Approximate Methods for Nuclei II

RPA, MEC, 2p2h... *(pionic modes of excitation in nuclei)*

Juan Nieves (IFIC, CSIC & UV)
Bibliography:


Juan Nieves, IFIC (CSIC & UV)
Outline:

• Introduction: Properties of the Coulomb interaction in an electron gas: screening of the interaction
• Particle and particle-hole propagators in a Fermi sea: occupation number and the Lindhard function
• Nuclear matter. Pion propagation in a nuclear medium: \( \pi N \) interaction at intermediate energies
• Induced spin-isospin NN interaction in a nuclear medium
• Examples:
  • inclusive muon capture in nuclei
  • pion-nucleus interactions: pionic atoms, pion nucleus scattering, ...
  • inclusive electron-nucleus scattering

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One electron inside of an electron gas (metal) polarizes the medium in such a way that, in a region around it, the negative charges are slightly displaced away from the electron, leaving behind the background charge of positive ions.

\[
\frac{1}{4\pi r} \rightarrow \frac{1}{4\pi} e^{-\mu(\rho)r}
\]

\(\rho\) is the electron density of the gas and \(\mu\) a certain function of it. The effect of the polarizations has been to convert the infinite range interaction into one of finite range. The positive charge around one electron, coming from the polarization of the medium, cancels the electron negative charge, and a at large distances we see an effective charge zero.

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In momentum space (static photon propagator, $q^0=0$)

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2 + \mu^2(p)}$$

Photon acquires an effective mass: interaction becomes of shorter range!

The physical mechanism for the polarization consists in a transfer of some electrons from occupied states of a Fermi sea to some unoccupied states: producing **particle-hole excitations**:

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Many Body diagram:

\[ \Pi^{\mu\nu}(q) = \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi(q) \]

\textbf{photon selfenergy corresponding to a single ph excitation}

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Solving the Dyson equation in the Landau gauge,

\[ \Pi^{\mu\nu}(q) = \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi(q) \]

\[ iD_0^{\mu\nu}(q^2) = -i \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \frac{1}{q^2 + i\epsilon} \]

\[ iD^{\mu\nu}(q) = iD_0^{\mu\nu}(q) + iD_0^{\mu\rho}(q) i\Pi^{\rho\sigma}(q) iD_0^{\sigma\nu}(q) + \cdots \]

\[ iD^{\mu\nu}(q) = -i \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \frac{1}{q^2 + \Pi(q)} \]

\[ \frac{1}{4\pi} \frac{1}{r} \rightarrow \frac{1}{4\pi} \frac{e^{-\mu(\rho)r}}{r} \]

Coordinate space (Coulomb static, \( q^0 = 0 \))

\[ \text{Determined by } \Pi(q)! \]

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Modifies the propagation of the photon inside of the Fermi sea.
Dirac sea:

all the states of negative energy are filled by electrons, a hole in the Dirac sea behaves like a positive charge particle e⁺

A particle-antiparticle excitation is represented by a transition of one electron from an occupied state of negative energy to an occupied state of positive energy

e⁺e⁻ excitation: free photon selfenergy renormalize electron mass and charge and the γe⁺e⁻ coupling

particle-hole excitation: excite occupied states to other unoccupied states of the Fermi sea. These transitions are additional to those from the negative energy states to the positive energy ones.

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Fermi gas particle-hole propagator: Lindhard function

For the electromagnetic interaction
In the static limit...

\[ H_{em}(x) = -e \bar{\Psi}(x) \gamma^\mu \Psi(x) A_\mu(x) \]

\[ -i \Pi(q^0, \vec{q}) = (-ie)(-ie)(-2) \int \frac{d^4k}{(2\pi)^4} iG_0(k)iG_0(k+q) \]

\[ G_0(k) = \frac{1 - n(\vec{k})}{k^0 - \epsilon(\vec{k}) + i\eta} + \frac{n(\vec{k})}{k^0 - \epsilon(\vec{k}) - i\eta} \]

electron propagator in a Fermi sea (non relativistic)

\[ n(k): \text{occupation number for a free Fermi gas.} \]

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Fermion Propagator

A) Ground state = vacuum $\equiv |0\rangle$ (Contains no particles)

