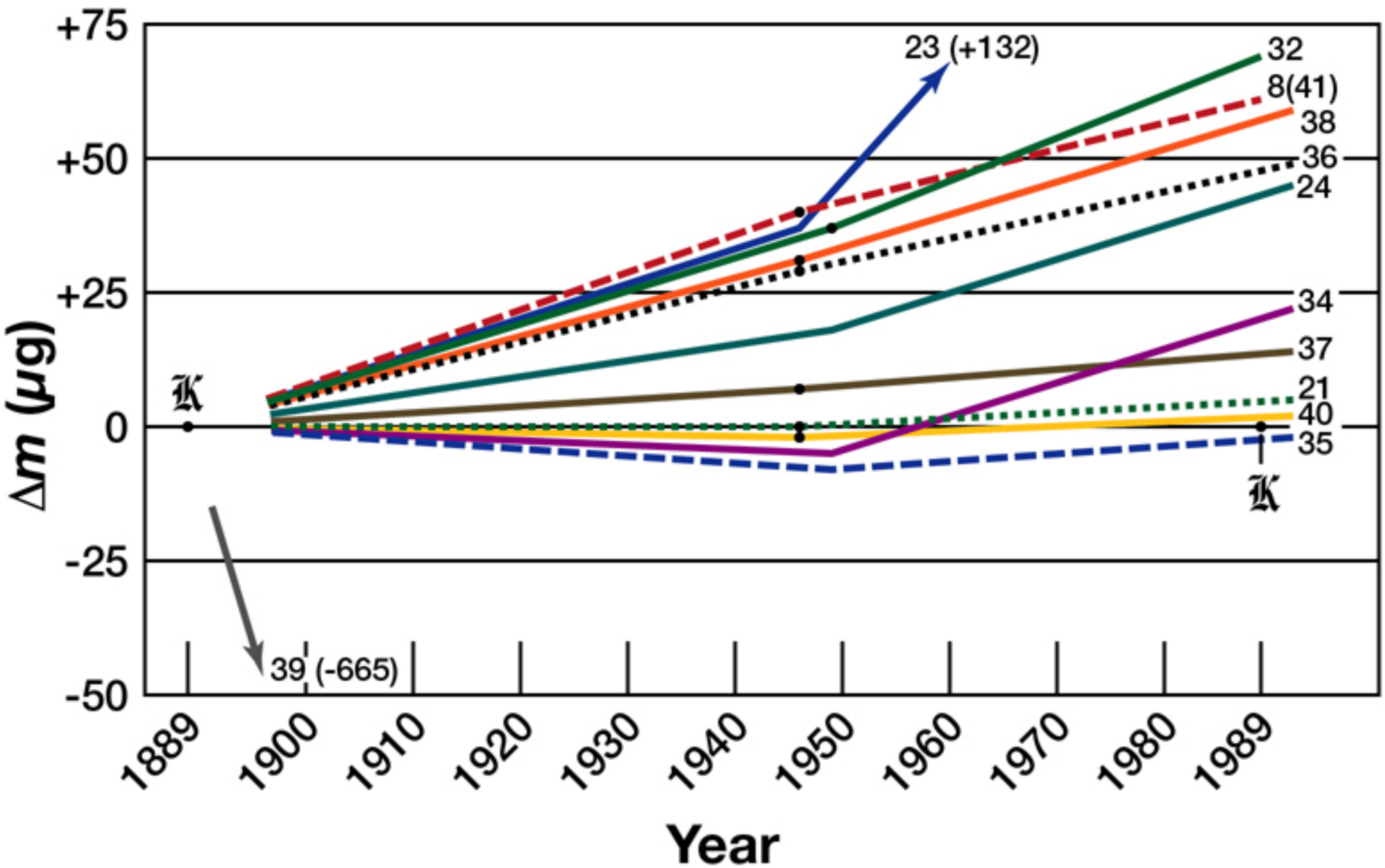
Advantage: no more IPK

physics.nist.gov/cuu/Units/

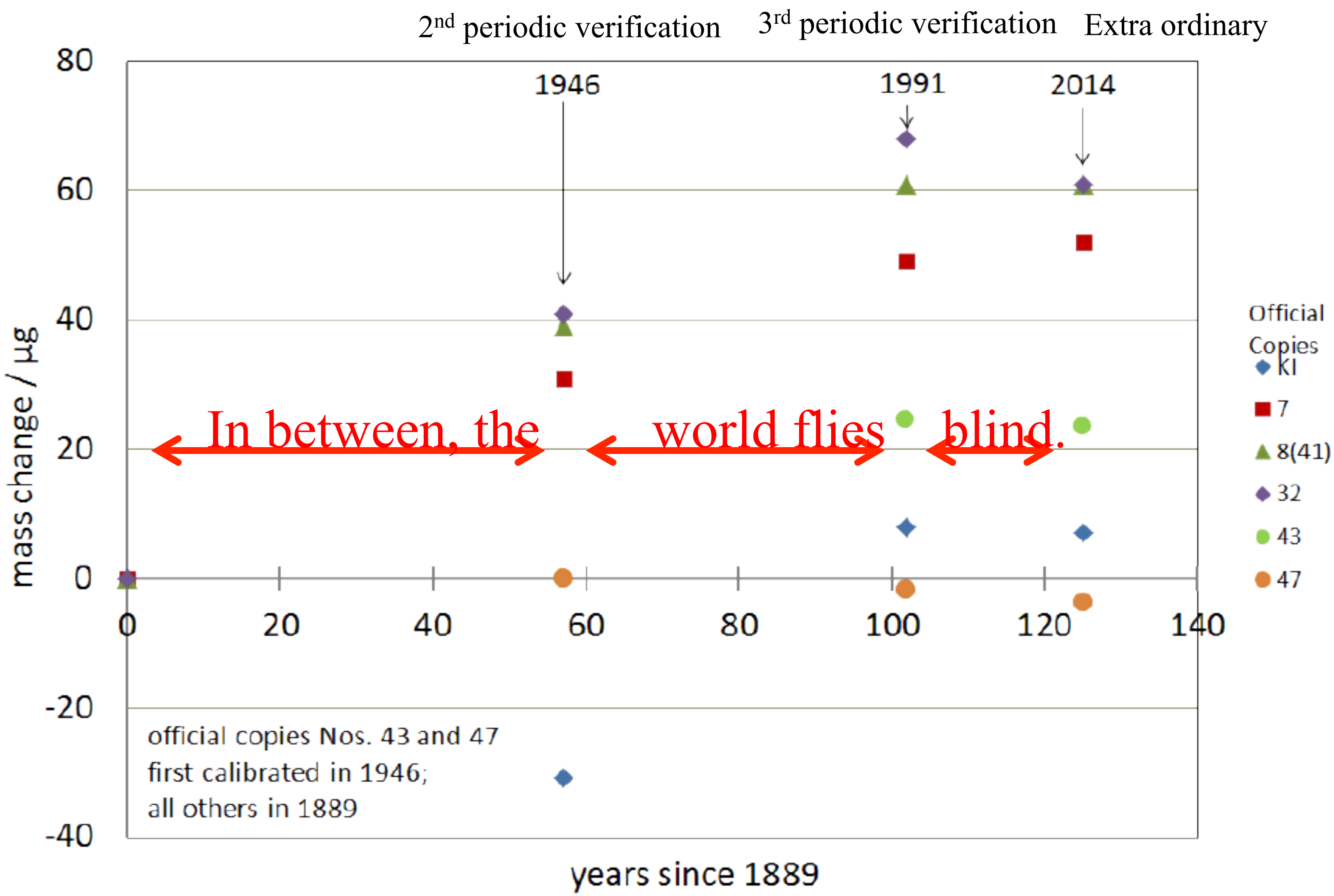
THE KILOGRAM (IPK)



Drift of the IPK

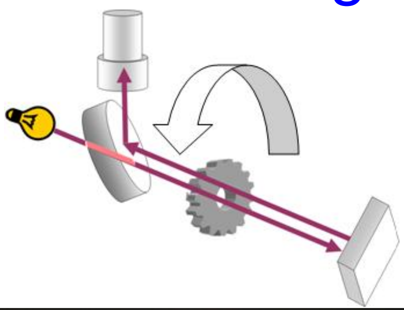


Not available to everyone, anytime, anywhere



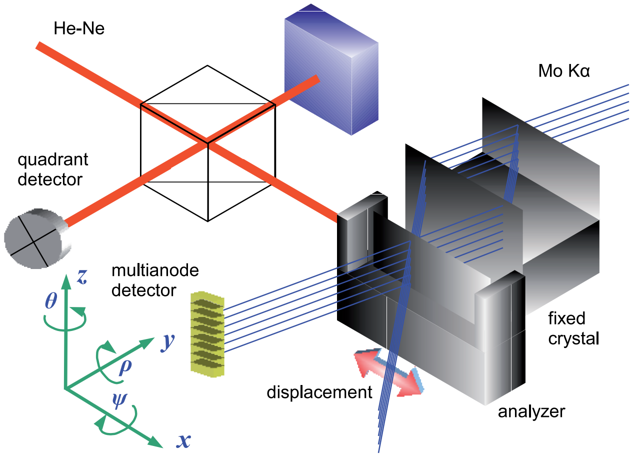
Advantage: Scaling with the new SI - Length

