NuSTEC Training in Neutrino-Nucleus Scattering Physics

Approximate Methods for Nuclei II

RPA, MEC, 2p2h… (**pionic modes of excitation in nuclei**)

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- Oset E., Toki H. and Weise W.: Phys. Rep. 83 (1982) 281
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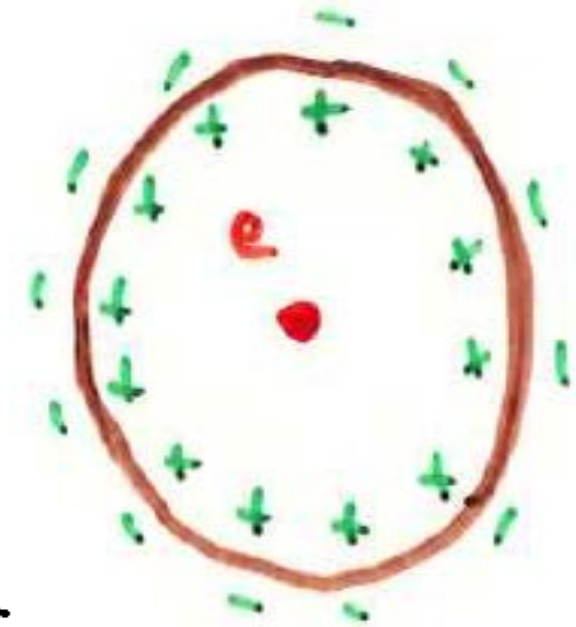
Outline:

- Introduction: Properties of the Coulomb interaction in an electron gas: screening of the interaction
- Particle and particle-hole propagators in a Fermi sea: occupation number and the Lindhard function
- Nuclear matter. Pion propagation in a nuclear medium: πN interaction at intermediate energies
- Induced spin-isospin NN interaction in a nuclear medium
- Examples:
  - inclusive muon capture in nuclei
  - pion-nucleus interactions: pionic atoms, pion nucleus scattering, …
  - inclusive electron-nucleus scattering

One electron inside of an electron gas (metal) **polarizes the medium** in such a way that, in a region around it, the negative charges are slightly displaced away from the electron, leaving behind the background charge of positive ions

SCREENING of the Coulomb Interaction

$$\frac{1}{4\pi}\frac{1}{r} \longrightarrow \frac{1}{4\pi}\frac{e^{-\mu(\rho)r}}{r}$$



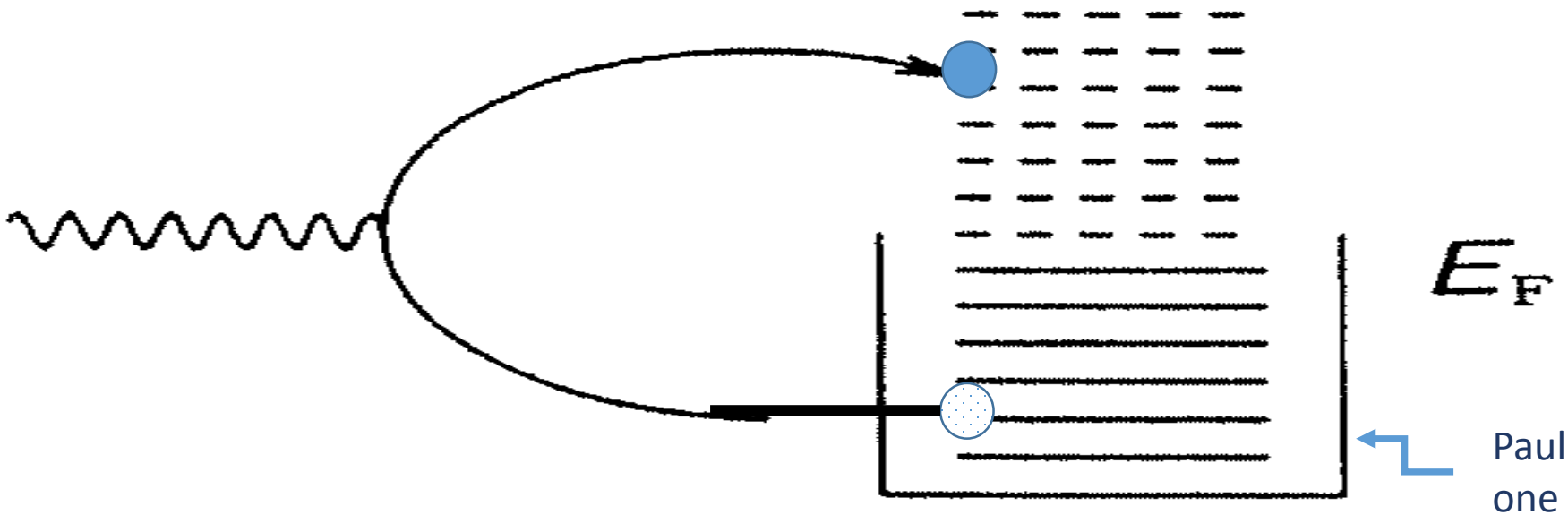
ρ is the electron density of the gas and μ a certain function of it. The effect of the polarizations has been to **convert the infinite range interaction into one of finite range.** The positive charge around one electron, coming from the polarization of the medium, cancels the electron negative charge, and a at large distances we see an effective charge zero.

In momentum space (static photon propagator, $q^0=0$)

$$\frac{1}{\vec{q}^{\,2}} \longrightarrow \frac{1}{\vec{q}^{\,2} + \mu^2(\rho)}$$

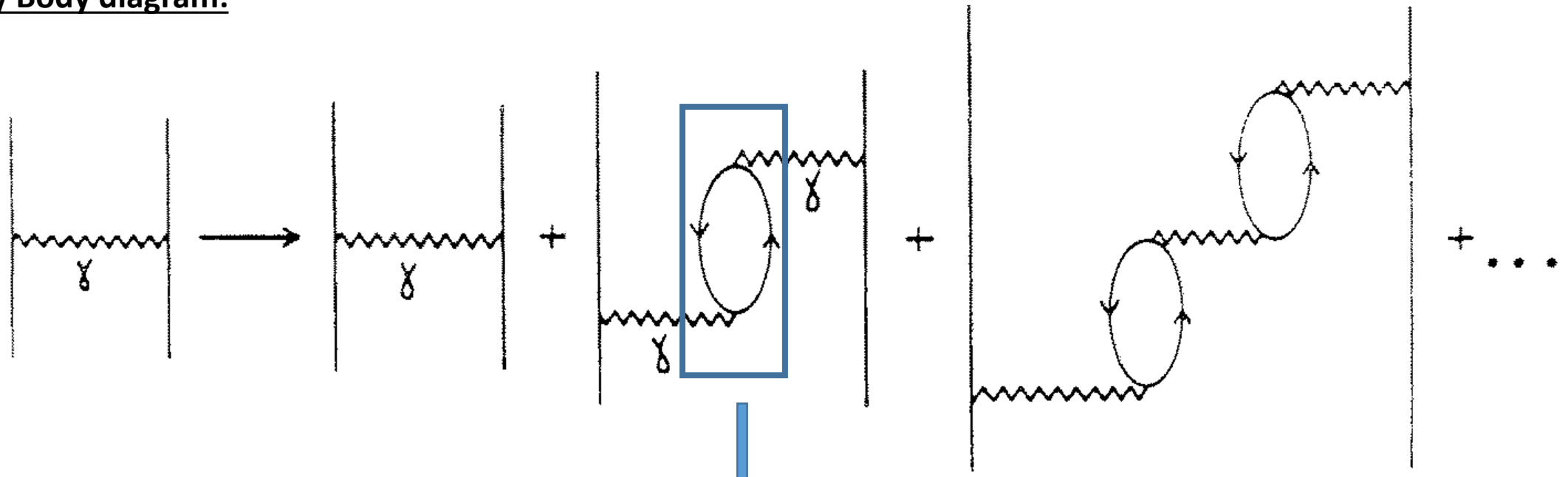
Photon acquires an effective mass: interaction becomes of shorter range !

The physical mechanism for the polarization consists in a transfer of some electrons from occupied states of a Fermi sea to some unoccupied states: **producing particle-hole excitations**:



Pauli-Fermi statistics: only one fermion per state

Many Body diagram:



gauge invariance

$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi(q)$$

photon selfenergy corresponding to a single ph excitation

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Solving the Dyson equation in the Landau gauge,

$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi(q)$$

$$iD_0^{\mu\nu}(q^2) = -i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{q^2+i\epsilon}$$

$$iD^{\mu\nu}(q) = iD_0^{\mu\nu}(q) + iD_0^{\mu\rho}(q)\, i\Pi^{\rho\sigma}(q)\, iD_0^{\sigma\nu}(q) + \cdots$$

$$iD^{\mu\nu}(q) = -i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{q^2+\Pi(q)}$$

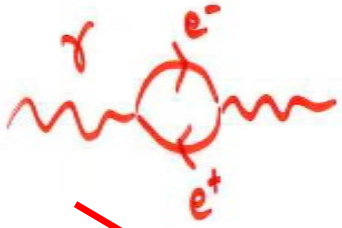
Modifies the propagation of the photon inside of the Fermi sea

$$\frac{1}{4\pi}\frac{1}{r} \longrightarrow \frac{1}{4\pi}\frac{e^{-\mu(\rho)r}}{r}$$

Coordinate space (Coulomb static, $q^0=0$)

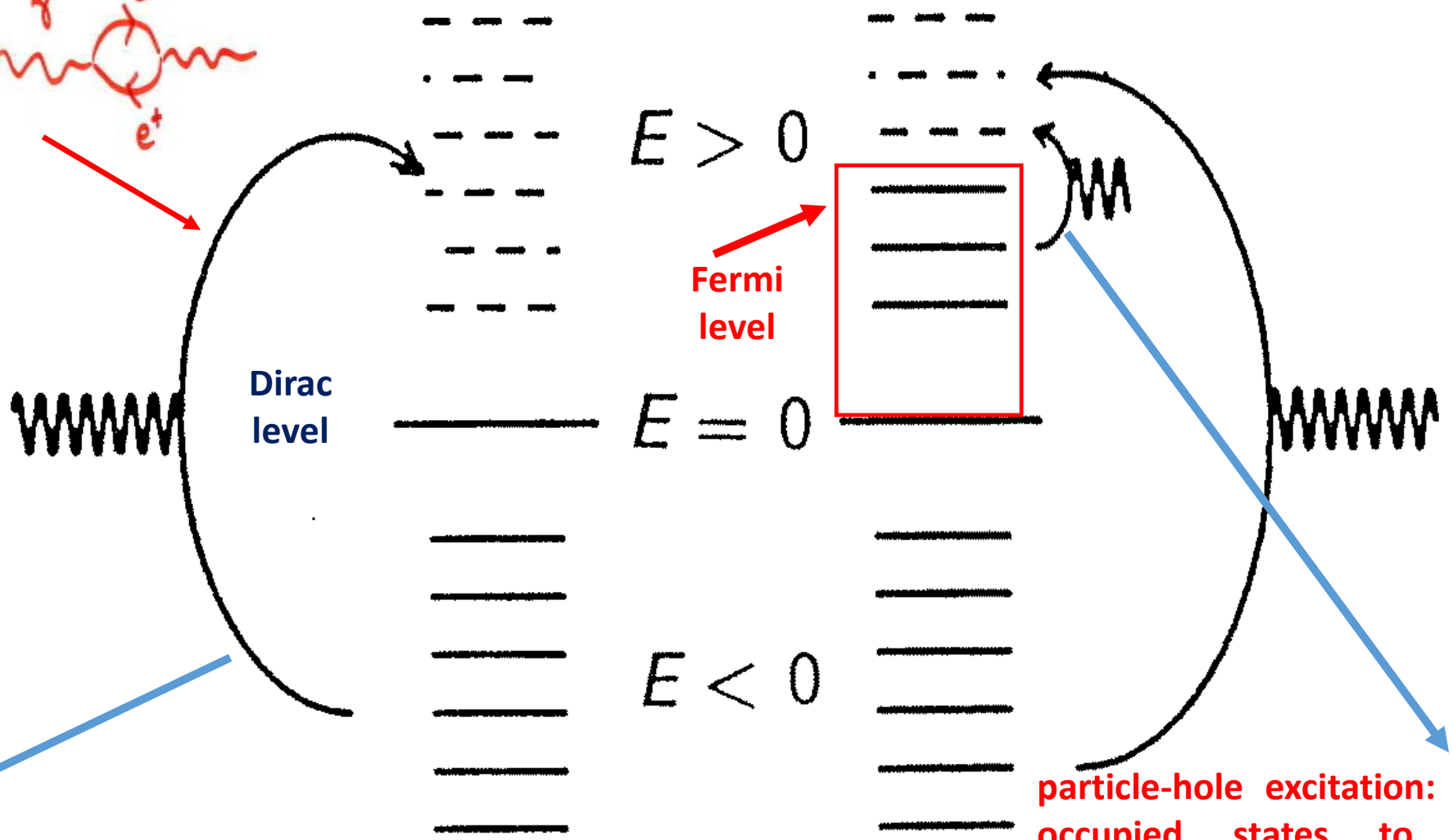
Determined by $\Pi(q)$!

Dirac sea:



all the states of negative energy are filled by electrons, a hole in the Dirac sea behaves like a positive charge particle e^+

A particle-antiparticle excitation is represented by a transition of one electron from an occupied state of negative energy to an occupied state of positive energy



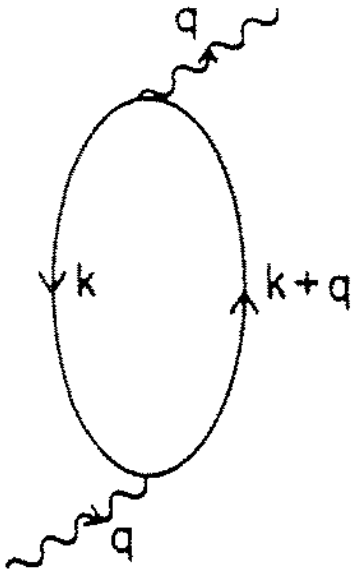
e^+e^- excitation: free photon selfenergy renormalize electron mass and charge and the γe^+e^- coupling

particle-hole excitation: excite occupied states to other unoccupied states of the Fermi sea. *These transitions are additional to those from the negative energy states to the positive energy ones.*

Fermi gas particle-hole propagator: Lindhard function

For the electromagnetic interaction
In the static limit…

$$H_{\rm em}(x) = -e\,\overline{\Psi}(x)\gamma^\mu\,\Psi(x)A_\mu(x)$$



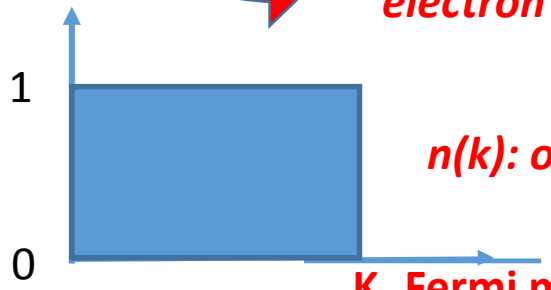
vertex
non-relat.

spin

$$-\mathrm{i}\Pi(q^0,\vec{q}) = (-\mathrm{i}e)(-\mathrm{i}e)(-2)\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{i}G_0(k)\,\mathrm{i}G_0(k+q)$$

$$G_0(k) = \frac{1-n(\vec{k})}{k^0-\epsilon(\vec{k})+\mathrm{i}\eta} + \frac{n(\vec{k})}{k^0-\epsilon(\vec{k})-\mathrm{i}\eta}$$

electron propagator in a Fermi sea (non relativistic)



n(k): occupation number for a free Fermi gas.

electron kinetic energy

K_F Fermi momentum

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FERMION PROPAGATOR

$px \equiv p^\mu x_\mu = p^0 x_0 - \vec{p}\vec{x}$

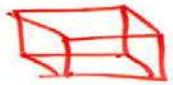
A) GROUND STATE = VACUUM $\equiv |0\rangle$ (CONTAINS NO PARTICLES)

$$\psi(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(c_r(\vec{p})\, u_r(\vec{p})\, e^{-ip\cdot x} + d_r^+(p)\, v_r(\vec{p})\, e^{ipx} \right)$$

$$\bar{\psi}(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(d_r(p)\, \bar{v}_r(p)\, e^{-ip\cdot x} + c_r^+(\vec{p})\, \bar{u}_r(\vec{p})\, e^{ipx} \right)$$

$(p^0 = E(\vec{p}) = \sqrt{\vec{p}^2 + m^2})$

FINITE VOLUME V



V $\quad \frac{1}{V}\sum_{\vec{p}} \longrightarrow \int \frac{d^3p}{(2\pi)^3}$, $\quad \vec{p}$: momentum.
r: helicity.

$c_r^+(\vec{p})$ CREATES A FERMION WITH MOMENTUM $\vec{p}$ AND HELICITY r:

$$c_r^+(\vec{p})|0\rangle = |\text{fermion}, \vec{p}, r\rangle$$

$d_r^+(\vec{p})$ CREATES A ANTI-FERMION WITH MOMENTUM $\vec{p}$ AND HELICITY r

$$d_r^+(\vec{p})|0\rangle = |\overline{\text{fermion}}, \vec{p}, r\rangle$$

$c_r(\vec{p})$, $d_r(\vec{p})$ ASSOCIATED ANNIHILATION OPERATORS

$$c_r(\vec{p})|0\rangle = d_r(\vec{p})|0\rangle = 0.$$

$$i\,\tilde{G}(x',x) = i\int \frac{d^4p}{(2\pi)^4}\, e^{-ip(x'-x)}\, \tilde{G}(p) \equiv \langle 0|T(\psi_\alpha(x')\bar{\psi}_\beta(x))|0\rangle =$$

$$= \Theta(x'^0 - x^0)\langle 0|\psi_\alpha(x')\bar{\psi}_\beta(x)|0\rangle - \Theta(x^0 - x'^0)\langle 0|\bar{\psi}_\beta(x)\psi_\alpha(x')|0\rangle$$

$$\theta(x'^0-x^0)\left\{\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E(p)}\, e^{iE(p)(x^0-x'^0)}\, e^{-i\vec{p}(\vec{x}-\vec{x}')}\,(\not{p}+m)\right\}$$

$$+\,\theta(x^0-x'^0)\left\{\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E(p)}\, e^{iE(p)(x'^0-x^0)}\, e^{i\vec{p}(\vec{x}'-\vec{x})}\,(-E\gamma^0-\vec{p}\vec{\gamma}+m)\right\}$$

$$= i\int \frac{d^4p}{(2\pi)^4}\, e^{-ip(x'-x)}\, G(p)$$

$$\Rightarrow\; G(p) = \frac{\not{p}+m}{p^2-m^2+i\epsilon}\,.$$

B) GROUND STATE $\equiv$ FERMI SEA $\equiv$

$|0\rangle \longrightarrow |\Psi_0\rangle$

$d_r(p)\,|\Psi_0\rangle = 0$

$c^\dagger_r(p)\,|\Psi_0\rangle = 0 \qquad (p<K_F)$ (CAN NOT CREATE A PARTICLE BELOW THE FERMI SURFACE)

$c_r(p)\,|\Psi_0\rangle \neq 0 \qquad (p<K_F)$ (CAN ANNIHILATE A PARTICLE BELOW THE FERMI SURFACE) $\rightarrow$ $\rightarrow$ CREATES A HOLE.

$c_r(p)$

$\equiv$

$p<P_F$.

$$c_r(p) = \begin{cases} b_r(p)\,; & p>K_F \quad \text{annihilates particles} \\ b^\dagger_r(p)\,; & p<K_F \quad \text{creates holes.} \end{cases}$$

WE CAN REWRITE THE FIELD $\psi(x)$ IN TERMS OF OPERATORS WHICH DESTROY THE VACCUUM.

$\Downarrow$

$$c_r(\vec{p}) = \begin{cases} b_r(\vec{p}), & p > p_F \\ b_r^\dagger(\vec{p}), & p \le p_F \end{cases} \qquad c_r^\dagger(\vec{p}) = \begin{cases} b_r^\dagger(\vec{p}), & p > p_F \\ b_r(\vec{p}), & p \le p_F \end{cases}$$

$$\begin{aligned}
\{c_r(\vec{p}), c_s(\vec{p}\,')\} &= \{d_r(\vec{p}), d_s(\vec{p}\,')\} = 0 \\
\{c_r(\vec{p}), d_s(\vec{p}\,')\} &= \{c_r^\dagger(\vec{p}), d_s^\dagger(\vec{p}\,')\} = 0 \\
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\{c_r(\vec{p}), d_s^\dagger(\vec{p}\,')\} &= \{c_r^\dagger(\vec{p}), d_s(\vec{p}\,')\} = 0 \\
\{c_r(\vec{p}), c_s^\dagger(\vec{p}\,')\} &= \{d_r(\vec{p}), d_s^\dagger(\vec{p}\,')\} = \delta_{rs}\delta_{\vec{p}\vec{p}\,'}
\end{aligned}$$

$$\begin{aligned}
\{b_r(\vec{p}), b_s(\vec{p}\,')\} &= \{b_r^\dagger(\vec{p}), b_s^\dagger(\vec{p}\,')\} = 0 \\
\{b_r(\vec{p}), b_s^\dagger(\vec{p}\,')\} &= \delta_{rs}\delta_{\vec{p}\vec{p}\,'}
\end{aligned}$$

$$\psi(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(d_r^\dagger(p)\, v_r(p)\, e^{ipx} + \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} e^{-ipx} \left(\Theta(|\vec{p}|-p_F)\, b_r(\vec{p})\, u_r(p) + \Theta(p_F-|p|)\, b_r^\dagger(p)\, u_r(p) \right) \right.$$

$$\bar{\psi}(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(d_r(p)\, \bar{v}_r(p)\, e^{-ipx} + e^{ipx} \cdot \left[\Theta(|\vec{p}|-p_F)\, b_r^\dagger(p)\, \bar{u}_r(p) + \Theta(p_F-|\vec{p}|)\, b_r(p)\, \bar{u}_r(p) \right) \right.$$

PROPAGADOR:

$$\langle \Psi_0 | T(\psi(x')\bar{\psi}(x)) | \Psi_0 \rangle = \cdots\cdots \langle 0 | T(\psi(x')\bar{\psi}(x)) | 0 \rangle + \left[- \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E(p)} \boxed{\Theta(p_F-p)} (\not{p}+m)\, e^{-iE(p)(x'^0-x^0)} e^{i\vec{p}(\vec{x}'-\vec{x})} \right]$$

$$\Rightarrow \boxed{G^0(p) = \frac{\not{p}+m}{p^2-m^2+i\epsilon} + 2\pi i\, n(p) \frac{\not{p}+m}{2E(p)} \delta(p^0-E(p))}$$

$$E(\vec{p}) = \sqrt{\vec{p}^{\,2}+m^2}$$

$$n(p) = \Theta(p_F-p)$$

Blocking only affects to particles on-shell.

NON-RELATIVISTIC REDUCTION

$$G^0(p) = \frac{1}{p^0-\epsilon(p)+i\epsilon} + 2\pi i\, n(p)\, \delta(p^0-\epsilon(p)) \equiv \frac{1-n(p)}{p^0-\epsilon(p)+i\epsilon} + \frac{n(p)}{p^0-\epsilon(p)-i\epsilon},$$

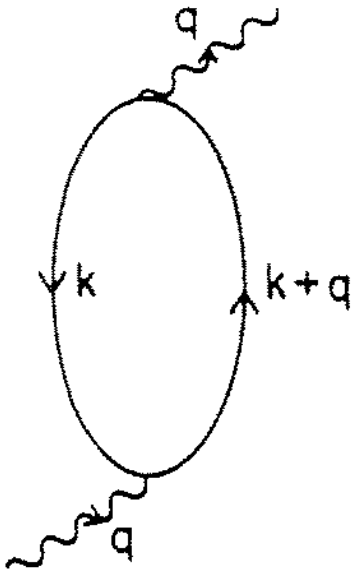
$$\epsilon(p) = m + \vec{p}^{\,2}/2m$$

$$\frac{1}{p^0-\epsilon(\vec{p})\pm i\epsilon} = \mathcal{P}\left(\frac{1}{p^0-\epsilon(\vec{p})}\right) \mp i\pi\delta(p^0-\epsilon(\vec{p}))$$

particle-hole propagator: Lindhard function

For the electromagnetic interaction
In the static limit…

$$H_{\rm em}(x) = -e\,\overline{\Psi}(x)\gamma^\mu\,\Psi(x)A_\mu(x)$$



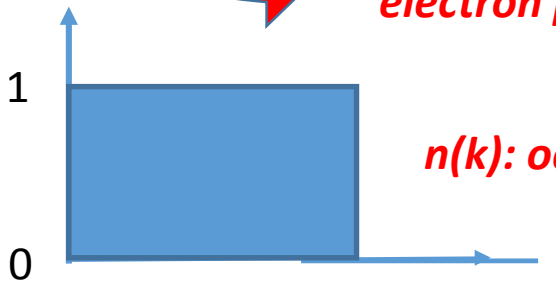
vertex
non-relat.

spin

$$-\mathrm{i}\Pi(q^0,\vec{q}) = (-\mathrm{i}e)(-\mathrm{i}e)(-2)\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{i}G_0(k)\,\mathrm{i}G_0(k+q)$$

$$G_0(k) = \frac{1-n(\vec{k})}{k^0-\epsilon(\vec{k})+\mathrm{i}\eta} + \frac{n(\vec{k})}{k^0-\epsilon(\vec{k})-\mathrm{i}\eta}$$

electron propagator in a Fermi sea (non-relativistic)



n(k): occupation number for a free Fermi gas.

electron kinetic energy

K_F Fermi momentum

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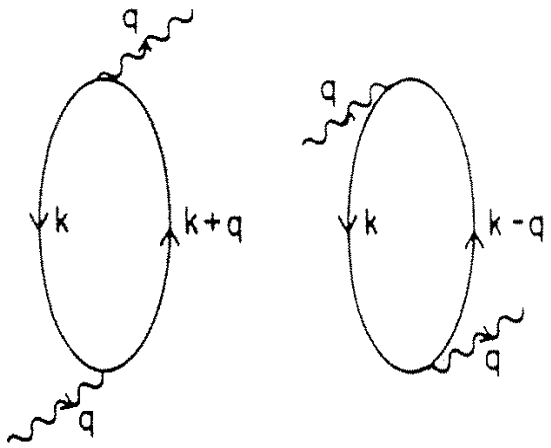
The k^0 integration can be done in the complex plane…

$$\Pi(q^0, \vec{q}) = e^2 U_e(q^0, \vec{q}),$$

where $U_e(q^0, \vec{q})$, called the Lindhard function is given by

$$U_e(q^0, \vec{q}) = 2 \int \frac{d^3k}{(2\pi)^3} \left[\frac{n(\vec{k})(1 - n(\vec{k} + \vec{q}))}{q^0 - \epsilon(\vec{k} + \vec{q}) + \epsilon(\vec{k}) + i\eta} + \frac{n(\vec{k} + \vec{q})(1 - n(\vec{k}))}{-q^0 + \epsilon(\vec{k} + \vec{q}) - \epsilon(\vec{k}) + i\eta} \right].$$



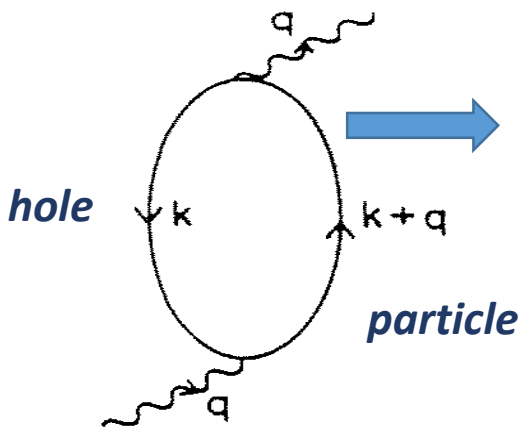
Direct and cross terms of the ph excitation

$U_e(q)$ can have both real and imaginary parts. The imaginary part comes from situations in the intermediate states integration, where the are placed on shell [***Cutkowsky's rule*** (Itzykson & Zuber, Quantum Field Theory, McGraw-Hill, New York, 194)].

