Introduction to the SM

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Last time

- Gauge interactions: QED, QCD and EW sectors
- SSB: Must have it
- Charged current interactions

This lecture: Neutral current interaction and flavor

Answer to HW

Write NR operators that violate B and/or L

- For L we get dim 5

$$\frac{HHLL}{\Lambda}$$

Gives neutrino masses. We will talk on it later

- For B we get dim 6

$$\frac{QQQL}{\Lambda^2}$$

Give proton decay. Plan to talk on it tomorrow

Neutral currents

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\,\bar\psi \not{Z} \psi\,,$$

- Photon and Z. The Z is the extra stuff
- Both LH and RH coupling. Still Z is parity violating
- Diagonal couplings. No flavor violation at tree level
- Processes involving the Z can be used to measure $\sin^2\theta_W$
- Together with m_W and $G_F = \sqrt{2}g^2/8M_W^2$ we can get the two parameters of the model, g and g'

Experimental tests

Of course, the model was built from experimental data...

- Calculate the ratios (use $\sin^2\theta_W = 0.25$)

$$R_1 = \frac{\Gamma(Z \to e_L \bar{e}_L)}{\Gamma(Z \to e_R \bar{e}_R)} \qquad R_2 = \frac{\Gamma(Z \to e\bar{e})}{\Gamma(Z \to \nu\bar{\nu})}$$

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- We get, in agreement with the data

$$R_1 \approx \frac{(1/2 - 1/4)^2}{(1/4)^2} = 1 \qquad R_2 \approx \frac{(1/4)^2 + (1/4)^2}{3 \times (1/2)^2} = \frac{1}{6}$$

- In fact, these Z decays gives $\sin^2\theta_W \approx 0.23$

More tests

- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see for

$$\frac{\Gamma(W^+ \to e^+\nu)}{\Gamma(W^+ \to \mu^+\nu)} \qquad \frac{\Gamma(Z \to e^+e^-)}{\Gamma(Z \to \mu^+\mu^-)}$$

- Low energy data: For example,
  - tau decays

$$\frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})}$$

  - $\Gamma(\pi \to \ell\nu)$ proof of spin one nature of the weak interaction
  - neutrino scattering: proof of the left-handedness of it

Neutrino scattering

$$\sigma(\nu e^- \to \nu e^-) = \frac{G_F^2 s}{\pi} \qquad \sigma(\bar{\nu} e^- \to \bar{\nu} e^-) = \frac{G_F^2 s}{3\pi}$$

- Note the factor of 3
- Think about backward scattering:
  - νe: Both LH and thus, $J_Z = 0$ before and after. Can go
  - $\bar{\nu} e$: One LH and one RH: $J_Z = +1$ before and $J_Z = -1$ after. Cannot go

Gauge sector: summary

- 3 groups, very different
- A lot tests on the model. Pass them all
- Electroweak precision measurements. A lot was done at LEP and Tevatron

Fermions

Lepton masses

- In a chiral theory fermions are massless
- In the SM they get mass from the interactions with the Higgs
- For leptons only the charged leptons get a mass. We need both LH and RH fields for a mass

$$Y_{ij} \left(\bar{L}_L\right)_i \phi \left(E_R\right)_j \to Y_{ij}\, v\, \bar{e}_L e_R + ...$$

- The mass is proportional to the Yukawa coupling and the vev $m = Yv$
- For leptons we can choose Y to be diagonal in flavor space and we get the known lepton masses

Quarks

$$Y_{ij}^{D}\left(\bar{Q}_L\right)_i \phi \left(D_R\right)_j + Y_{ij}^{U}\left(\bar{Q}_L\right)_i \tilde{\phi} \left(U_R\right)_j$$

- The Yukawa matrix, Y_{ij}^{F}, is a general complex matrix
- After the Higss acquires a vev, the Yukawa terms give masses to the fermions. Also, after the breaking we can talk about U_L and D_L, not about Q_L
- If Y is not diagonal, flavor is not conserved (soon we will go over the subtleties here)
- If Y carries a phase, CP is violated (soon we will understand). C and P is violated to start with

CP violation

A simple “hand wave” argument of why CP violation is given by a phase

- It is all in the $+h.c.$ term

$$Y_{ij}\left(\bar{Q}_L\right)_i \phi\,(D_R)_j + Y^*_{ji}\left(\bar{D}_R\right)_j \phi^\dagger\,(Q_L)_i$$

- Under CP

$$Y_{ij}\left(\bar{D}_R\right)_j \phi^\dagger\,(Q_L)_i + Y^*_{ji}\left(\bar{Q}_L\right)_j \phi\,(D_R)_i$$

- CP is conserved if $Y_{ij} = Y^*_{ij}$
- Not a full proof, since there is still a basis choice...