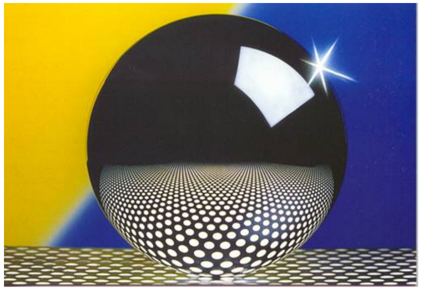
Time of Flight



4×10^{8} m measured to 1 part in 10^{11}

Interferometry





2×10^{-10} m measured to 5 parts in 10^{9}

Advantage: Scaling with the new SI – Mass

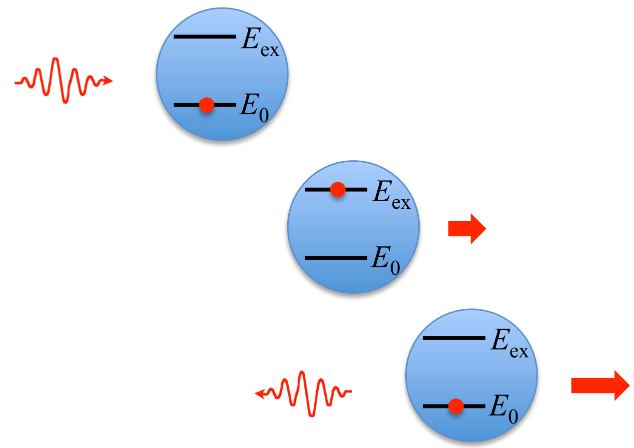
Watt Balance

$$m = \frac{h}{4gv}$$

1 kg measured to 2 parts in 10^8

Atom recoil

$$m = \frac{2h}{c^2}\frac{\nu_0^2}{\Delta\nu}\left(1 - \frac{\Delta\nu}{2\nu_0} + \dots\right)$$



2×10^{-25} kg measured to 2 parts in 10^9

Impact: Improved values of other constants

Quantity	Symbol	Present SI $u_r \times 10^9$	New SI $u_r \times 10^9$
Planck constant	h	44	0
Elementary charge	e	22	0
Boltzmann constant	k	910	0
Avogadro constant	N_A	44	0
Josephson constant	K_J	22	0
von Klitzing constant	R_K	0.32	0
Electron mass	m_e	44	0.64
Atomic mass unit	m_u	44	0.70
Mass of carbon-12	$m(^{12}C)$	44	0.70
Molar gas constant	R	910	0
Faraday constant	F	22	0
Stefan-Boltzmann constant	σ	3600	0
Fine-structure constant	α	0.32	0.32

Impact: No uncertainty in energy conversions

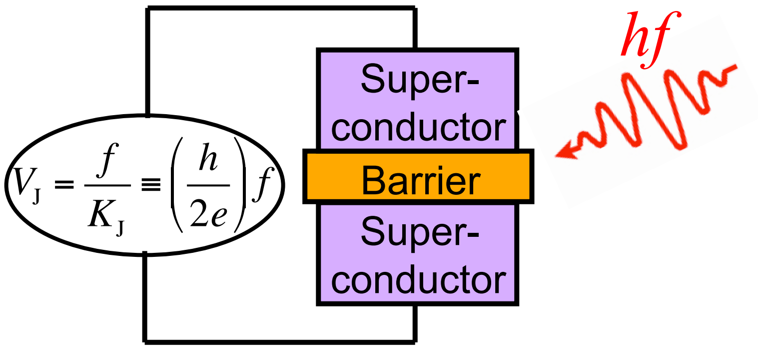
Quantity	Symbol	Present SI $u_r \times 10^9$	New SI $u_r \times 10^9$
Planck constant	h	44	0
Elementary charge	e	22	0
Boltzmann constant	k	910	0
Avogadro constant	N_A	44	0
$E=mc^2$ energy equivalent	J ↔ kg	0	0
$E=hc/\lambda$ energy equivalent	J ↔ m^{-1}	44	0
$E=h\nu$ energy equivalent	J ↔ Hz	44	0
$E=kT$ energy equivalent	J ↔ K	910	0
1 J =1 (C/e) eV energy equivalent	J ↔ eV	22	0

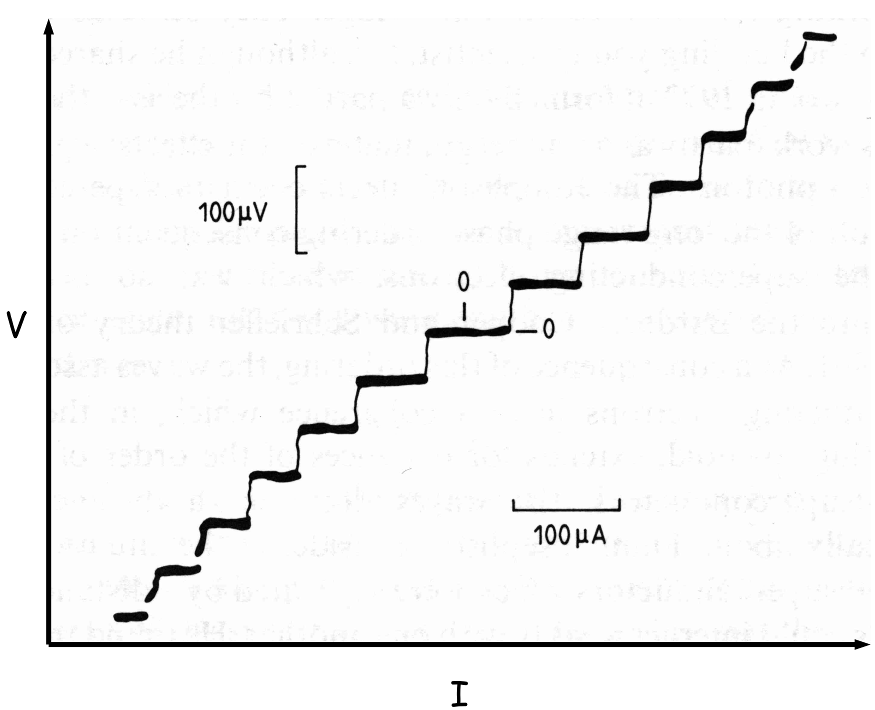
Consequences: Presently exact values no longer exact

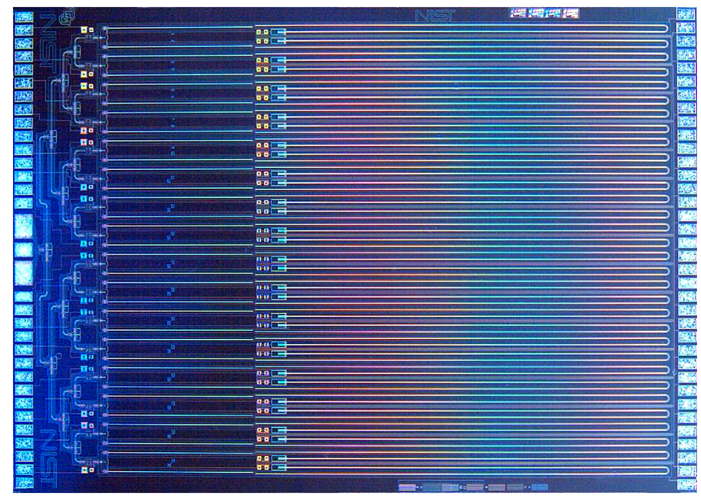
Quantity	Symbol	Present SI $u_r \times 10^9$	New SI $u_r \times 10^9$
Planck constant	h	44	0
Elementary charge	e	22	0
Boltzmann constant	k	910	0
Avogadro constant	N_A	44	0
International prototype of the kilogram	$m(K)$	0	44
Permeability of free space	μ_0	0	0.32
Permittivity of free space	ε_0	0	0.32
Triple point of water	T_{TPW}	0	910
Molar mass of carbon-12	$M(^{12}C)$	0	0.70