The imaginary part can be obtained using

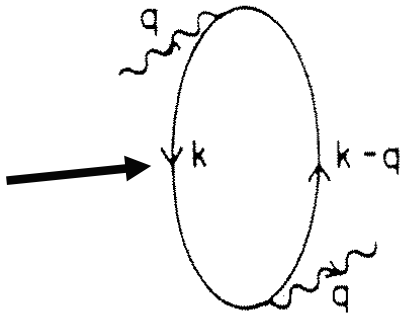
$$\frac{1}{p^0 - \epsilon(\vec{p}) \pm i\epsilon} = \mathcal{P}\left(\frac{1}{p^0 - \epsilon(\vec{p})}\right) \mp i\pi\delta(p^0 - \epsilon(\vec{p}))$$

For **$q^0 > 0$**, only the **direct term** gives rise to an imaginary part



$$\mathrm{Im}\, U_e(q) = -2 \int \frac{d^3k}{(2\pi)^3} \pi\delta\left[q^0 - \epsilon(\vec{k}+\vec{q}) + \epsilon(\vec{k})\right] n(\vec{k}) \left[1 - n(\vec{k}+\vec{q})\right]$$

For $q^0 < 0$, the crossed term gives rise to an imaginary part



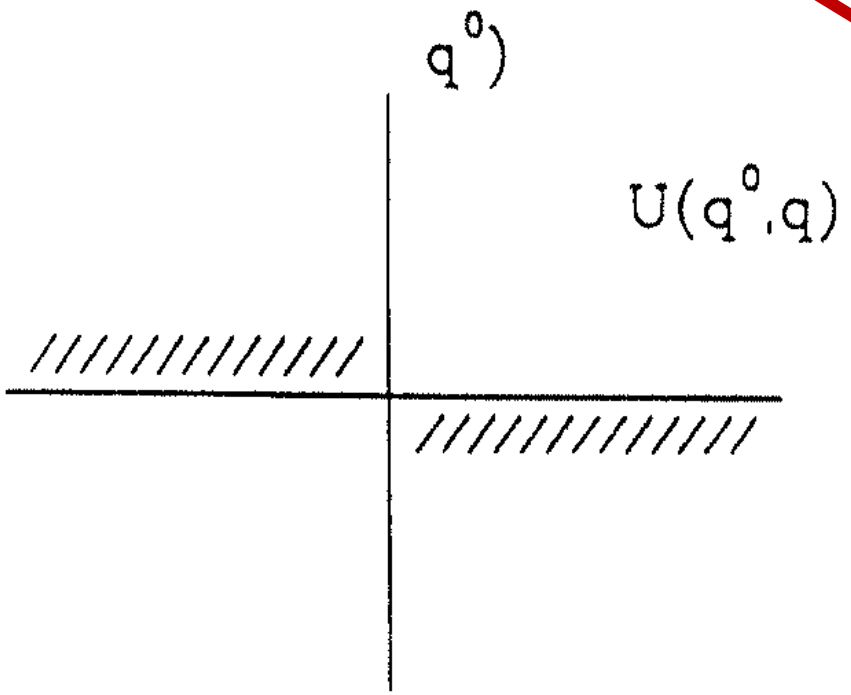
$$U_e(q^0,\vec{q}) = 2\int \frac{d^3k}{(2\pi)^3}\left[\frac{n(\vec{k})(1-n(\vec{k}+\vec{q}))}{q^0-\epsilon(\vec{k}+\vec{q})+\epsilon(\vec{k})+i\eta} + \frac{n(\vec{k}+\vec{q})(1-n(\vec{k}))}{-q^0+\epsilon(\vec{k}+\vec{q})-\epsilon(\vec{k})+i\eta}\right].$$

Analytical structure: In the complex q^0 plane, it has a continuous set of poles ***(cut) in the four quadrant***

$$q^0 = \epsilon_{par} - \epsilon_{hole} - i\eta$$

and in the second quadrant

$$q^0 = \epsilon_{hole} - \epsilon_{par} + i\eta$$



Analytical cuts in the second and fourth quadrants, and it has an imaginary part for real values of q^0 situated in the analytical cuts.

For complex values of q^0

$$2U_e(\nu,\hat{q}) = \frac{mk_F}{\pi^2}\left\{-1+\frac{1}{2\hat{q}}\left[1-\left(\frac{\nu}{\hat{q}}-\frac{\hat{q}}{2}\right)^2\right]\ln\frac{\nu/\hat{q}-\hat{q}/2+1}{\nu/\hat{q}-\hat{q}/2-1}\right.$$
$$\left.-\frac{1}{2\hat{q}}\left[1-\left(\frac{\nu}{\hat{q}}+\frac{\hat{q}}{2}\right)^2\right]\ln\frac{\nu/\hat{q}+\hat{q}/2+1}{\nu/\hat{q}+\hat{q}/2-1}\right\}$$

$$\nu = \frac{q^0 m}{k_F^2}, \qquad \hat{q} = \frac{q}{k_F}$$

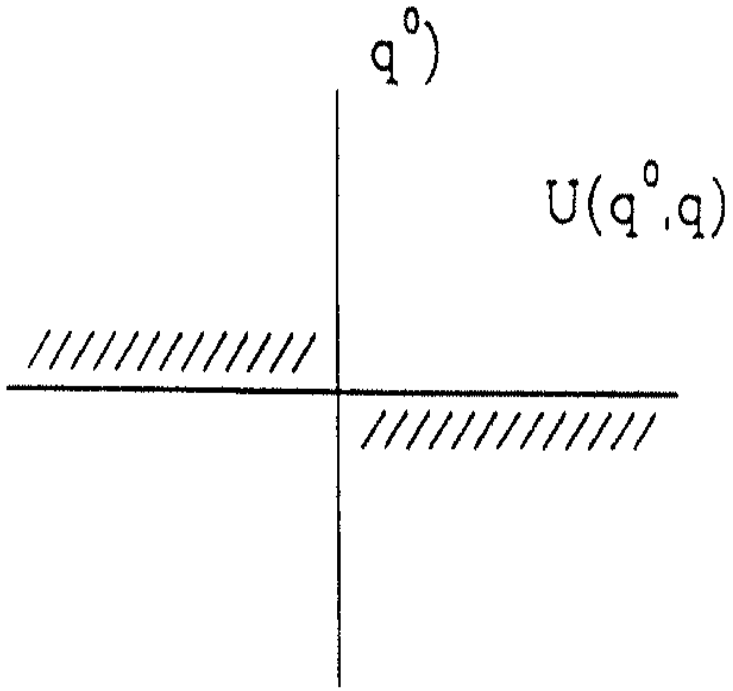
Analytical structure: In the complex q^0 plane, it has a continuous set of poles ***(cut) in the four quadrant and in the second quadrant***

$$2ImU_e(\nu,\hat{q}) = \frac{-2mk_F}{4\pi\hat{q}}\left[1-\left(\frac{|\nu|}{\hat{q}}-\frac{\hat{q}}{2}\right)^2\right]$$

for $\hat{q} > 2$ and $\frac{1}{2}\hat{q}^2+\hat{q} \geq |\nu| \geq \frac{1}{2}\hat{q}^2-\hat{q}$ or $\hat{q} < 2$ and $\frac{1}{2}\hat{q}^2+\hat{q} \geq |\nu| \geq \hat{q}-\frac{1}{2}\hat{q}^2$,

$$2ImU_e(\nu,\hat{q}) = -\frac{mk_F}{\pi\hat{q}}|\nu|$$

for $\hat{q} < 2$ and $0 \leq |\nu| \leq \hat{q}-\frac{1}{2}\hat{q}^2$, and $\mathrm{Im}\,U_N(\nu,\hat{q}) = 0$ otherwise.



Analytical cuts in the second and fourth quadrants, and it has an imaginary part for real values of q^0 situated in the analytical cuts.

Fully relativistic expressions:

$$S(p) = \left(p^\mu \gamma_\mu + m\right)\left(\frac{1}{p^2 - m^2 + i\,\varepsilon} + i\,\frac{\pi}{E(\vec{p}\,)}\,\delta\left(p^0 - E(\vec{p}\,)\right) n(\vec{p}\,)\right)$$

$$E(\vec{p}) = \sqrt{\vec{p}^{\,2} + m^2}$$

$$G(p) = \left(\frac{1}{p^2 - m^2 + i\,\varepsilon} + i\,\frac{\pi}{E(\vec{p}\,)}\,\delta\left(p^0 - E(\vec{p}\,)\right) n(\vec{p}\,)\right)$$

$$U_e(q) = -2i\int \frac{d^4p}{(2\pi)^4}\,G(p)G(p+q) = 2\int \frac{d^3p}{(2\pi)^3}\frac{m}{E(\vec{p}\,)}\frac{m}{E(\vec{p}+\vec{q})}\frac{n(\vec{p}\,)\left(1 - n(\vec{p}+\vec{q})\right)}{q^0 + E(\vec{p}) - E(\vec{p}+\vec{q}) + i\varepsilon} + (q \leftrightarrow -q)$$

low density limit

$$ImU_e(q) = -m^2\,\frac{\Theta(q^0)\Theta(-q^2)}{2\pi|\vec{q}|}\,\Theta(E_F - \varepsilon_R)\,(E_F - \epsilon_R) \approx -\pi\rho\,\frac{m}{E(\vec{q}\,)}\,\delta\left(q^0 + M - E(\vec{q})\right)$$

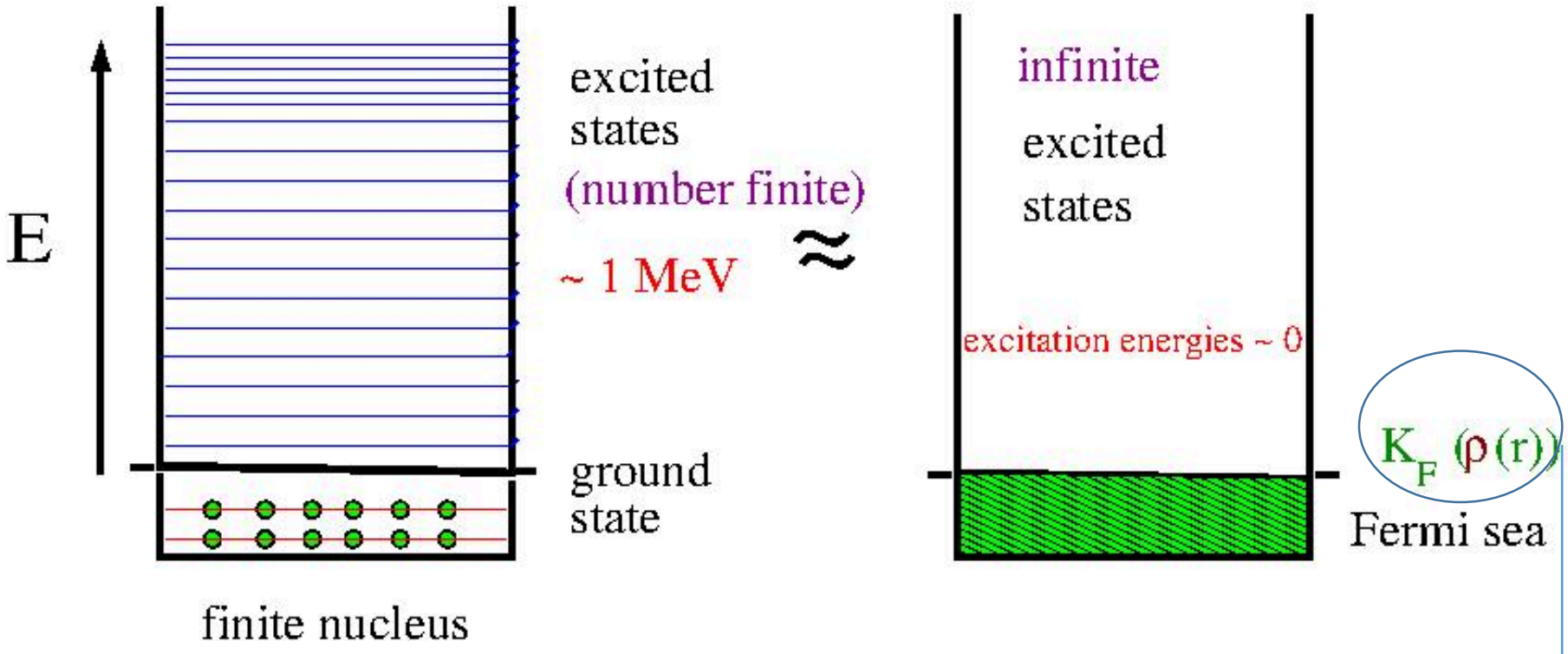
$$E_F(\vec{p}) = \sqrt{k_F^2 + m^2}, \qquad \varepsilon_R = Max\left\{m, E_F - q^0, \frac{-q^0 + |\vec{q}|\sqrt{1 - 4m^2/q^2}}{2}\right\}$$

Dictionary:

- Propagation of photons through an electron gas (metal)
- Electric charge of the electron and screening
- Photon carrier of the electromagnetic interaction

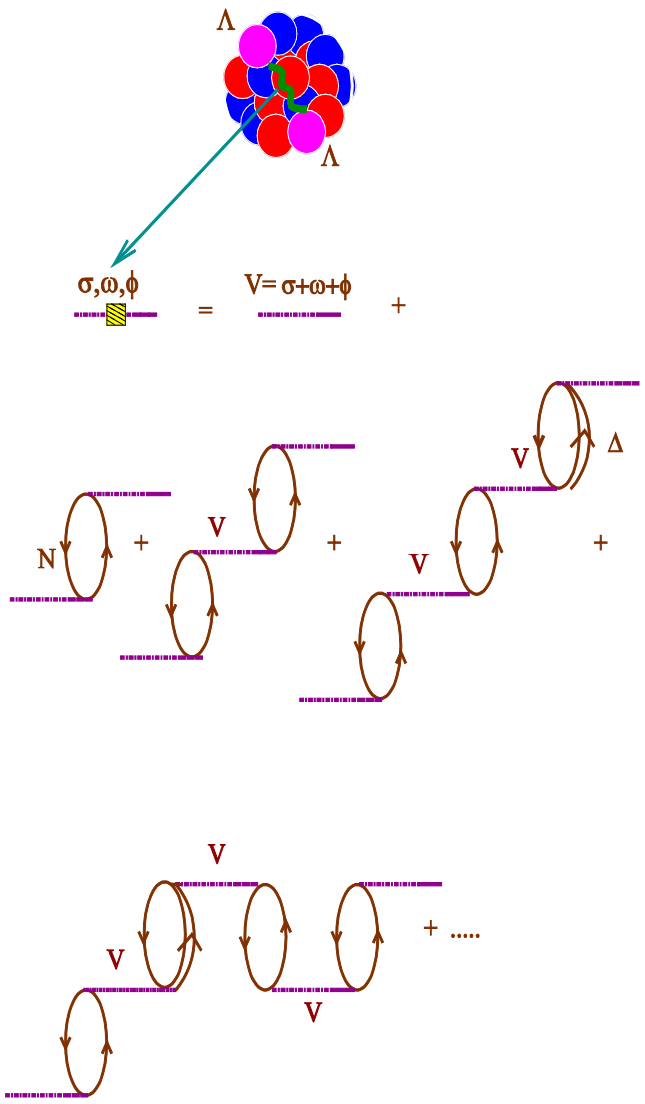
→

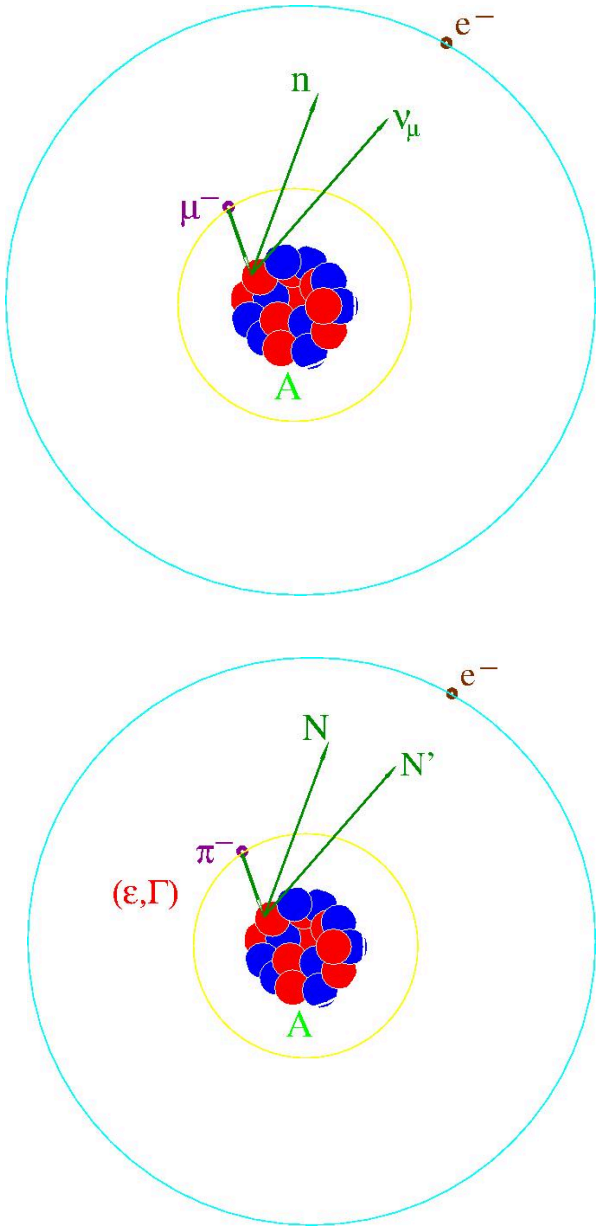
- Propagation of γ, W, Z^0 or pions (mesons in general) through a nuclear medium
- Axial charge of the nucleon and axial polarization
- π,ρ,… carriers of the NN interaction

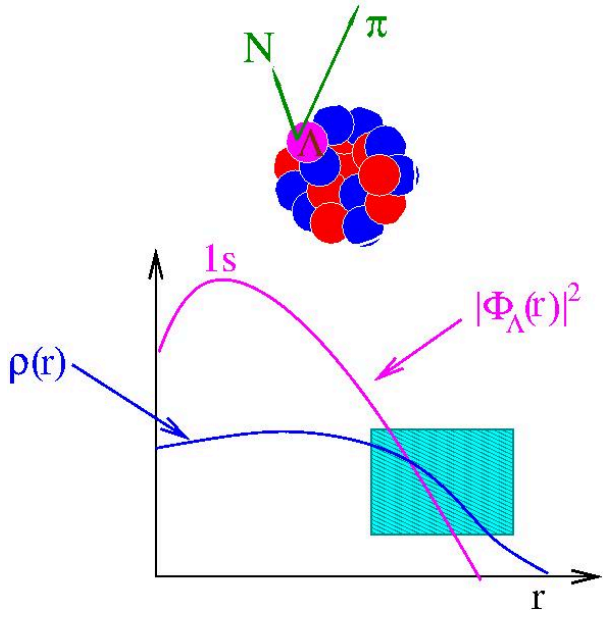


For inclusive processes , with large excitation energies (> 50 MeV) and spherical nuclei…

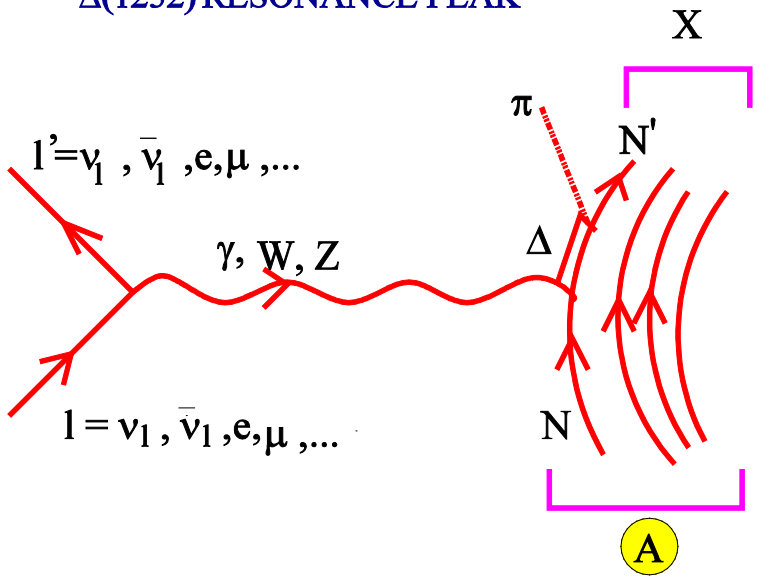
Local density approximation (LDA)







Δ(1232) RESONANCE PEAK



EXCITATION OF Δ (1232) DEGREES OF FREEDOM

Nuclear medium: Nuclear matter (approximation)

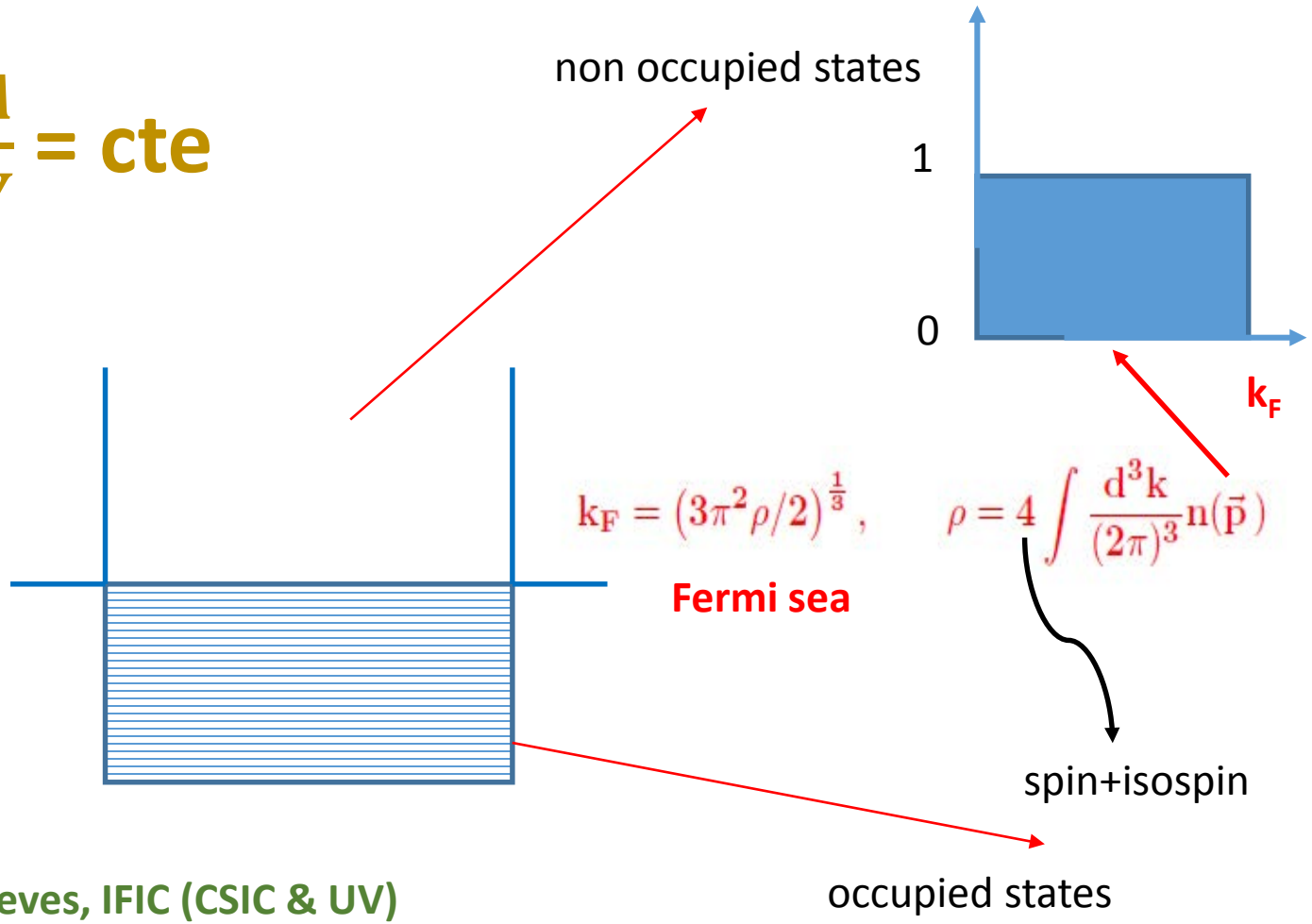
- Infinite system homogeneous and isotropic
- Infinite volume with an infinite number of nucleons, but with a constant density

$$\rho = \frac{A}{V} = \text{cte}$$

- *Momentum and energy are conserved*
- Symmetric Z=N=A/2 (no necessary)
- *Pauli's exclusion principle*
- *Finite nuclei: LDA (local density appx)*

ρ_{NM} (cte) $\rightarrow \rho(\vec{r})$ (s-wave)

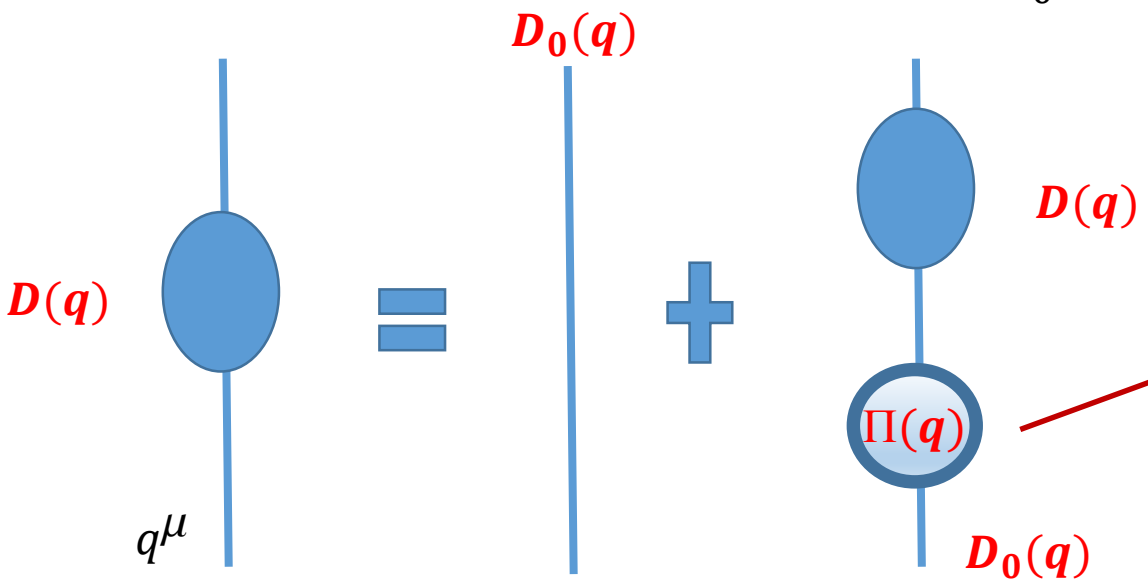
$\vec{q}^{\,2}\, \rho_{NM} \rightarrow \vec{\nabla}\rho(\vec{r})\, \vec{\nabla}$ (p-wave)



propagation of photons in the electron gas → propagation of pions in the nuclear medium
(carriers of the interaction)

Dyson equation

$$iD = iD_0 + iD_0\, i\Pi\, iD \Longrightarrow \boldsymbol{D(q)} = \frac{\boldsymbol{D_0}}{\boldsymbol{1 - D_0\Pi}} = \frac{\boldsymbol{1}}{\boldsymbol{q^2 - m_\pi^2 - \Pi(q)}}$$

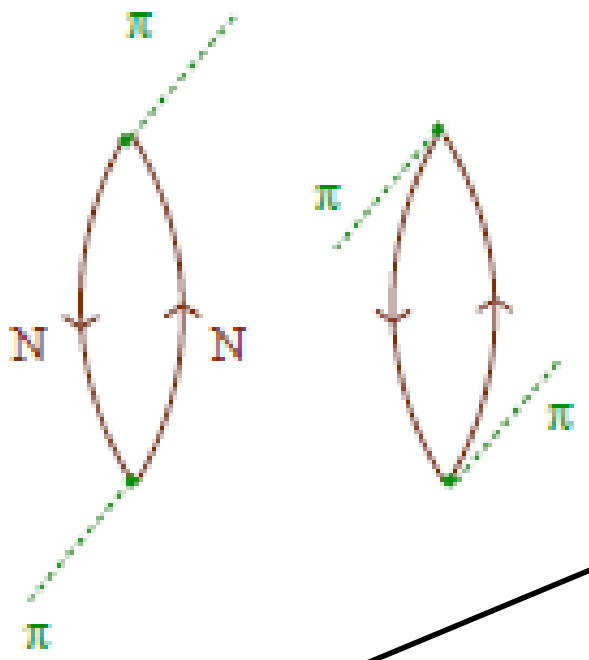


pion selfenergy: contains only irreducible diagrams

Full pion propagator $D(q)$

Free pion propagator $D_0(q)$

Pion selfenergy: first approximation πNN vertex



Chiral symmetry

$$\mathcal{L}^{\sigma}_{\text{int}} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma_\mu\vec{\tau}(\vec{\phi}\times\partial^\mu\vec{\phi})\Psi$$

P-wave *S-wave (Weinberg-Tomozawa)*

$$f = \frac{m_\pi}{2f_\pi}g_A$$

$\psi(x)$: SU(2) ⇒ bi-SPINOR : $\begin{bmatrix}\phi_p(x)\\ \phi_m(x)\end{bmatrix}$

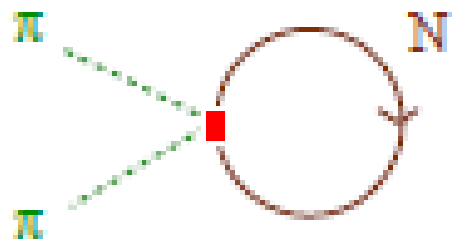
$\phi_+(x) = \phi_-^+(x) = \frac{1}{\sqrt{2}}(\phi_1(x) - i\,\phi_2(x))$; $\phi_0(x) = \phi_3(x)$

$\phi_-^+(x)$ CREATES A π^- AND ANNIHILATES A π^+ MESON.

[J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, New York, 1965]

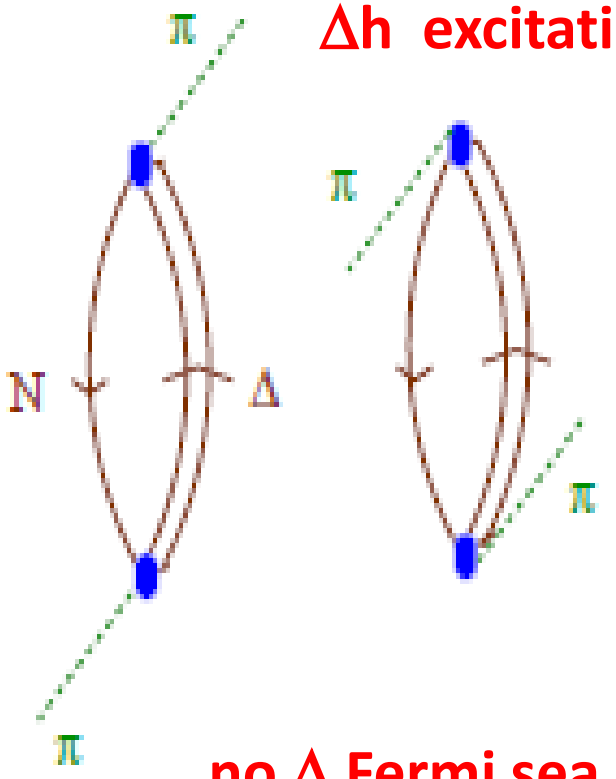
$$\Pi(q) = \frac{f^2}{m_\pi^2}\vec{q}^{\,2}\,U_N(q)$$

$$U_N(q) = 2U_e(q), \quad (m_e \to m_N)$$



S-wave selfenergy: small for isoscalar nuclei !