The CKM matrix

It is all about moving between bases...

- We can diagonalize the Yukawa matrices

$$Y_{diag} = V_L Y V_R^\dagger, \qquad V_L, V_R \text{ are unitary}$$

- The mass basis is defined as the one with Y diagonal, and this is when

$$(d_L)_i \to (V_L)_{ij}(d_L)_j, \qquad (d_R)_i \to (V_R)_{ij}(d_R)_j$$

- The couplings to the photon is not modifies by this rotation

$$\mathcal{L}_\gamma \sim \bar{d}_i \delta_{ij} d_i \;\to\; \bar{d}_i V \delta_{ij} V^\dagger d \sim \bar{d}_i \delta_{ij} d_i$$

CKM, W couplings

- For the W the rotation to the mass basis is important

$$\mathcal{L}_W \sim \bar{u}_L^i \delta_{ij} d_L^i \;\to\; \bar{u}_i V_L^U \delta_{ij} V_L^{D\dagger} d \sim \bar{u}_i V_{CKM} d_i$$

  where

$$V_{CKM} = V_L^U V_L^{D\dagger}$$

- The point is that we cannot have Y_U, Y_D and the couplings to the W diagonal at the same basis
- In the mass basis the W interaction change flavor, that is flavor is not conserved

CKM: Remarks

$$V_{CKM} = V_L^U V_L^{D\dagger}$$

- V_{CKM} is unitary
- The CKM matrix violates flavor only in charge current interactions, for example, in transition from u to d

$$V_{us}\,\bar{u}\,s\,W^+,$$

- In the lepton sector without RH neutrinos $V = 1$ since V_L^ν is arbitrary. This is in general the case with degenerate fermions
- When we add neutrino masses the picture is the same as for quarks. Yet, for leptons it is usually not the best to work in the mass basis

FCNC

FCNC=Flavor Changing Neutral Current

- Very important concept in flavor physics
- Important: Diagonal couplings vs universal couplings

FCNCs

In the SM there are no FCNCs at tree level. Very nice! In Nature FCNC are highly suppressed

- Historically, $K \to \mu\nu$ vs $K_L \to \mu\mu$
- The suppression was also seen in charm and B
- In the SM we have four neutral bosons, g, γ, Z, h. Their couplings are diagonal
- The reasons why they are diagonal, and what it takes to have FCNC, is not always trivial
- Of course we have FCNC at one loop (two charged current interactions give a neutral one)

Photon and gluon tree level FCNC

- For exact gauge interactions the couplings are always diagonal. It is part of the kinetic term

$$\partial_\mu \delta_{ij} \to (\partial_\mu + iqA_\mu)\delta_{ij}$$

- Symmetries are nice...
- In any extension of the SM the photon couplings are flavor diagonal

Higgs tree level FCNC

- The Higgs is a possible source of FCNC. With one Higgs doublet, the mass matrix is align with the Yukawa

$$\mathcal{L}_m \sim Y\, v\, \bar{d}_L d_R \qquad \mathcal{L}_{int} \sim Y H \bar{d}_L d_R$$

- With two doublets we have tree level FCNC

$$\mathcal{L}_m \sim \bar{d}_L (Y_1 v_1 + Y_2 v_2) d_R \qquad \mathcal{L}_{int} \sim H_1 \bar{d}_L Y_1 d_R$$

- There are “ways” to avoid it, by imposing extra symmetries

Z exchange FCNC

- For broken gauge symmetry there is no FCNC when: "All the fields with the same irreps in the unbroken symmetry also have the same irreps in the broken part"
- In the SM the Z coupling is diagonal since all $q = -1/3$ RH quarks are $(3,1)_{-1/3}$ under $SU(2) \times U(1)$
- What we have in the couplings is

$$\bar{d}_i \, (T_3)_{ij} \, d_j \;\to\; \bar{d} \, V \, (T_3)_{ij} \, V^\dagger d_j, \qquad VT_3V^\dagger \propto I \text{ if } T_3 \propto I$$

- Adding quarks of different irreps generate tree level FCNC Z couplings
- It is the same for new neutral gauge bosons (usually denoted by Z')

A little conclusion

- In the SM flavor is the issue of the 3 generations of quarks
- Flavor is violated by the charged current weak interactions only
- There is no FCNC at tree level. Not trivial, and very important
- All flavor violation is from the CKM matrix

Parameter counting

How many parameters we have?