$\Psi(x) = \sum_{\mathbf{p}, \sigma} \frac{1}{V 2\pi^3} \left( C_b(\mathbf{p}) \psi_\sigma(\mathbf{p}) e^{-i\mathbf{p}\cdot x} + \psi^\dagger_\sigma(\mathbf{p}) C_b^\dagger(\mathbf{p}) e^{i\mathbf{p}\cdot x} \right)$

$\Phi(x) = \sum_{\mathbf{p}, \sigma} \frac{1}{V 2\pi^3} \left( d_b^{\dagger}(\mathbf{p}) d_\sigma^b(\mathbf{p}) e^{-i\mathbf{p}\cdot x} + d_\sigma^b(\mathbf{p}) d_b^{\dagger}(\mathbf{p}) e^{i\mathbf{p}\cdot x} \right)$

Finite volume $V$

$\int \frac{1}{V} \frac{d^3p}{(2\pi)^3}$ \quad $\mathbf{p}$: momentum.

$\mathbf{r}$: helicity.

$C_b^\dagger(\mathbf{p})$ creates a fermion with momentum $\mathbf{p}$ and helicity $\uparrow$

$C_b^\dagger(\mathbf{p}) |0\rangle = 1$ fermion, $\mathbf{p}, \uparrow$

$d_b^{\dagger}(\mathbf{p})$ creates a anti-fermion with momentum $\mathbf{p}$ and helicity $\downarrow$

$d_b^{\dagger}(\mathbf{p}) |0\rangle = 1$ fermion, $\mathbf{p}, \downarrow$

$C_b(\mathbf{p}), d_b(\mathbf{p})$ associated annihilation operators

$C_b^\dagger(\mathbf{p}) |0\rangle = d_b^{\dagger}(\mathbf{p}) |0\rangle = 0.$

$i G(x', x) = i \int \frac{d^3p}{(2\pi)^3} e^{-i\mathbf{p}\cdot(x' - x)} G_b^\dagger(\mathbf{p}) \equiv \langle 0 | T \left( \Psi(x', \mathbf{p}) \overline{\Phi}(x, \mathbf{p}) \right) | 0 \rangle$

$= \Theta(x' - x) \langle 0 | \Psi(x', \mathbf{p}) \overline{\Phi}(x, \mathbf{p}) | 0 \rangle - \Theta(-x' - x') \langle 0 | \overline{\Phi}(x', \mathbf{p}) \Psi(x, \mathbf{p}) | 0 \rangle$
\[
\begin{align*}
\Theta(x^0, x^i) & \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E(p)}} e^{i p \cdot (x^0, x^i)} e^{-i p \cdot (x^0, x^i)} (\gamma^0 + \gamma^4) \\
\theta(x^0 - x'^0, x^i - x'^i) & \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E(p)}} e^{i p \cdot (x^0 - x'^0, x^i - x'^i)} e^{-i p \cdot (x^0 - x'^0, x^i - x'^i)} (\gamma^0 - \gamma^4)
\end{align*}
\]

\[\Rightarrow (g p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon}\]

2) **Ground State = Fermion Sea**

107 → 140

\[d_r(p) 140^+ = 0\]

\[c^+(p) 140^- = 0 \quad (p < k_F) \quad (\text{can not create a particle below the Fermi surface})\]

\[c_r(p) 140^- = 0 \quad (p > k_F) \quad (\text{can annihilate a particle below the Fermi surface})\]

We can rewrite the field \(\Psi(x)\) in terms of operators which destroy the vacuum.

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particle-hole propagator: Lindhard function

For the electromagnetic interaction

In the static limit...

\[ H_{em}(x) = -e \overline{\psi}(x) \gamma^\mu \psi(x) A_\mu(x) \]

\[ -i \Pi(q^0, \vec{q}) = (-ie)(-ie)(-2) \int \frac{d^4 k}{(2\pi)^4} iG_0(k) iG_0(k + q) \]

\[ G_0(k) = \frac{1 - n(\vec{k})}{k^0 - \epsilon(\vec{k}) + i\eta} + \frac{n(\vec{k})}{k^0 - \epsilon(\vec{k}) - i\eta} \]

Electron propagator in a Fermi sea (non-relativistic)

\( n(k): \) occupation number for a free Fermi gas.