The pion cannot only excite nucleons above the Fermi sea, but it can also excite the internal degrees of freedom of the nucleon since it is a composite particle made out of quarks. Hence a nucleon can be converted into a $\Delta, N^*, \Delta^*, etc..$



Δh excitation

no Δ Fermi sea !

Δ(1232) [spin 3/2 and isospin 3/2] plays an important role at intermediate energies because of its lower mass and strong coupling → contribution to ▯

$$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^\dagger (\partial^\mu \vec{\phi}) \Psi + \text{h.c.}$$

where Ψ_μ is a Rarita-Schwinger $J^\pi = 3/2^+$ field, $\vec{T}^\dagger$ is the isospin transition operator from isospin 1/2 to 3/2, and $f^* = 2.13 \times f = 2.14.$

Some technical aspects about the Δh excitation

- $\left\langle \frac{3}{2}\, M_T \left| T_\nu^\dagger \right| \frac{1}{2}\, m_T \right\rangle = \left(\frac{1}{2}, 1, \frac{3}{2} \middle| m_T, \nu, M_T \right) \cancel{\left\langle \frac{3}{2} \left\| T_\nu^\dagger \right\| \frac{1}{2} \right\rangle}$ → 1 (Wigner-Eckart)

- Δ propagator (unstable particle)

$P^{\mu\nu}$: spin 3/2 projector

$$G^{\mu\nu}(p_\Delta) = \frac{P^{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta},$$

$$P^{\mu\nu}(p_\Delta) = -(\not{p}_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu \gamma^\nu - p_\Delta^\nu \gamma^\mu}{M_\Delta} \right],$$

$$\Gamma_\Delta(s) = \frac{1}{6\pi}\left(\frac{f^*}{m_\pi}\right)^2 \frac{M}{\sqrt{s}}\left[\frac{\lambda^{1/2}(s, m_\pi^2, M^2)}{2\sqrt{s}}\right]^3 \times \Theta(\sqrt{s} - M - m_\pi), \qquad s = p_\Delta^2,$$

$(\pi N\ CM\ momentum)^3$

$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$

Some technical aspects about the Δh excitation: non-relativistic expressions

- πNΔ transition: $$H_{\pi N\Delta} = \frac{f^*}{\mu}\,\Psi_N^\dagger(x)\,S_i\,\partial_i\phi^\lambda(x)\,T^\lambda\,\Psi_\Delta(x) + \text{h.c.},$$

spin transition operator.

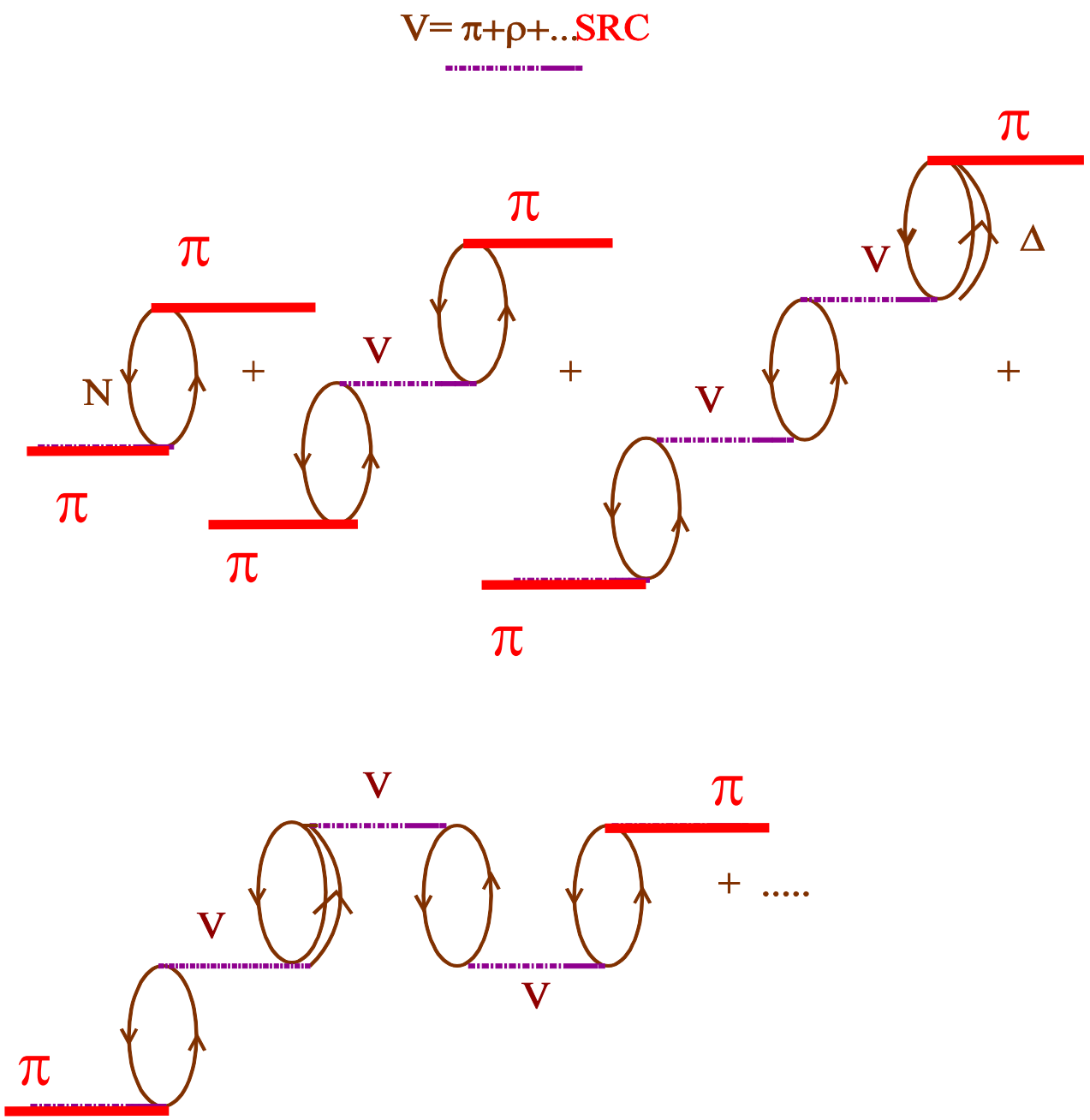
- Δ propagator

$$G_\Delta(k) = \frac{1}{k^0 - w_R - T_\Delta + \frac{1}{2}\mathrm{i}\Gamma_\Delta}$$

with $w_R = M_\Delta - M_N$, T_Δ the Δ kinetic energy a

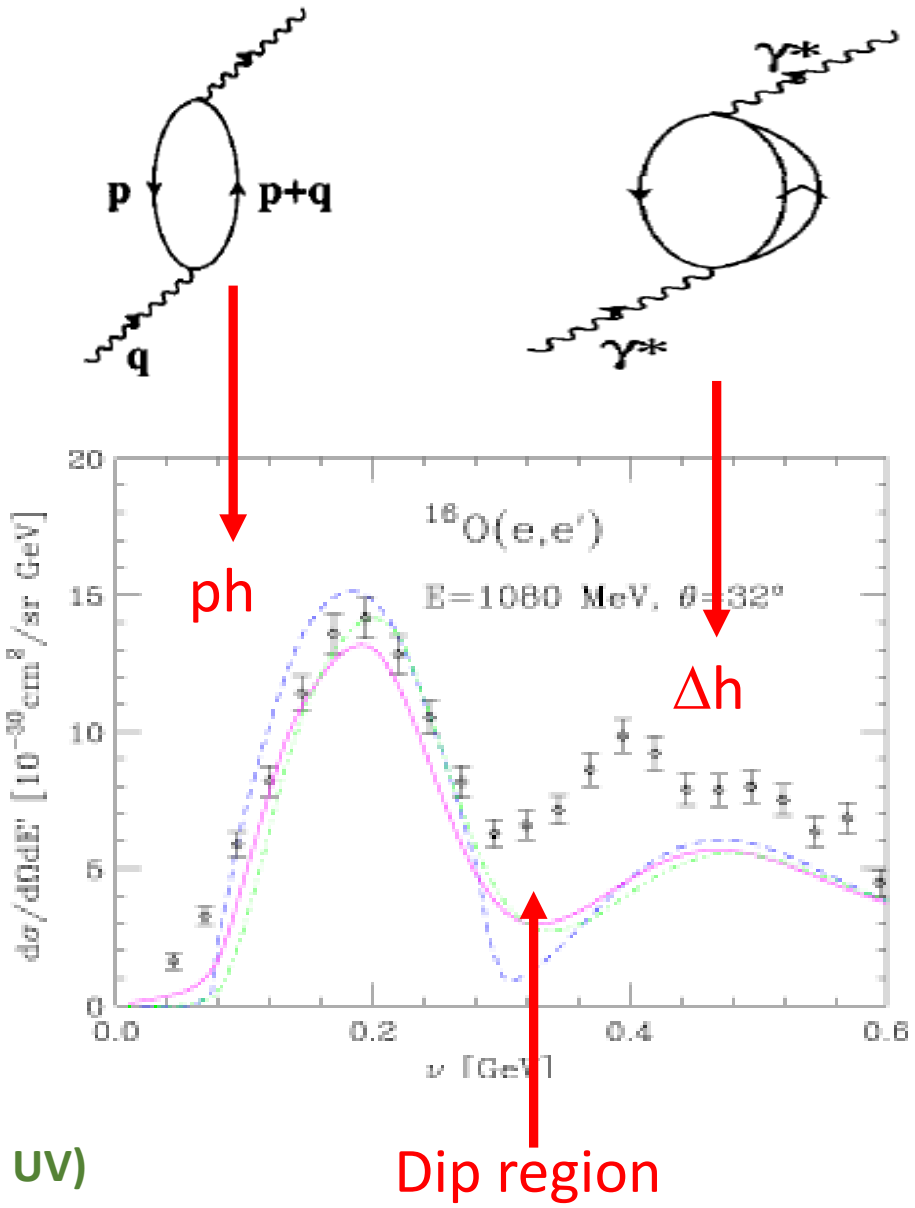
The pion selfenergy reads: $\Pi(q) = \frac{f^2}{m_\pi^2}\,\vec{q}^{\,2}\,U(q)$ *with* $U(q) = U_N(q) + U_\Delta(q)$

$$U_\Delta(q) = -\mathrm{i}\left(\frac{4}{3}\right)^2\left(\frac{f^*}{f}\right)^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left[G^0(k)G_\Delta(k+q) + G^0(k)G_\Delta(k-q)\right]$$



Remarks:

- External γ can also excite Δh components



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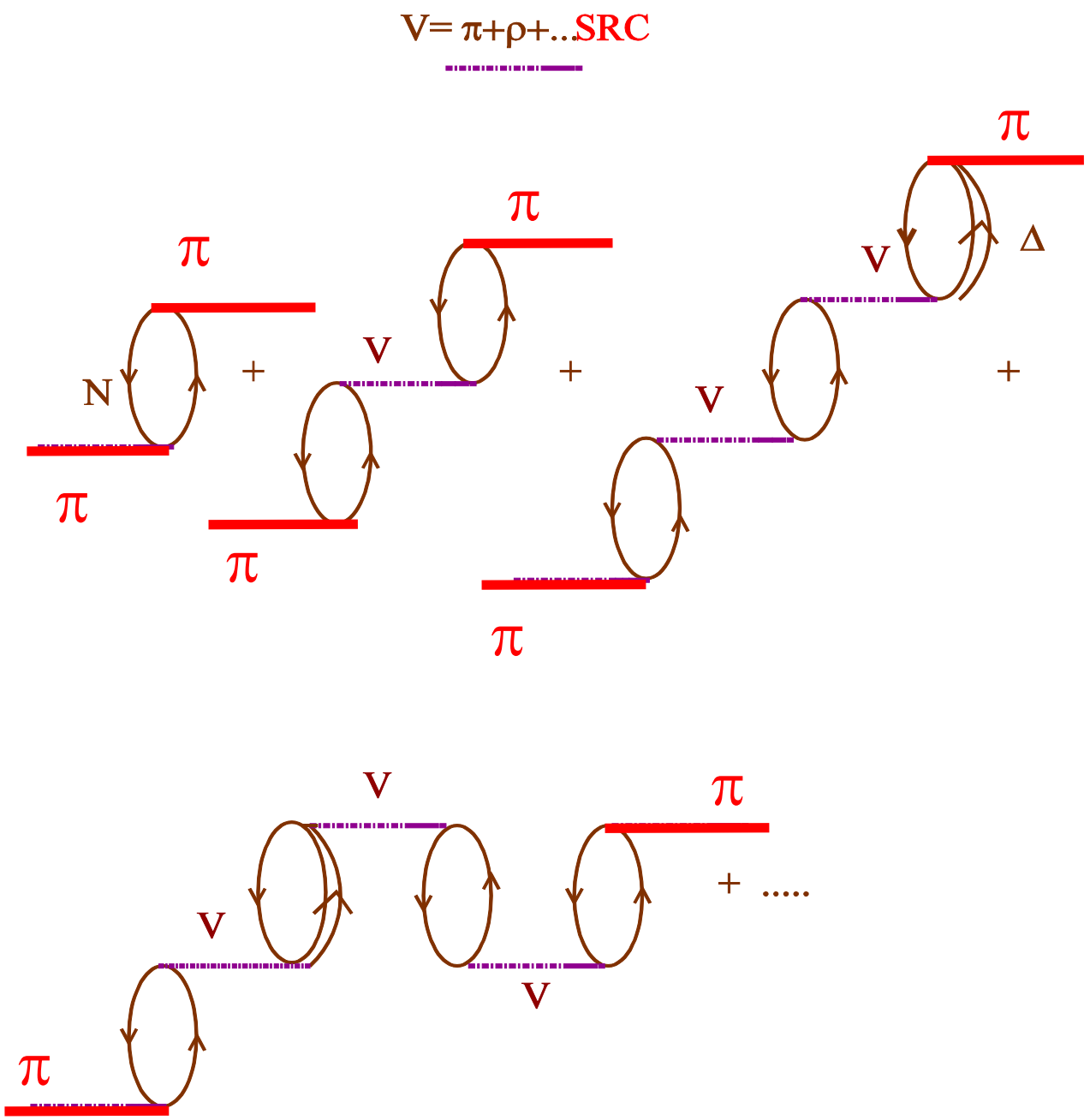
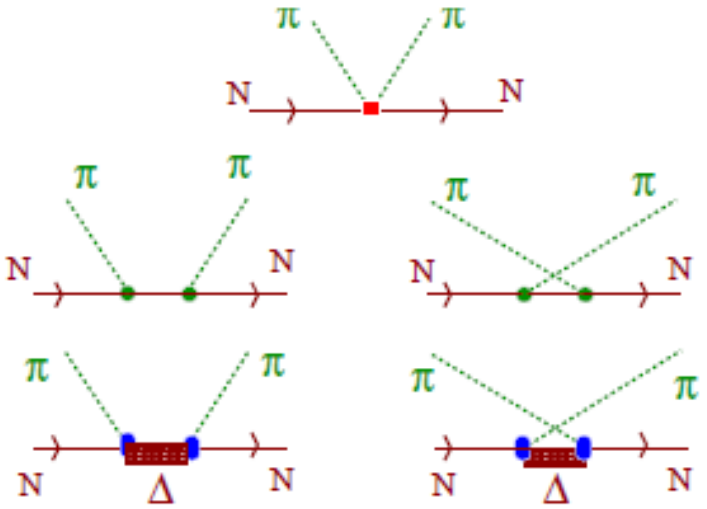


Remarks:

- *V is an effective ph-ph, ph-Δh and Δh-Δh interaction in the nuclear medium*
- *V=π + ρ+ other mesons (Short Range Correlations). Starting point πN → πN in free space (πNN and πNΔ couplings)* [Ericson+Weise, *Pions in nuclei]* *and NN → NN* ***Bonn potential***



NN INTERACTION: SIMPLE MODEL: (Oset + Weise, 1979)

$\pi + \rho$ exchanges + nuclear correlations

- $H_{\rho NN}(q) = i \frac{f_\rho}{m_\rho} (\vec{\sigma} \times \vec{q}) \vec{\epsilon}\, \tau^\alpha$

$\left(\mathcal{L}(x) = -g_{\rho NN} \bar{\psi} \left(\gamma^\mu - \frac{\kappa}{2m} \sigma^{\mu\nu} \partial_\nu\right) \vec{\tau}\, \vec{\rho}_\mu \psi\right)$

- $H_{\rho N\Delta}(q) = \frac{i f^*_\rho}{m_\rho} (\vec{S}^+ \times \vec{q}) \vec{\epsilon}\, T^\lambda$

with $C_\rho \equiv \frac{f_\rho^2 / m_\rho^2}{f^2 / m_\pi^2} \simeq 2$, $\quad f^*_\rho / f_\rho = f^* / f$. ← QUARK MODEL.

Now: $\pi + \rho$ EXCHANGES

- $\hat{V}_{s-i} = \hat{V}_\pi(q) + \hat{V}_\rho(q)$

$\hat{V}_\pi(q) = \sigma_1^i \sigma_2^j \vec{\tau}_1 \vec{\tau}_2 \left(\frac{f}{m_\pi}\right)^2 V^\pi_{ij}(q)$; $\quad V^\pi_{ij} = F_\pi^2 \vec{q}^{\,2} D_\pi(q)\, \hat{q}_i \hat{q}_j$

(Longitudinal !!)

$\hat{V}_\rho(q) = \sigma_1^i \sigma_2^j \vec{\tau}_1 \vec{\tau}_2 \left(\frac{f}{m_\pi}\right)^2 V^\rho_{ij}(q)$; $\quad V^\rho_{ij} = F_\rho^2 \vec{q}^{\,2} D_\rho(q)\, C_\rho \times$

!! (Transversal $\Longrightarrow$ $\times (\delta_{ij} - \hat{q}_i \hat{q}_j)$

where the form factors (off-shell extrapolations)

$$F_i(q^2) = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q^2} \qquad (q^2 = q^{0\,2} - \vec{q}^{\,2})$$

$\Lambda_\pi = 1250$ MeV $= \Lambda_\pi^*$

$\Lambda_\rho = 2500$ MeV $= \Lambda_\rho^*$

- BECAUSE $C_\rho = C_\rho^*$, $\Lambda_\pi^* = \Lambda_\pi$ AND $\Lambda_\rho^* = \Lambda_\rho$ THE FORMER POTENTIALS DESCRIBE $\Delta N \to NN$, $NN \to \Delta N$, $\Delta N \to \Delta N$ AND $\Delta\Delta \to NN$ INTERACTION WITH THE FOLLOWING REPLACEMENTS

$$\frac{f}{m_\pi}\,\sigma\tau \to \frac{f^*}{m_\pi}\,ST \quad \text{or} \quad \frac{f^*}{m_\pi}\,S^+T^+$$

- V^π_{ij} AND V^ρ_{ij} ARE ORTHOGONALS

$\Longrightarrow$ SHORT RANGE CORRELATIONS

- ATTRIBUTED TO THE EXCHANGE OF THE ω-MESON.
- CORRELATED POTENTIAL IN COORDINATE SPACE

$$\tilde{V}(r) = V(r)\,g(r) \;;\quad g(r) = 1 - j_0(q_c r) \;;\quad q_c \simeq m_\omega = 783$$

$\Rightarrow \frac{1}{m_\omega}$ defines the range of the correlations

- MOMENTUM SPACE

$$\tilde{V}(q) = \int \frac{d^3k}{(2\pi)^3}\, g(\vec{k}-\vec{q})\, V(\vec{k})$$

$$g(\vec{k}-\vec{q}) = (2\pi)^3\,\delta^3(\vec{k}-\vec{q}) - 2\pi^2\,\delta(|\vec{q}-\vec{k}| - q_c)\,/\,q_c^2$$

NN potential V(q)= $c_0\{f_0(\rho)+f_0'(\rho)\vec{\tau}_1\vec{\tau}_2+g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2\}+\vec{\tau}_1\vec{\tau}_2\sum_{i,j=1}^{3}\sigma_1^i\sigma_2^j V_{ij}^{\sigma\tau}$

$$V_{ij}^{\sigma\tau}=(\hat{q}_i\hat{q}_j V_l(q)+(\delta_{ij}-\hat{q}_i\hat{q}_j)V_t(q))$$

with $\hat{q}_i = {}^{q_i}/_{|\vec{q}|}$

V_π

$$V_l(q^0,\vec{q})=\frac{f^2}{m_\pi^2}\left\{\left(\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2-q^2}\right)^2\frac{\vec{q}^2}{q^2-m_\pi^2}+g_l'(q)\right\},$$

V_ρ

$$\frac{f^2}{4\pi}=0.08,\ \Lambda_\pi=1200\ \text{MeV},$$

$$V_t(q^0,\vec{q})=\frac{f^2}{m_\pi^2}\left\{C_\rho\left(\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2-q^2}\right)^2\frac{\vec{q}^2}{q^2-m_\rho^2}+g_t'(q)\right\},$$

$$C_\rho=2,\ \Lambda_\rho=2500\ \text{MeV},\ m_\rho=770\ \text{MeV}.$$

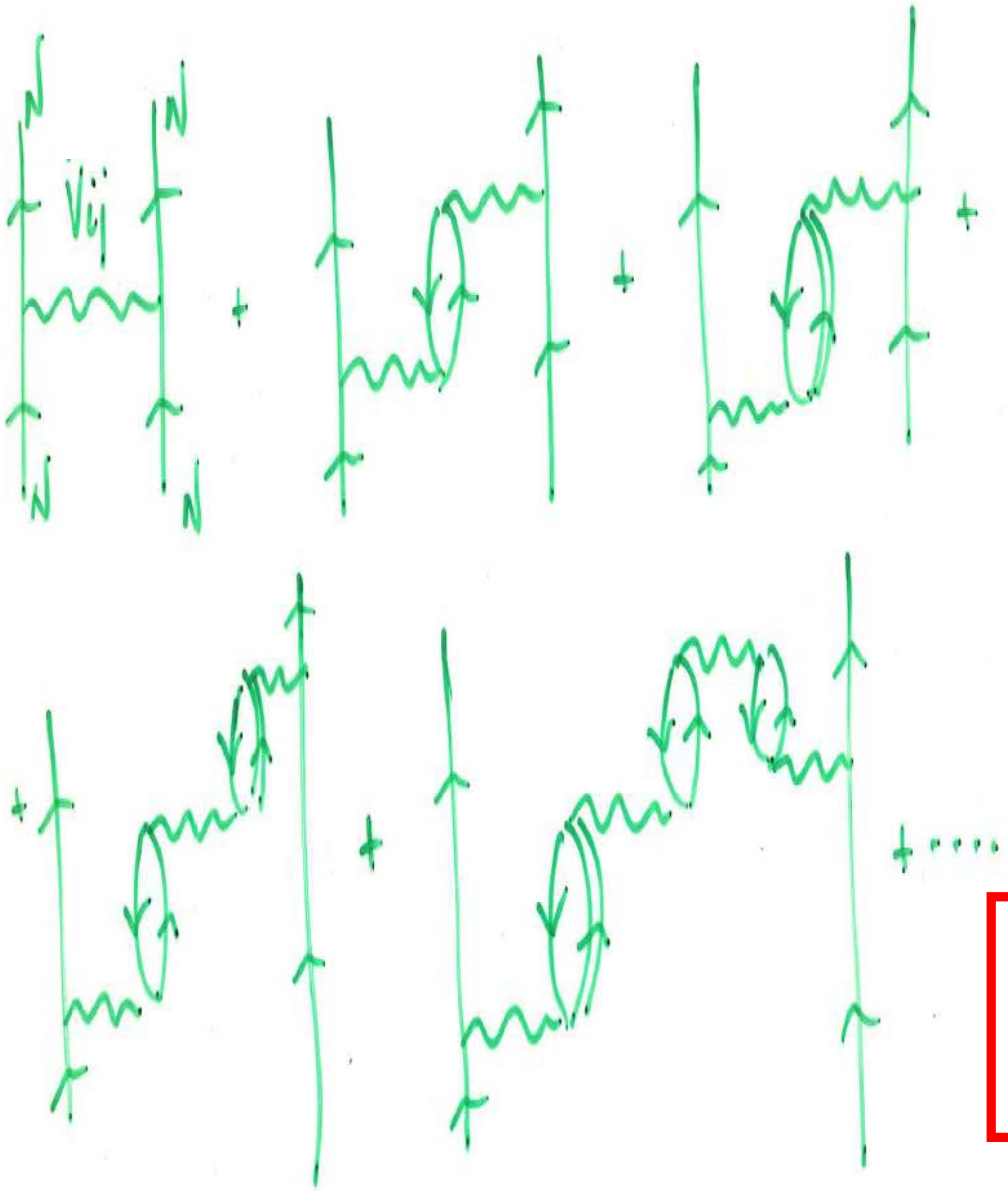
zero range Landau force
J. Speth et al., Phys. Rep. 33 (1977) 127

The $N\Delta$ and the $\Delta\Delta$ potentials are obtained from V_l and V_t by replacing

$$\vec{\sigma}\rightarrow\vec{S},\ \vec{\tau}\rightarrow\vec{T}.$$

$$f\rightarrow f^*$$

SRC



The spin-isospin part of the interaction, taking into account the propagation of the mesons through the medium

$$W_{\sigma\tau}(q) = \sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 V_{ij}^{\sigma\tau}(q) +$$
$$\sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 \{ V_{ik}^{\sigma\tau}(q) \, U(q) V_{kj}^{\sigma\tau}(q) \} +$$
$$\sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 \{ V_{ik}^{\sigma\tau}(q) \, U(q) V_{km}^{\sigma\tau}(q) U(q) V_{mj}^{\sigma\tau}(q) \} +$$
$$\ldots \; = \sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 W_{ij}^{\sigma\tau}(q)$$

$$U(q) = U_N(q) + U_\Delta(q)$$

(direct + crossed terms)

$$W_{ij}^{\sigma\tau}(q) = \frac{V_l(q)}{1 - U(q) V_l(q)} \, \hat{q}_i \hat{q}_j + \frac{V_t(q)}{1 - U(q) V_t(q)} \left(\delta_{ij} - \hat{q}_i \hat{q}_j \right)$$