Consequence: shift in quantum electrical standards

Josephson Voltage Standard





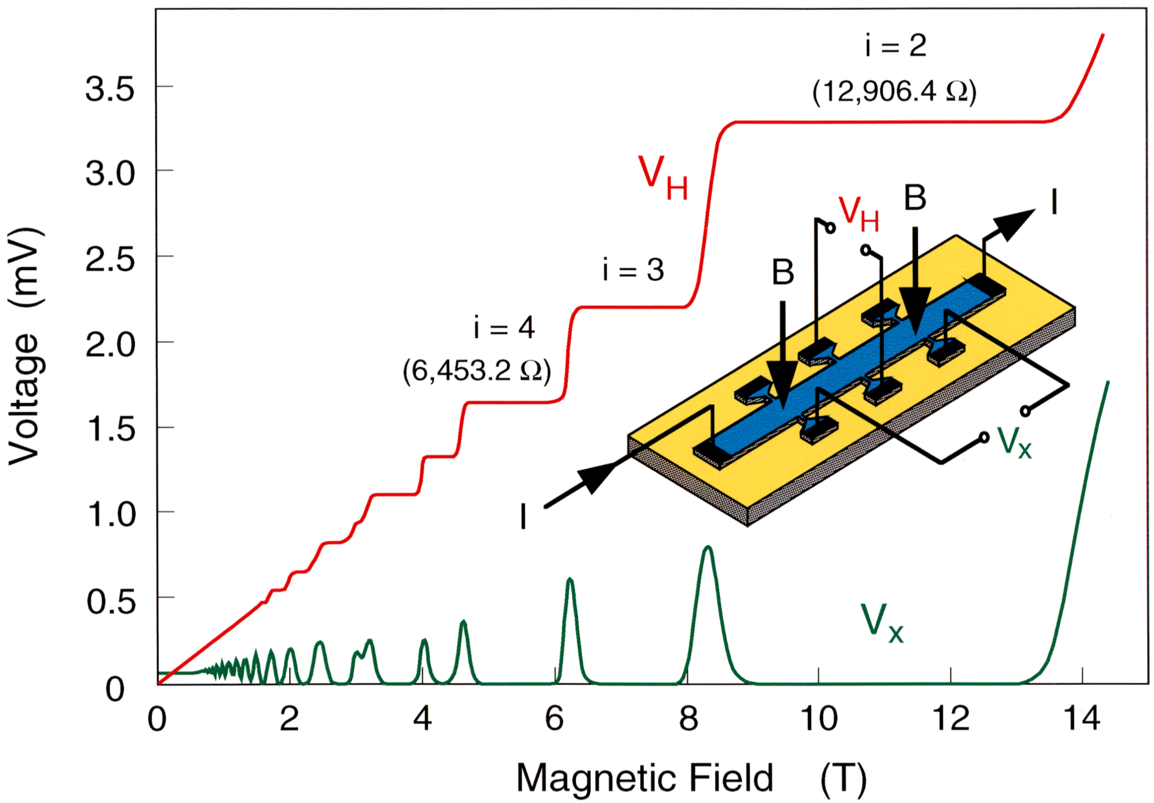


NIST Programmable Josephson Voltage Array Standard

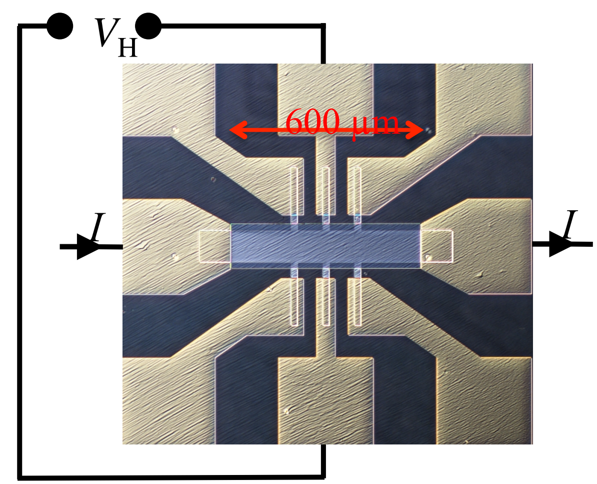
$$V_J = \left(\frac{h}{2e}\right) nf$$

Consequence: shift in quantum electrical standards

Quantum Hall Resistance

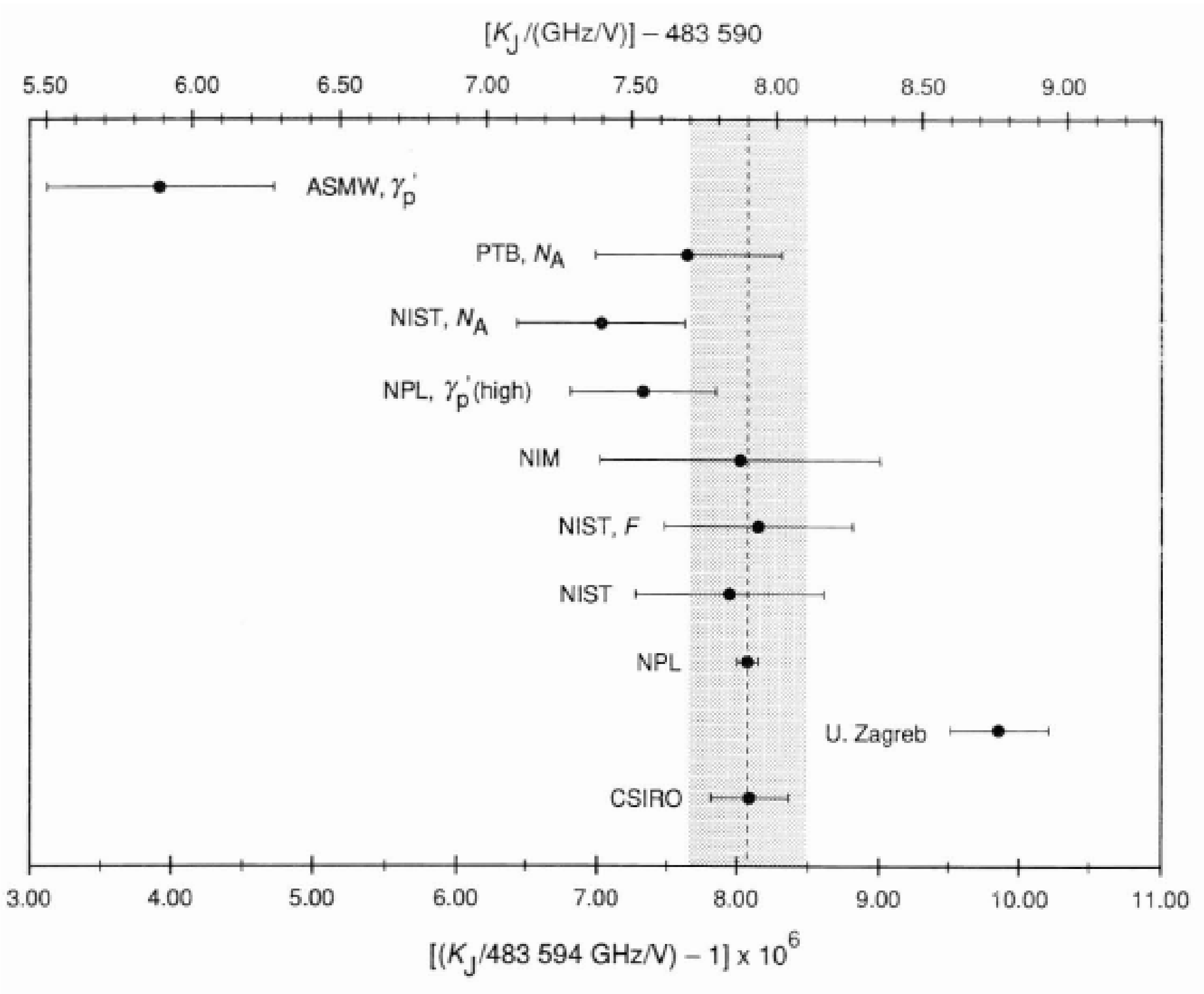


$$R_H = \frac{V_H}{I} = \frac{R_K}{i} \equiv \frac{1}{i}\left(\frac{h}{e^2}\right)$$



NIST SiC graphene quantum Hall resistance standard

1990 Conventional Electrical Unit of the volt



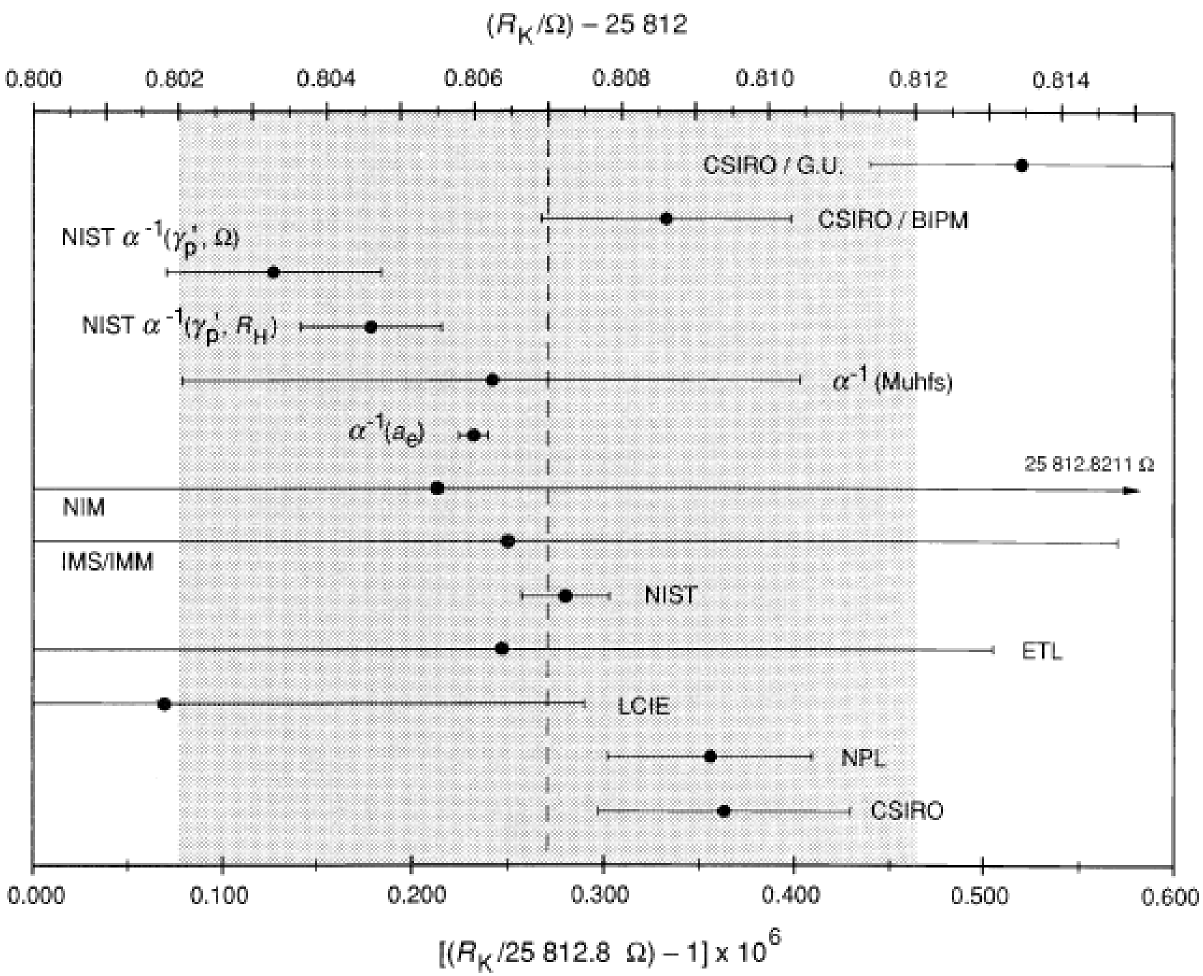
$$V_J = \left(\frac{h}{2e}\right)nf = \frac{nf}{K_J}$$

$$K_{J\text{-}90} = 483597.9 \text{ GHz/V}$$

Quinn, T.J., Metrologia **26**(1), 69-74 (1989)

Taylor, B.N., and T.J. Witt, Metrologia **26**(1), 47-62 (1989)

1990 Conventional Electrical Unit of the ohm



$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

$R_{K\text{-}90} = 25812.807\ \Omega$

Quinn, T.J., Metrologia **26**(1), 69-74 (1989)

Taylor, B.N., and T.J. Witt, Metrologia **26**(1), 47-62 (1989)

Electric current – the ampere (cont.d)

Consequence:

The resulting shifts in the new SI electrical units with respect to the 1990 conventional electrical units are:

$$\mathrm{V} = V_{90}[1 - (100 \times 10^{-9})]; \quad (-100 \text{ ppb})$$
$$\Omega = \Omega_{90}[1 - (17 \times 10^{-9})]; \quad (-17 \text{ ppb})$$
$$\mathrm{A} = A_{90}[1 - (83 \times 10^{-9})]; \quad (-83 \text{ ppb})$$
$$\mathrm{C} = C_{90}[1 - (83 \times 10^{-9})]; \quad (-83 \text{ ppb})$$
$$\mathrm{W} = W_{90}[1 - (183 \times 10^{-9})]; \quad (-183 \text{ ppb})$$
$$\mathrm{F} = F_{90}[1 + (17 \times 10^{-9})]; \quad (17 \text{ ppb})$$
$$\mathrm{H} = H_{90}[1 - (17 \times 10^{-9})]; \quad (-17 \text{ ppb})$$

It should be noted these shifts are within the assigned uncertainties by the Consultative Committee for Electricity and Magnetism (CCEM) of the CIPM (Quinn, 1989, 2001):

$$\mathrm{V} = V_{90}[1 \pm (400 \times 10^{-9})]; \quad (\pm 400 \text{ ppb})$$
$$\Omega = \Omega_{90}[1 \pm (100 \times 10^{-9})]; \quad (\pm 100 \text{ ppb})$$

Outline

- Brief history of the SI
- The new SI
- **Communication about New SI**

Communication: general public

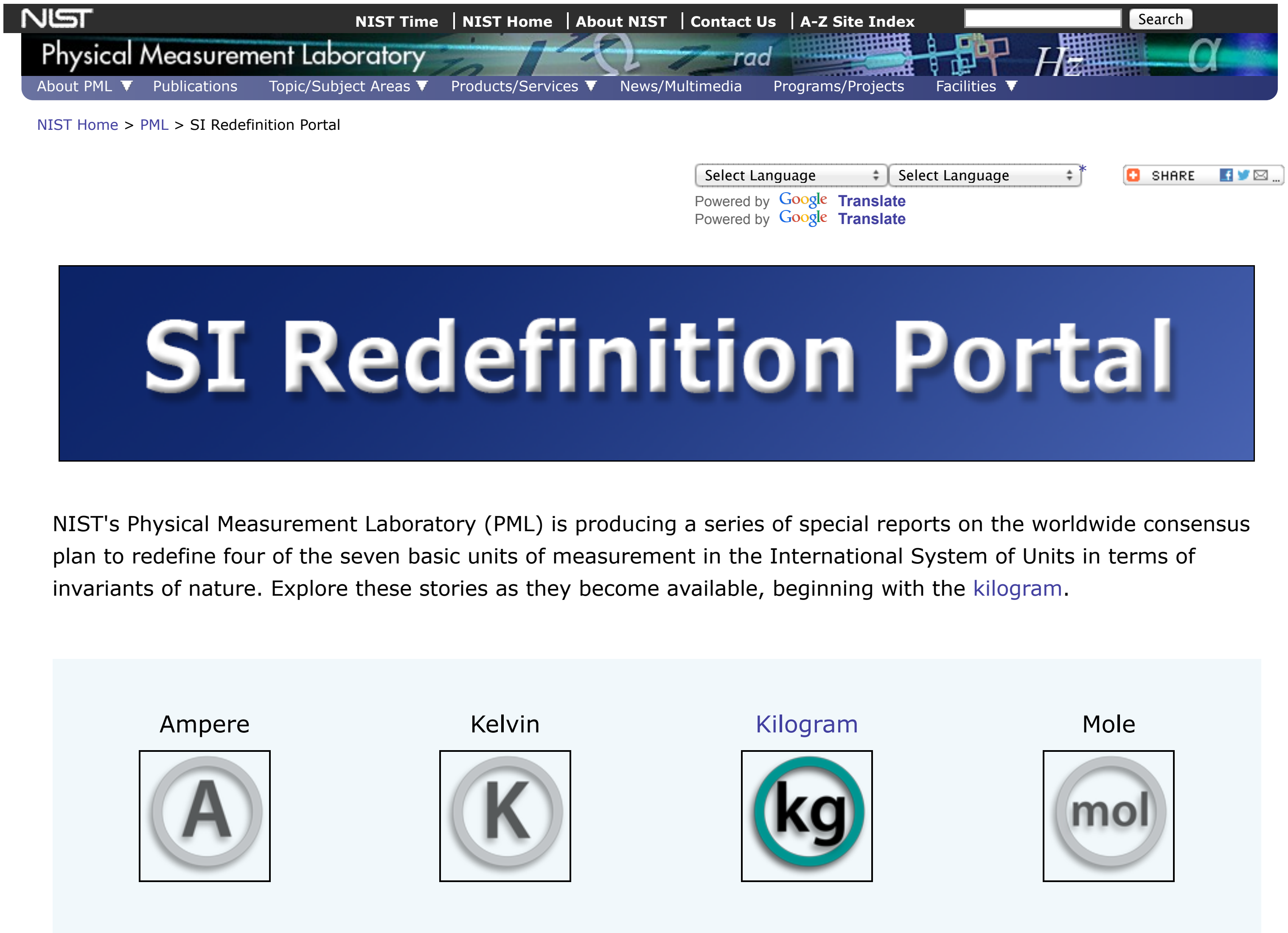
oz.wikia.com

Communication: general public

- **MLB Official Rules:**
  – The ball . . . shall weigh not less than five nor more than 5 1/4 ounces avoirdupois
  – The bat shall be . . . not more than 42 inches in length

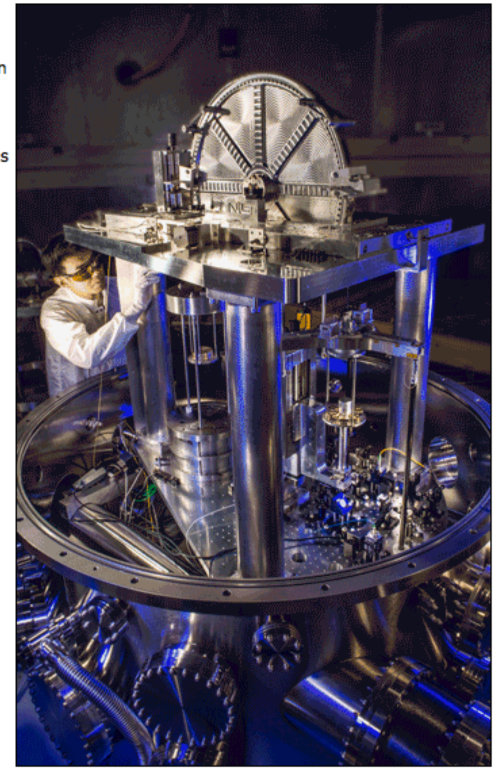
Communication: public outreach

http://nist.gov/pml/si-redef/



Redefining the Kilogram

Main Menu



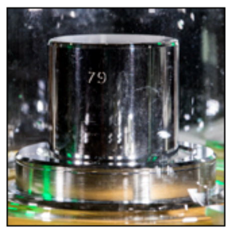
For more than a century, the kilogram (kg) – the fundamental unit of mass in the International System of Units (SI) – has been defined as exactly equal to the mass of a small polished cylinder, cast in 1879 of platinum and iridium, which is kept in a triple-locked vault on the outskirts of Paris.

That object is called the International Prototype of the Kilogram (IPK), and the accuracy of every measurement of mass or weight worldwide, whether in pounds and ounces or milligrams and metric tons, depends on how closely the reference masses used in those measurements can be linked to the mass of the IPK.

That situation is about to change. The world metrology community plans to redefine the kilogram soon, freeing it from its embodiment in one golf-ball-sized artifact at one location, and basing it instead on a constant of nature. That transformation will be as profound as any in the history of measurement. **MORE**

Past

Before it became the international mass standard in 1875, the kg had a restive birth during the French Revolution.



Present

An artifact kg requires complicated periodic re-calibrations and coping with drifting mass standards.

Future

The new kg will be based on a constant of nature and thus will be measureable anywhere in the world.