Electron kinetic energy

\( K_F: \) Fermi momentum

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The $k^0$ integration can be done in the complex plane...

$$\Pi(q^0, \bar{q}) = e^2 U_e(q^0, \bar{q}),$$

where $U_e(q^0, \bar{q})$, called the Lindhard function is given by

$$U_e(q^0, \bar{q}) = 2 \int \frac{d^3 k}{(2\pi)^3} \left[ \frac{n(\vec{k})(1 - n(\vec{k} + \vec{q})))}{q^0 - \epsilon(\vec{k} + \vec{q}) + \epsilon(\vec{k}) + i\eta} \right.$$\[\begin{align*} &+ \frac{n(\vec{k} + \vec{q})(1 - n(\vec{k}))}{-q^0 + \epsilon(\vec{k} + \vec{q}) - \epsilon(\vec{k}) + i\eta} \end{align*} \right].$$

Direct and cross terms of the ph excitation

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\( U_e(q) \) can have both real and imaginary parts. The imaginary part comes from situations in the intermediate states integration, where the are placed on shell \([C\text{utkowsky’s rule}}\) (Itzykson & Zuber, Quantum Field Theory, McGraw-Hill, New York, 194)].

\[
\frac{1}{p^0 - \epsilon(p)} \pm i\epsilon \int \frac{1}{p^0 - \epsilon(p)} \pm i\delta(p^0 - \epsilon(p))
\]

For \( q^0 > 0 \), only the \textbf{direct term} gives rise to an imaginary part.

For \( q^0 < 0 \), the \textbf{crossed term} gives rise to an imaginary part.

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Analytical structure: In the complex $q^0$ plane, it has a continuous set of poles (cut) in the four quadrants

$q^0 = \epsilon_{\text{par}} - \epsilon_{\text{hole}} - i\eta$

and in the second quadrant

$q^0 = \epsilon_{\text{hole}} - \epsilon_{\text{par}} + i\eta$

Analytical cuts in the second and fourth quadrants, and it has an imaginary part for real values of $q^0$ situated in the analytical cuts.
For complex values of $q^0$

$$2U_e(\nu, \hat{q}) = \frac{mk_F}{\pi^2} \left\{ -1 + \frac{1}{2\hat{q}} \left[ 1 - \left( \frac{\nu}{\hat{q}} - \frac{\hat{q}/2 + 1}{\nu/\hat{q} - \hat{q}/2 - 1} \right) \ln \frac{\nu/\hat{q} - \hat{q}/2 + 1}{\nu/\hat{q} + \hat{q}/2 - 1} \right] \ln \frac{\nu/\hat{q} - \hat{q}/2 + 1}{\nu/\hat{q} + \hat{q}/2 - 1} \right\}$$

$$\nu = \frac{q^0 m}{k_F^2}, \quad \hat{q} = \frac{q}{k_F}$$

Analytical structure: In the complex $q^0$ plane, it has a continuous set of poles (cut) in the four quadrant and in the second quadrant.

$$2ImU_e(\nu, \hat{q}) = \frac{-2mk_F}{4\pi\hat{q}} \left[ 1 - \left( \frac{|\nu|}{\hat{q} - \frac{\hat{q}/2 + 1}{\nu/\hat{q} - \hat{q}/2 - 1}} \right)^2 \right]$$

for $\hat{q} > 2$ and $\frac{1}{2} \hat{q}^2 + \hat{q} \geq |\nu| \geq \frac{1}{2} \hat{q}^2 - \hat{q}$ or $\hat{q} < 2$ and $\frac{1}{2} \hat{q}^2 + \hat{q} \geq |\nu| \geq \hat{q} - \frac{1}{2} \hat{q}^2$.

$$2ImU_e(\nu, \hat{q}) = -\frac{mk_F}{\pi\hat{q}} |\nu|$$

for $\hat{q} < 2$ and $0 \leq |\nu| \leq \hat{q} - \frac{1}{2} \hat{q}^2$, and $Im U_N(\nu, \hat{q}) = 0$ otherwise.