Induced spin-isospin NN interaction in a nuclear medium

$$\hat{q}_i \hat{q}_j \perp \left(\delta_{ij} - \hat{q}_i \hat{q}_j \right)$$

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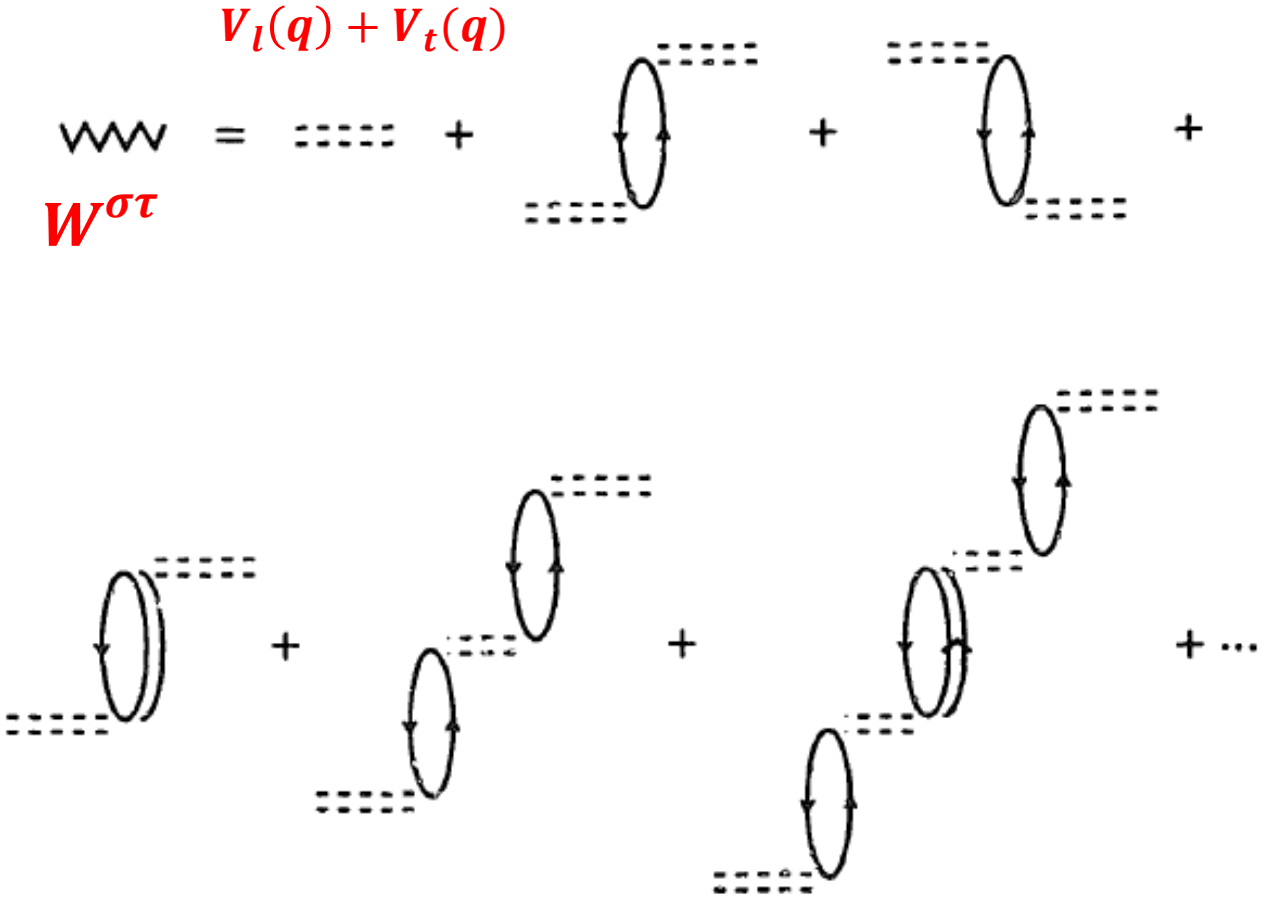
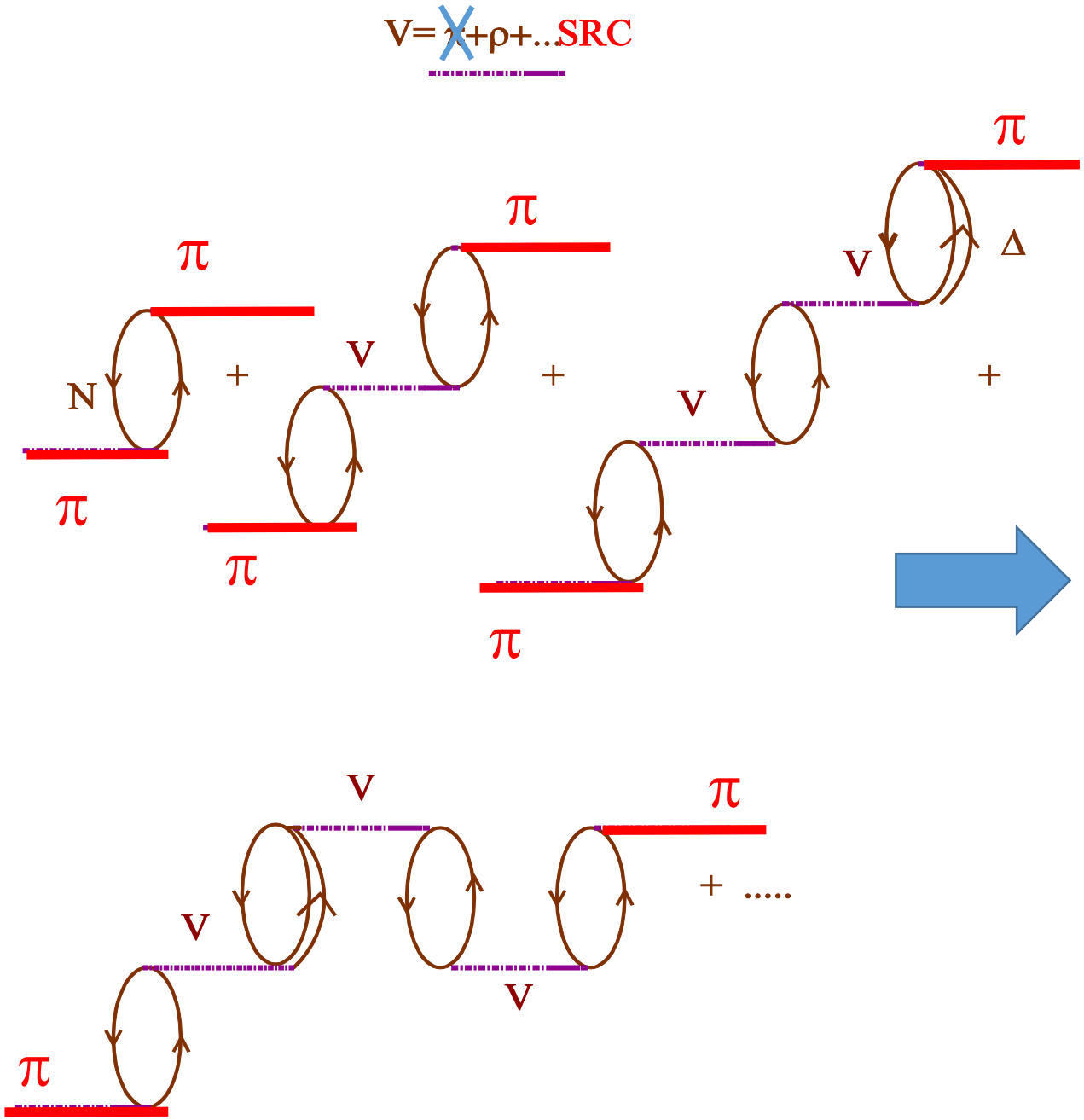
Diagrammatically,

$$W_{ij}^{\sigma\tau}(q) = \frac{V_l(q)}{1-U(q)V_l(q)}\,\hat{q}_i\hat{q}_j + \frac{V_t(q)}{1-U(q)V_t(q)}\left(\delta_{ij} - \hat{q}_i\hat{q}_j\right)$$

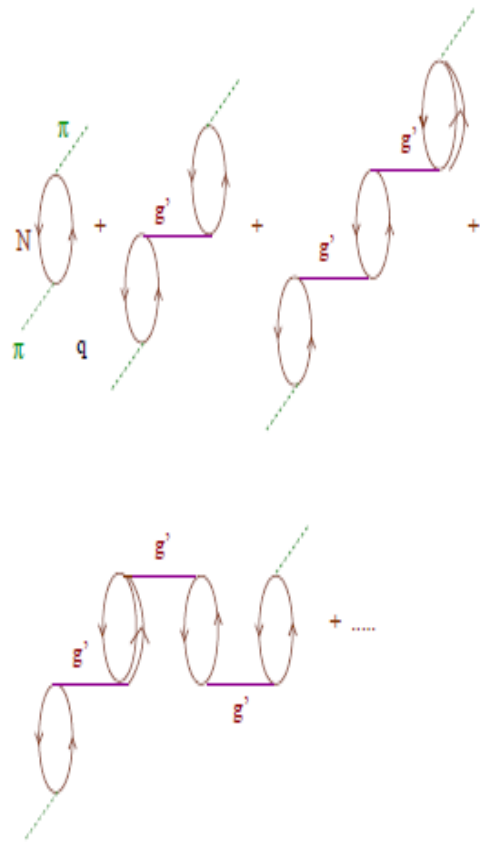
From the spin-isospin interaction, we construct the induced interaction by exciting ph and Δh components in a RPA sense

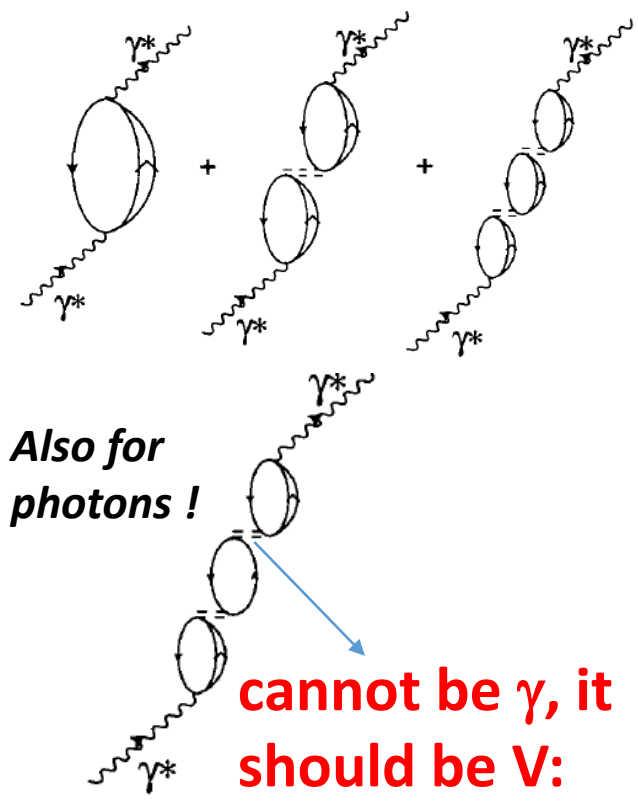




Remarks:

- To compute the pion selfenergy, only irreducible diagrams should be considered: V → ρ + SRC (g') [the initial pion selects the longitudinal channel]





Also for photons !

cannot be γ, it should be V: ph-ph, ph-Δh interaction

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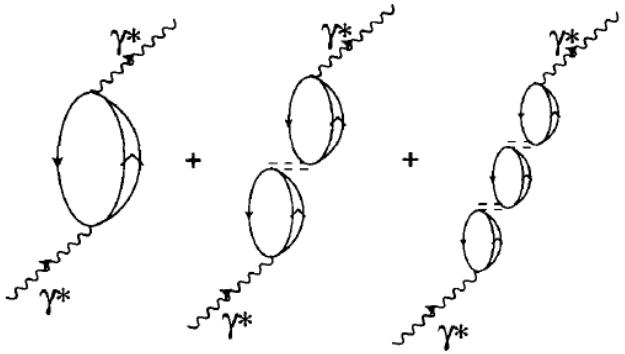
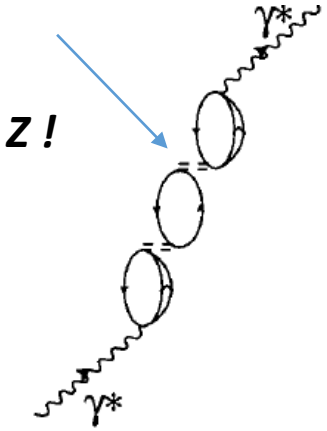
Remarks:



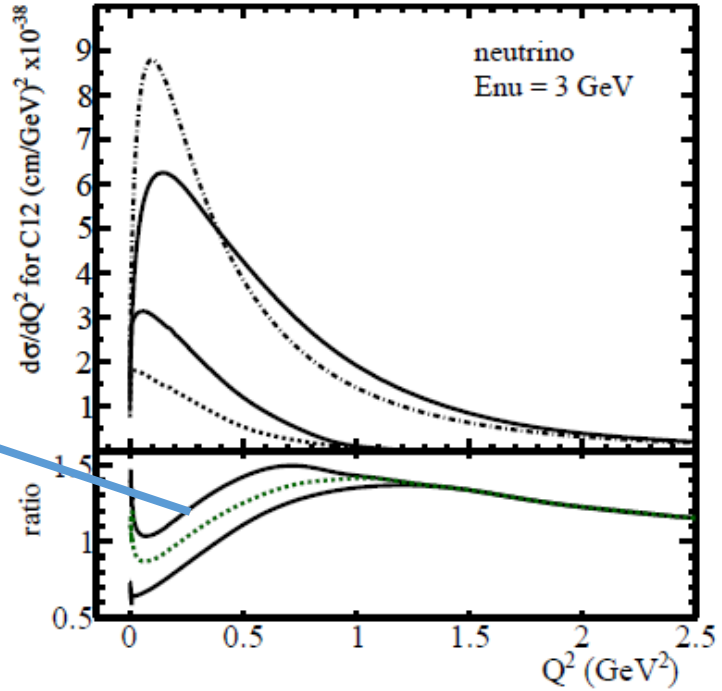
RPA corrections should disappear (ratio goes to 1.0) **at very large Q^2 values, because this is a collective effect which strength decreases when sizes larger than one nucleon are no longer being probed**. Hence in any realistic model, one should expect a qualitative Q^2 behavior similar to that exhibited by the QE_{RPA}/QE_{noRPA} ratio line depicted in the figure.: **low Q^2 suppression, followed by an enhancement that could even give rise to a net increase of the cross section, and finally all RPA effects should disappear for sufficiently high Q^2 values**

Also for photons, W, Z !



cannot be γ, *W, Z,* it should be V: ph-ph, ph-Δh interaction

QE_{RPA}/QE_{noRPA}



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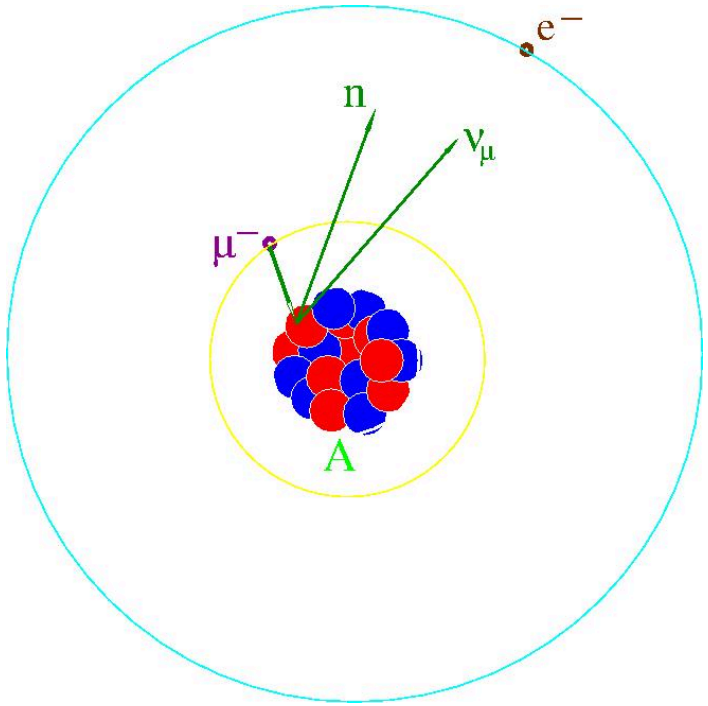
Inclusive muon capture in nuclei

[E. Oset et al., NPA 510 (1990) 591; J. Nieves et al., PRC 70 (2004) 055503; J.E. Amaro et al., EJPA 24 (2005) 343]



- Hydrogen-like atom, but $R_{\mu^-} \ll R_{e^-}$, since $m_\mu \gg m_e$
- There are screening and relativistic effects (solve Dirac equation)
- μ^- can be absorbed by the nucleus

$$(\mu^- A_Z)_{\text{bound}} \to \nu_\mu X \qquad [\mu^- p \to n\, \nu_\mu]$$

$$\boxed{\Gamma = -2\, Im\, \Pi_{\mu^-}}$$

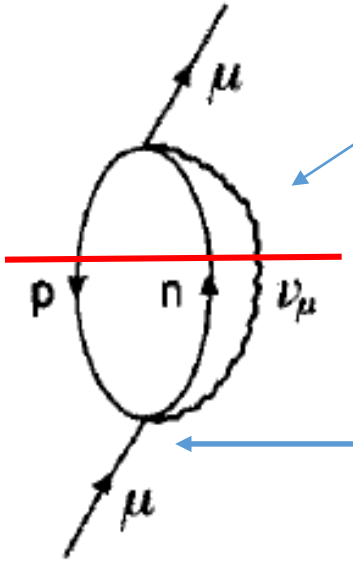
Leptonic current

$$L(x) = \sqrt{\tfrac{1}{2}}\, G\, J^\mu(x) L^+_\mu(x)$$

$$\tilde{L}^{\mu^+} \to \bar{u}_\nu(p_\nu)\gamma^\mu(1-\gamma_5)u_\mu(p_\mu)\,,$$

$$\tilde{J}^\mu \to \bar{u}_n(p_n)[g_V\gamma^\mu + i\frac{g_M}{2m_p}\sigma^{\mu\nu}q_\nu + g_A\gamma^\mu\gamma_5 + \frac{g_P}{m_\mu}q^\mu\gamma_5]u_p(p_p)$$

Hadron current



Lowest order contribution: ***imaginary part of the Lindhard function***

$$\Gamma = -2 \int \frac{d^3 p_\nu}{(2\pi)^3} \frac{2m_\nu}{2E_\nu} \frac{2m_\mu}{2E_\mu} \frac{2m_p}{2E_p} \frac{2m_n}{2E_n} \overline{\sum} \sum |T|^2 \, \mathrm{Im}\, \bar{U}(p_\nu - p_\mu)$$

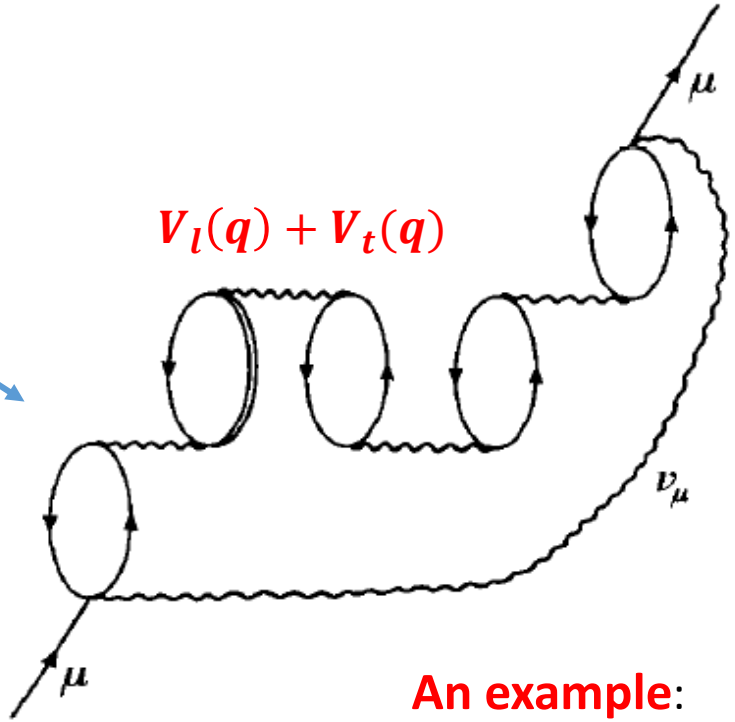
- ***Finite nuclei*: LDA (local density appx)**

$$\Gamma = \int d^3 r \, |\Phi_{1s}(\boldsymbol{r})|^2 \tilde{\Gamma}(\rho_p(\boldsymbol{r}), \rho_n(\boldsymbol{r}))$$

Strong renormalization effects:

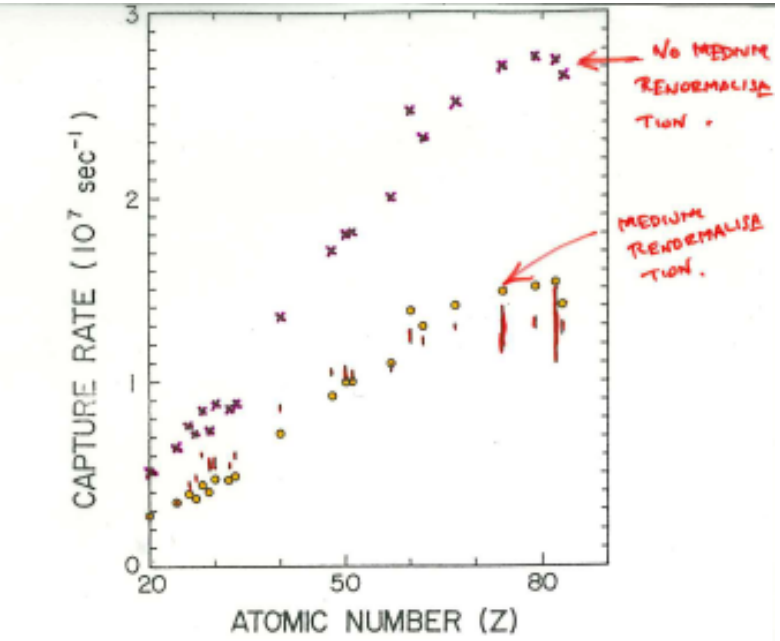
	Pauli $[10^4\, s^{-1}]$	RPA $[10^4\, s^{-1}]$	Exp $[10^4\, s^{-1}]$	$\left(\Gamma^{\rm Exp} - \Gamma^{\rm Th}\right)/\Gamma^{\rm Exp}$
^{12}C	5.42	3.21	3.78 ± 0.03	0.15
^{16}O	17.56	10.41	10.24 ± 0.06	−0.02
^{18}O	11.94	7.77	8.80 ± 0.15	0.12
^{23}Na	58.38	35.03	37.73 ± 0.14	0.07
^{40}Ca	465.5	257.9	252.5 ± 0.6	−0.02
^{44}Ca	318	189	179 ± 4	−0.06
^{75}As	1148	679	609 ± 4	−0.11
^{112}Cd	1825	1078	1061 ± 9	−0.02
^{208}Pb	1939	1310	1311 ± 8	0.00

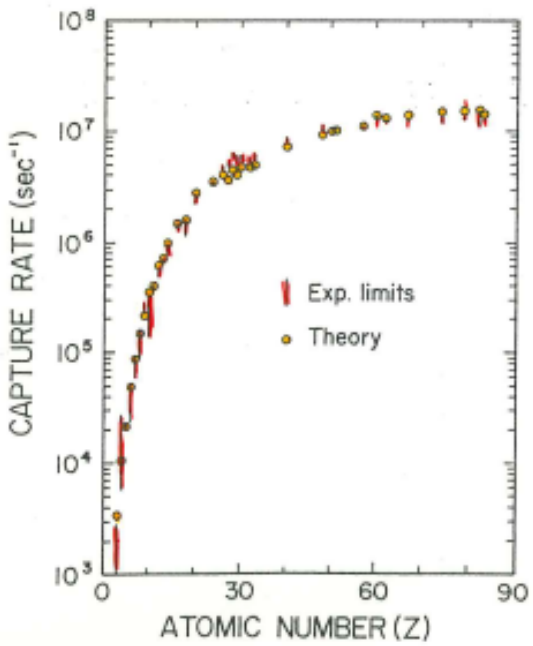
$V_l(q) + V_t(q)$



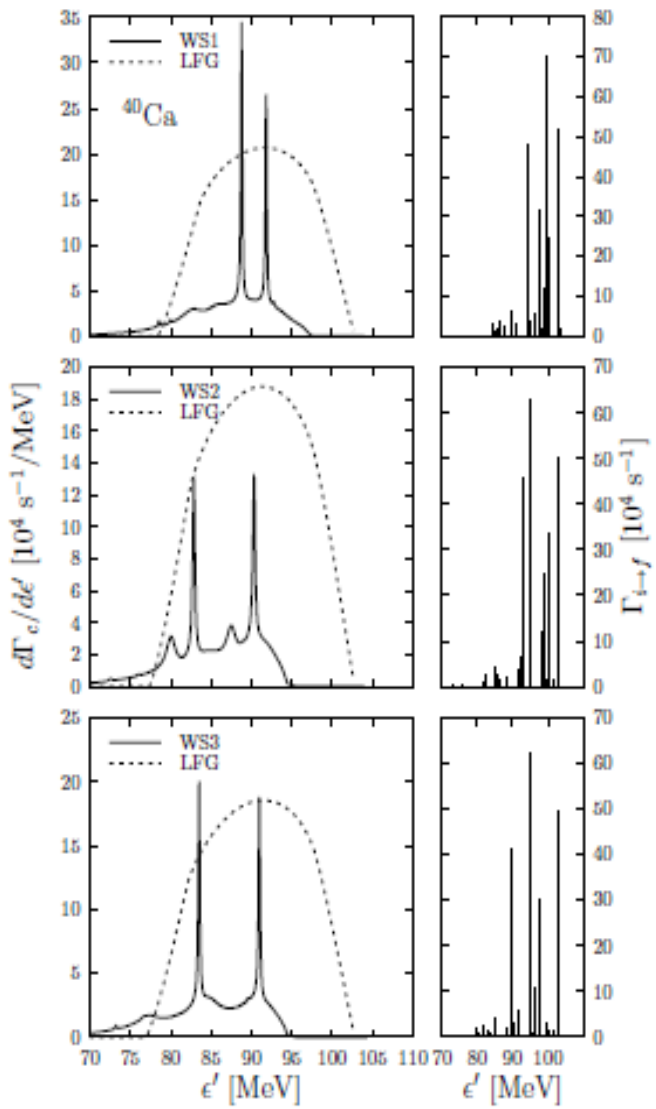
An example:

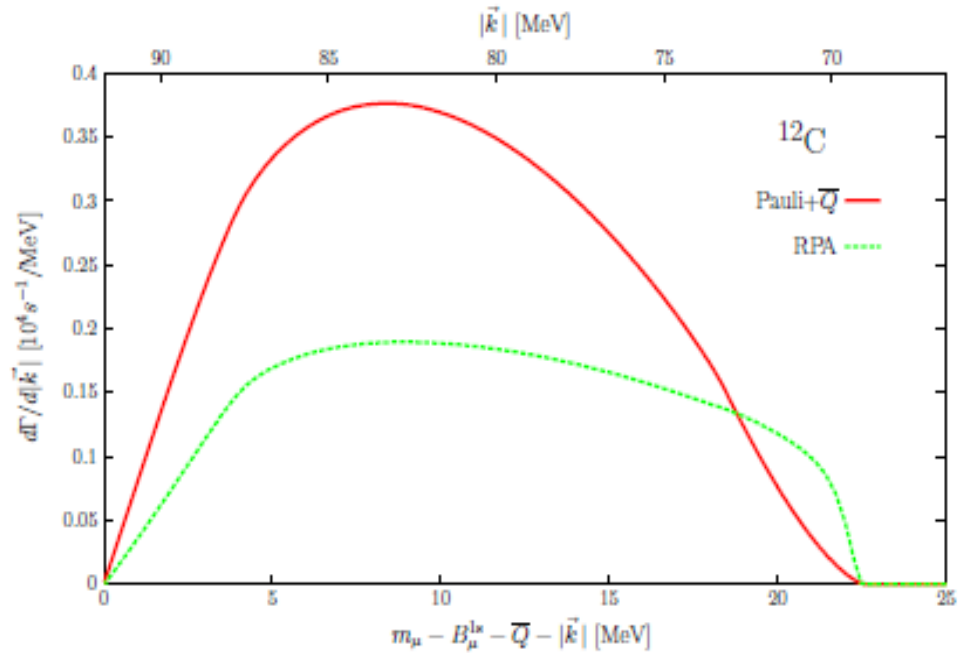
$$g_A^2 \, \mathrm{Im}\, 2\bar{U} \to g_A^2 \left\{ \frac{1}{3} \frac{\mathrm{Im}\, 2\bar{U}}{|1 - UV_\ell|^2} + \frac{2}{3} \frac{\mathrm{Im}\, 2\bar{U}}{|1 - UV_t|^2} \right\}$$





Shell model





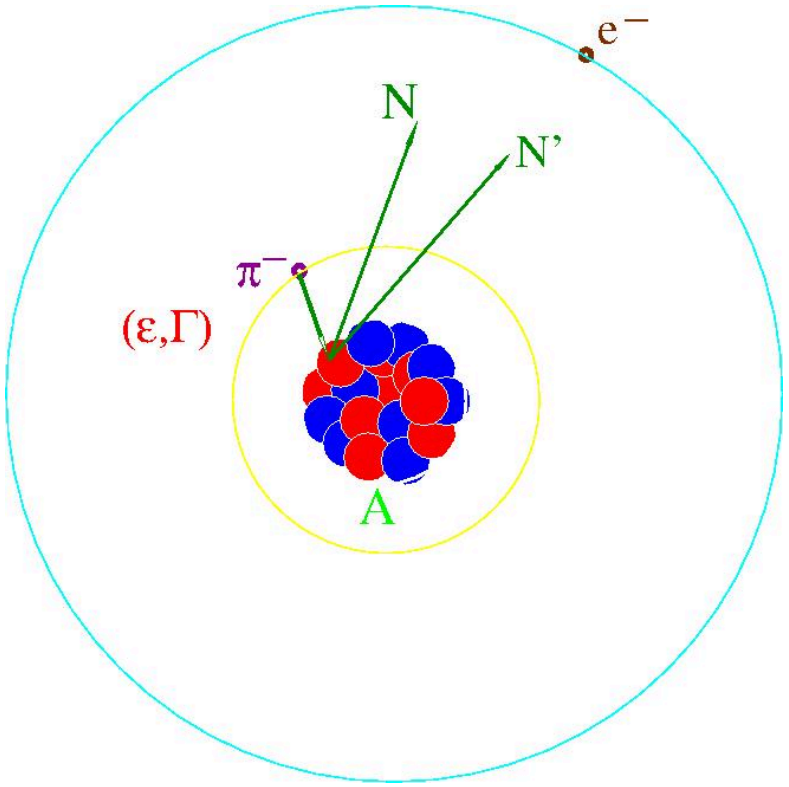
	discrete	total	LFG	%
WS1	29.10	37.12	36.73	−1.1
WS2	27.79	33.79	34.90	3.3
WS3	26.28	32.73	35.03	7.0

Nuclear Effects? SM vs FG

(Amaro, Nieves, Maieron and Valverde EPJA 24, 343)

Pionic Atoms

[J. Nieves et al., NPA 554 (1993) 509]



- Precise experimental measurements (spectroscopy techniques): **shifts $\varepsilon = B_{exp} - B_{em}$ and widths Γ** → information on the pion-nucleus interaction

- $\Pi(q^0, \vec{q}, \rho(r)) = 2q^0\, V_{opt}(\mathrm{r})$, we solve the Klein-Gordon equation
- Impulse approximation

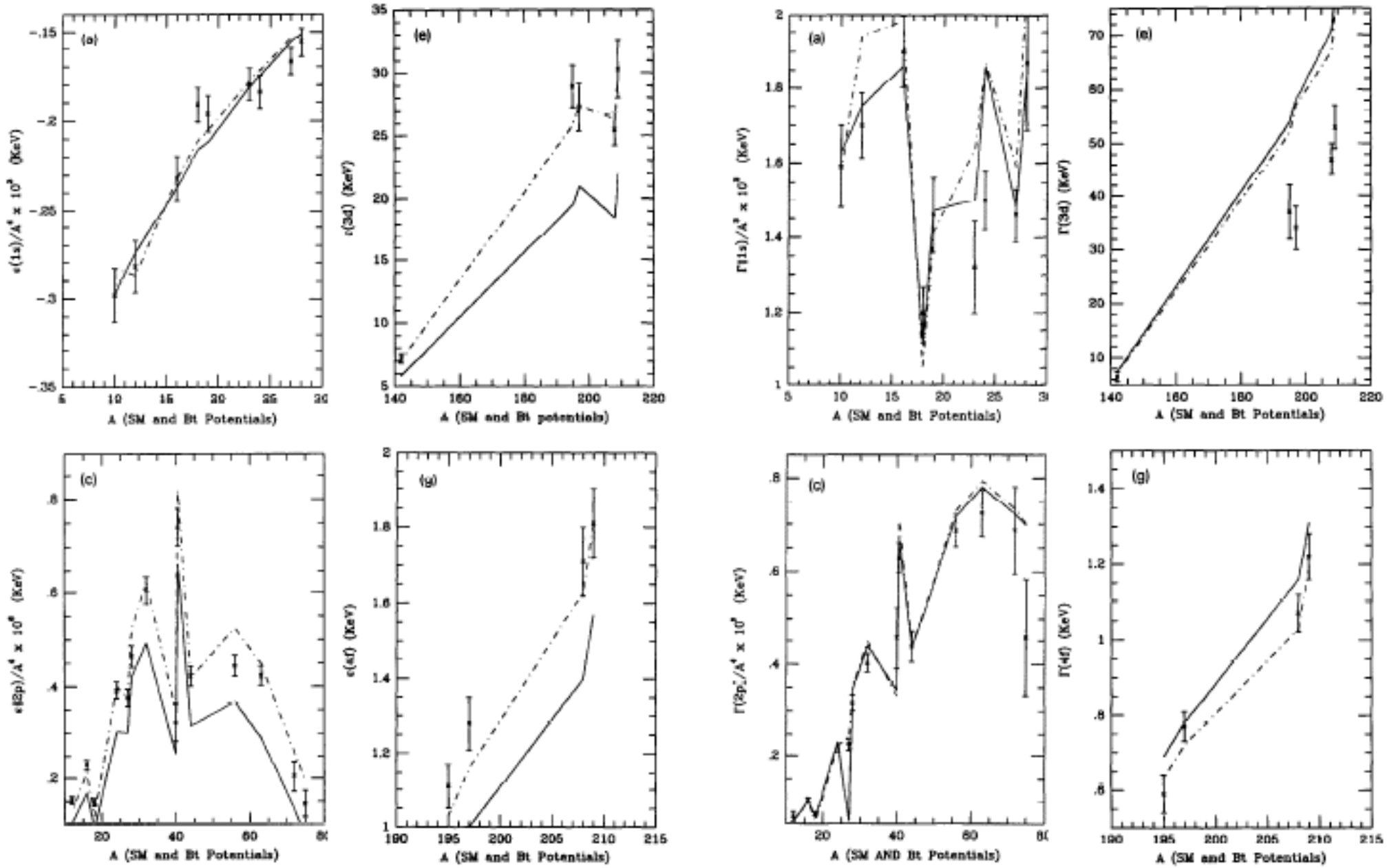
$$\Pi(q^0, \vec{q}) \approx T(q^0, \vec{q})\rho \Rightarrow \Gamma = 0$$

- Widths come from absorption by two nucleons: $\pi^- NN \rightarrow NN$ that give rise to complex terms, proportional to ρ^2 in $\Pi(q^0, \vec{q}, \rho(r))$.