News

- NIST's Latest Value for *h* Using NIST-3
- The Great Gravity Showdown

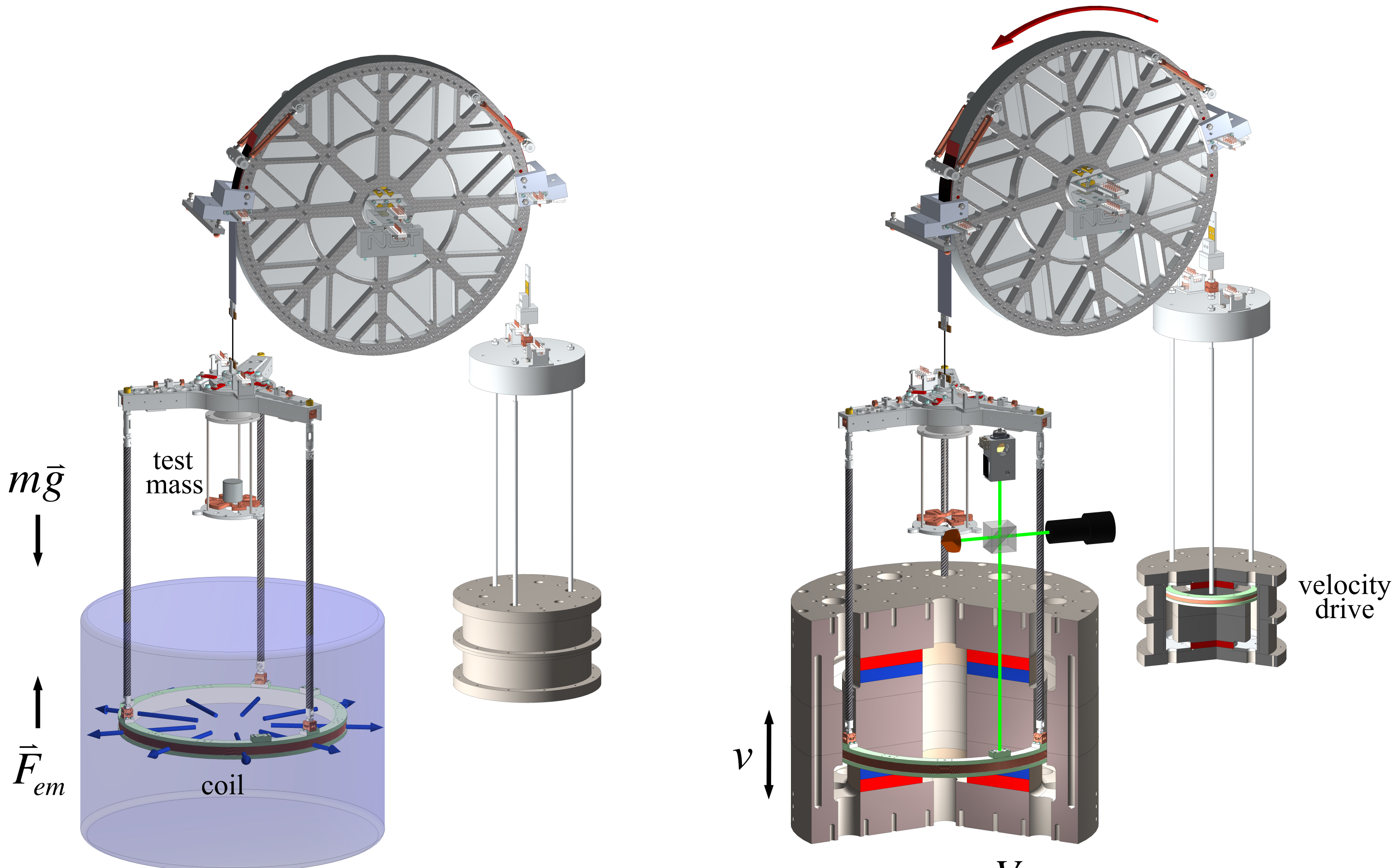
Resources

- How to Weigh Everything from Atoms to Apples Using the Revised SI
- BIPM on Mass and Related Quantities
- NIST's Electronic Kilogram Project
- NIST 2014 Technical Paper on Determination of *h*

Operating Principles of the NIST-4 Watt Balance

NIST Physical Measurement Laboratory

Force Mode **Velocity Mode**

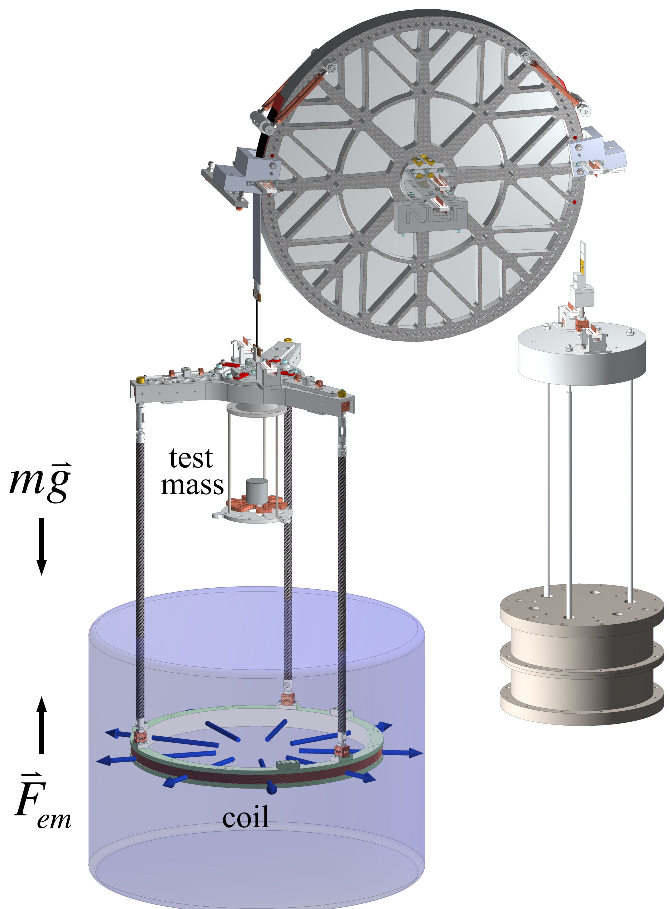


$$mg = I\int\left(d\vec{l}\times\vec{B}\right)_z = IBl \rightarrow \frac{mg}{I} = Bl$$

$$V = vBl \rightarrow \frac{V}{v} = Bl$$

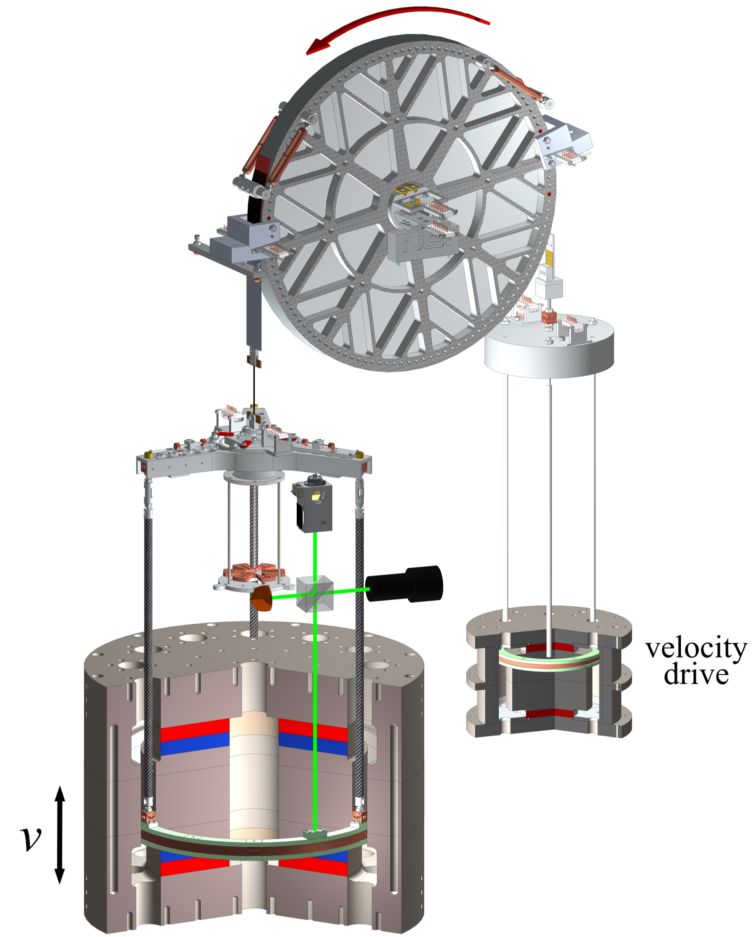
$$\boxed{mgv = VI}$$

Force Mode



$$mg = I\int\left(d\vec{l}\times\vec{B}\right)_z = IBl \rightarrow m = \frac{IBl}{g} = \frac{V_1}{R}\frac{Bl}{g}$$

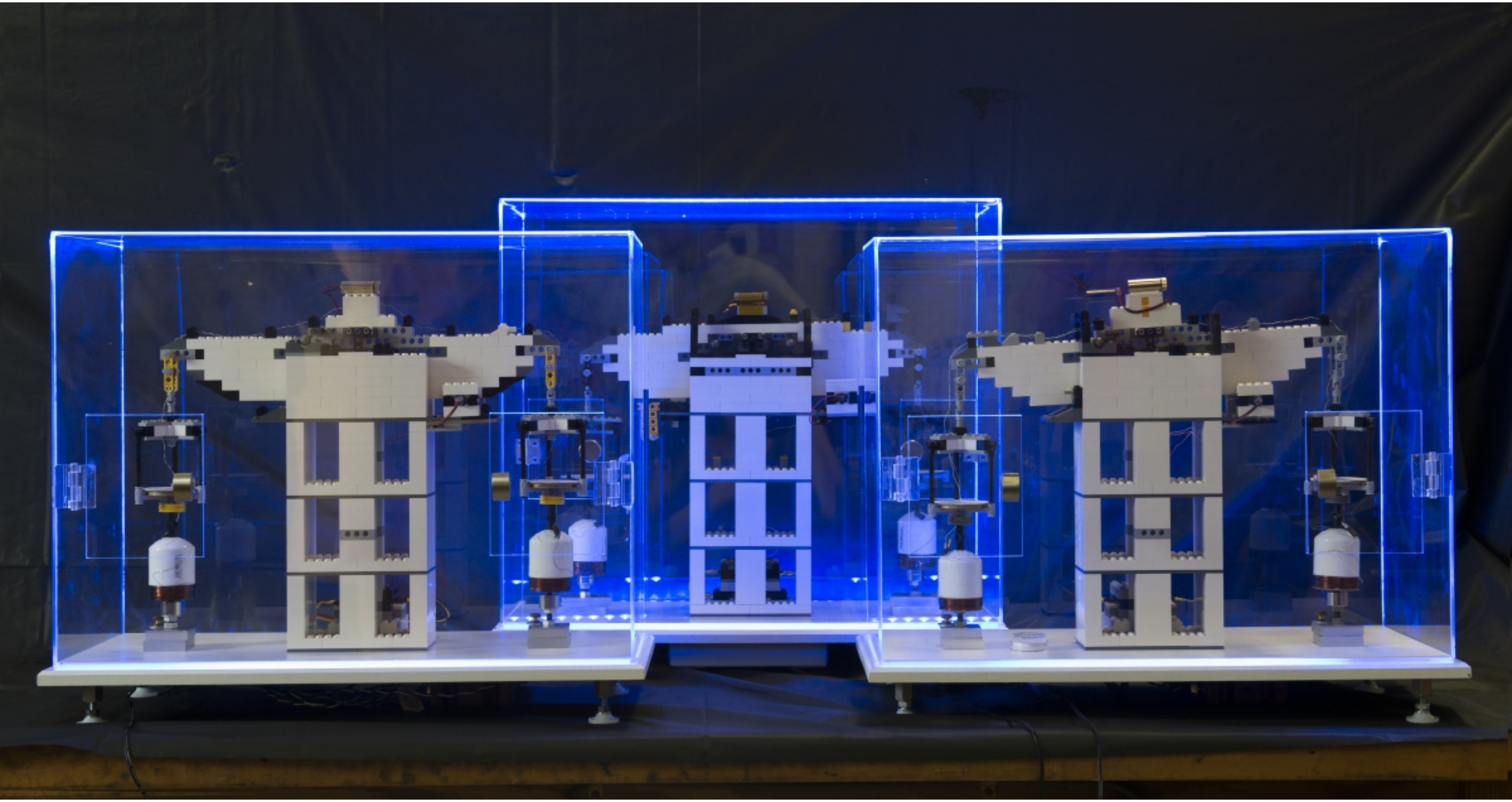
Velocity Mode



$$V_2 = vBl \rightarrow \frac{V_2}{v} = Bl$$

$$m = \frac{IBl}{g} = \frac{V_1V_2}{Rgv} = h\left(\frac{in_1n_2}{4}\right)\frac{f_1f_2}{gv}$$

