Ab initio methods for nuclei

Lecture III

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Outline

- ⋆ Lecture I: Nuclear Many-Body Theory
- ⋆ Lecture II: Nucleon Green's function and nuclear response at low to moderate momentum transfer
- ⋆ Lecture III: Electron and neutrino cross section in the impulse approximation and beyond
  - The impulse approximation regime
  - The factorizarion scheme
  - Including Final State Interactions (FSI)
  - Including Meson Exchange Currents (MEC)
  - Comparison to electron scattering data

Lepton-nucleus scattering

- ⋆ Double differential cross section of the process $\ell + \mathrm{A} \to \ell' + X$

$$\frac{d\sigma_A}{d\Omega_{\ell'} dE_{\ell'}} \propto L_{\mu\nu} W_A^{\mu\nu}$$

- ⋆ the tensor $L_{\mu\nu}$ depends on lepton kinematics only. Same as in lepton-nucleon scattering
- ⋆ the determination of the target response tensor

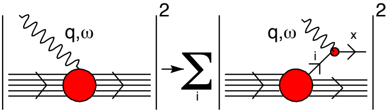
$$W_A^{\mu\nu} = \sum_n \langle 0|J_A^\mu|n\rangle\langle n|J_A^\nu|0\rangle \delta^{(4)}(P_0 + k_e - P_n - k_{e'})$$

requires the description of the target initial and final states, as well as of the nuclear current

- ⋆ While the initial state can always treated within the non relativistic approximation, the final state and the current depend on momentum transfer $|\mathbf{q}|$. At large $|\mathbf{q}|$ further approximations are required

The impulse approximation

- The IA, believed to be applicable at $|\mathbf{q}| \gtrsim 2\pi/d_{NN}$, where d_{NN} is the average NN distance, amounts to replacing (note that x can be *any* hadronic final state



$$J_A^\mu = \sum_i j_i^\mu \quad , \quad |X\rangle \to |x, \mathbf{p}_x\rangle \otimes |n, \mathbf{p}_n\rangle \ .$$

★ Factorization of the final state allows to decouple the nuclear dynamics from the elementary interaction vertex are decoupled

$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E)$$

The factorization *ansatz*

- ⋆ Using the factorized final state one obtains

$$\sum_X |X\rangle\langle X| \to \sum_x \int d^3p_x |x, \mathbf{p}_x\rangle\langle \mathbf{p}_x, x| \sum_n \int d^3p_n |n, \mathbf{p}_n\rangle\langle \mathbf{p}_n, n|$$

- ⋆ Insertion of a complete set of free nucleon states, satisfying

$$\int d^3k |N, \mathbf{k}\rangle\langle \mathbf{k}, N| = \mathbb{1}$$

leads to the factorization of the current matrix element according to

$$\langle 0|J^\mu|X\rangle = \left(\frac{m}{E_{p_n}}\right)^{1/2} \langle 0|\{|n, \mathbf{p}_n\rangle \otimes |N, -\mathbf{p}_n\rangle\} \sum_i \langle -\mathbf{p}_n, N|j_i^\mu|x, \mathbf{p}_x\rangle \ ,$$

- ⋆ The nuclear matrix element appearing in the above equation, being independent of **q**, can be safely computed using nuclear many-body theory

The factorization *ansatz* (continued)

- Within the factorization *ansatz* the target tensor can be written in the simple form

$$W_A^{\mu\nu} = \int d^3k \, dE \, \frac{M}{E_k} P(\mathbf{k}, E) \mathcal{W}^{\mu\nu}(k, k + \tilde{q}) \, ,$$

- $\mathcal{W}^{\mu\nu}$ is the tensor describing the interaction of a free nucleon of momentum $\mathbf{k}$ at four momentum transfer

$$\tilde{q} \equiv (\tilde{\omega}, \mathbf{q}) \quad , \quad \tilde{\omega} = E_x - E_k = \omega + m - E - E_k$$

- The substitution $\omega \to \tilde{\omega}$ is needed to take into account the fact that a fraction $\delta\omega$ of the energy transfer goes into excitation energy of the spectator system.
- Note that, in the case of electromagnetic interactions, using $\tilde{\omega}$ leads to a violation of gauge invariance.

