Ultracold Atoms
How Quantum Field Theory invaded Atomic Physics

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Ultracold Atoms

How QFT Invaded Atomic Physics

- Ultracold Atoms

aside: $X(3872)$ meson

- Quantum Field Theory

- Fermions with two spin states
  phase diagram, contact

- Identical bosons
  trimer spectrum, unitary Bose gas

aside: quark-mass dependence in nuclear physics
Cold Atom Physics

Atoms trapped and cooled using lasers

Nobel Prize 1997: Chu, Cohen-Tannoudji, Phillips
Temperature of trapped atoms decreased further by evaporative cooling.
Bose-Einstein condensation of atoms!

$^{87}$Rb atoms
JILA (Cornell, Wieman) 1995

$^7$Li atoms
Rice (Hulet) 1995

$^{23}$Na atoms
MIT (Ketterle) 1995

Nobel Prize 2001: Cornell, Wieman, Ketterle
Cold Atom Physics

ground state of many-atom system

bosons

Fermi sea

fermions

BEC

Fermi sea
Cold Atom Physics

Cooling of fermions to quantum degeneracy!

$^40$K atoms  JILA (Jin)  Jan 2001
$^6$Li atoms  Ecole Normale (Salomon)  July 2001
$^6$Li atoms  Rice (Hulet)  Aug 2001

$^7$Li  (boson)

$^6$Li  (fermion)
Interactions between Atoms

**Size** of atoms: \( R_{eq} \approx 0.4 \) nm \hspace{1cm} \text{(for Rb)}

**Interaction range:** \( R_6 = \left(\frac{mC_6}{\hbar^2}\right)^{1/4} \approx 8 \) nm \hspace{1cm} \text{(for Rb)}

\( -C_6/r^6 \)
Interactions between Atoms

thermal de Broglie wavelength: $\lambda_{\text{th}} = (2\pi \hbar^2 / mkT)^{1/2}$

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\lambda_{\text{th}}$ (for Rb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 K</td>
<td>0.2 nm</td>
</tr>
<tr>
<td>1 mK</td>
<td>6 nm</td>
</tr>
<tr>
<td>1 µK</td>
<td>20 nm</td>
</tr>
<tr>
<td>1 nK</td>
<td>600 nm</td>
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</table>

size of atoms: $R_{\text{eq}} \sim 0.4$ nm (for Rb)

interaction range: $R_6 \sim 8$ nm

$T < 1$ K: atoms behave like point particles.
$T < 1$ mK: atoms behave as if they had zero-range interactions.
scattering cross section: area of beam that intercepts as many particles as are scattered

generically, scattering cross section is comparable to \((\text{range})^2\)
Interactions between Atoms

convenient measure of interaction strength for low-energy atoms:

- scattering cross section at zero energy: \[ \sigma = 4\pi a^2 \]
- OR scattering length \( a \)

generically, \( a \) is comparable to interaction range
Helium atoms (\(^4\)He)
range: 0.7 nm
scattering length: \(a = +8\) nm

Neutrons
range: 3 fm
scattering length: \(a = -20\) fm

But quantum mechanics allows scattering of particles far beyond the interaction range!
Universal properties determined by $a$

binding energy: $\frac{\hbar^2}{(m \ a^2)}$

mean separation: $a/2$

Large Scattering Length

Quantum mechanics allows bound states whose constituents spend most of their time beyond their interaction range!

Deuteron

$p\ n$ range: 1.8 fm

mean separation: $\langle r \rangle = 2.7$ fm

$^4$He dimer

range: 0.7 nm

mean separation: $\langle r \rangle = 4$ nm

Interactions between Atoms
**X(3872) Meson**

- discovered in $B^+$ decay
- confirmed in $p\bar{p}$ collisions

- decays into $J/\psi \pi^+\pi^-$ like $\psi(2S) = c \bar{c}$ meson

- decay is into $J/\psi \rho^*$ which has isospin $1$

$\Rightarrow$ cannot be $c \bar{c}$ meson which has isospin $0$

What is the $X(3872)$?

CDF II discovered in $B^+$ decay
Belle (September 2003)
CDF II confirmed in $p\bar{p}$ collisions
CDF II (December 2003)

![Graph](chart.png)
X(3872) Meson

- quantum numbers $I^{++}$  
  LHCb 2014
  $\Longrightarrow$ S-wave coupling to charm mesons $D^{*0} \bar{D}^0$

- mass is extremely close to the threshold
  for the charm mesons $D^{*0} \bar{D}^0$
  mass measured most accurately by CDF2, Belle, LHCb, Babar, BES3
  threshold measured most accurately by Babar, CLEO, LHCb, KEDR
  $\Longrightarrow$ binding energy is only $0.2 \pm 0.3$ MeV

$\Longrightarrow$ must be weakly bound molecule of $D^{*0} \bar{D}^0$
  with universal properties
  determined by binding energy
"X(3872)" Meson

loosely bound charm meson molecule comparable in size to the largest nuclei!