Analytical cuts in the second and fourth quadrants, and it has an imaginary part for real values of $q^0$ situated in the analytical cuts.

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Fully relativistic expressions:

\[
S(p) = (p^\mu \gamma_\mu + m) \left( \frac{1}{p^2 - m^2 + i \varepsilon} + i \frac{\pi}{E(\vec{p})} \delta(p^0 - E(\vec{p})) n(\vec{p}) \right)
\]

\[
E(\vec{p}) = \sqrt{\vec{p}^2 + m^2}
\]

\[
G(p) = \left( \frac{1}{p^2 - m^2 + i \varepsilon} + i \frac{\pi}{E(\vec{p})} \delta(p^0 - E(\vec{p})) n(\vec{p}) \right)
\]

\[
U_e(q) = -2i \int \frac{d^4 p}{(2\pi)^4} G(p)G(p+q) = 2 \int \frac{d^3 p}{(2\pi)^3} \frac{m}{E(\vec{p}) E(\vec{p}+\vec{q})} \frac{m}{q^0 + E(\vec{p}) - E(\vec{p}+\vec{q}) + i\varepsilon} + (q \leftrightarrow -q)
\]

\[
\text{Im} U_e(q) = -m^2 \frac{\Theta(q^0) \Theta(-q^2)}{2\pi |\vec{q}|} \Theta(E_F - \varepsilon_R) (E_F - \varepsilon_R) \approx -\pi \rho \frac{m}{E(\vec{q})} \delta(q^0 + M - E(\vec{q}))
\]

\[
E_F(\vec{p}) = \sqrt{k_F^2 + m^2}, \quad \varepsilon_R = \text{Max} \left\{ m, E_F - q^0, -\frac{q^0 + |\vec{q}| \sqrt{1-4m^2/q^2}}{2} \right\}
\]

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**Dictionary.**

- Propagation of photons through an electron gas (metal)
- Electric charge of the electron and screening
- Photon carrier of the electromagnetic interaction

- Propagation of $\gamma$, $W$, $Z^0$ or pions (mesons in general) through a nuclear medium
- Axial charge of the nucleon and axial polarization
- $\pi, \rho, \ldots$ carriers of the NN interaction
For inclusive processes, with large excitation energies (> 50 MeV) and spherical nuclei...

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Nuclear medium: Nuclear matter (approximation)

- Infinite system homogeneous and isotropic
- Infinite volume with an infinite number of nucleons, but with a constant density
  \[ \rho = \frac{A}{V} = \text{cte} \]
- Momentum and energy are conserved
- Symmetric Z=N=A/2 (no necessary)
- Pauli’s exclusion principle
- Finite nuclei: LDA (local density approx)

\[ \rho_{\text{NM}} \text{ (cte)} \rightarrow \rho (\vec{r}) \text{ (s-wave)} \]

\[ \overrightarrow{q}^2 \rho_{\text{NM}} \rightarrow \nabla \rho (\vec{r}) \nabla \text{ (p-wave)} \]

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propagation of photons in the electron gas $\rightarrow$ propagation of pions in the nuclear medium (carriers of the interaction)

**Dyson equation**

\[
q^\mu iD = iD_0 + iD_0 i\Pi iD \implies D(q) = \frac{D_0}{1 - D_0\Pi} = \frac{1}{q^2 - m_\pi^2 - \Pi(q)}
\]

Full pion propagator $D(q)$  
Free pion propagator $D_0(q)$

\[
P(q) = D(q) - D_0(q)
\]

Pion selfenergy: contains only irreducible diagrams

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Pion selfenergy: first approximation $\pi NN$ vertex

$$\Pi(q) = \frac{f^2}{m_{\pi}^2} \hat{q}^2 U_N(q)$$

$$U_N(q) = 2U_e(q), \quad (m_e \rightarrow m_N)$$

Chiral symmetry

$$\mathcal{L}_{\text{int}} = \frac{g_A}{f_{\pi}} \bar{\Psi} \gamma^\mu \gamma_5 \frac{\tau^a}{2} (\partial_\mu \Phi) \Psi - \frac{1}{4f_{\pi}^2} \bar{\Psi} \gamma_\mu \tau^a (\Phi \times \partial_\mu \Phi) \Psi$$

P-wave

S-wave (Weinberg-Tomozawa)

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The pion cannot only excite nucleons above the Fermi sea, but it can also excite the internal degrees of freedom of the nucleon since it is a composite particle made out of quarks. Hence a nucleon can be converted into a $\Delta$, $N^*$, $\Delta^*$, etc.