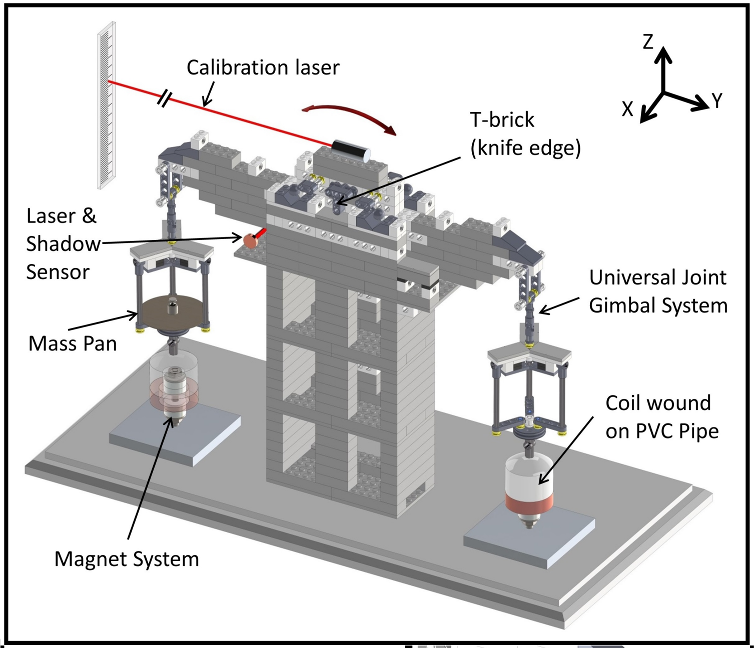
An army of LEGO Watt Balances!

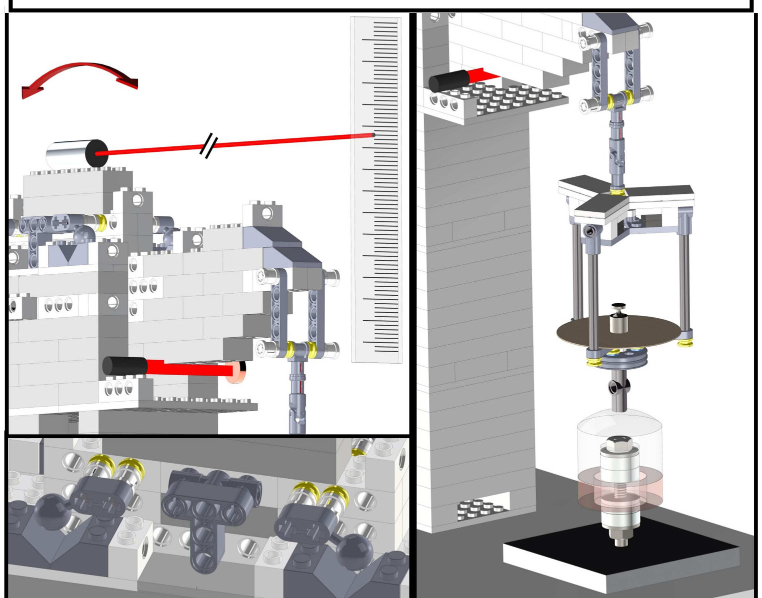


"A LEGO Watt Balance: An Apparatus to demonstrate the definition of mass based on the new SI" (submitted to AJP)

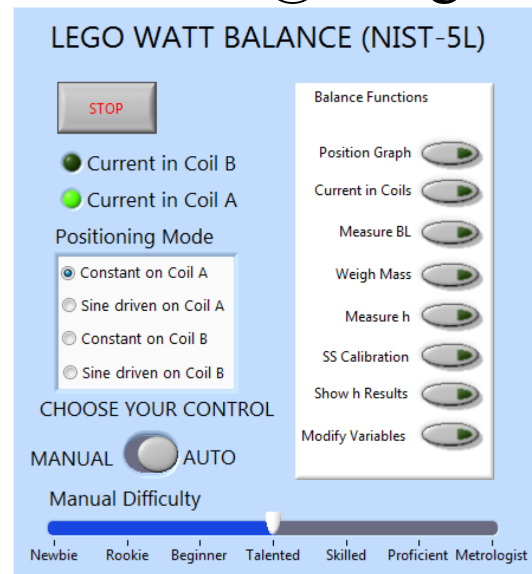
Part Name	Part No.	Quantity	Total Price ($)
Custom LEGO Watt Balance Software	Contact leon.chao@nist.gov	1	Free
Order from http://shop.lego.com/en-US/Pick-A-Brick-ByTheme			
Brick 2x4	300101	65	19.50
Brick 2x8	6033776	73	36.50
Brick 1x2 with cross hole	4233487	12	4.20
T-Beam 3x3 w/hole O4.8	4552347	2	0.60
Technic Brick 1x2 O4.9	370026	16	2.40
Technic Brick 1x4 O4.9	4211441	48	12.00
Technic Brick 1x6 O4.9	389426	2	0.80
Technic Brick 1x8 O4.9	4211442	2	1.00
Technic Ang. Beam 3x5 90 Deg.	4211713	2	0.60
Plate 8x8	4210802	9	9.90
Plate 1x2	4211398	15	1.50
Plate 1x4	4211445	10	1.50
Plate 2x3	4211396	6	1.20
Cross Axle 2M W. Groove	4109810	8	0.80
Cross Axle 3M	4211815	6	0.14
Cross Axle 5M	4211639	6	1.20
Cross Axle 8M	370726	8	1.60
Bush for Cross Axle	4211622	14	2.10
1/2 Bush for Cross Axle	4211573	28	2.80
Double Bush 3M O4.9	4560175	6	1.20
Roof Tile 2x2/45 deg	303926	4	0.80
Roof Tile 2x2/45 deg Inv.	366026	2	0.40
Roof Tile 2x3/25 deg	4211106	2	0.40
Roof Tile 2X3/25 deg Inv.	374726	4	0.80
Connector Peg W. Friction 3M	4514553	8	2.00
Connector Peg/Cross Axle	4666579	6	0.60
Catch w. Cross Hole	4107081	8	1.60
Flat Tile 2x4	4560178	4	1.20
Hinge 1x2 Lower Part	383101	6	1.50
Hinge 1x2 Upper Part	6011456	6	1.50
Double Conical Wheel Z12 1M	4177431	2	0.60
Angle Element, 180 Degrees [2]	4107783	2	0.40
Order from http://atrbricks.brickowl.com/			
Technic Beam 1 x 4 x 0.5 with Boss	2825 / 32006	6	0.30
Technic Beam 2 Beam w. Angled Ball Joint	50923 / 59141	2	0.13
Beam 5 x 0.5	32017	4	0.76
Wedge Belt Wheel	2786 / 4185	4	1.00
Gear with 8 Teeth (Narrow)	3647	2	0.20
Order from http://thatspecialbrick.brickowl.com/			
Universal Joint	61903	2	0.94
Gear with 16 teeth	94925	2	0.34
Bevel Gear with 12 teeth	6589	2	0.26
Order from http://www.labjack.com/u6			
Multifunction DAQ with USB - 16 Bit	U6	1	299.00
Order from http://www.phidgets.com			
PhidgetAnalog 4 Output	1002_0	1	90.00
Order from http://www.apinex.com			
Focus. Line Red Laser Module <1mW	YCHG-650	1	15.00
Line Laser Module (650nm) - <1mW	LN60-650	1	15.00
Order from http://www.mouser.com			
Photodiode 7.98mm Dia Area	718-PC50-7-TO8	1	61.63
USB Cables USB A - MINI-B	538-88732-8702	1	1.65
USB A-TO-B Shielded 2.09m	538-88732-9200	1	3.58
Low signal Relay	769-TXS2-4.5V	1	4.58
Resistors 240ohms	291-240-RC	1	0.10
Resistors 330ohms	291-330-RC	4	0.40
Resistors 1500ohms	291-1.5k-RC	1	0.10
Linear Voltage Regulators	511-LM317T	1	0.72
Order from http://www.magnet4less.com			
N48 grade - 3/4 (OD) x 1/4 (ID) x 3/4 in. ring magnet	NR011-1	4	15.96
Order from http://www.mcmaster.com			
Brass Threaded Rod - 1/4"-20 Thread, 1' length	98812A039	1	2.65
White PVC Pipe Fitting	4880K53	2	1.00
White PVC Unthreaded Pipe	48925K93	1	5.27
Total			633.77