Hole state spectral function

- Definition

$$P_h(\mathbf{k}, E) = \sum_n |\langle n|a_{\mathbf{k}}|0\rangle|^2 \, \delta(E_0 + E - E_n)$$

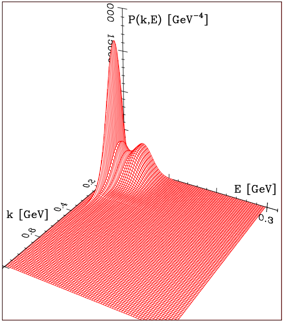
- Exact calculations have been carried out for $A = 2, \; 3$. Accurate results, obtained using correlated wave functions, are also available for nuclear matter.
- For medium-haevy nuclei, approximated spectral functions have been constructed combining nuclear matter results and experimental information from $(e, e'p)$ experiments in the local density approximation (LDA)

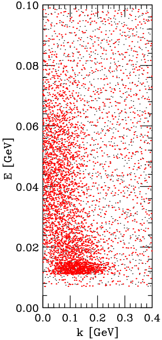
$$P_h(\mathbf{k}, E) = P_{\rm exp}(\mathbf{k}, E) + P_{\rm corr}(\mathbf{k}, E)$$

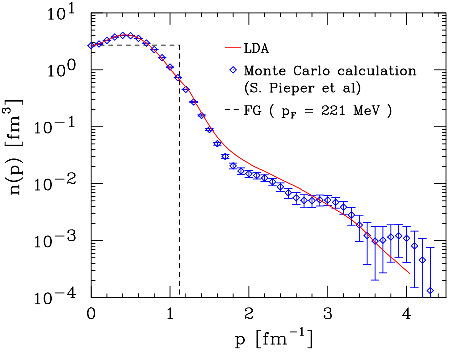
$$P_{\rm exp}(\mathbf{k}, E) = \sum_n Z_n |\phi_n(\mathbf{k})|^2 \, F_n(E - E_n)$$

$$P_{\rm corr}(\mathbf{k}, E) = \int d^3r \, \rho_A(r) \, P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

Spectral function and momentum distribution of Oxygen



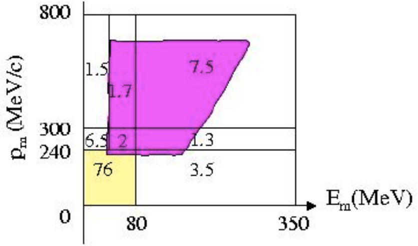


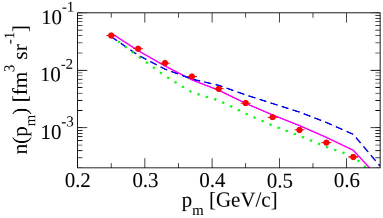


- FG model: $P_h(\mathbf{k}, E) \propto \theta(k_F - |\mathbf{k}|)\, \delta(E - \sqrt{|\mathbf{k}|^2 + m^2} + \epsilon)$
- shell model states account for $\sim 80\%$ of the strenght
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum *and* large removal energy ($\mathbf{k} \gg k_F, E \gg \epsilon$)

Measured correlation strength

- the correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target
- strong energy-momentum correlation: $E \sim E_{thr} + \frac{A-2}{A-1} \frac{\mathbf{k}^2}{2m}$

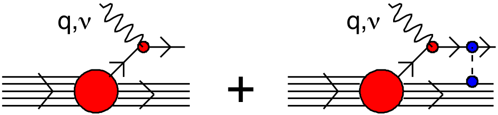




- Measured correlation strength 0.61 ± 0.06, to be compared with the theoretical predictions 0.64 (CBF) and 0.56 (G-Matrix)

Beyond IA: final state interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the importance of FSI



$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ with the particle spectral function of the non interacting system

FSI (continued)

- effects of FSI on the inclusive cross section
  - (A) shift in energy transfer, $\omega \to \omega' + U(\mathbf{k} + \mathbf{q})$, arising from interactions with the mean field of the spectators
  - (B) redistributions of the strenght, arising from the coupling of $1p - 1h$ final state to $np - nh$ final states
- high energy approximation
  - (A) the struck nucleon moves along a straight trajectory with constant velocity
  - (B) the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers.

$$\delta(\omega - E - \sqrt{|\mathbf{k} + \mathbf{q}|^2 + m^2}) \to \sqrt{T}\delta(\omega' - E - \sqrt{|\mathbf{k} + \mathbf{q}|^2 + m^2})$$

$$+(1 - \sqrt{T})f(\omega' - E - \sqrt{|\mathbf{k} + \mathbf{q}|^2 + m^2}))$$

- the nuclear transparency T and the folding function f can be computed within nuclear many-body theory using the *measured* nucleon-nucleon scattering amplitude

MEC contribution within in the factorization scheme

- ⋆ Highly accurate and consistent calculations can be carried out in the non relativistic regime
- ⋆ Fully relativistic MEC used within the Fermi gas model
- ⋆ Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to two-nucleon emission amplitudes
  - ▹ Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|n\rangle \to |\mathbf{p},\mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)},\mathbf{p},\mathbf{p}'\rangle$$

$$\langle X|j_{ij}{}^{\mu}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k},\mathbf{k}') \langle \mathbf{p}\mathbf{p}'|j_{ij}{}^{\mu}|\mathbf{k}\mathbf{k}'\rangle \, \delta(\mathbf{k}+\mathbf{k}'+\mathbf{q}-\mathbf{p}-\mathbf{p}')$$