Uranium nucleus

$D^*$

$D^0$
Scattering length $a$
for ultracold atoms, can be controlled experimentally!

$a$ changes slowly with magnetic field $B$
except near Feshbach resonance where $a$ diverges to $\pm \infty$
Large Scattering Length

scattering length $a$ can be controlled by magnetic field can be made much larger than range $4|a|$
Trapped Atoms

Number density of atoms $n$
OR Fermi wavenumber $k_F = (3\pi^2 n)^{1/3}$

typical spacing between atoms: $l/k_F$

Inter-atom spacing $l/k_F$: controlled by number of trapped atoms and by trapping potential (even at center of trap, $l/k_F \gg$ range)
Universality

particles with short-range interactions and large scattering length $|a| \gg \text{range}$
have identical behavior at low temperature, low density
(if expressed in terms of dimensionless variables $k_F a$, $k_F \lambda_{th}$)

neutrons with $T \ll 10 \text{ MeV}, \ n \ll 10^{-3}/\text{fm}^3$
can be studied experimentally using ultracold atoms
($^6\text{Li atoms}$ in lowest two hyperfine spin states)
even though length scales differ by orders of magnitude
If $a = \pm \infty$ (unitary limit), scattering cross section at zero energy is infinite!

**Unitarity bound from quantum mechanics:**

$$\sigma \leq \frac{4\pi \hbar^2}{mE}$$

At nonzero energy, scattering cross section saturates unitarity bound:

$$\sigma(E) = \frac{4\pi \hbar^2}{mE}$$

no length scale $\implies$ **scale-invariant** interactions!
QFT for Ultracold Atoms

(sufficiently) Fundamental Theory

many-body Schroedinger equation for atoms in a trapping potential $V(r)$ interacting through interatomic potential $U(r-r')$

(atomic may have multiple spin states)
QFT for Ultracold Atoms

equivalent formulation:
Nonlocal Quantum Field Theory
for atoms in a trapping potential \( V(r) \)
interacting through potential \( U(r-r') \)

particles: atoms

interaction at a distance!
However ultracold atoms behave like point particles with zero-range interactions. The thermal wavelength $\lambda_{th}$ is much larger than the interatom spacing $1/k_F$, which can be described by local quantum field theory.
Ultracold Atoms

Local Quantum Field Theory (zero-range interactions)

Particles: atoms
(Perhaps with multiple spin states)

Point interaction

Interaction strength: scattering length $a$
(Perhaps different scattering length for each pair of spin states)
**QFT for Ultracold Atoms**

**Ultracold Atoms** can be described by **Local Quantum Field Theory**

**Advantages**

- **zero-range limit** is taken from beginning
- allows different calculational methods
  - integral equations
  - lattice Monte Carlo
  - operator product expansion

interactions can be weak: \( k_F |a| \ll l \)

or strong: \( k_F |a| \sim l \)

or infinitely strong: \( a = \pm \infty \) **(unitary limit)**
Local Quantum Field Theory

Loop diagrams involve integrals over momenta of virtual particles that are often ultraviolet divergent.

Divergences can be controlled by "renormalization"
Quantum Field Theory

**Local Quantum Field Theory**

Stephen Weinberg

“What is quantum field theory and what did we think it is?”

hep-th/9702027

**general framework for interacting particles**

consistent with

- quantum mechanics
- Lorentz invariance
- Galilean invariance
- cluster decomposition

weakly interacting QFT

can be defined in terms of Feynman diagrams
Strongly-coupled Quantum Field Theory can be defined by Renormalization Group flow to ultraviolet fixed point. Ken Wilson.

RG defines flow in abstract theory space of equivalent theories at increasingly shorter distances.