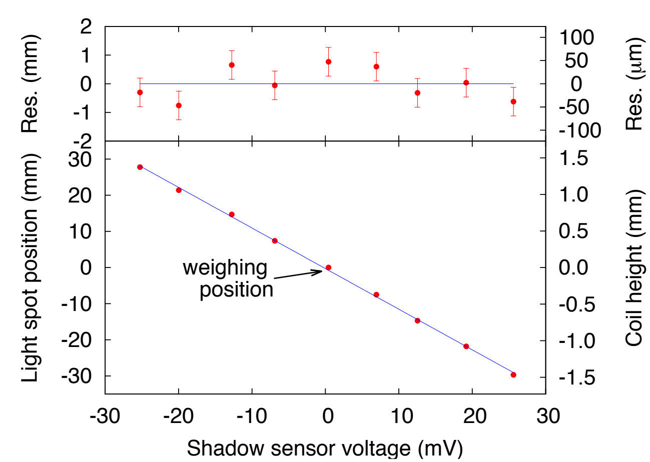
Total cost: $634





Contact:
leon.chao@nist.gov



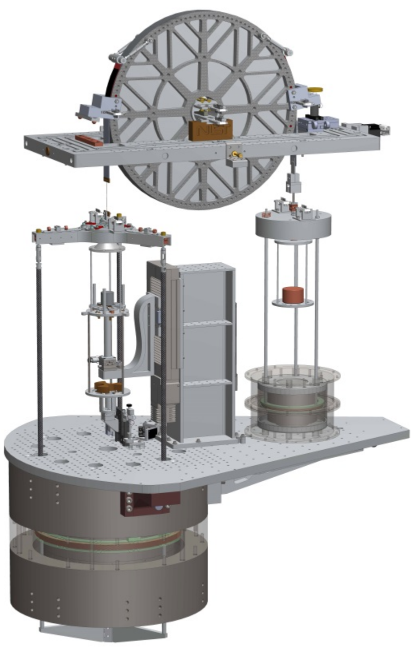


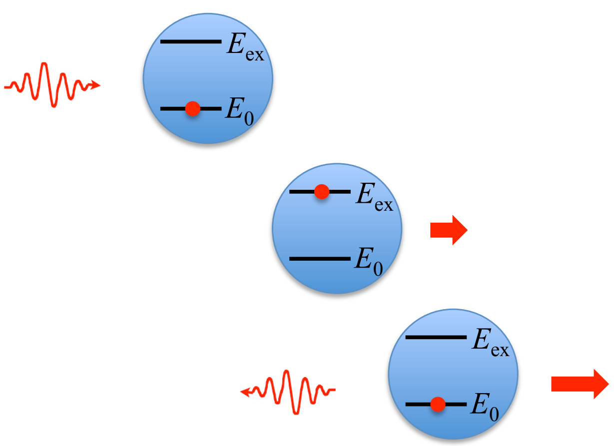
Communication: metrological and scientific communities

- **Metrological and scientific communities:**
  - System with no uncertainty over 25 orders of magnitude

$$m = \frac{h}{4g\nu}$$

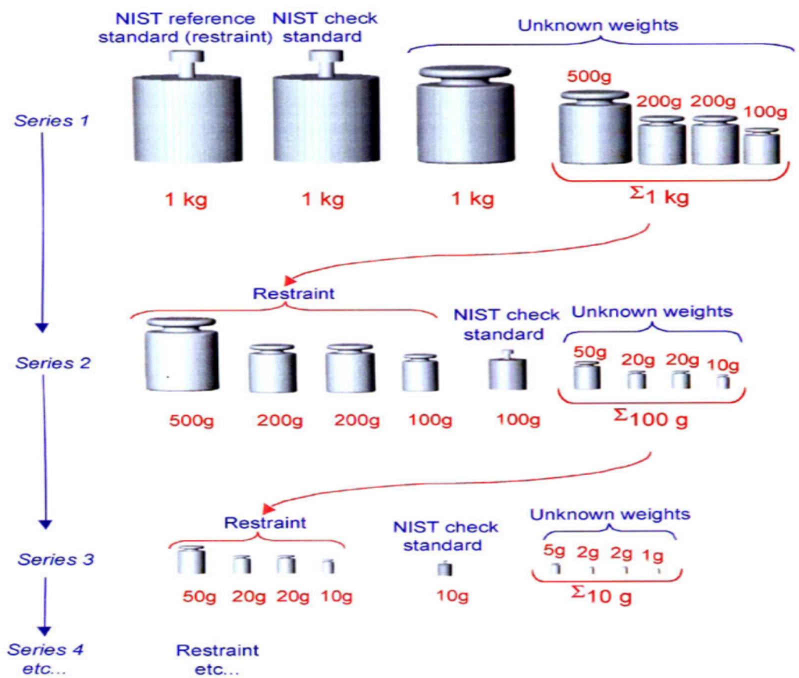
$$m = \frac{2h}{c^2}\frac{\nu_0^2}{\Delta\nu}\left(1 - \frac{\Delta\nu}{2\nu_0} + \dots\right)$$

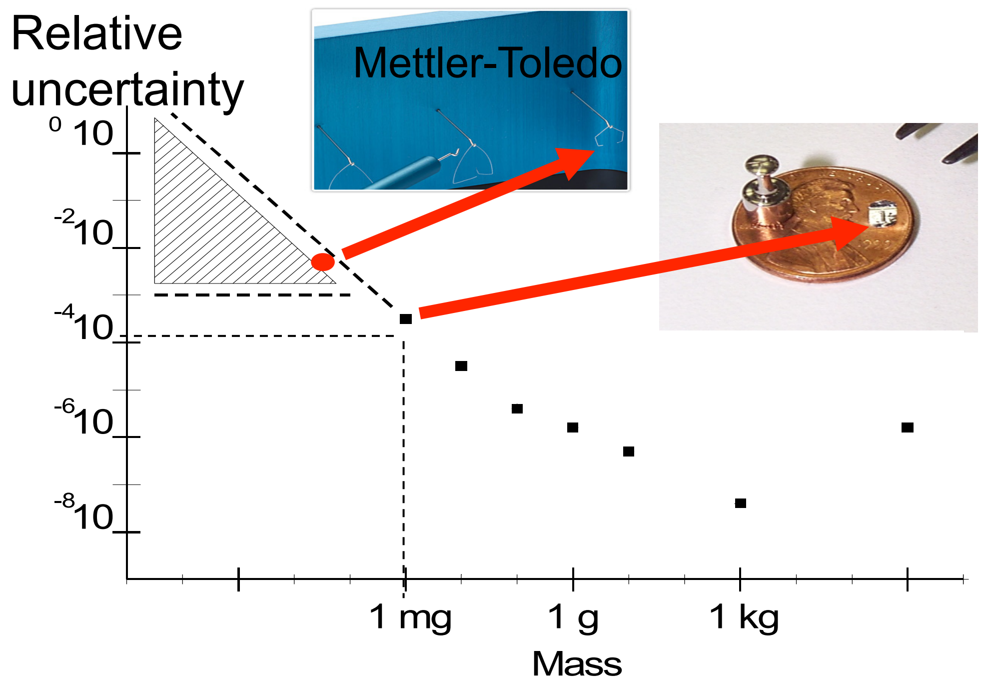




Communication: manufacturers of small mass dissemination

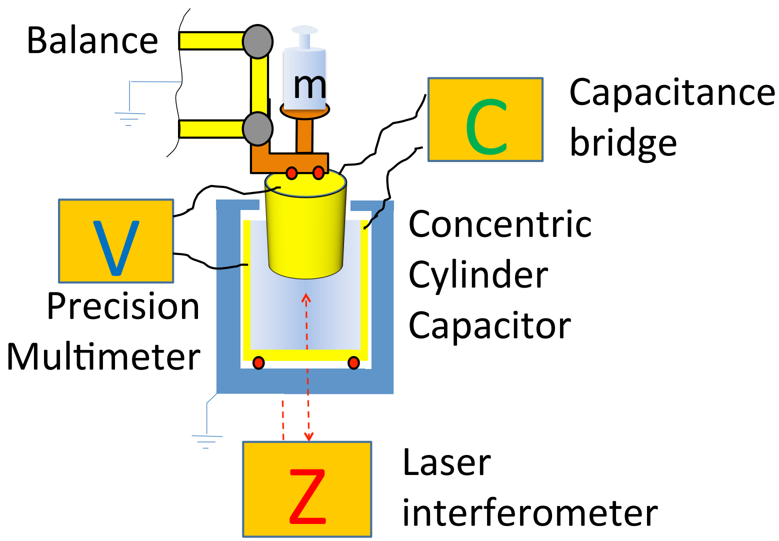
Dissemination to submultiples through weighing design