The amplitude

$$M_n(\mathbf{k},\mathbf{k}') = \langle n_{(A-2)},\mathbf{k},\mathbf{k}'|0\rangle$$

is independent of q and can be obtained from non relativistic many-body theory

Two-nucleon spectral function

- ⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter

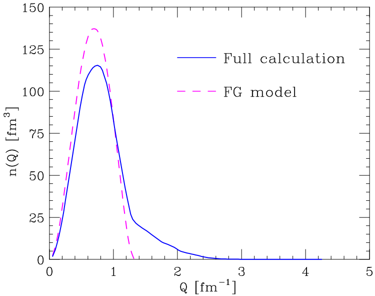
$$P(\mathbf{k}_1, \mathbf{k}_2, E) = \sum_n |M_n(k_1, k_2)|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}_1, \mathbf{k}_2) = \int dE \, P(\mathbf{k}_1, \mathbf{k}_2, E)$$

- ⋆ Relative momentum distribution

$$n(\mathbf{Q}) = 4\pi |\mathbf{Q}|^2 \int d^3q \, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 \quad , \quad \mathbf{Q} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$



Excitation of two particle-two hole (2p2h) final states

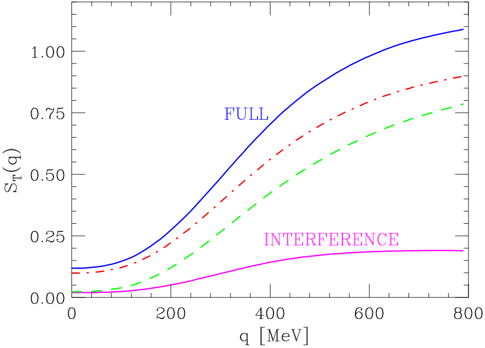
- ⋆ Within the independent particle model, two particle-two hole (2p2h) final states can only be produced through two-nucleon current interactions. In this context, 2p2h and MEC contributions are the same thing.
- ⋆ In the presence of correlations, 2p2h final states can be produced through different additional mechanisms:
  - ▹ Initial state correlations leading to the appearance of the high energy tail of the QE cross section. Can be included in the IA scheme using a realistic target spectral function
  - ▹ Final state interactions
- ⋆ The different mechanisms connecting the ground state to a 2p2h final state should be treated in a consistent fashion. Interference should also be taken into account

Role of interference effects

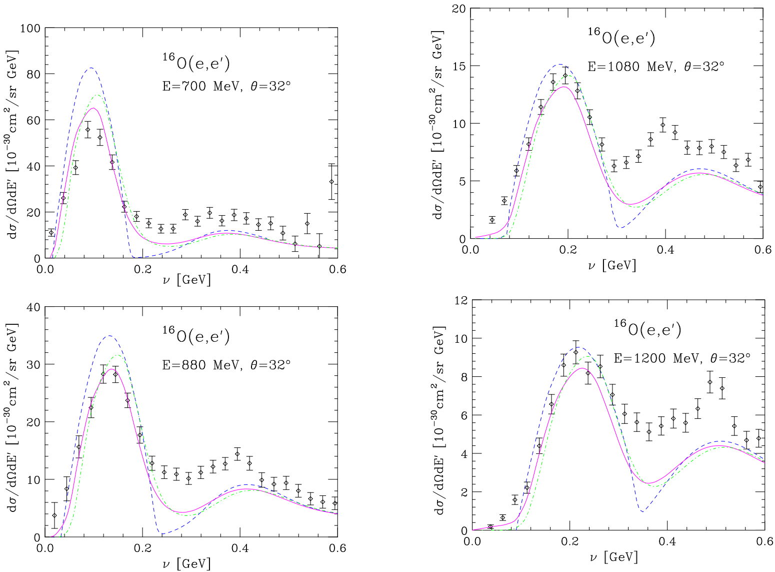
★ GFMC calculation of the electromagnetic sum rule in the transverse channel

$$S_T(\mathbf{q}) = \int d\omega [S^{xx}(\mathbf{q},\omega) + S^{yy}(\mathbf{q},\omega)]$$