- There exists experimental information on 1S and 2P levels in light nuclei and 3D and 4F in heavy nuclei. Phenomenological potentials fail to describe these data: **problem of the anomalies in pionic atoms.**

Precisions of the order of 5% through the whole periodic table.

Range: eV to 20-30 KeV



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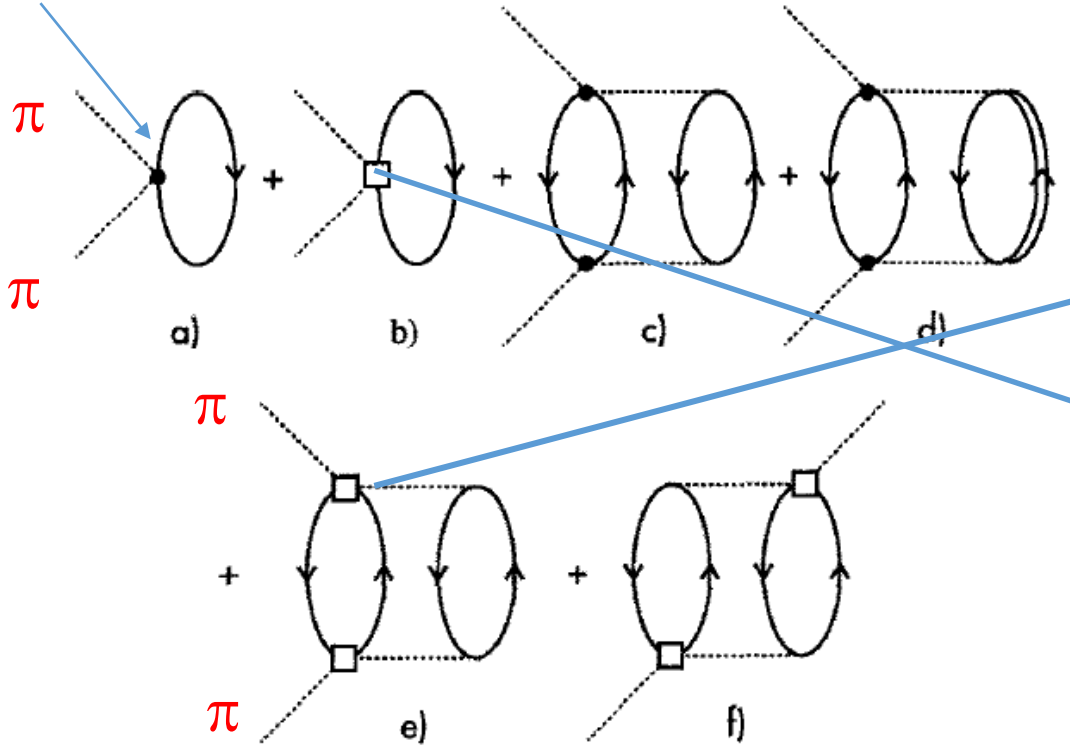
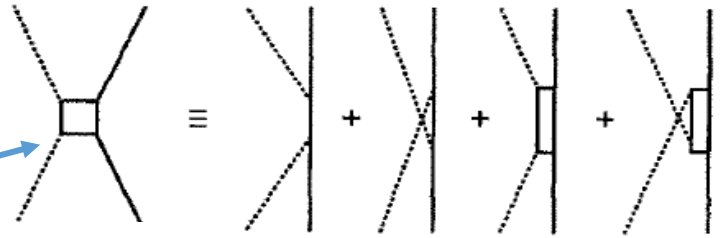
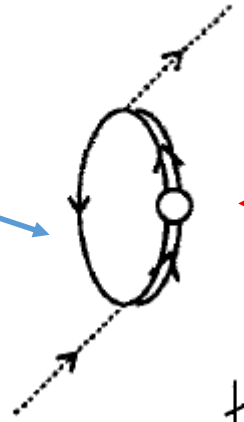
S-wave



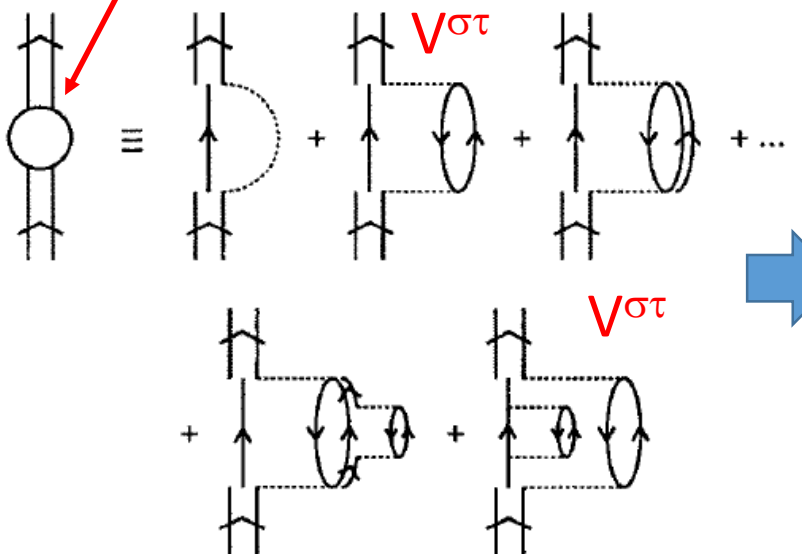
P-wave

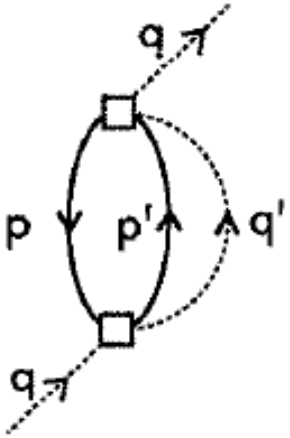


one of the contributions

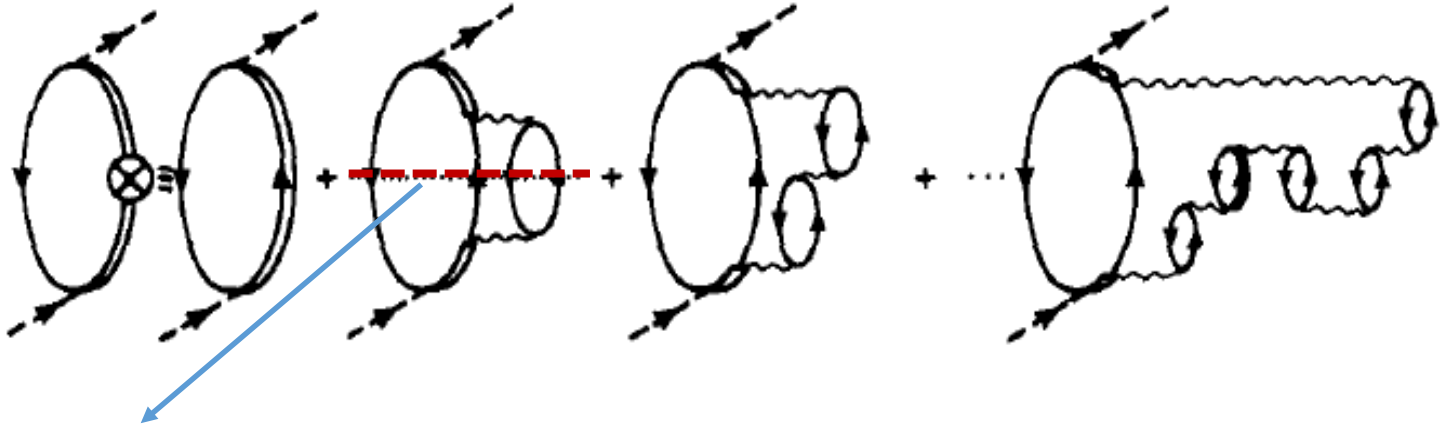


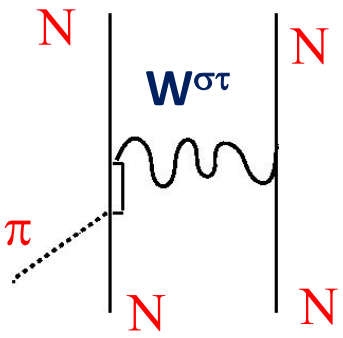
Δ selfenergy: Σ_Δ





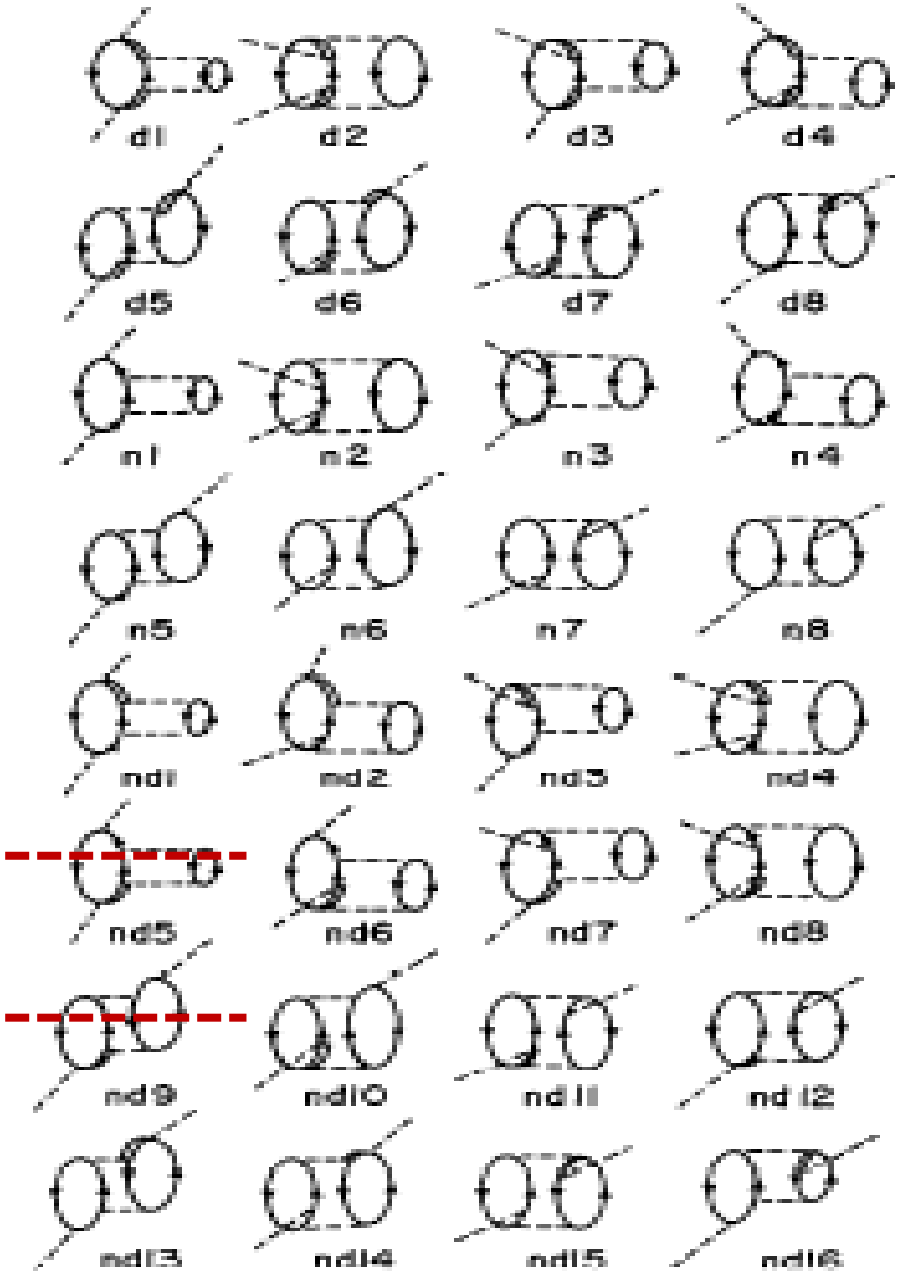
Many body (density) expansion in the number of hole lines!





$\pi NN \rightarrow NN$

2p2h



$$2\omega V_1^{(s)}(\boldsymbol{r}) = -4\pi[(1+\varepsilon)(b_0+\Delta b_0(\boldsymbol{r}))f(T)\rho + (1+\varepsilon)b_1(\rho_n-\rho_p)$$
$$+ i(\mathrm{Im}\, B_0\,(1+\tfrac{1}{2}\varepsilon)2(\rho_p^2+\rho_p\rho_n) + \mathrm{Im}\, B_0^Q(T)(1+\tfrac{1}{2}\varepsilon)\rho^2)]$$

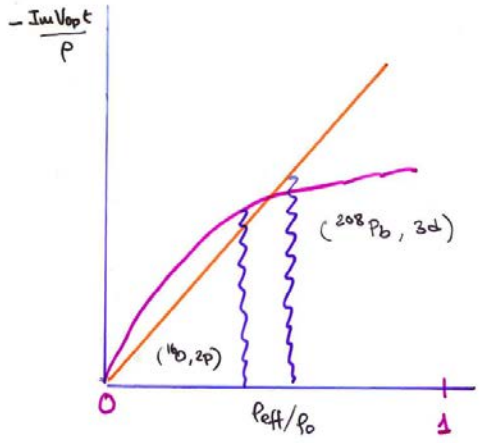
$$2\omega V_{\mathrm{opt}}^{(p)}(\boldsymbol{r}) = 4\pi\frac{M_N}{s}\left[\nabla\frac{P(r)}{1+4\pi g' P(r)}\nabla - \tfrac{1}{2}\varepsilon\Delta\left(\frac{P(r)}{1+4\pi g' P(r)}\right)\right]$$

Juan Nieves, IFIC (CSIC & UV)

Theoretical potential

$\mathrm{Im}\, V_{opt}^{STANDARD} \propto \rho^2$

$\mathrm{Im}\, V_{opt}^{NEW} \propto \frac{\rho_0}{2.72}\, \rho \,\mathrm{arctg}\, \frac{2.72\,\rho}{\rho_0}$ + L.L. Effect.



low densities

ANOMALIES: SIMULTANEOUS DESCRIPTION OF BOTH LIGHT AND HEAVY ATOMS

$\Downarrow$

$\Gamma_{2p} \uparrow$ and $\Gamma_{3d} \downarrow$

$$\left.\frac{\Gamma_{3d}^{HEAVY}}{\Gamma_{2p}^{LIGHT}}\right|_{NEW} < \left.\frac{\Gamma_{3d}^{Heavy}}{\Gamma_{2p}^{light}}\right|_{Standard}$$

QUENCHING PROPERTIES

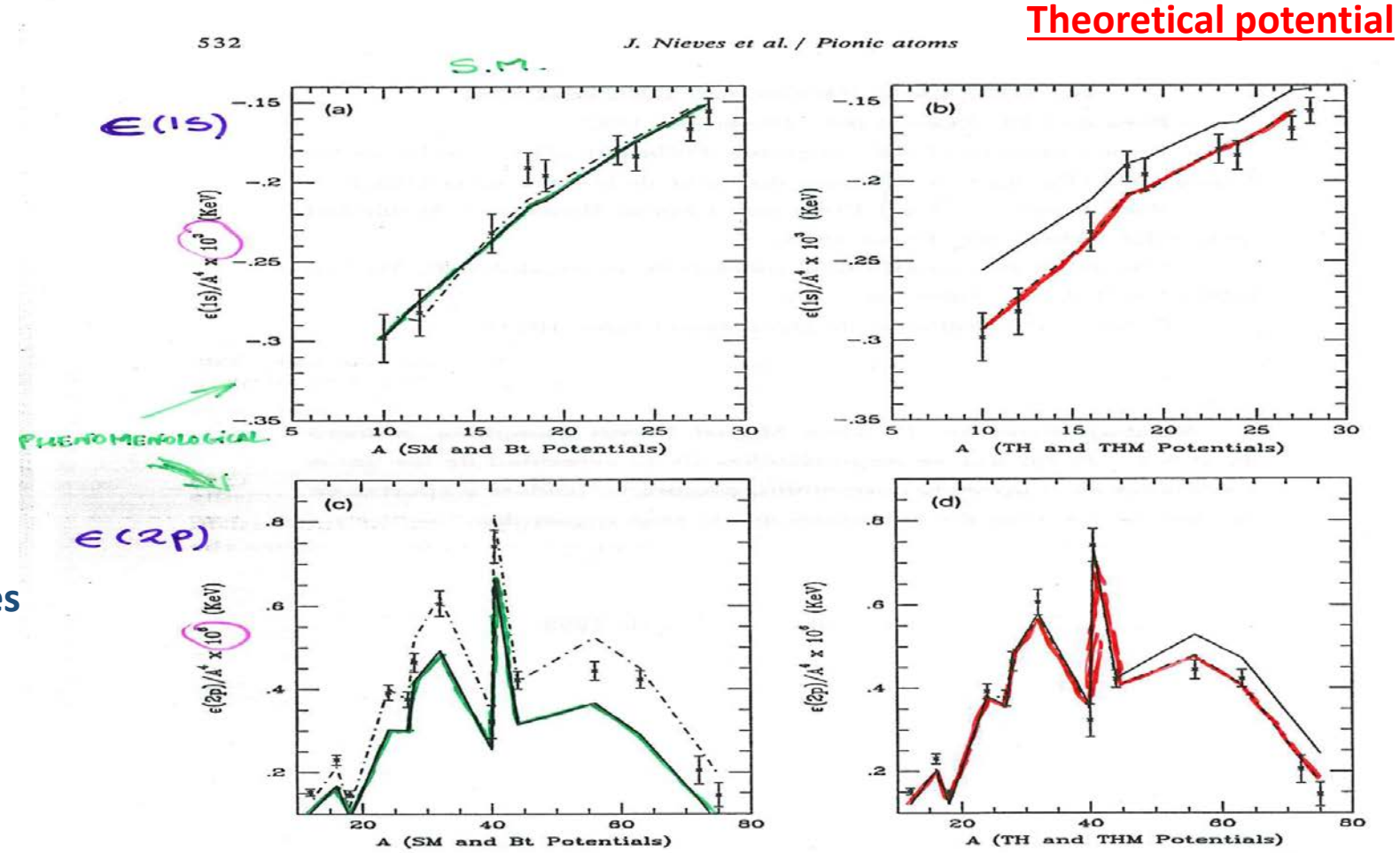


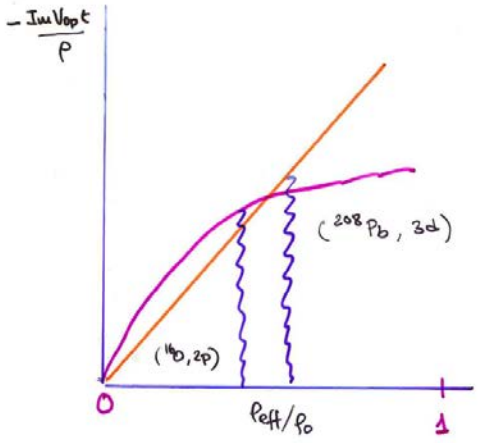
Fig. 8. Results for the shifts of different pionic states with different potentials. (a), (c), (e), (g): SM potential (solid line) of ref. [42]) given by eqs. (57) and (58), and Bt potential (dashed line) of ref. [1]) given by eqs. (59) and (60); (b), (d), (f), (h): Our potentials, theoretical potential (TH, solid line) and the semiphenomenological potential (THM, dashed line). The experimental values and the errors are statistical averages [see ref. [67]) for detailed values of the experimental results]. The lines between data points are just to guide the eye.

THEORETICAL.

Theoretical potential

$\mathrm{Im}\, V_{opt}^{STANDARD} \propto \rho^2$

$\mathrm{Im}\, V_{opt}^{NEW} \propto \frac{\rho_0}{2.72}\, \rho \arctan \frac{2.72\,\rho}{\rho_0}$ + L.L. Effect.



ANOMALIES: SIMULTANEOUS DESCRIPTION OF BOTH LIGHT AND HEAVY ATOMS

$\Downarrow$

$\Gamma_{2p} \uparrow$ and $\Gamma_{3d} \downarrow$

$$\left.\frac{\Gamma_{3d}^{HEAVY}}{\Gamma_{2p}^{LIGHT}}\right|_{NEW} < \left.\frac{\Gamma_{3d}^{Heavy}}{\Gamma_{2p}^{light}}\right|_{Standard}$$

QUENCHING PROPERTIES

medium densities

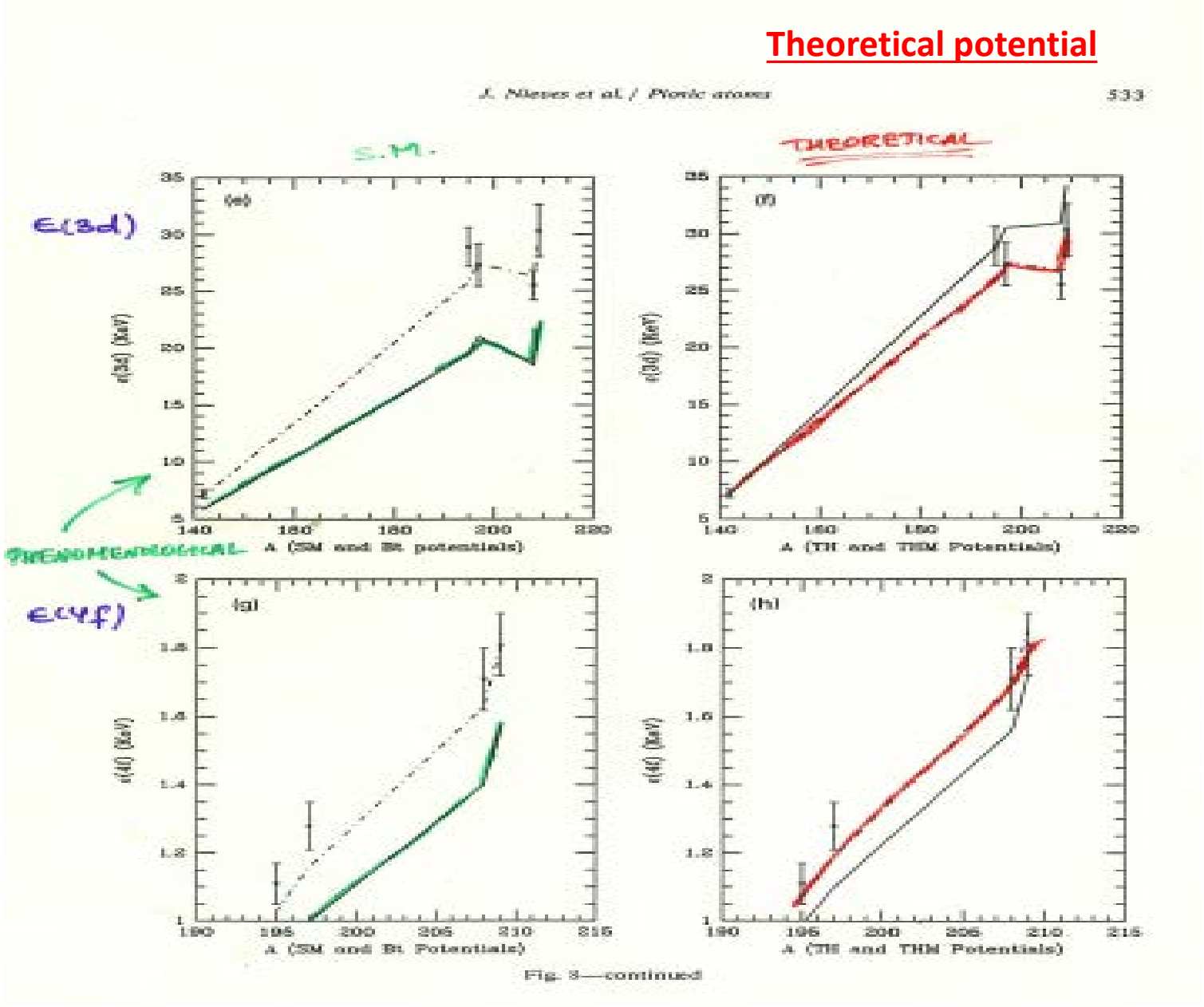


Fig. 3—continued

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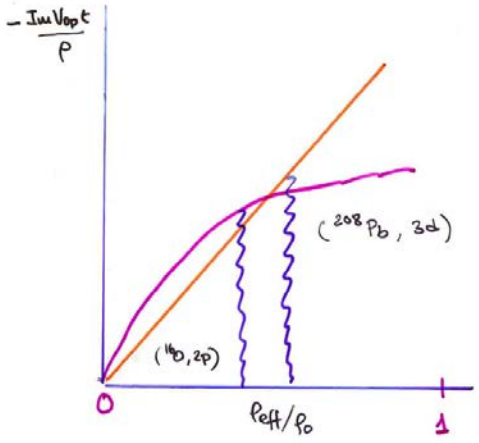
$\mathrm{Im}\, V_{opt}^{STANDARD} \propto \rho^2$

$\mathrm{Im}\, V_{opt}^{NEW} \propto \frac{\rho_0}{2.72}\, \rho \,\mathrm{arctg}\, \frac{2.72\,\rho}{\rho_0}$ + L.L. Effect.



ANOMALIES: SIMULTANEOUS DESCRIPTION OF BOTH LIGHT AND HEAVY ATOMS

$\Downarrow$

$\Gamma_{2p} \uparrow$ and $\Gamma_{3d} \downarrow$

$$\left.\frac{\Gamma_{3d}^{HEAVY}}{\Gamma_{2p}^{LIGHT}}\right|_{NEW} < \left.\frac{\Gamma_{3d}^{Heavy}}{\Gamma_{2p}^{light}}\right|_{Standard}$$

QUENCHING PROPERTIES

medium densities

Theoretical solution to the problem of the anomalies in pionic atoms !