RG fixed point $\implies$ scale invariance!
Fermion with Two Spin States

simplest QFT for ultracold atoms

particles
fermionic atoms (spin states 1 and 2)

point interaction

interaction strength: scattering length $a$
Fermion with Two Spin States

fermionic quantum fields: $\psi_1, \psi_2$

Lagrangian density

$$L = L_{\text{kinetic}} - \mathcal{H}_{\text{int}} - \mathcal{H}_{\text{trap}}$$

$$L_{\text{kinetic}} = \sum_{i=1,2} \psi_i^\dagger \left( i \frac{\partial}{\partial t} + \frac{1}{2m} \nabla^2 \right) \psi_i$$

$$\mathcal{H}_{\text{int}} = \frac{g_0}{m} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$$

$$\mathcal{H}_{\text{trap}} = V(\vec{r}) \left( \psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2 \right)$$
Fermion with Two Spin States

**Weak coupling**

quantum fields: $\Psi_i$ scaling dimension $3/2$

interaction operator: $\Psi_1^\dagger \Psi_2^\dagger \Psi_2 \Psi_1$ scaling dimension 6

($>5 \Rightarrow$ irrelevant)

perturbatively nonrenormalizable!

$$g_0 = 4 \pi a \text{ (+ counterterms)}$$

RG fixed point: free field theory
QFT for Ultracold Atoms

Strong coupling
quantum fields: $\Psi_i$ scaling dimension $3/2$
interaction operator: $\Psi_1^\dagger \Psi_2^\dagger \Psi_2 \Psi_1$ scaling dimension $4$
($<5 \implies$ relevant)

nonperturbatively renormalizable!
anomalous scaling dimensions!

RG fixed point: scale-invariant interacting theory (unitary limit!)
Fermion with 2 Spin States

**2-Body Problem**

can be solved analytically

Cross section
\[ \sigma \rightarrow 4\pi a^2 \] at low energy
\[ \rightarrow 4\pi \frac{\hbar^2}{(m E)} \] at high energy

Diatomc molecule if \( a > 0 \)

binding energy: \( \frac{\hbar^2}{(m a^2)} \)
mean radius: \( a/2 \)
Fermion with 2 Spin States

3-Body Problem
can be solved exactly numerically

\[ \text{where} \quad = \quad \]

4-Body Problem
can be solved exactly numerically

5-Body Problem
frontier of few-body physics
Fermion with 2 Spin States

**Fermi Gas with Two Spin States**

- balanced gas ($n_1 = n_2$)
- weak interactions: $k_F |a| \ll 1$

Ground state ($T=0$) is a **Superfluid**!

**attractive interaction** $a < 0$
- pairs of fermions
  - with momenta near Fermi surface
  - form Cooper pairs, which condense

**repulsive interaction** $a > 0$
- pairs of fermions bind to form diatomic molecules,
  - which condense

What happens in the **unitary limit**?
Fermion with 2 Spin States

**Fermi Gas with Two Spin States**

Phase diagram for homogeneous balanced gas \((n_1 = n_2)\)

- BCS superfluid
- Unitary superfluid
- BEC superfluid

Smooth crossover through unitary limit! Leggett 1980
Fermion with 2 Spin States

**Fermi Gas with Two Spin States**

signature of superfluidity: vortices!

Ketterle group (MIT) using $^6$Li atoms 2005

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Diagram showing the BEC superfluid, unitary superfluid, and BCS superfluid phases with varying interaction parameter $1/k_F a$. The scale on the x-axis ranges from -0.7 to 1.6.
Fermi Gas with Two Spin States

spin-imbalanced gas \((n_1 > n_2)\)

Phase diagram?
dimensionless variables: \(l/k_F a, T/E_F\), and \(n_2/(n_1+n_2)\)

homogeneous phases
- normal
- superfluid with gapped fermions
- superfluid with gapless fermions? Sarma?

nonhomogeneous phases
- Fulde-Ferrel?
- Larkin-Ovchinnikov?
In 2005, a graduate student at the University of Chicago named Shina Tan introduced a new concept into many-body physics called the “Contact”

The contact appears in many “Universal Relations” that hold for any state of the system (few-body or many-body, trapped or homogeneous, normal or superfluid, ...)

The contact relates the thermodynamics to the tails of correlation functions.

The contact plays a central role in many of the most important probes of ultracold atoms (photoassociation, rf spectroscopy, photoemission spectroscopy...)

Contact
What is the Contact?