$\Delta$(1232) [spin 3/2 and isospin 3/2] plays an important role at intermediate energies because of its lower mass and strong coupling \( \to \text{contribution to } \mathcal{L} \).

\[
\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_{\pi}} \bar{\Psi}_\mu \tilde{T}^\dagger (\partial^\mu \tilde{\phi}) \Psi + \text{h.c.}
\]

where $\Psi_\mu$ is a Rarita-Schwinger $J^\pi = 3/2^+$ field, $\tilde{T}^\dagger$ is the isospin transition operator from isospin 1/2 to 3/2, and $f^* = 2.13 \times f = 2.14$.

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Some technical aspects about the $\Delta h$ excitation

- $\left\langle \frac{3}{2} M_T \left| T_\nu^+ \right| \frac{1}{2} m_T \right\rangle = \left( \frac{1}{2}, 1, \frac{3}{2} \right| m_T, \nu, M_T \right\rangle \left\langle \frac{3}{2} \right| T_\nu^+ \left| \frac{1}{2} \right\rangle$

- $\Delta$ propagator (unstable particle)

$$G^{\mu\nu}(p_{\Delta}) = \frac{P^{\mu\nu}(p_{\Delta})}{p_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}G_{\Delta}}, \quad P^{\mu\nu}(p_{\Delta}) = -(p_{\Delta} + M_{\Delta}) \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{p_{\Delta}^\mu p_{\Delta}^\nu}{M_{\Delta}^2} \right] + \frac{1}{3} \frac{p_{\Delta}^\mu \gamma^\nu - p_{\Delta}^\nu \gamma^\mu}{M_{\Delta}}$$

$$\Gamma_{\Delta}(s) = \frac{1}{6\pi (m_\pi)} \frac{2 M}{\sqrt{s}} \left[ \frac{\lambda^{1/2}(s, m_\pi^2, M^2)}{2\sqrt{s}} \right]^3 \times \Theta(\sqrt{s} - M - m_\pi), \quad s = p_{\Delta}^2,$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

$P^{\mu\nu}$: spin $3/2$ projector

$(\pi N \text{ CM momentum})^3$

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Some technical aspects about the $\Delta h$ excitation: non-relativistic expressions

- $\pi N \Delta$ transition: \[ H_{\pi N \Delta} = \frac{f^*}{\mu} \Psi^\dagger_N(x) S_i \partial_i \phi^\lambda(x) T^\lambda \Psi_\Delta(x) + \text{h.c.} , \]

- $\Delta$ propagator

\[ G_\Delta(k) = \frac{1}{k^0 - w_R - T_\Delta + \frac{1}{2} i \Gamma_\Delta} \]

with $w_R = M_\Delta - M_N$, $T_\Delta$ the $\Delta$ kinetic energy.

The pion selfenergy reads: \[ \Pi(q) = \frac{f^2}{m^2_\pi} \bar{q}^2 U(q) \quad \text{with} \quad U(q) = U_N(q) + U_\Delta(q) \]

\[ U_\Delta(q) = -i \left( \frac{4}{3} \right)^2 \left( \frac{f^*}{f} \right)^2 \int \frac{d^4 k}{(2\pi)^4} \left[ G^0(k) G_\Delta(k + q) + G^0(k) G_\Delta(k - q) \right] \]

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Remarks:
• External $\gamma$ can also excite $\Delta h$ components

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Remarks:

- $V$ is an effective $ph-ph$, $ph-\Delta h$ and $\Delta h-\Delta h$ interaction in the nuclear medium.
- $V = \pi + \rho + \text{other mesons (Short Range Correlations)}$. Starting point $\pi N \to \pi N$ in free space ($\pi NN$ and $\pi N\Delta$ couplings) [Ericson+Weise, Pions in nuclei] and $NN \to NN$ Bonn potential.