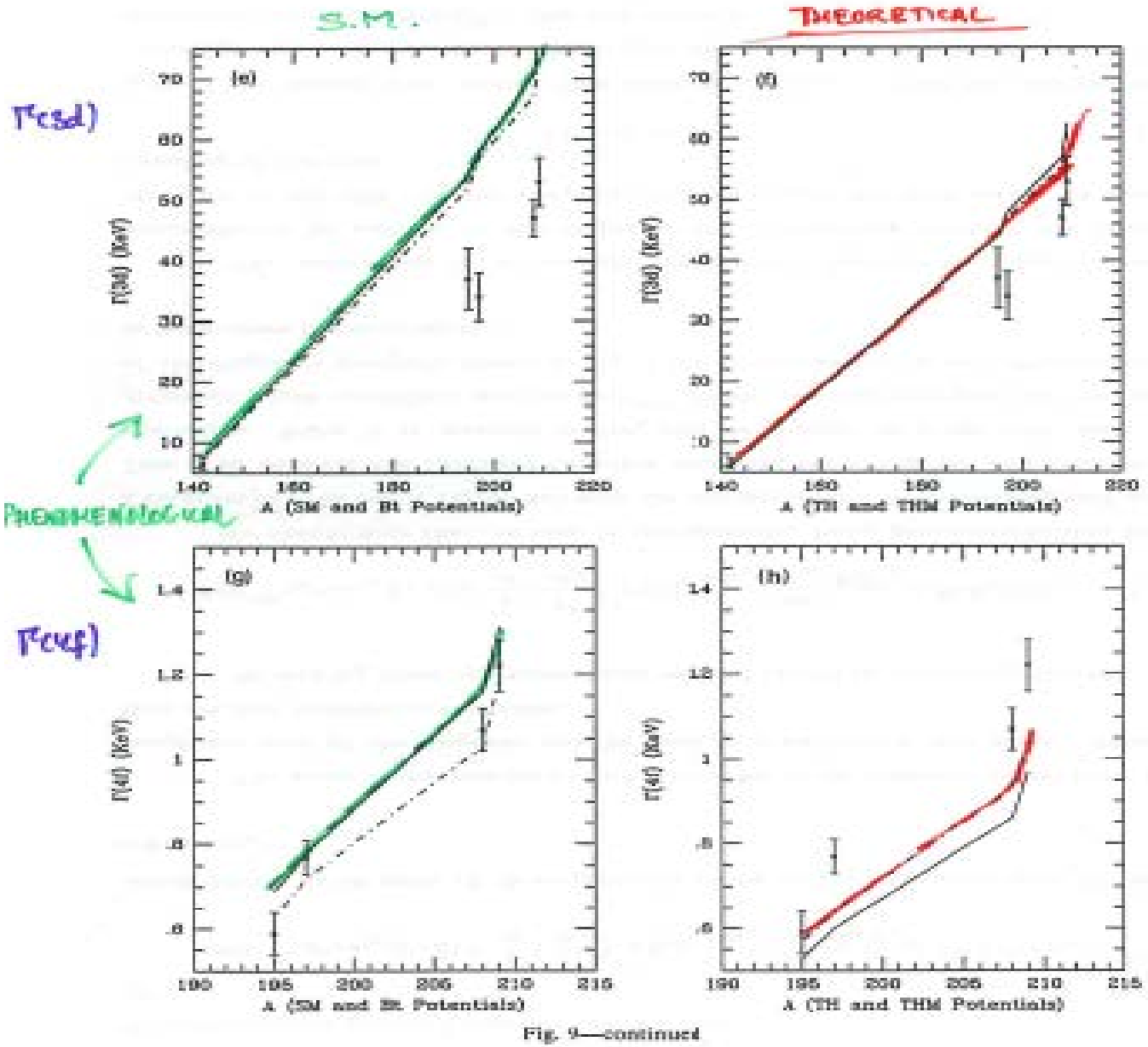


Fig. 9—continued

Theoretical potential

$\pi^{\pm}$ – nucleus reactions

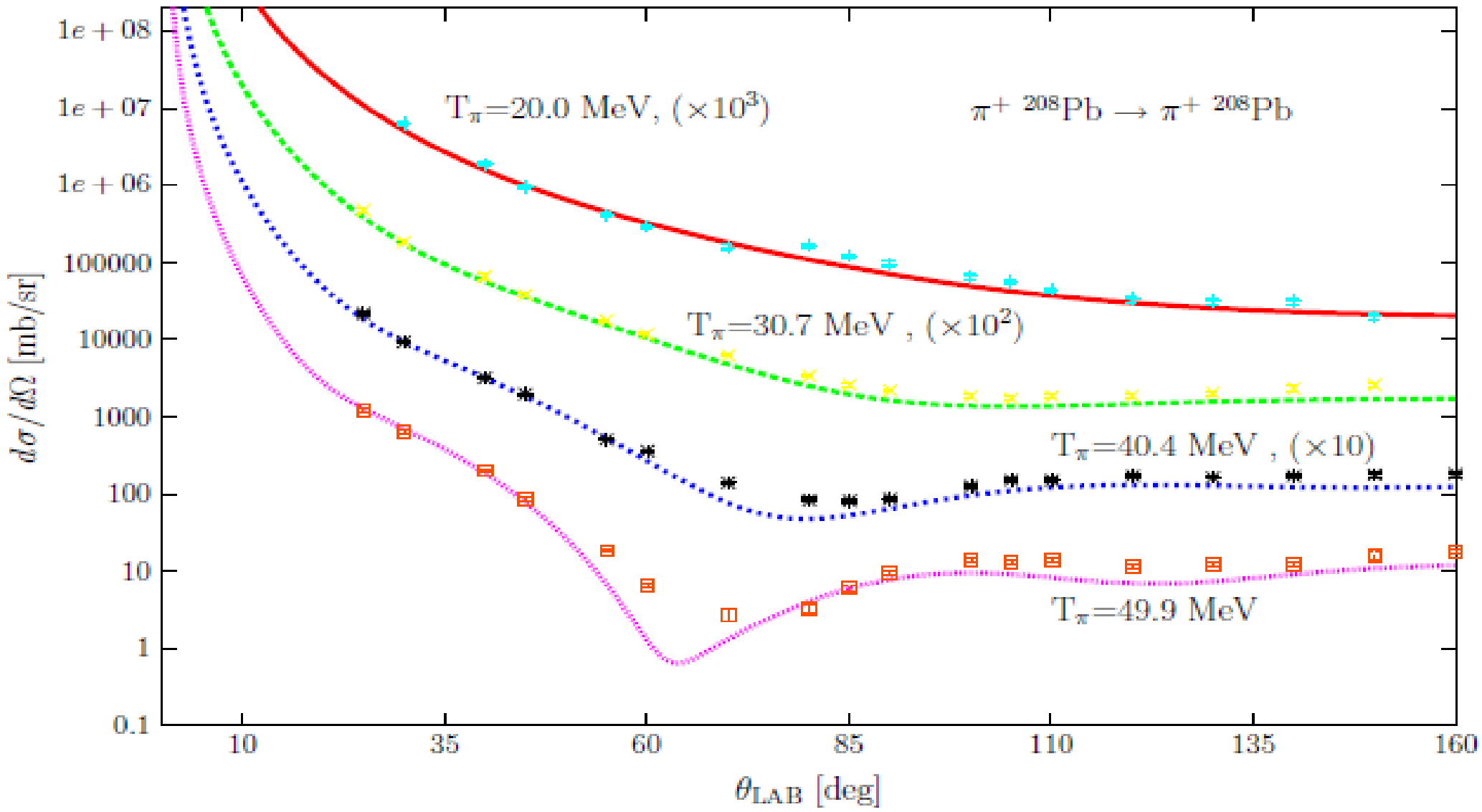
- $\pi^{\pm}$ – nucleus reactions [J. Nieves et al., NPA 554 (1993) 554]
  - ✓ $\pi^{\pm} A_Z \rightarrow \pi^{\pm} A_Z$ [elastic]
  - ✓ $\pi^{\pm} A_Z \rightarrow \pi' X$ [quasielastic]
  - ✓ $\pi^{\pm} A_Z \rightarrow X$ (no pions) [absorption]

- Determination of neutron distributions from pionic atom data [C. García-Recio et al., NPA 547 (1992) 473]
-
- Radiative pion capture [H.C. Chiang et al., NPA 510 (1990) 573]

  $$(\pi^- A_Z)_{\text{bound}} \rightarrow \gamma X$$

- Chiral symmetry restoration [C. García-Recio et al., PLB 541 (2002) 64]

$$f_\pi(\rho)/f_\pi \rightarrow 0, \rho >> 0$$

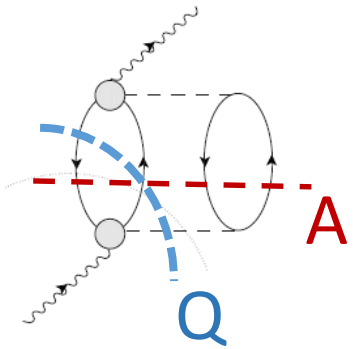


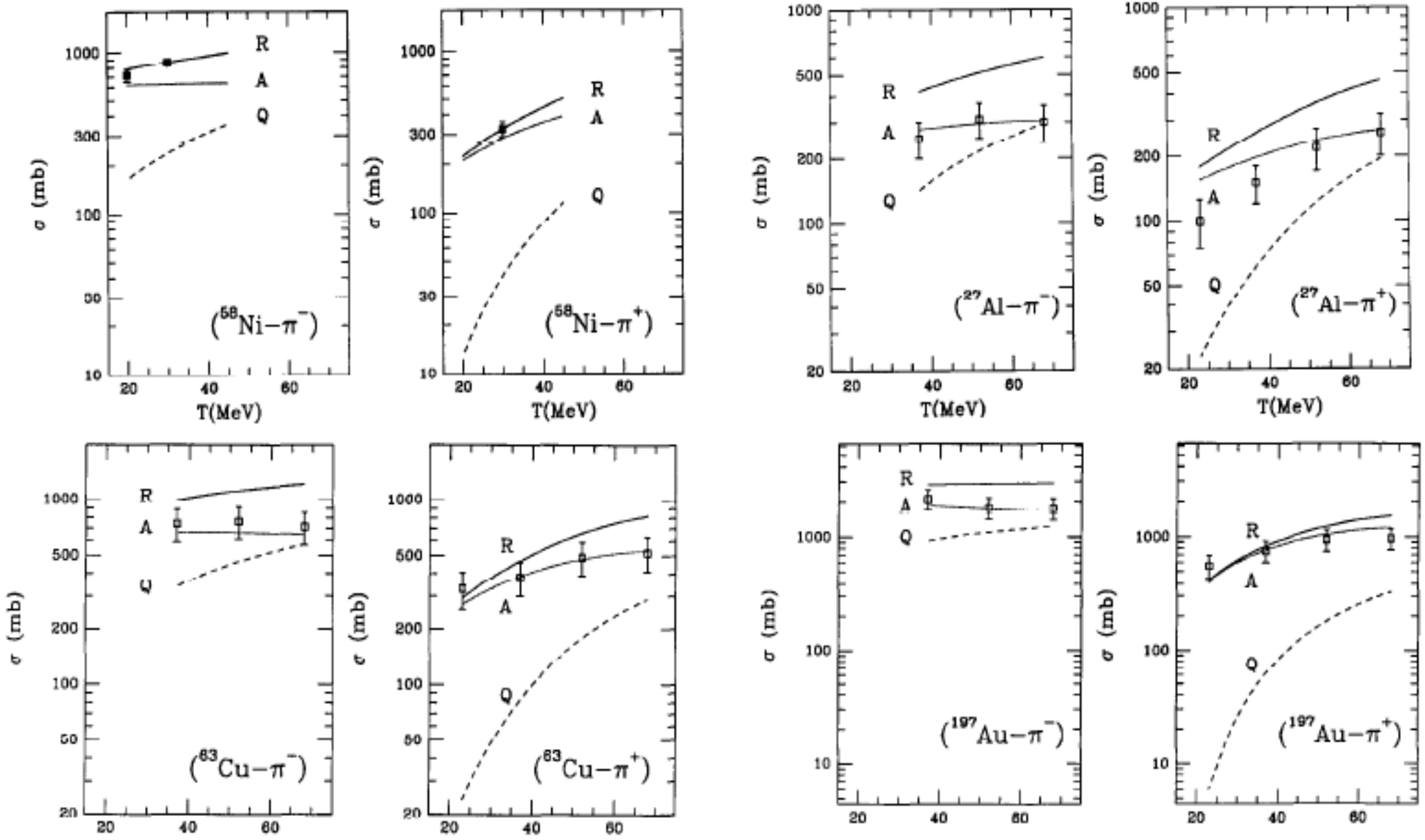
Juan Nieves, IFIC (CSIC & UV)

Q+R = A

Absorption + Quasielastic= Reaction

Q= pions which have changed either charge, energy or momentum





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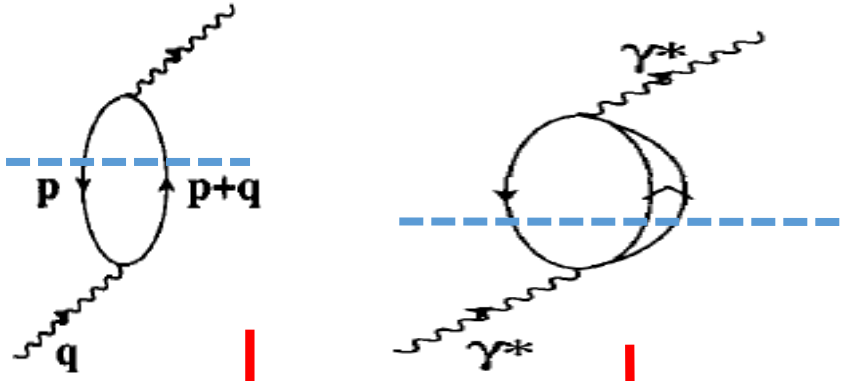
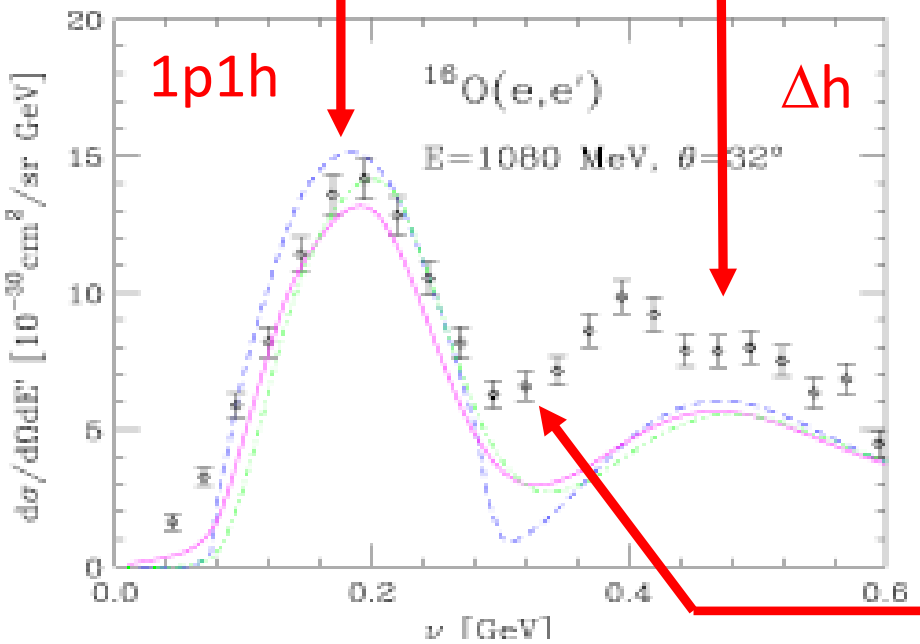
Inclusive electron-nucleus scattering

[A. Gil et al.,NPA 627 (1997) 543; NPA 627 (1997) 599]





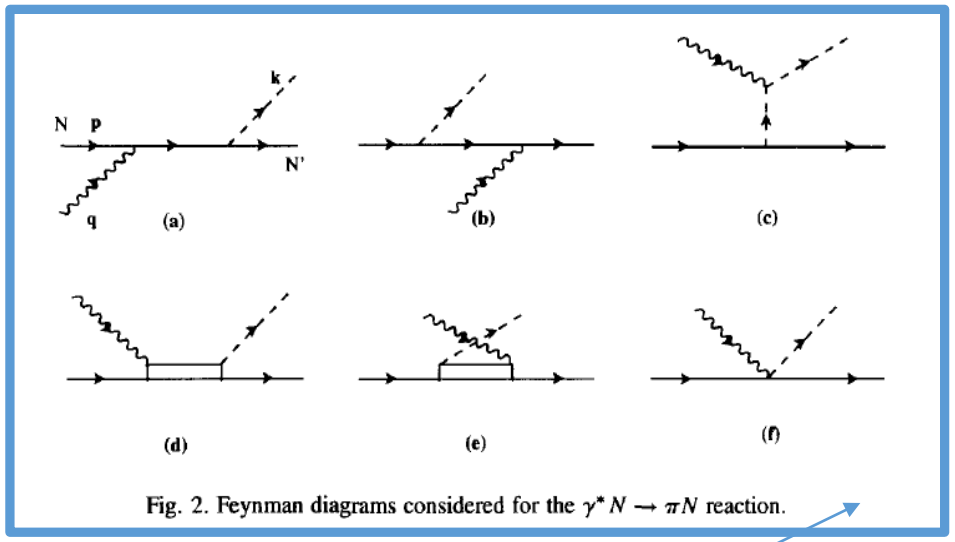
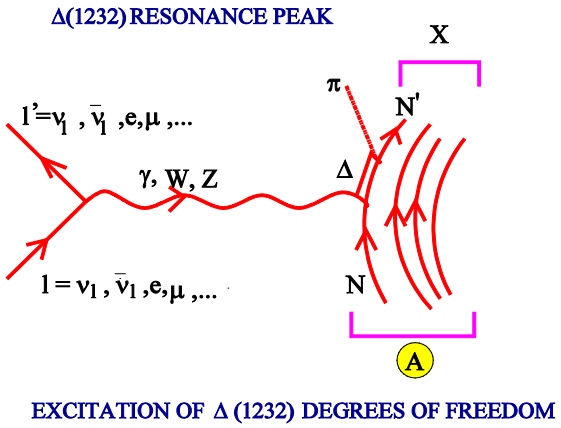
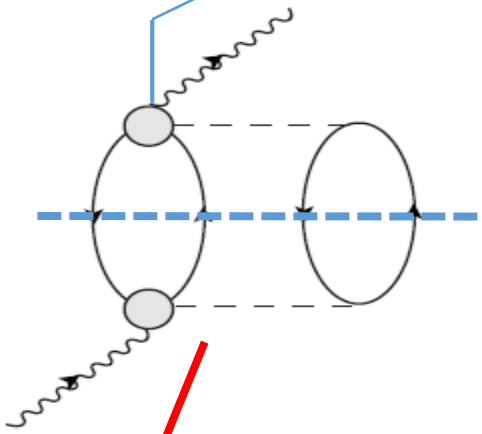


Fig. 2. Feynman diagrams considered for the $\gamma^* N \to \pi N$ reaction.





Dip region 2p2h

Juan Nieves, IFIC (CSIC & UV)

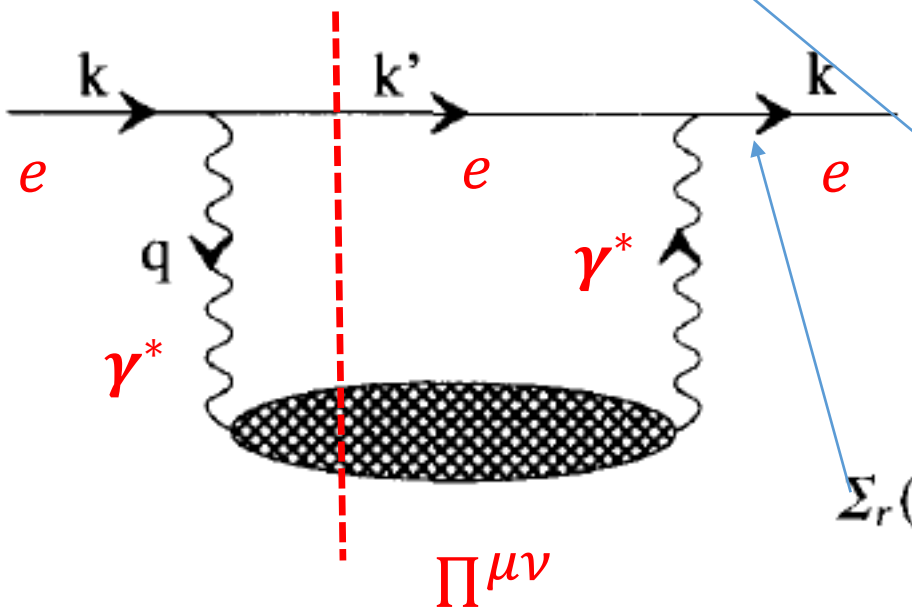
$$e + A_Z \to e' X$$

$$\frac{d^2\sigma}{d\Omega'_e dE'_e} = \frac{\alpha^2}{q^4}\frac{|\boldsymbol{k}'|}{|\boldsymbol{k}|} L^{\mu\nu} W_{\mu\nu}$$

$$L_{\mu\nu}(e,e') = 2\left(k'_{e\mu}k_{e\nu} + k'_{e\nu}k_{e\mu} + \frac{q^2}{2}g_{\mu\nu}\right)$$

$$W^{\mu\nu} = -\frac{1}{\pi e^2}\int d^3r \frac{1}{2}\left(\mathrm{Im}\,\Pi^{\mu\nu} + \mathrm{Im}\,\Pi^{\nu\mu}\right)$$

virtual photon selfenergy in the nuclear medium



$$\Gamma(k) = -2\frac{m_e}{E_e}\mathrm{Im}\,\Sigma$$

$$\Sigma_r(k) = ie^2\int \frac{d^4q}{(2\pi)^4}\bar{u}_r(k)\gamma_\mu\frac{(\not{k}' + m_e)}{k'^2 - m_e^2 + i\epsilon}\gamma_\nu u_r(k)\frac{\Pi_\gamma^{\mu\nu}(q)}{(q^2+i\epsilon)^2}$$

$$d\sigma = \Gamma(k)\,dt\,dS = -\frac{2m}{E_e}\mathrm{Im}\,\Sigma\,dl\,dS = -\frac{2m}{|\boldsymbol{k}|}\mathrm{Im}\,\Sigma\,d^3r \Rightarrow \boxed{\sigma = -\int d^3r\frac{2m}{|\boldsymbol{k}|}\mathrm{Im}\,\Sigma(k,\rho(\boldsymbol{r}))}$$

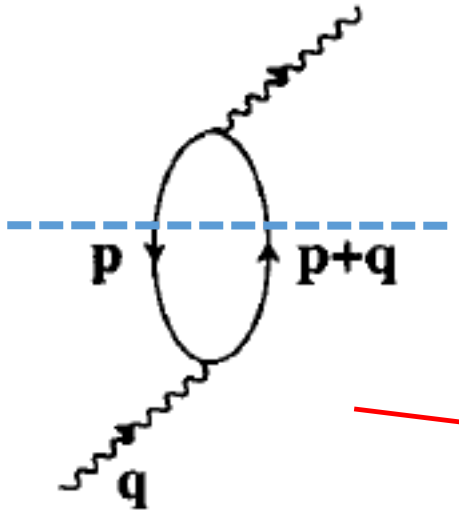
$$\Sigma(k) \to 2i\mathrm{Im}\,\Sigma(k)\Theta(k^0),$$
$$\Xi(k') \to 2i\mathrm{Im}\,\Xi(k')\Theta(k'^0),$$
$$\Pi^{\mu\nu}(q) \to 2i\mathrm{Im}\,\Pi^{\mu\nu}(q)\Theta(q^0),$$

where

$$\Xi(k') = \frac{1}{k'^2 - m_e^2 + i\epsilon}$$

Technically Cutkowsky's rules are used to obtain the imaginary part of the many body Feynman diagrams

Quasielastic peak



(lowest oder: imaginary part of the Lindhard function)

$$\mathrm{Im}\,\Pi^{00} = \tfrac{1}{2}\mathrm{Im}\,\bar{U}(q,\rho)\,\langle \mathrm{Tr}(V^0 V^{\dagger 0})\rangle,$$

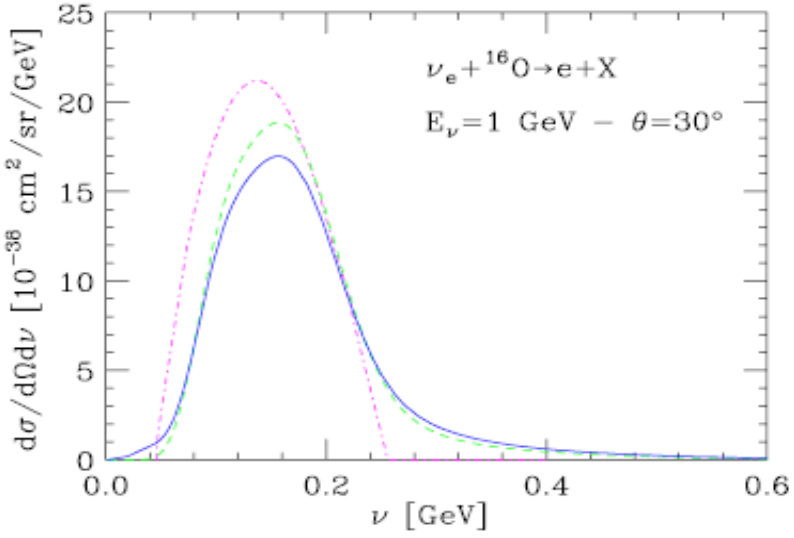
$$\mathrm{Im}\,\Pi^{xx} = \tfrac{1}{2}\mathrm{Im}\,\bar{U}(q,\rho)\,\langle \mathrm{Tr}(V^x V^{\dagger x})\rangle.$$

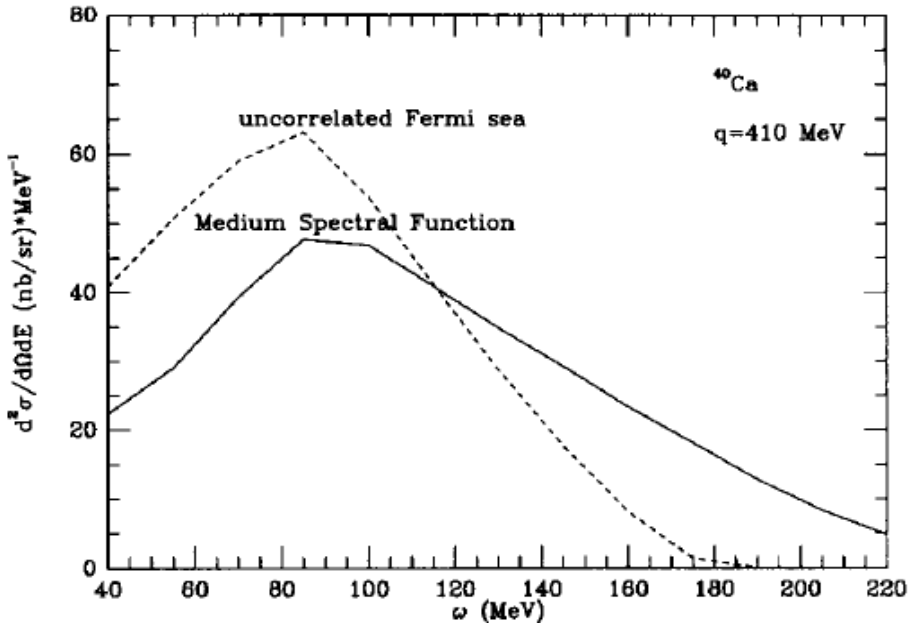
neglecting Fermi motion

$$p^2 = (p+q)^2 = M^2 \rightarrow q^2 = -2pq \rightarrow q^0 = -\,{}^{q^2}/_{2M}$$

Cutkowsky's rules

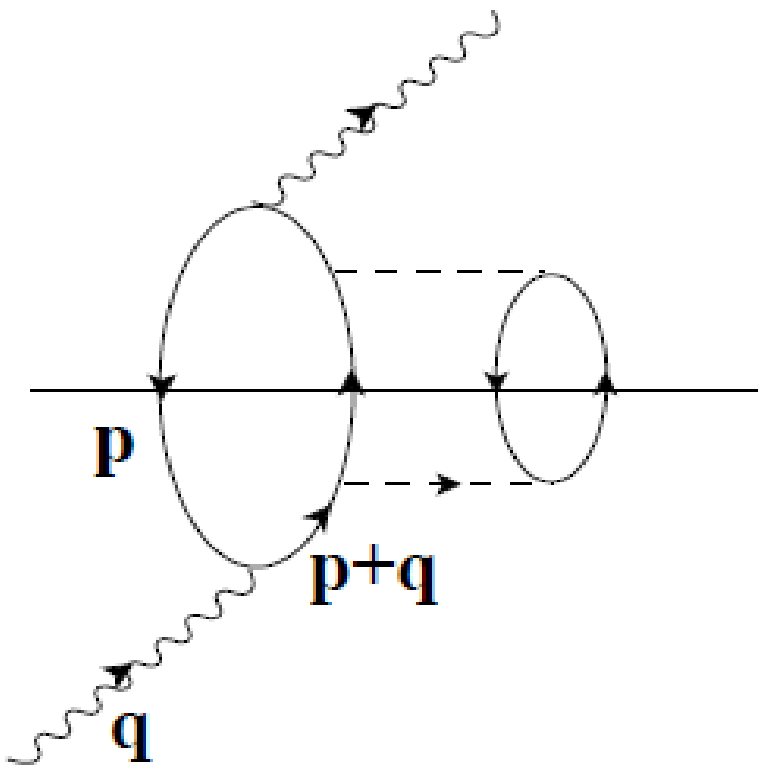
spectral function ?





Juan Nieves, IFIC (CSIC & UV)

- **Spectral Function (SF) + Final State Interaction (FSI):** dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the ph excitation



– Change of nucleon dispersion relation:
  * hole ⇒ Interacting Fermi sea (SF)
  * particle ⇒ Interaction of the ejected nucleon with the final nuclear state (FSI)

$$G(p) \to \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \vec{p}\,)}{p^0 - \omega - i\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega, \vec{p}\,)}{p^0 - \omega + i\epsilon}$$

The hole and particle spectral functions are related to nucleon self-energy $\boxed{\Sigma}$ in the medium,

$$S_{p,h}(\omega,\vec{p}\,) \;=\; \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{[\omega^2-\vec{p}^{\,2}-M^2-\mathrm{Re}\Sigma(\omega,\vec{p}\,)]^2+[\mathrm{Im}\Sigma(\omega,\vec{p}\,)]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

To take into account SF+FSI → replace $\mathrm{Im}\bar{U}_R^N(q)$ by the response function:

$$-\frac{1}{2\pi}\int_0^{+\infty} dp p^2 \int_{-1}^{+1} dx \int_{\mu-q^0}^{\mu} d\omega \mathbf{S_h}(\omega,\vec{\mathbf{p}})\mathbf{S_p}(\mathbf{q^0}+\omega,\mathbf{t})$$

with $t^2 = \vec{p}^{\,2} + \vec{q}^{\,2} + 2|\vec{p}\,||\vec{q}\,|x$.

This nuclear effect is additional to those due to RPA (long range) correlations !!

$$S_{p,h}(\omega,\vec{p}\,) \;=\; \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{\left[\omega^2-\vec{p}^{\,2}-M^2-\mathrm{Re}\Sigma(\omega,\vec{p}\,)\right]^2+\left[\mathrm{Im}\Sigma(\omega,\vec{p}\,)\right]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

For non interacting fermions $\boxed{\Sigma = 0}$,

$$S_p(\omega,\vec{p}\,) \;=\; \frac{\theta(|\vec{p}\,|-k_F)}{2E(\vec{p}\,)}\delta(\omega-E(\vec{p}\,))$$

$$S_h(\omega,\vec{p}\,) \;=\; \frac{\theta(k_F-|\vec{p}\,|)}{2E(\vec{p}\,)}\delta(\omega-E(\vec{p}\,))$$

and only Pauli blocking is incorporated!!