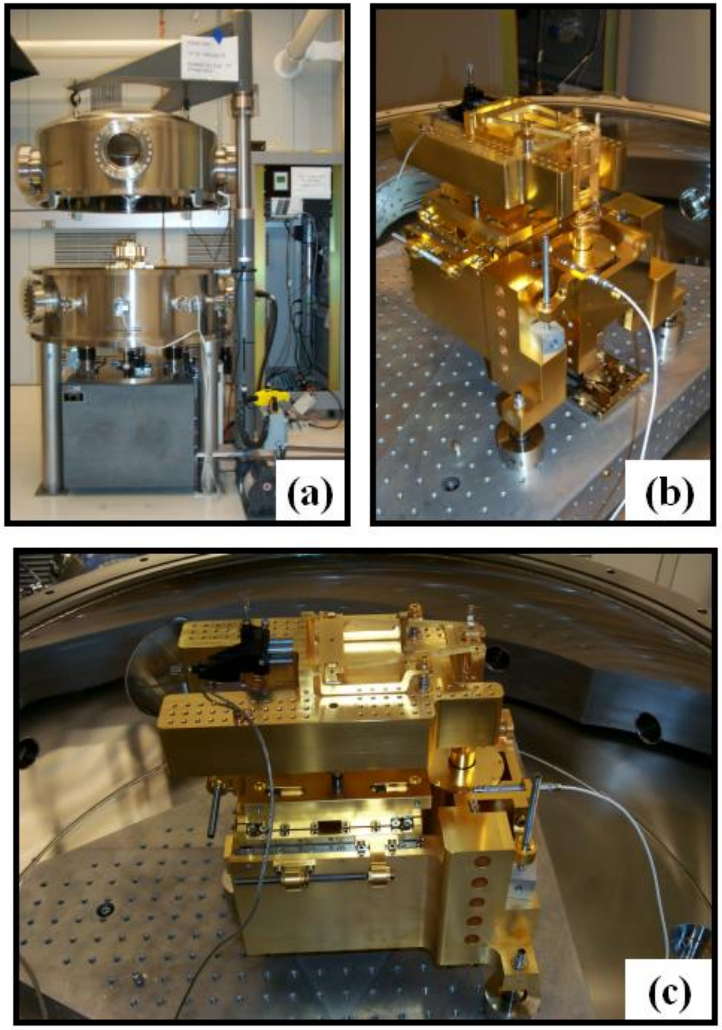




Future small mass dissemination

NIST Electrostatic force balance



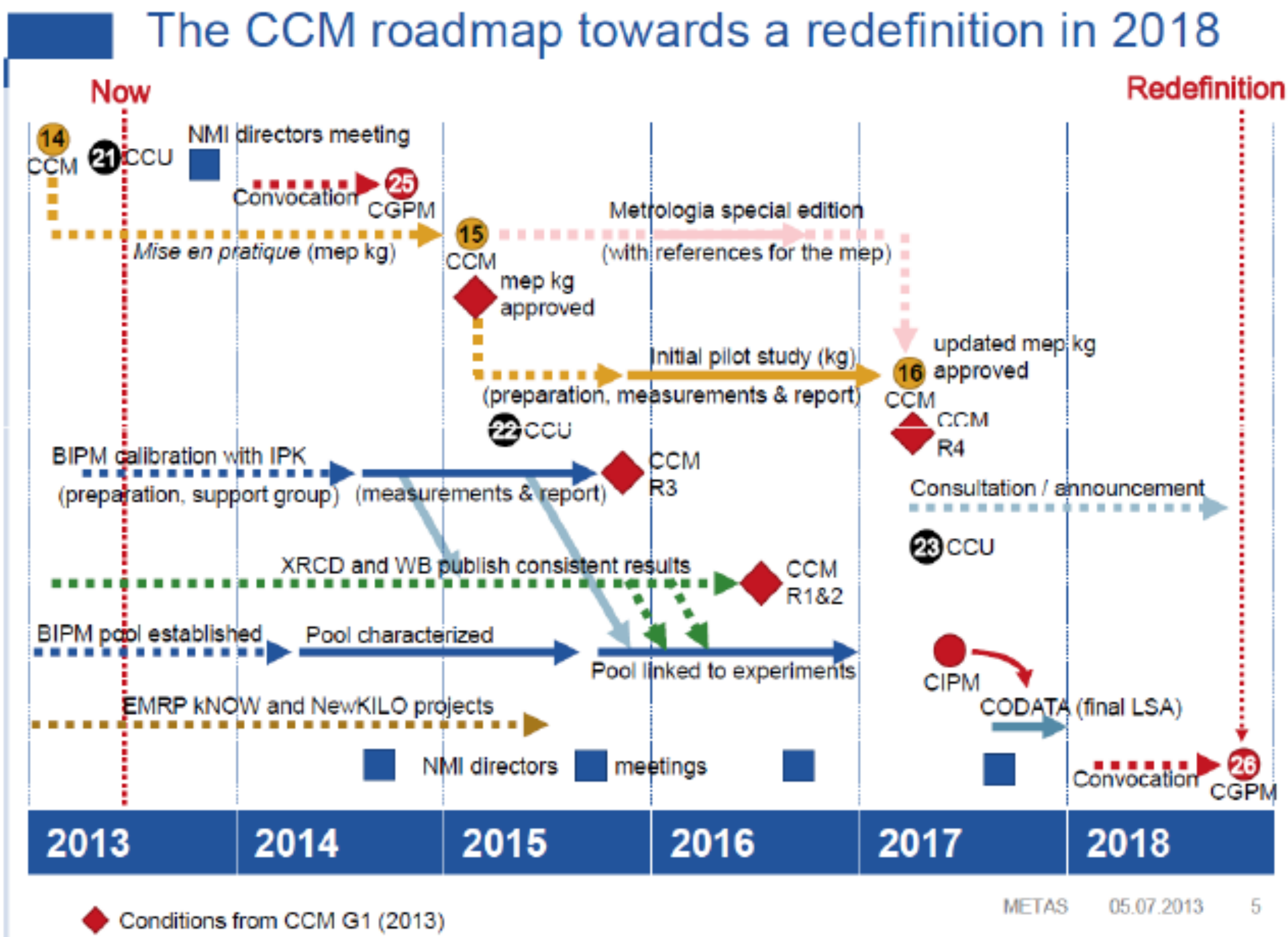


$$F = mg = \frac{1}{2}\, dC/dz\, V^2$$

- 20 mg realized to 20 ppm at IMEKO 2006
- With recent upgrades, realize 1 mg to a few ppm

  (50 times better than present mass dissemination)

Near future: on the road to redefinition



Far future: change yet again?

Quantity	=	{Quantity}	[Quantity]
$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	=	9 192 631 770	Hz
c	=	299 792 458	m/s
h	=	$6.626\ldots \times 10^{-34}$	J s
e	=	$1.602\ldots \times 10^{-19}$	C
k	=	$1.380\ldots \times 10^{-23}$	J/K
N_{A}	=	$6.022\ldots \times 10^{-23}$	1/mol
K_{cd}	=	683	lm/W

Far future: change yet again?

Quantity	=	{Quantity}	[Quantity]
R_∞	=	10 973 731.568 5	m^{-1}
c	=	299 792 458	m/s
h	=	$6.626\ldots \times 10^{-34}$	J s
e	=	$1.602\ldots \times 10^{-19}$	C
k	=	$1.380\ldots \times 10^{-23}$	J/K
N_A	=	$6.022\ldots \times 10^{-23}$	1/mol
K_{cd}	=	683	lm/W

Far future: change yet again?

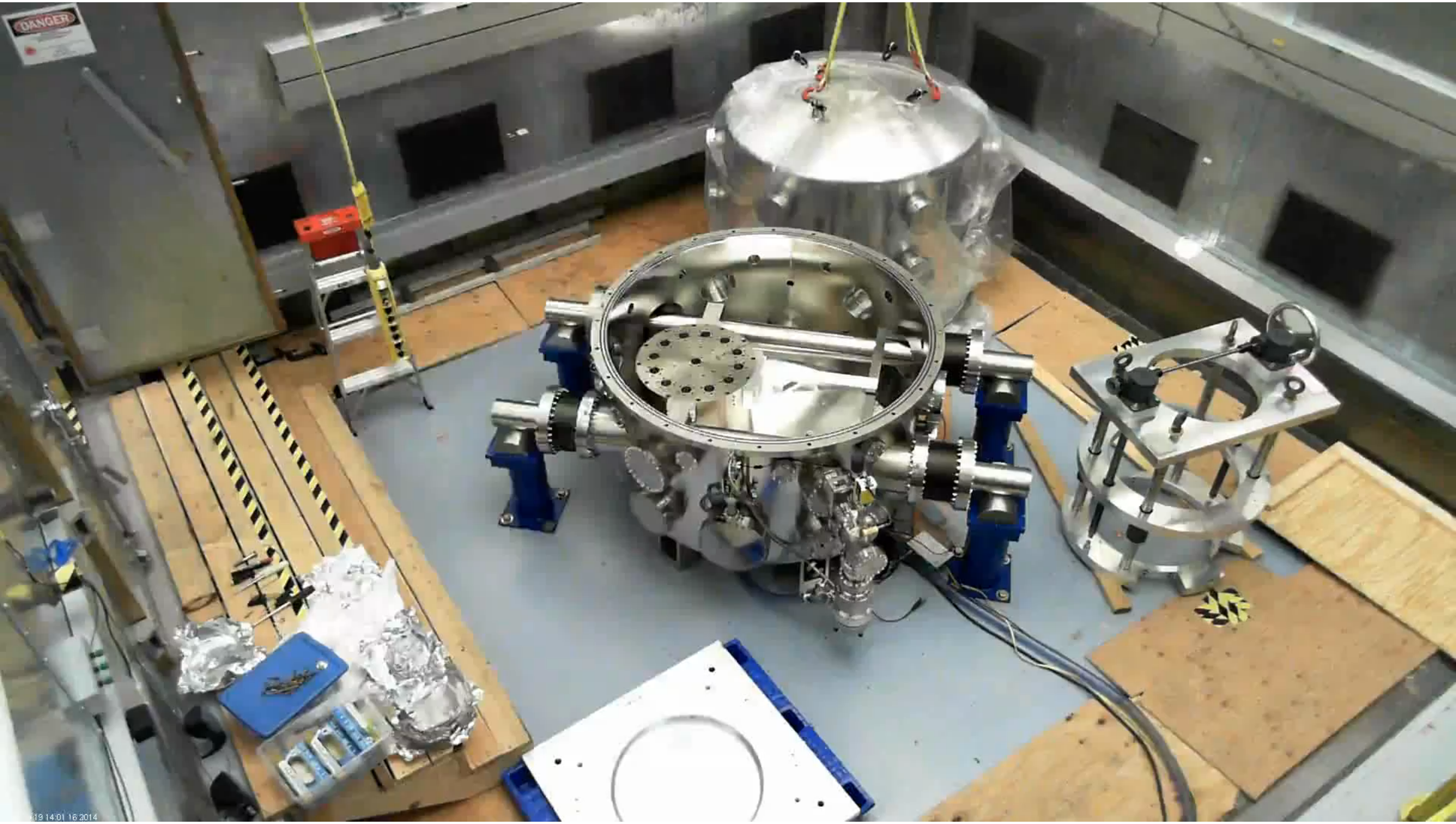
Quantity	=	{Quantity}	[Quantity]
$\Delta\nu(X)_?$	=	# × 10^{14}	Hz
c	=	299 792 458	m/s
h	=	6.626 . . × 10^{-34}	J s
e	=	1.602 . . × 10^{-19}	C
k	=	1.380 . . × 10^{-23}	J/K
N_A	=	6.022 . . × 10^{-23}	1/mol
K_{cd}	=	683	lm/W

Summary

- The SI will continue to evolve with new knowledge about our physical world

- Fundamental constants are a true universal measure available to everyone, anytime, anywhere

- The SI is expected to be redefined in terms of exact values of a set of fundamental constants in 2018


DANGER