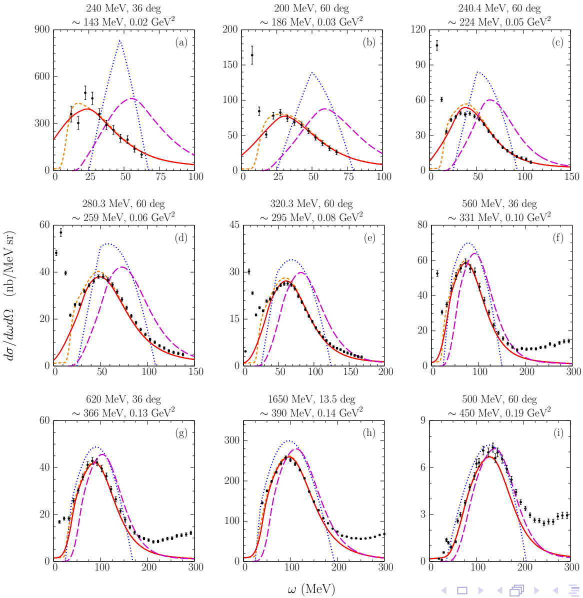
$$S^{\alpha\beta}(\mathbf{q},\omega) = \sum_N \langle 0|J_A^\alpha|N\rangle\langle N|J_A^\beta|0\rangle\delta(E_0+\omega-E_N)\ .$$



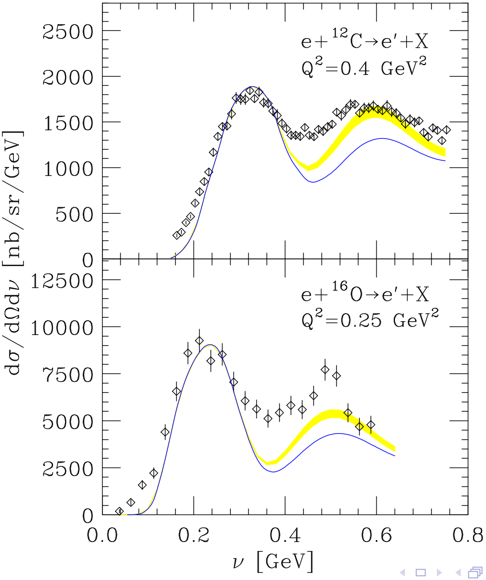
Comparison to Oxygen data @ $0.2 \lesssim Q^2 \lesssim 0.6$ GeV^2



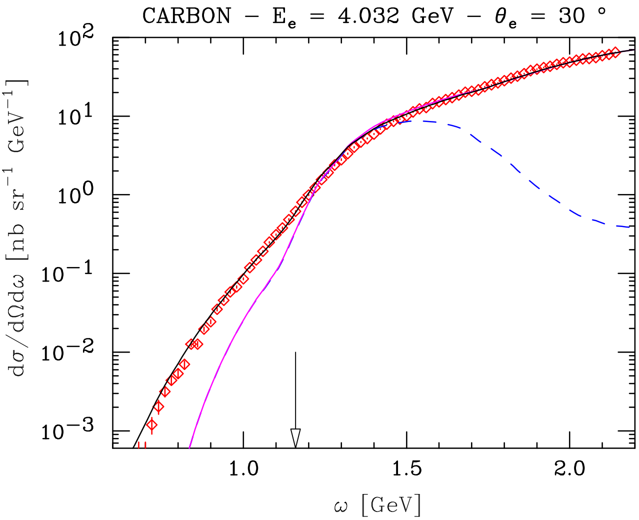
Ciomparison to carbon data



Improving the description of the Δ-production region



Including DIS



Summary of Lecture III

- ⋆ The factorization scheme, which appears to be justified in the impulse approximation regime, allows for a consistent extension of the approach based on NMBT to the region in which the non relativistic approximation breaks down
- ⋆ The available results suggest that the spectral function approach can be generalized to take into account final state interaction (FSI) effects
- ⋆ The extension to the treatment of Meson Exchange Currents (MEC) is needed, to treat one- and two-body current on the same footing and take into account interference effects
- ⋆ The large database of electron scattering data should be fully exploited to validate the proposed theoretical models

Final summary and outlook

- ⋆ In spite of the fact that no truly *ab initio* approach is available, a consistent description of a variety of nuclear properties can be obtained from approaches based on effective degrees of freedom and effective interactions.
- ⋆ The GFMC has the potential to provide exact calculations of the nuclear response, carried out using a realistic nuclear hamiltonian. However, it is limited to the quasi elastic channel in the non relativistic regime.
- ⋆ The approach based on the factorization scheme and the spectral function formalism, thoroughly tested against electron scattering data, can be applied at large momentum transfer in all channels
- ⋆ The GFMC and spectral function formalisms, based on the same dynamical model, should be seen as complementary. The synergy between these two approaches may lead to the development to a reliable description of the nuclear response in the broad kinematical range relevant to neutrino experiments.