How many parameters are physical?

- "Unphysical" parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

  - $N(\text{Phys})$, number of physical parameters
  - $N(\text{tot})$, total number of parameters
  - $N(\text{broken})$, number of broken generators

Example: Zeeman effect

A hydrogen atom with weak magnetic field

- The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \qquad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- But there are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \to SO(2)$
- 2 broken generators, can be "used" to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \quad \Rightarrow \quad 1 = 3 - 2$$

Back to the flavor sector

Without the Yukawa interaction, a model with N copies of the same field has a $U(N)$ global symmetry

- It is just the symmetry of the kinetic term

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, ..., N$$

- $U(N)$ is the general rotation in N dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators

Lepton sector

First example, two generation SM

- Two Yukawa matrices: Y^D, Y^U, $N_T = 16$
- Global symmetries of the kinetic terms: $U(2)_Q \times U(2)_D \times U(2)_U$, 12 generators
- Exact accidental symmetries: $U(1)_B$ 1 generator
- Broken generators due to the Yukawa: $N_B = 12 - 1 = 11$
- Physical parameters: $N_P = 16 - 11 = 5$. They are the 4 quarks masses and the Cabibbo angle

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T =$
- Symmetry generators of kinetic terms: $N_G =$
- Unbroken global generators$N_U =$
- Broken generators: $N_B =$
- Physical parameters: $N_P =$

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T = 2 \times 18 = 36$
- Symmetry generators of kinetic terms: $N_G = 3 \times 9 = 27$
- Unbroken global generators$N_U = 1$
- Broken generators: $N_B = 27 - 1 = 26$
- Physical parameters: $N_P = 36 - 26 = 10$
- 6 quark masses, 3 mixing angles and one CPV phase

Remark: The broken generators are 17 Im and 9 Re. We have 18 real and 18 imaginary to “start with” so the physical ones are $18 - 17 = 1$ and $18 - 9 = 9$

The CKM matrix

The flavor parameters

- The 6 masses. We kind of know them. There is a lot to discuss, but I will not do it in these lectures
- The CKM matrix has 4 parameters
  - 3 mixing angles (the orthogonal part of the mixing)
  - One phase (CP violating)
- A lot to discuss on how to determined and check them. I will be brief here

The CKM matrix

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} V \gamma^\mu D_L W_\mu^+ + \text{h.c.}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- CKM is unitary

$$\sum V_{ij} V_{ik}^* = \delta_{jk}$$

- Experimentally, $V \sim 1$. Off diagonal terms are small
- Many ways to parametrize the matrix

CKM parametrization

- The standard parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

- In general there are 5 entries that carry a phase
- Experimentally:

$$|V| \approx \begin{pmatrix} 0.97383 & 0.2272 & 3.96 \times 10^{-3} \\ 0.2271 & 0.97296 & 4.221 \times 10^{-2} \\ 8.14 \times 10^{-3} & 4.161 \times 10^{-2} & 0.99910 \end{pmatrix}$$

The Wolfenstein parametrization

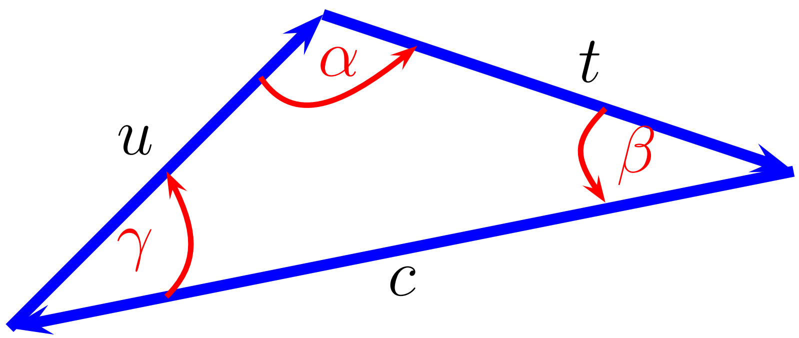
- Since $V \sim 1$ it is useful to expand it

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- One small parameter $\lambda \sim 0.2$, and three (A, ρ, η) that are roughly $O(1)$
- As always, be careful (unitarity...)
- Note that to this order only V_{13} and V_{31} have a phase

The unitarity triangle

A geometrical presentation of $V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0$



Rescale by the c size and rotated

$$A\lambda^3 \left[(\rho + i\eta) + (1 - \rho - i\eta) + (-1)\right] = 0$$