The simplest description ⇒ relativistic Fermi Gas with non interacting fermions $\boxed{\Sigma = 0}$,

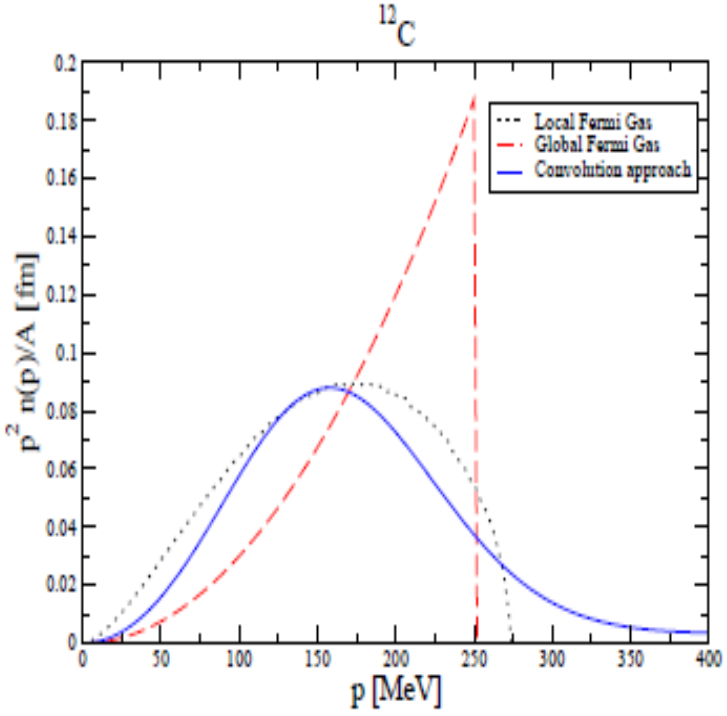
$$S_p(\omega,\vec{p}) = \frac{\theta(|\vec{p}| - k_F)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

$$S_h(\omega,\vec{p}) = \frac{\theta(k_F - |\vec{p}|)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

and only Pauli blocking is incorporated!!

Local vs Global Fermi Gas ?

$$k_F^{p,n}(r) = \left[3\pi^2\rho^{p,n}(r)\right]^{1/3} \text{ vs } k_F^{p,n} = \text{cte ?}$$



Local vs Global Fermi Gas ?

$$k_F(r) = \left[3\pi^2\rho(r)/2\right]^{1/3} \text{ vs } k_F = \text{cte ?}$$

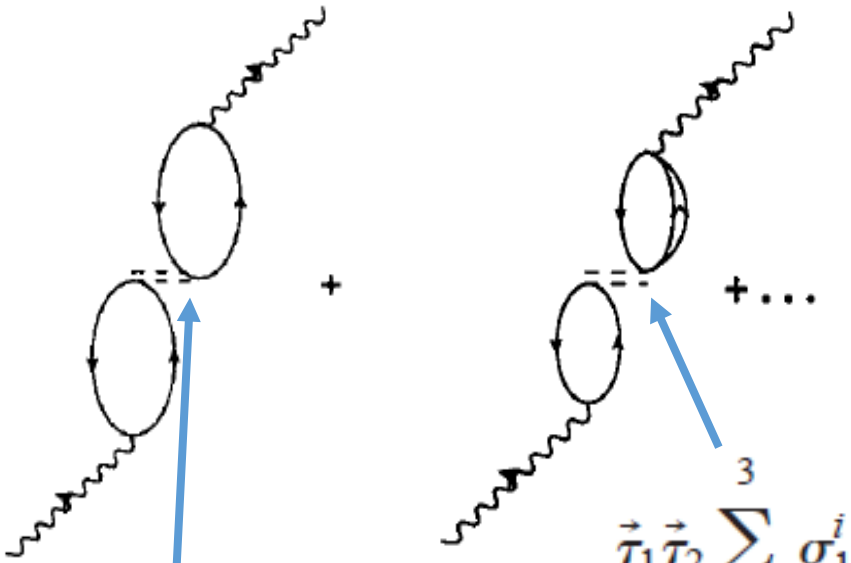
$$S_h(\omega,\vec{p}) = \delta(\omega - E(\vec{p}))\theta(k_F - |\vec{p}|)/2\omega$$

$$\begin{aligned} n^{\mathrm{RgFG}}(|\vec{p}|) &= \frac{4V}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) \\ &= \frac{3A}{4\pi k_F^3}\theta(k_F - |\vec{p}|) \\ n^{\mathrm{LDA}}(|\vec{p}|) &= 4\int \frac{d^3r}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) \\ &= 4\int \frac{d^3r}{(2\pi)^3}\theta(k_F(r) - |\vec{p}|) \end{aligned}$$

$$\left(\int d^3p\, n(|\vec{p}|) = A\right)$$

Convolution approach: C. Ciofi degli Atti, S. Liuti, and S. Simula, PRC 53, 1689 (1996), provide realistic distribution due to short-range correlations !

Polarization effects (RPA) at QE peak



$$\vec{\tau}_1\vec{\tau}_2\sum_{i,j=1}^{3}\sigma_1^i\sigma_2^j V_{ij}^{\sigma\tau}$$

$$c_0\{f_0(\rho)+f_0'(\rho)\vec{\tau}_1\vec{\tau}_2+g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2$$

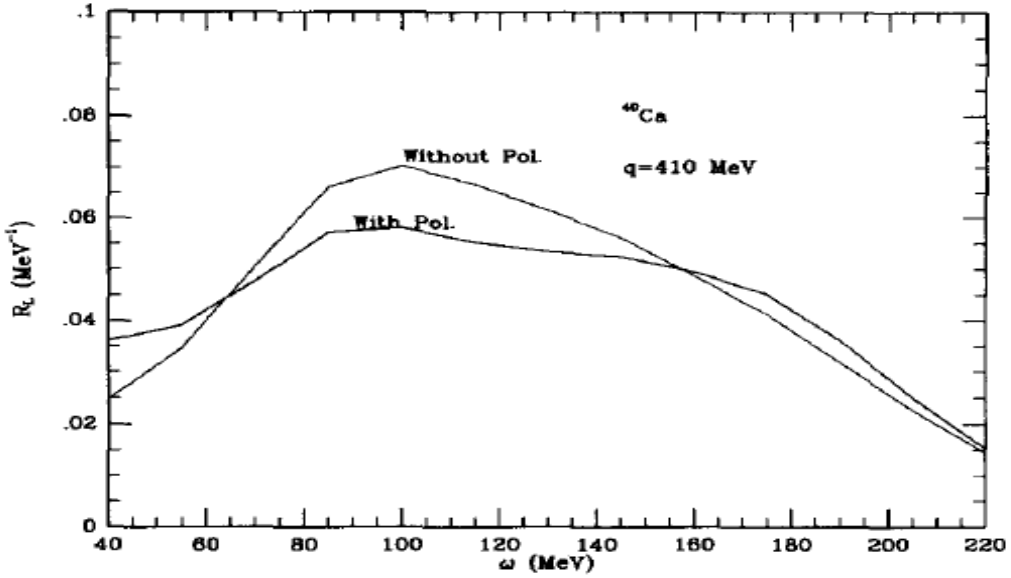
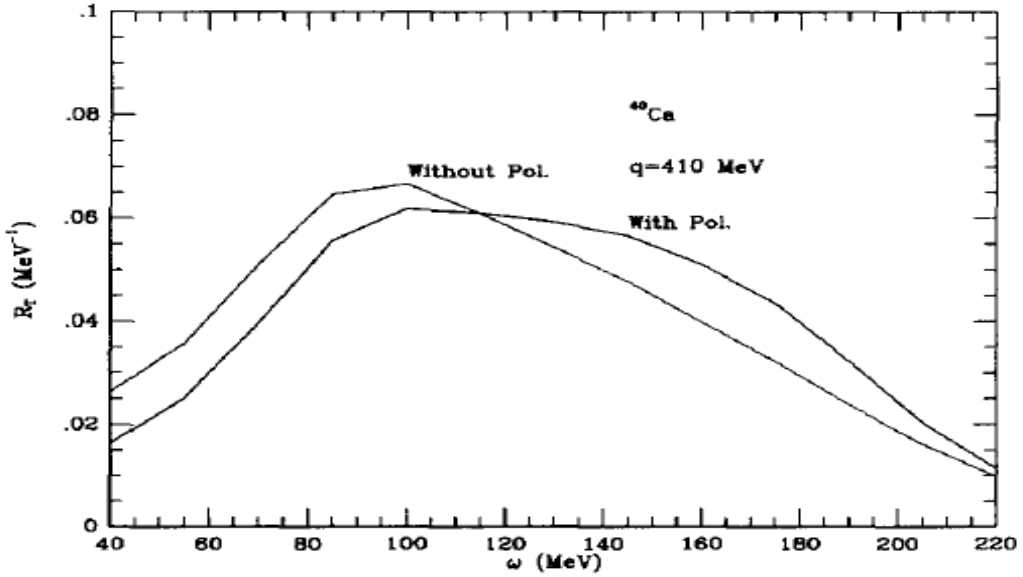
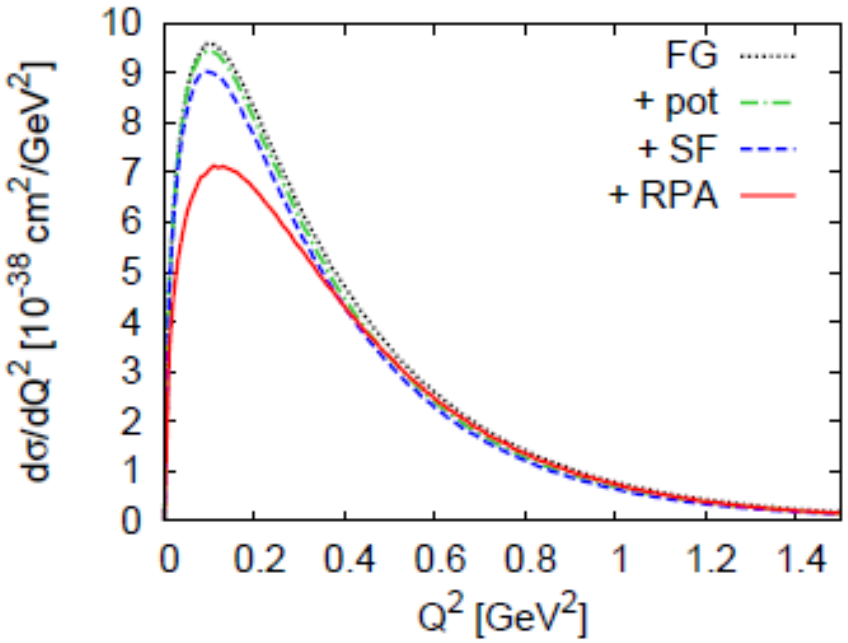


Fig. 37. Polarization (RPA) effect in the evaluation of R_L.



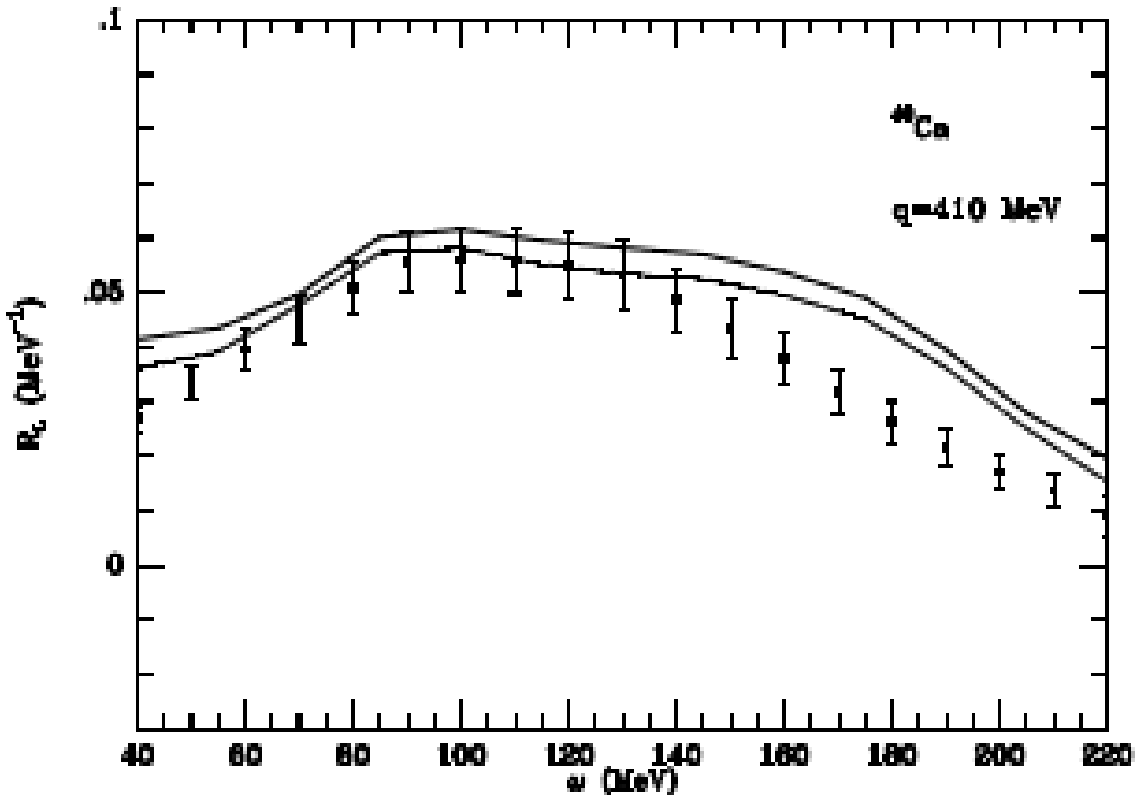


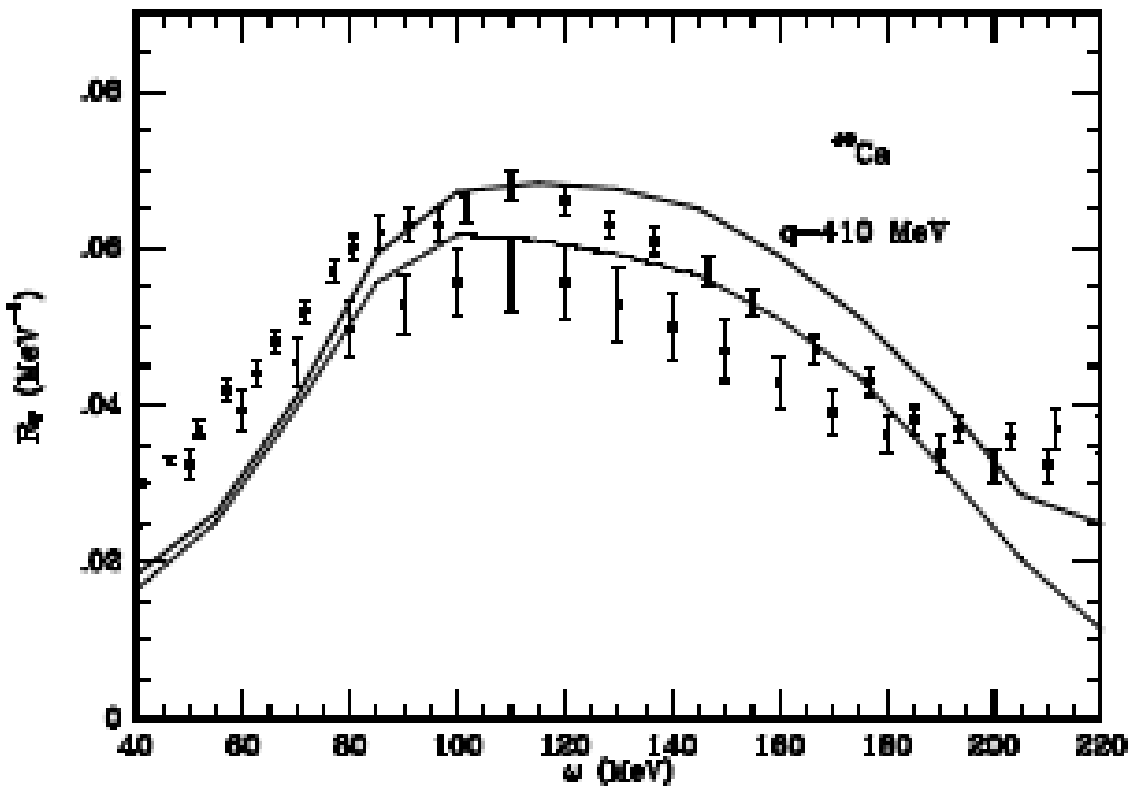
RPA vs SF effects: Differential cross sections for the CCQE reaction on ^{12}C averaged over the MiniBooNE flux

(Alvarez-Ruso L et al., 2009 AIP Conf. Proc. 1189 151)

$\boxed{\text{RPA} \gg \text{SF}}$

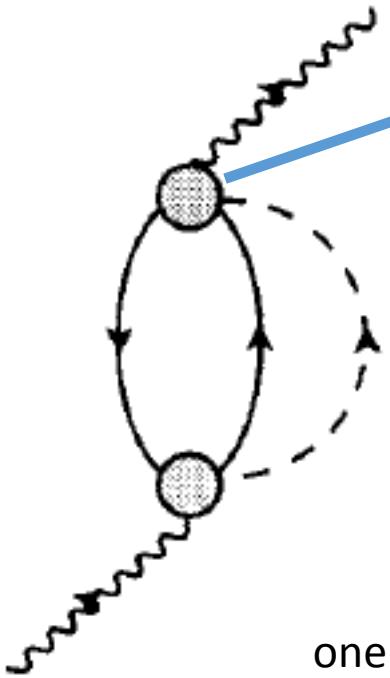
It depends on the specific kinematics and observable !





R_L and R_T QE response functions for $e + {}^{40}\mathrm{Ca} \to e' + X$

Pion production



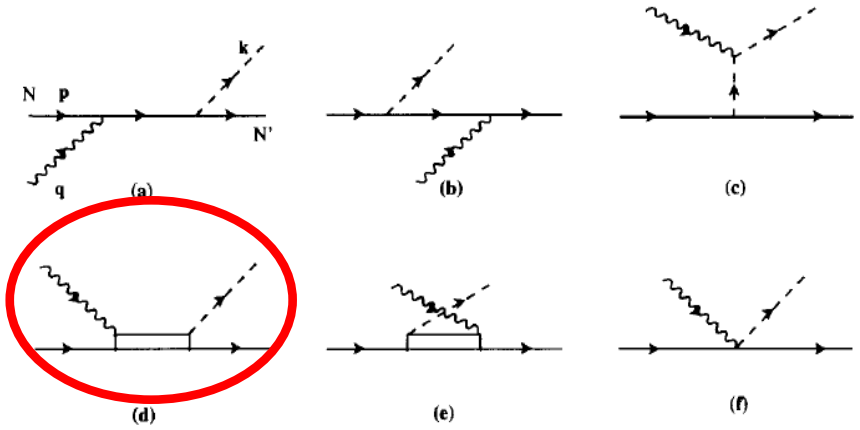
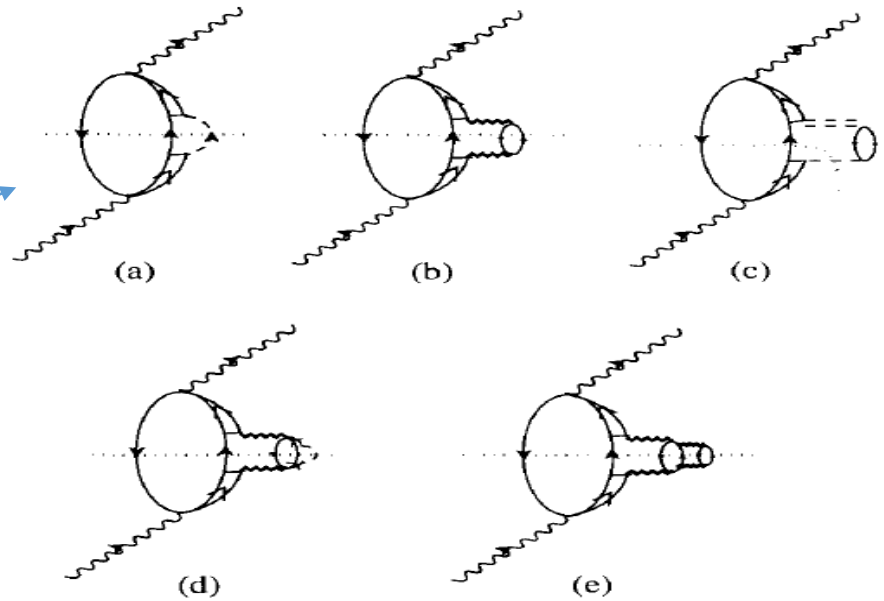
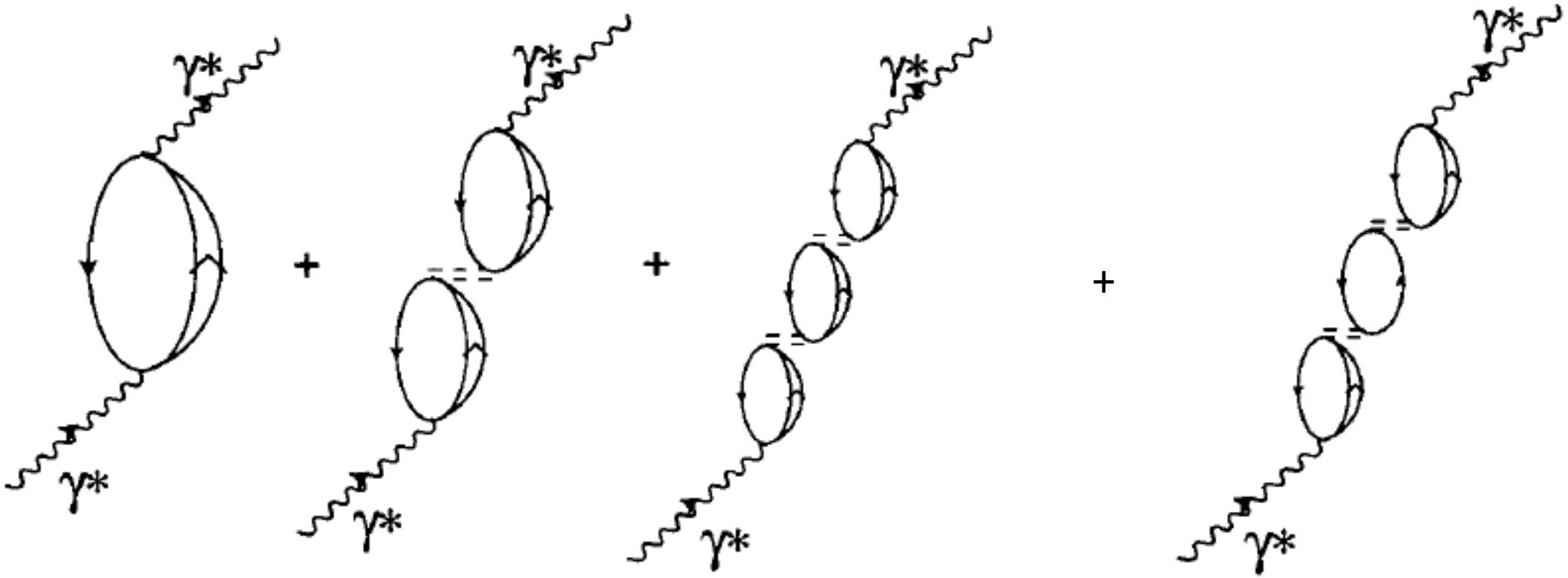


Fig. 2. Feynman diagrams considered for the $\gamma^* N \to \pi N$ reaction.

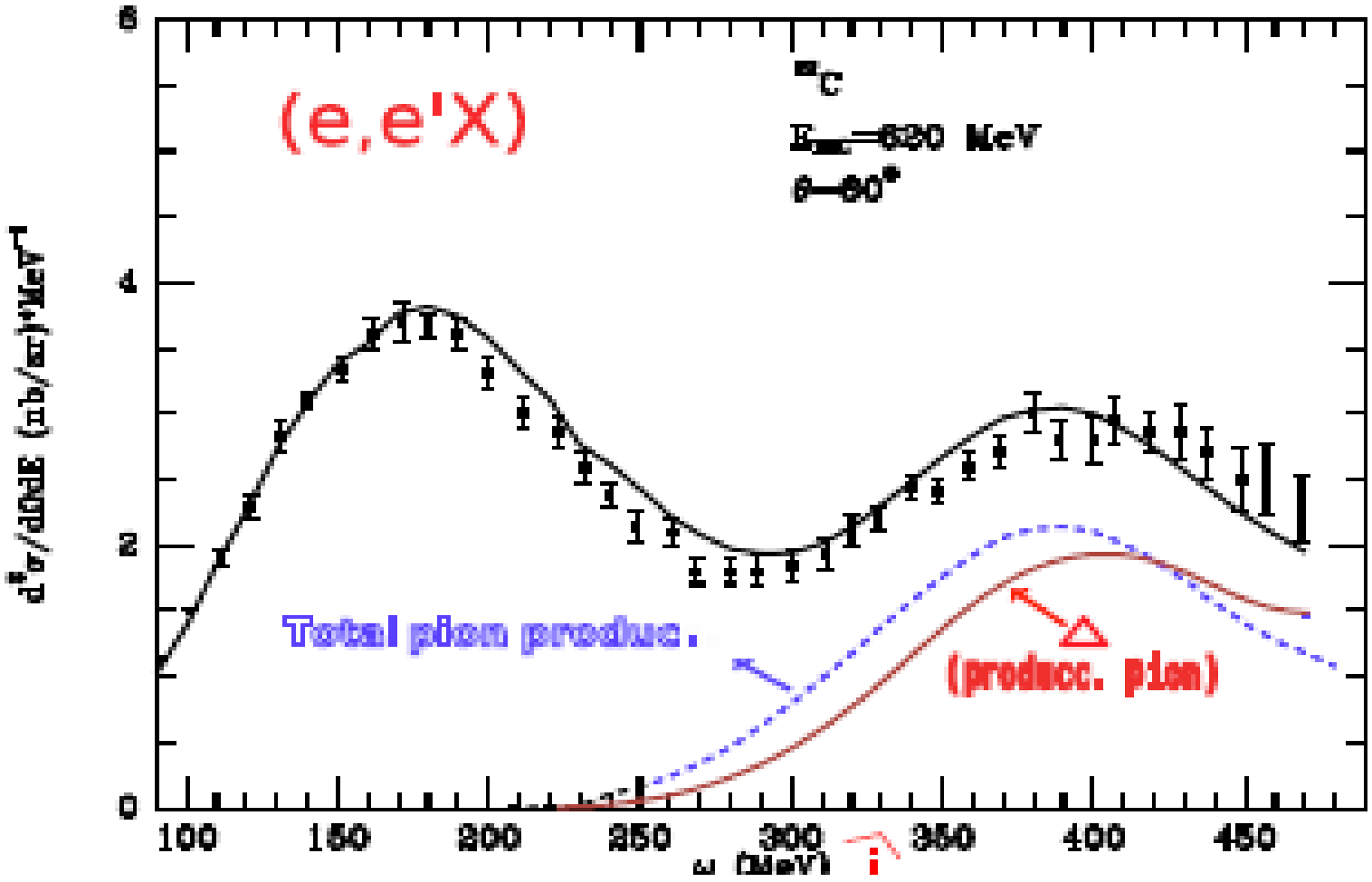
one of the terms generates the
Δ contribution





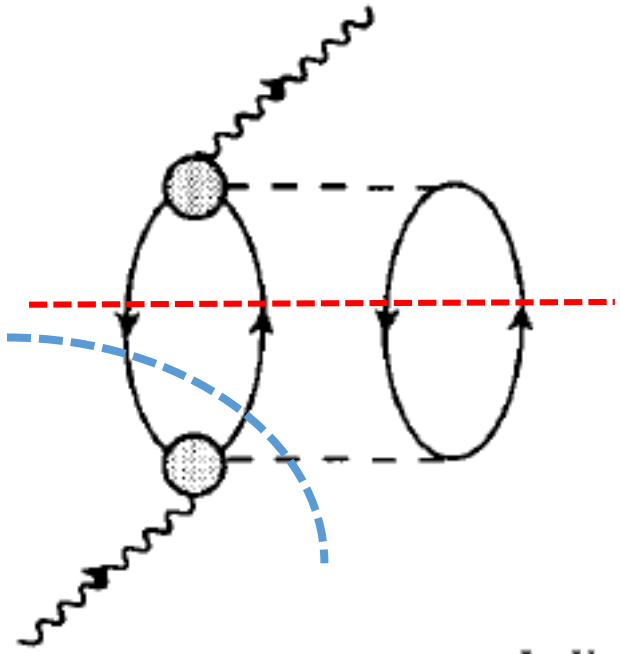
RPA corrections to the dominant Δh term

Juan Nieves, IFIC (CSIC & UV)



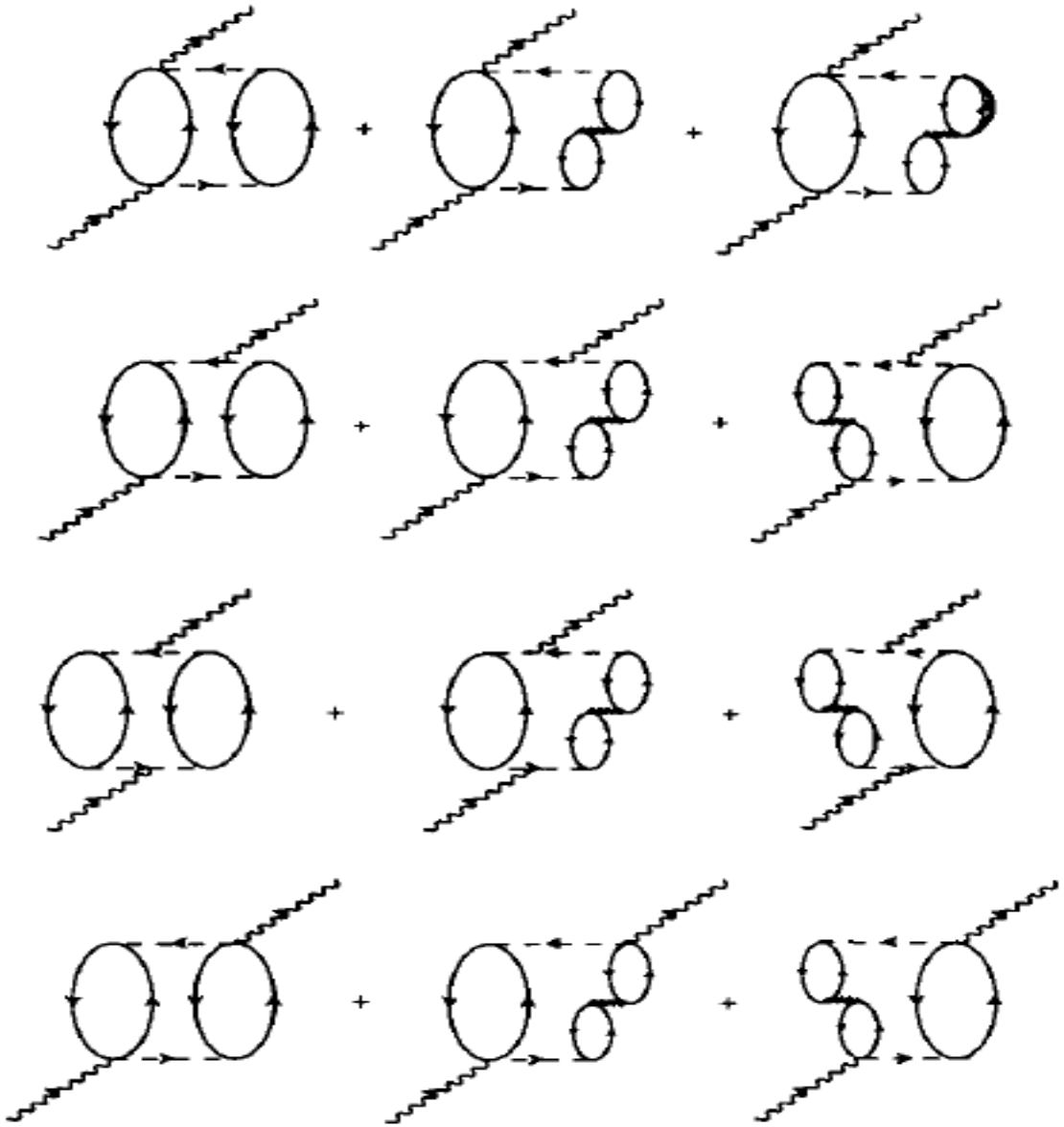
- Δ dominant component of the pion production contribution
- Missing strength both at the dip region and the Δ peak

2p2h (two body absorption) contributions



RPA corrections to
2p2h contributions

$$\mathrm{Im}\,U_N \to a\frac{\mathrm{Im}\,U_N}{|1-U_\lambda(q)V_l|^2} + b\frac{\mathrm{Im}\,U_N}{|1-U_\lambda V_t|^2}$$



+.....