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NN Interaction: Simple Model (Oller + Weise, 1979)

\[ \pi + p \text{ exchanges + nuclear correlations} \]

\[ 0 + \text{HeNN}(q) = \frac{i f_\pi}{m_\pi} (\hat{\sigma} \times \hat{q}) \cdot \hat{q} \]

\[ 2(\pi) + \gamma_{\text{NN}}(q) \left[ \hat{q} \cdot \hat{q'} \right] \]

\[ 0 + \text{HeNN}(q) = \frac{i f_\pi}{m_\pi} (\hat{\sigma} \times \hat{q}) \cdot \hat{q} \]

\[ \text{with } \quad C_\pi \equiv \frac{f_\pi^2}{f^2} \approx 2 \quad \frac{f_\pi}{f} \approx \frac{f^*}{f^*} \]

\text{Quark Model.}

Now: \( \pi + p \) Exchanges

\[ 0 + \hat{\Sigma} = \hat{\Sigma}_\pi (q) + \hat{\Sigma}_p (q) \]

\[ \hat{\Sigma}_\pi (q) = \frac{i}{4} \sigma_i \sigma_j \tilde{\epsilon}_i \tilde{\epsilon}_j \left( \frac{f_{\mu \nu}}{m_n} \right)^2 \mathbf{V}_{ij} (q^2) \quad \text{(longitudinal II)} \]

\[ \hat{\Sigma}_p (q) = \frac{i}{4} \sigma_i \sigma_j \tilde{\epsilon}_i \tilde{\epsilon}_j \left( \frac{f_{\mu \nu}}{m_p} \right)^2 \mathbf{V}_{ij} (q^2) \quad \text{(transversal II)} \]

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Where the form factors (off-shell extrapolations)

$$F_i(q^2) = \frac{\Lambda_i^2 - M_i^2}{\Lambda_i^2 - q^2} \quad (q^2 = q^2 - \frac{3}{4})$$

\[\Lambda^\pi = 1250 \text{ MeV} = \Lambda^\pi\]
\[\Lambda^\rho = 2500 \text{ MeV} = \Lambda^\rho\]

0 because $C^\pi = C^\rho$, $\Lambda^\pi = \Lambda^\pi$ and $\Lambda^\rho = \Lambda^\rho$ the former potentials describe $\Lambda N \to NN$, $NN \to \Lambda N$, $\Lambda N \to \Lambda N$ and $\Lambda N \to NN$ interaction with the following replacements

$$\frac{\rho}{\pi} \sigma T \to \frac{\rho}{\pi} \frac{T^*}{\pi^*}$$

0 $V^\pi_{ij}$ and $V^\rho_{ij}$ are orthogonal.

⇒ **Short range correlations**
0 attributed to the exchange of the $\pi$ - $\rho$ on $N$.
0 Correlated potential in coordinate space

$$\tilde{V}(r) = V(r) g(r); \quad g(r) = 1 - \delta_0 (\pi r); \quad \rho_c = \frac{\rho}{\pi^*}$$

⇒ $\frac{1}{\rho}$ defines the range of the correlations

⇒ **Momentum space**

$$\tilde{V}(q) = \int \frac{d^3 \pi}{(2\pi)^3} g(\vec{r}, \vec{q}) \tilde{V}(\vec{r})$$

$$g(\vec{r}, \vec{q}) = (2\pi)^3 \delta^3(\vec{r} - \vec{q}) - 2\pi^2 \frac{SU(\vec{r} - \vec{q}) \cdot \vec{T}}{q^2}$$

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NN potential $V(q) = c_0 \{ f_0(\rho) + f'_0(\rho) \hat{\tau}_1 \hat{\tau}_2 + g_0(\rho) \hat{\sigma}_1 \hat{\sigma}_2 \} + \hat{\tau}_1 \hat{\tau}_2 \sum_{i,j=1}^{3} \sigma_i^j \sigma_2^j V_{ij}^{\pi}$.  

with $\hat{q}_i = q_i / |q|$  

$V_l(q^0, \vec{q}) = \frac{f^2}{m^2_\pi} \left\{ \left( \frac{\Lambda^2_\pi - m^2_\pi}{\Lambda^2_\pi - q^2} \right)^2 \frac{\vec{q}^2}{q^2 - m^2_\pi} + g'(q) \right\}$.  