- contact is the thermodynamic variable conjugate to $1/a$

- the contact $C$ is extensive:
  integral over space of the contact density $C(R)$
  $$C = \int d^3R \ C(R)$$

- contact has dimensions $1/(\text{length})$
  contact density has dimensions $1/(\text{length})^4$

- contact density measures the number of 1-2 pairs per $(\text{volume})^{4/3}$
Tail of the momentum distribution

Shina Tan  cond-mat/0505200

momentum distribution has a power-law tail that falls like $1/k^4$

$$n_\sigma(k) \to \frac{1}{k^4} C$$

same coefficient $C$ for both spins: $\sigma = 1, 2$

$C$ is the contact

normalization: $$\int \frac{d^3 k}{(2\pi)^3} n_\sigma(k) = N_\sigma$$
change in free energy from small change in scattering length $a$

$$\frac{d}{da} F = \frac{\hbar^2}{4\pi ma^2} C$$

If $C$ is known as a function of $a$, it can be integrated to get $F$. $C$ determines all other thermodynamic functions!
Contact can be defined by tail of the momentum distribution

\[ n_\sigma(k) \rightarrow \frac{1}{k^4} C \]

Contact determines thermodynamics

\[ \frac{d}{da} F = \frac{\hbar^2}{4\pi ma^2} C \]

Tail wagging the dog?
interaction energy density $\mathcal{H}_{\text{int}} = \frac{g_0}{m} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$

contact density operator $C = g_0^2 \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$

- **Adiabatic relation**

$$\frac{d}{da} F = \frac{\hbar^2}{4\pi m a^2} C$$

from renormalization of effective field theory

- **Tail** of the momentum distribution

$$n_\sigma(k) \longrightarrow \frac{1}{k^4} C$$

from operator product expansion
Contact

Experimental Validation

Jin group (JILA) using $^{40}$K atoms  2010

- verified that momentum distribution has $1/k^4$ tail!

- measured contact using
  - momentum distribution
  - rf spectroscopy
  - virial theorem

- verified that universal relations are satisfied
Identical Bosons

not the simplest QFT for ultracold atoms!

point interactions

\[ 2 \rightarrow 2 \]
\[ 3 \rightarrow 3 \]

interaction parameters

scattering length \( a \)

Efimov parameter \( K_* \)  
momentum scale on which dependence can only be log-periodic
Identical Bosons

**Weak coupling**

quantum field: $\psi$  
interaction operator: $\psi^\dagger \psi^\dagger \psi \psi$  
$\psi^\dagger \psi^\dagger \psi^\dagger \psi \psi \psi$  
$\psi^\dagger \psi^\dagger \psi^\dagger \psi^\dagger \psi \psi \psi$  
$g_0 = 8 \pi a \ (+ \text{counterterms})$

perturbatively non-renormalizable!

RG fixed point: free field theory

scaling dimension $3/2$

$6$

$9$

$(>5 \Rightarrow \text{irrelevant})$
Strong coupling
quantum field: \( \psi \) scaling dimension 3/2
interaction operators: \( \psi \dagger \psi \dagger \psi \psi \) scaling dimension 4
\( (<5 \Rightarrow \text{relevant}) \)
\( \psi \dagger \psi \dagger \psi \dagger \psi \psi \psi \) scaling dimension 5
\( (=5 \Rightarrow \text{marginal}) \)

nonperturbatively renormalizable!
anomalous scaling dimensions!

two interaction parameters

scattering length \( a \)
3-body parameter
Strongly-coupled Quantum Field Theory can be defined by Renormalization Group flow to ultraviolet fixed point or limit cycle or ...

Ken Wilson

Identical Bosons

complete flow around the RG limit cycle changes scale by a discrete scaling factor $\lambda_0$ but returns to the same system

$\Rightarrow$ discrete scale invariance!
**Identical Bosons**

**strongly interacting QFT** can be defined by **RG limit cycle**

Renormalization of **local QFT** for identical bosons involves **RG limit cycle** with discrete scaling factor 22.7

Bedaque, Hammer, and van Kolck  1999

implies the existence of a physical momentum scale $K_*$ that is equivalent to $\lambda_0 K_*$

$\implies$ dependence on $K_*$ can only be **log-periodic**

(such as $\sin[s_0 \log(k/K_*)]$, where $\lambda_0 = e^{\pi/s_0}$)
Identical Bosons

2-Body Problem

can be solved analytically

\[ \sigma \rightarrow 8\pi \ a^2 \]  
\[ \rightarrow 8\pi \frac{\hbar^2}{(m \ E)} \]  
at low energy

at high energy

Cross section

Diatomic molecule if \( a > 0 \)

binding energy: \( \frac{\hbar^2}{(m \ a^2)} \)

mean radius: \( \frac{a}{2} \)

mean radius: \( a/2 \)
**3-Body Problem**

Can be solved exactly numerically

+ 3→3 interactions

Where

**4-Body Problem**

Can be solved exactly numerically

**5-Body Problem**

Frontier of few-body physics
Efimov Effect

Vitaly Efimov (1970)