Two cuts: $\gamma^* NN \to NN$

$\gamma^* N \to N\pi\ (dressed)$

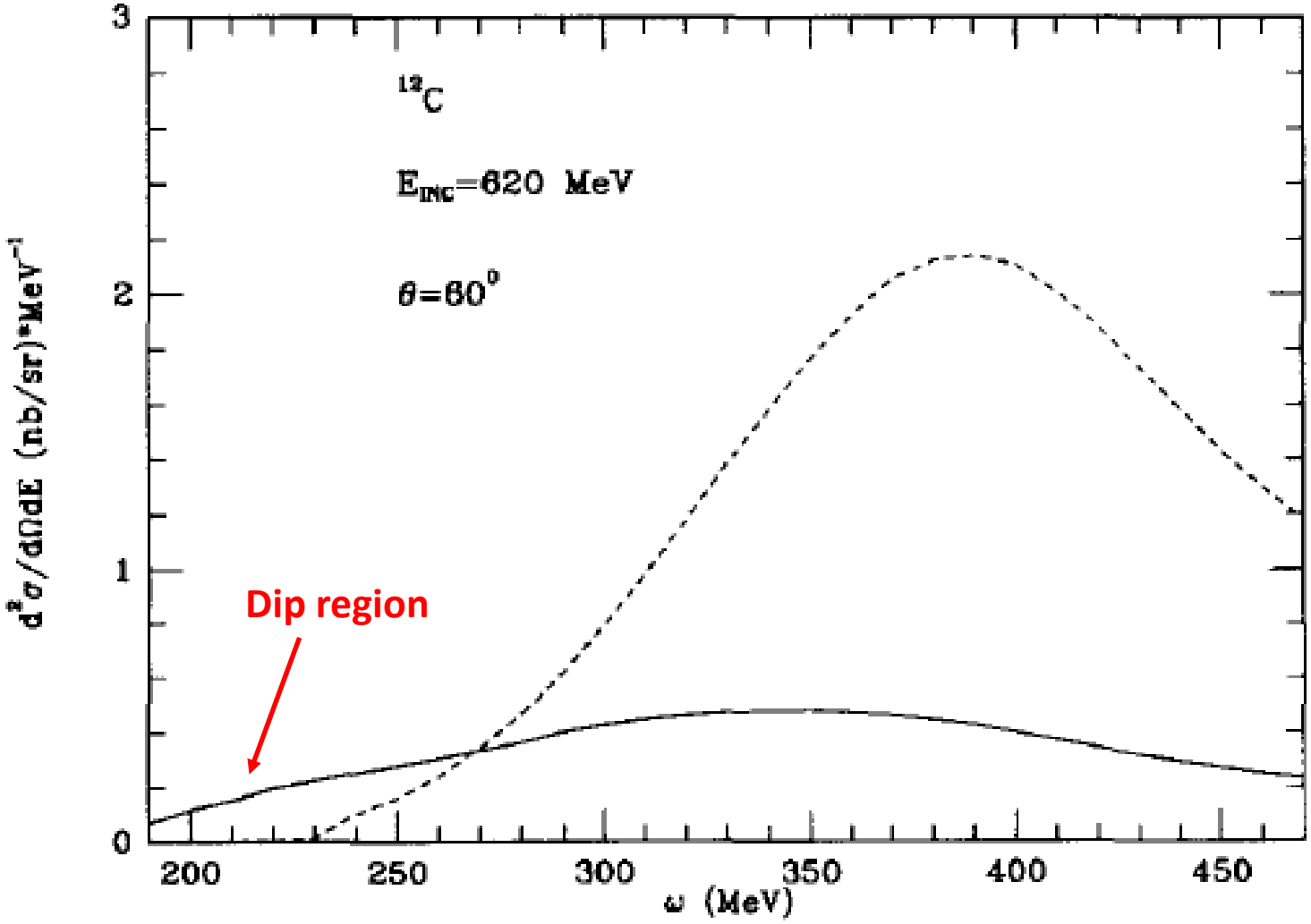


Fig. 44. Two-body photon absorption (solid line) versus pion production (dotted line) for ^{12}C.

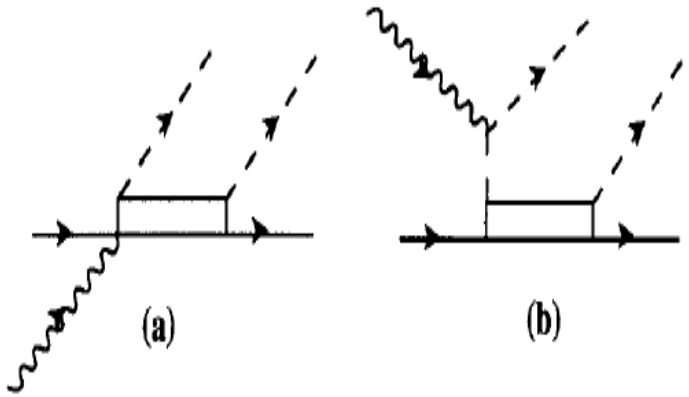
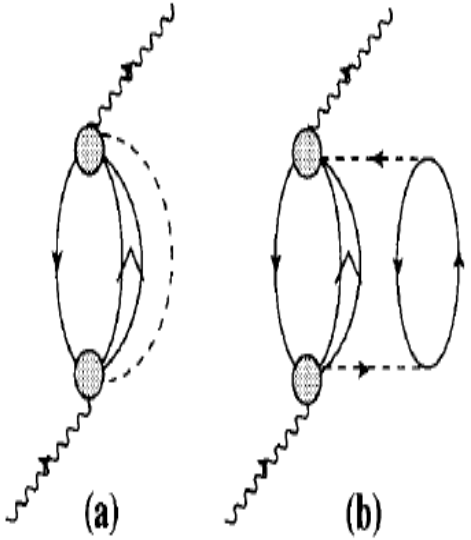
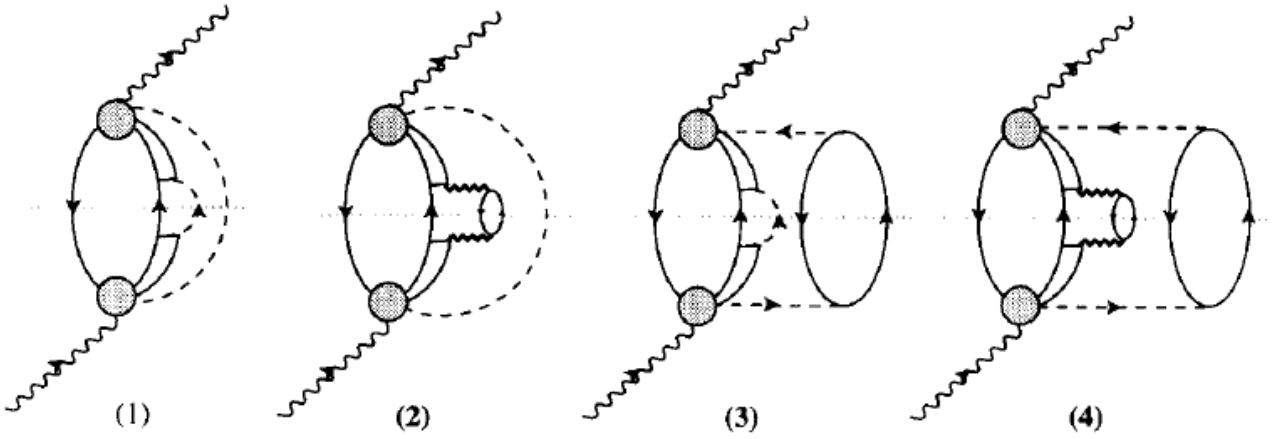


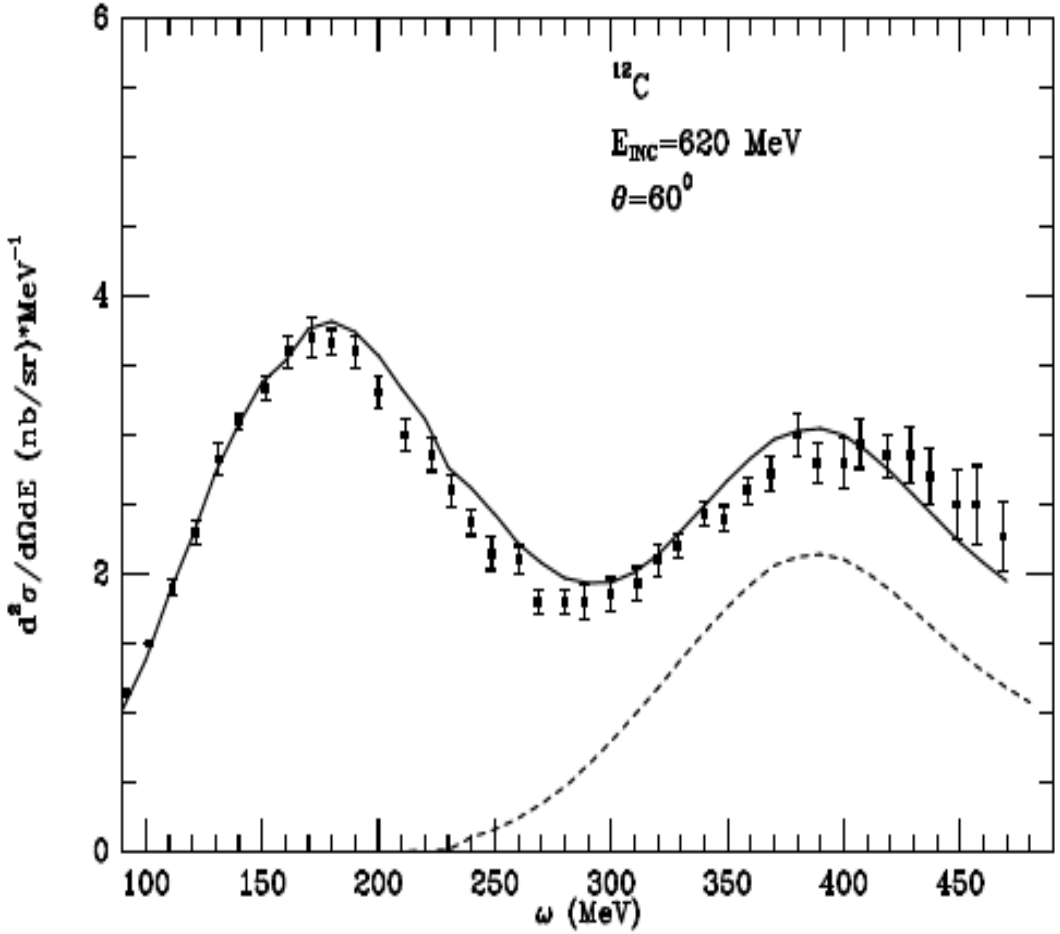
Fig. 23. Relevant Feynman diagrams that enter in the evaluation of the $\gamma^* N \to N2\pi$ cross section.

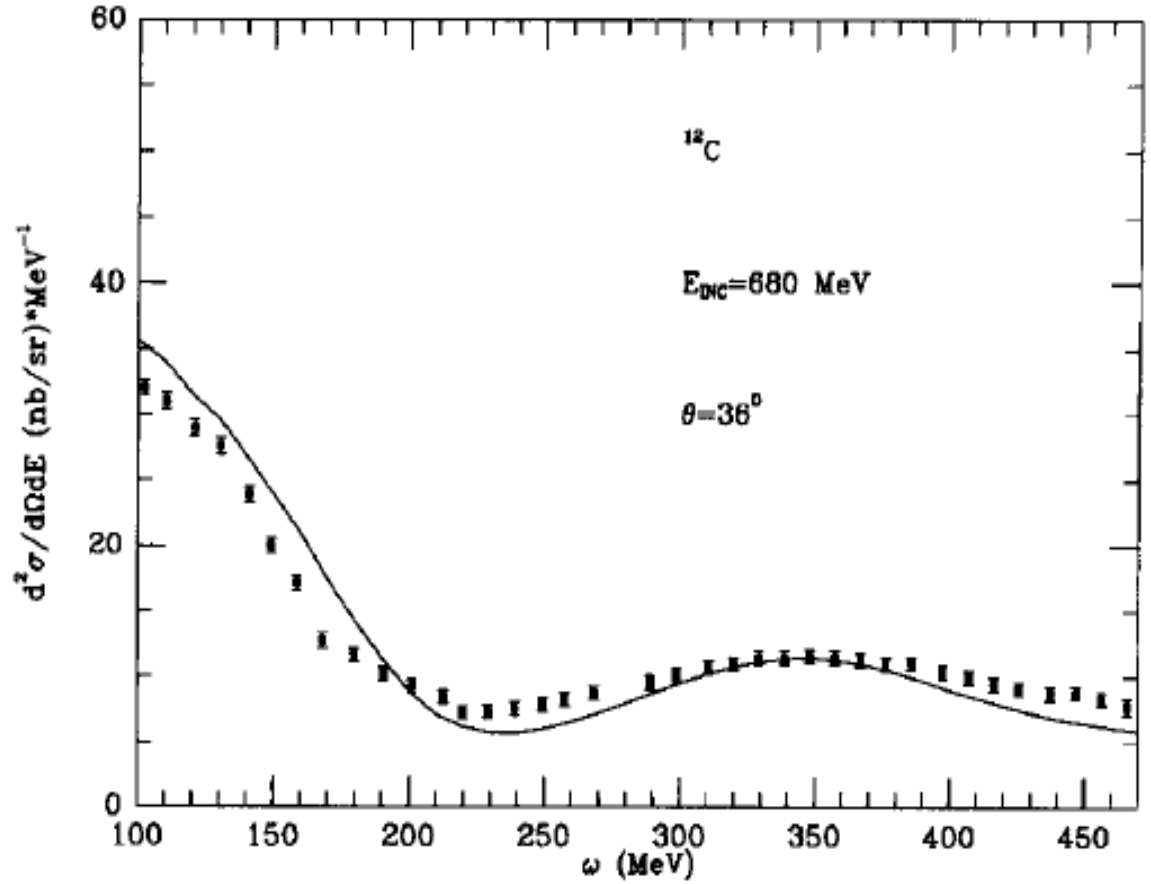
Additional contributions generated from the channel

$$\gamma^* N \to N\pi\pi$$



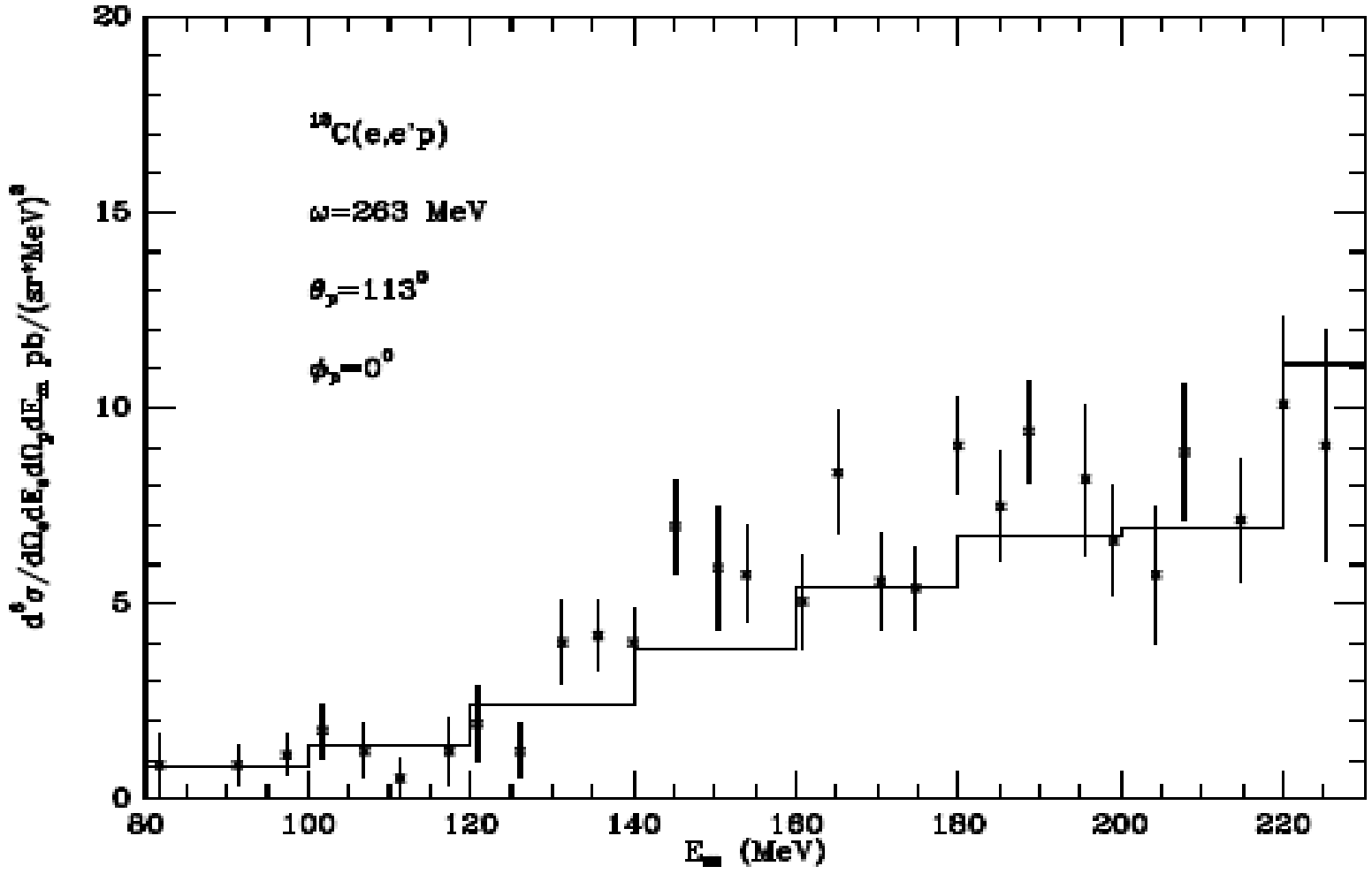






$$e + {}^{12}\mathrm{C} \to e' + X$$

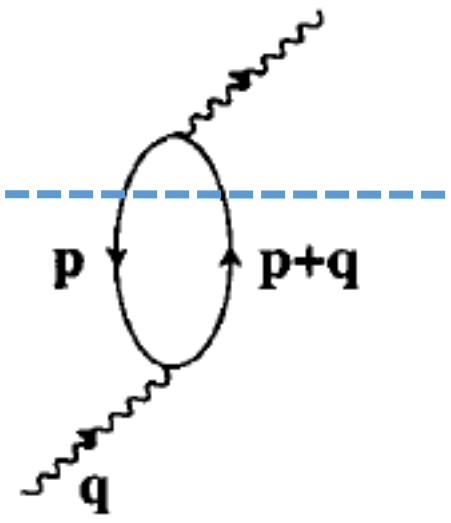
and by means of a Monte Carlo simulation we obtain cross sections for the processes $(e, e'N)$, $(e, e'NN)$, $(e, e'\pi)$, ...



Real Photon Results

Same formalism applied to the study of the interaction of Real Photons with Nuclei at Intermediate Energies: Total Photo-absorption cross section $\gamma A_Z \to X$ [Carrasco + Oset, NPA 536 (1992) 445] and Inclusive (γ,π), (γ,N), (γ,NN) and $(\gamma,N\pi)$ reactions [Carrasco + Oset + Salcedo NPA 541 (1992) 585 and Carrasco+Vicente-Vacas+ Oset NPA 570 (1994) 701]

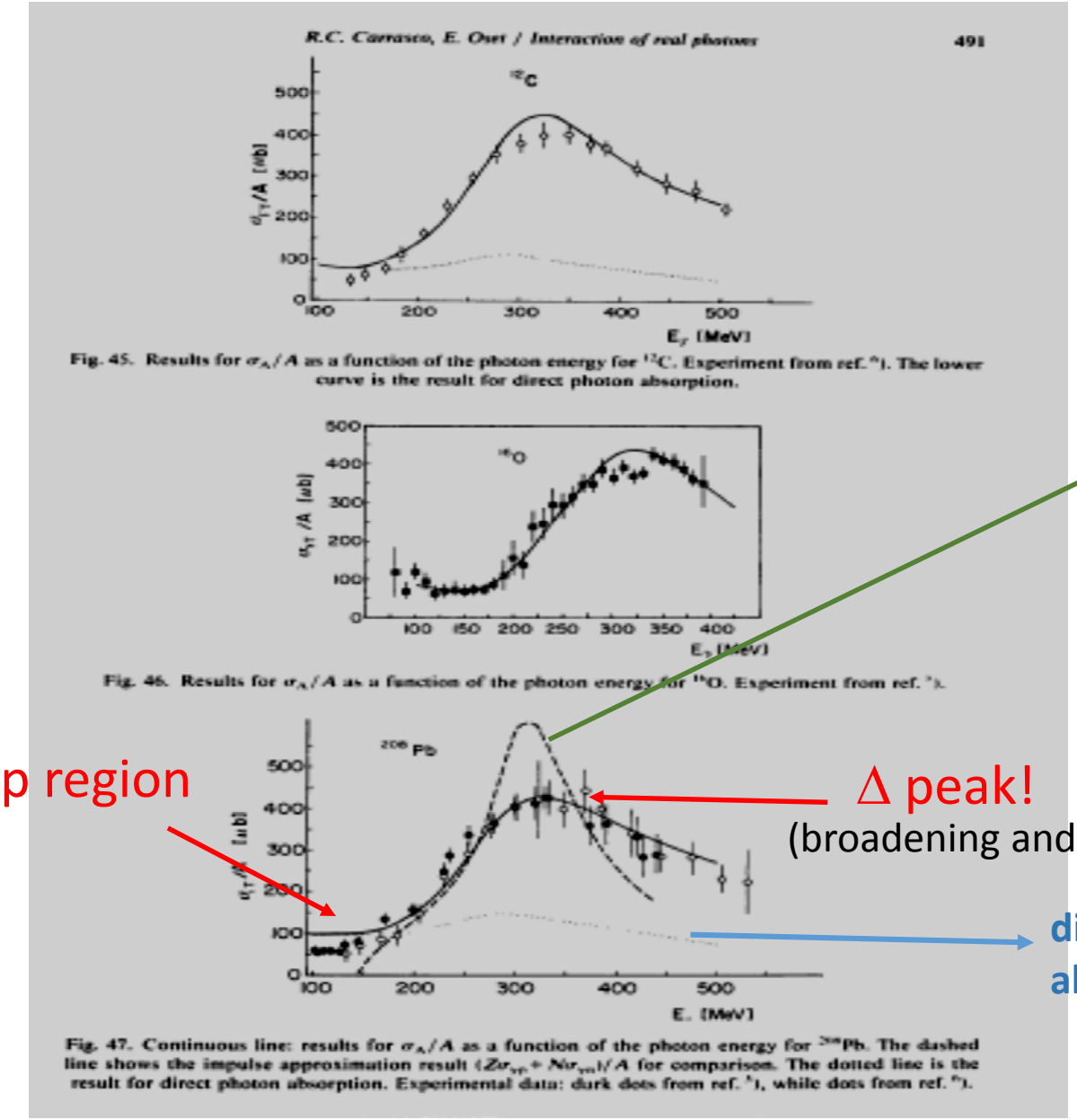
Important difference:



is forbidden !

$$p^2 = (p+q)^2$$

does not have solution when all particles are in the mass shell, i.e. $p^2 = (p+q)^2 = M^2$ and $q^2 = 0$.



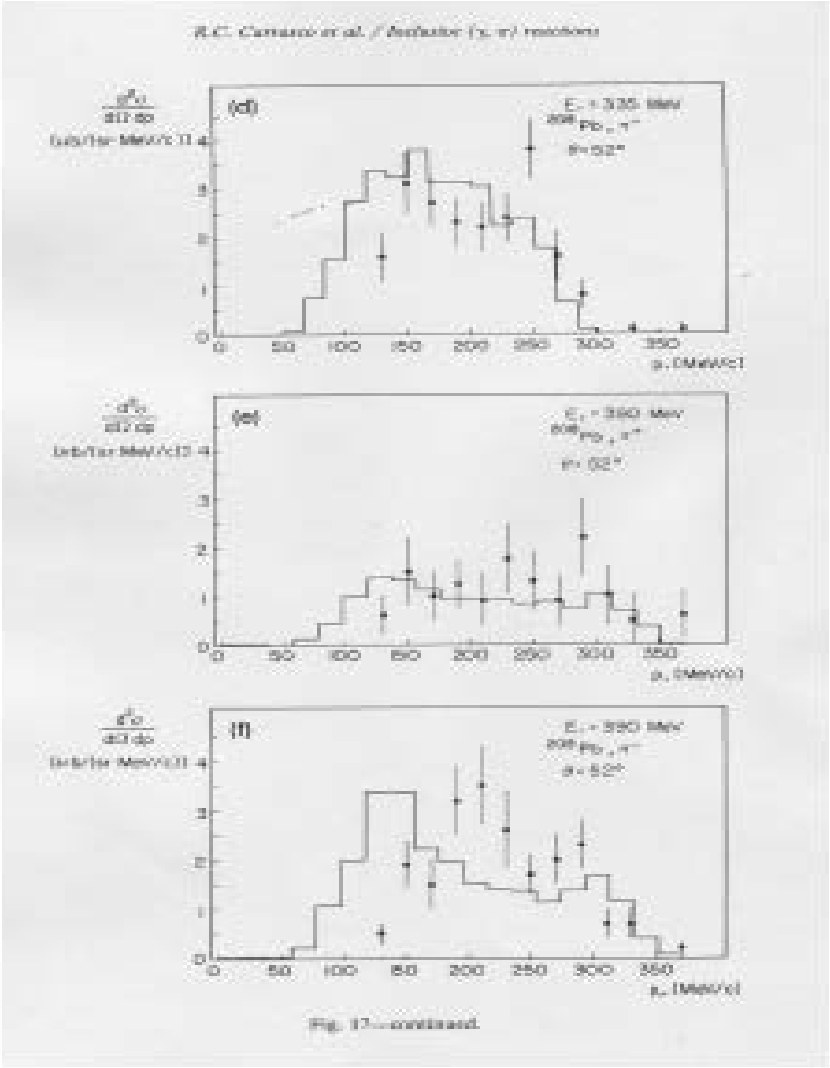
dip region

Δ peak!

(broadening and shift)

free Δ contribution

direct photon absorption



$$\gamma + {}^{208}\mathrm{Pb} \to X + \pi^{\pm}$$

$$\gamma + A \to X$$

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