In the unitary limit \( a \to \pm \infty \) there are infinitely many triatomic molecules

- binding energies differ by factors of \( 1/22.7^2 \)
- radii differ by factors of 22.7
Both $NN$ scattering lengths are large compared to the range

$$a_{i=1} = -21 \text{ fm} \quad a_{i=0} = +5.4 \text{ fm}$$

2-nucleon bound states

deuteron ($pn$): binding energy $\approx 2.2 \text{ MeV}$
[dineutron ($nn$): almost bound]

3-nucleon bound states

triton ($pnn$): binding energy $\approx 7.6 \text{ MeV}$
$^3\text{He}$ ($ppn$): $\approx 7.7 \text{ MeV}$

What would happen if you changed the up and down quark masses?
infrared RG limit cycle of QCD  

Braaten and Hammer (2003)

The physical up and down quark masses are close to critical values where the triton has infinitely many excited states!

- binding energies differ by factors of $\frac{1}{22.7^2}$
- radii differ by factors of $22.7$
Identical Bosons

3-Body Recombination

resonant enhancement from Efimov trimer near 3-atom threshold

- three low-energy atoms collide

- they form a virtual Efimov trimer

- trimer decays into atom and dimer with large kinetic energy
discovery of Efimov trimer in $^{133}\text{Cs}$ atoms through atom loss resonance

Grimm group (Innsbruck)  Nov 2005

Identical Bosons

universal line shape for $T=0$ from QFT

Braaten & Hammer 2003
Universal Relations for Identical Bosons
Braten, Kang, Platter 2011

• derived from Operator Product Expansion
• involve 2-body contact $C_2$ and 3-body contact $C_3$!

Tail of the momentum distribution

\( \frac{1}{k^4} \) tail plus log-periodic \( \frac{1}{k^5} \) tail

\[
\begin{align*}
  n(k) &\rightarrow \frac{1}{k^4} C_2 + \frac{F(k)}{k^5} C_3 \\
  F(k) &= 89.3 \sin[2s_0 \log(k/\kappa_*) - 1.34] \\
\end{align*}
\]

where \( s_0 = 1.00624 \)

\( \kappa_* = \text{binding momentum of Efimov trimer at } a = \pm \infty \)
(determined by position of Efimov loss resonance)
Identical Bosons

**Bose Gas**

phase diagram for homogeneous gas

Where is boundary of BEC superfluid phase? Does it extend to unitary limit?
Unitary Bose Gas

- Unitary Bose gas
- Bose-Einstein condensate
- Unstable to mechanical collapse

1st experiments in 2013
- Salomon group (Ecole Normale)
- Jin group (JILA)
Jin group (JILA) 2013

Weakly interacting BEC of $^{85}$Rb atoms
Ramped suddenly to unitary limit ($a = \pm \infty$)
Wait for a variable holding time $t$
Measure momentum distribution

High-momentum tail: grows, then saturates after 100 $\mu$s
Unitary Bose gas

**JILA** momentum distributions

- multiply by $k^4$
- scale by $k_F = (6\pi^2\langle n \rangle)^{1/3}$

![Graph showing momentum distributions with scaling and scaling violations.](image)
Tail of the momentum distribution

\[ \frac{1}{k^4} \text{tail plus log-periodic } \frac{1}{k^5} \text{tail} \]

\[ n(k) \longrightarrow \frac{1}{k^4} C_2 + \frac{F(k)}{k^5} C_3 \]

\[ F(k) = 89.3 \sin[2s_0 \log(k/\kappa^*_d) - 1.34] \]

where \( s_0 = 1.00624 \)

\( \kappa^*_d \) determined by position of Efimov loss resonance measured by JILA 2011

only unknowns are \( C_2 \) and \( C_3 \)
JILA momentum distributions can be fit very well by $1/k^4$ tail from 2-body contact plus log-periodic $1/k^5$ tail from 3-body contact. Smith, Kang, Platter, Braaten 2014

- 2 adjustable parameters: 2-body and 3-body contacts
- Positions of minima determined by JILA observation of Efimov trimer
Summary

**Ultracold atoms**
can be approximated by **point** particles
with **zero-range** interactions
can be described by a **local quantum field theory**

The relevant **strongly coupled QFT’s** can be defined
by an **RG fixed point** (fermions with 2 spin states)
or by an **RG limit cycle** (identical bosons)

Methods of **QFT** developed in particle physics
have powerful applications to **ultracold atoms**
(e.g. renormalization,
operator product expansion, ...)

Applications of **QFT to ultracold atoms**
can also provide insights into particle physics.