$V_t(q^0, \vec{q}) = \frac{f^2}{m^2_\pi} \left\{ C_\rho \left( \frac{\Lambda^2_\rho - m^2_\rho}{\Lambda^2_\rho - q^2} \right)^2 \frac{\vec{q}^2}{q^2 - m^2_\rho} - g'(q) \right\}$.  

$C_\rho = 2$, $\Lambda_\rho = 2500$ MeV, $m_\rho = 770$ MeV.  

$\frac{f^2}{4\pi} = 0.08$, $\Lambda_\pi = 1200$ MeV,  

The $N\Delta$ and the $\Delta\Delta$ potentials are obtained from $V_l$ and $V_t$ by replacing  

$\hat{\sigma} \to \hat{S}$, $\hat{\tau} \to \hat{T}$,  

$\hat{f} \to \hat{f}^*$.
The spin-isospin part of the interaction, taking into account the propagation of the mesons through the medium

\[
W_{\sigma\tau}(q) = \sigma_1^i \sigma_2^j \tilde{\tau}_1 \tilde{\tau}_2 V_{ij}^{\sigma\tau}(q) + \\
\sigma_1^i \sigma_2^j \tilde{\tau}_1 \tilde{\tau}_2 \{V_{ik}^{\sigma\tau}(q) U(q) V_{kj}^{\sigma\tau}(q)\} + \\
\sigma_1^i \sigma_2^j \tilde{\tau}_1 \tilde{\tau}_2 \{V_{ik}^{\sigma\tau}(q) U(q) V_{km}^{\sigma\tau}(q) U(q) V_{mj}^{\sigma\tau}(q)\} + \\
\ldots = \sigma_1^i \sigma_2^j \tilde{\tau}_1 \tilde{\tau}_2 W_{ij}^{\sigma\tau}(q)
\]

\[
U(q) = U_N(q) + U_{\Delta}(q) \\
(direct \ + \ crossed \ terms)
\]

Induced spin-isospin NN interaction in a nuclear medium

\[
W_{ij}^{\sigma\tau}(q) = \frac{V_{l}(q)}{1-U(q)V_{l}(q)} \hat{q}_i \hat{q}_j + \frac{V_{l}(q)}{1-U(q)V_{l}(q)} (\delta_{ij} - \hat{q}_i \hat{q}_j)
\]

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Diagrammatically,

\[ W_{ij}^{\sigma\tau}(q) = \frac{V_t(q)}{1-U(q)V_t(q)} \hat{q}_i \hat{q}_j + \frac{V_t(q)}{1-U(q)V_t(q)} (\delta_{ij} - \hat{q}_i \hat{q}_j) \]

From the spin-isospin interaction, we construct the induced interaction by exciting \( \text{ph} \) and \( \Delta h \) components in a RPA sense.

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To compute the pion selfenergy, only irreducible diagrams should be considered: $V \rightarrow \rho + \text{SRC} (g')$ [the initial pion selects the longitudinal channel].

Also for photons! it cannot be $\gamma$, it should be $V$: ph-ph, ph-$\Delta h$ interaction.
Remarks:

RPA corrections should disappear (ratio goes to 1.0) at very large $Q^2$ values, because this is a collective effect which strength decreases when sizes larger than one nucleon are no longer being probed. Hence in any realistic model, one should expect a qualitative $Q^2$ behavior similar to that exhibited by the $\frac{QE_{RPA}}{QE_{noRPA}}$ ratio line depicted in the figure. Low $Q^2$ suppression, followed by an enhancement that could even give rise to a net increase of the cross section, and finally all RPA effects should disappear for sufficiently high $Q^2$ values.

Also for photons, $W, Z$! cannot be $\gamma$, $W, Z$, it should be $V$: ph-ph, ph-$\Delta h$ interaction.

Juan Nieves, IFIC (CSIC & UV)
Inclusive muon capture in nuclei


- Hydrogen-like atom, but $R_{\mu^-} \ll R_e^-$, since $m_\mu \gg m_e$
- There are screening and relativistic effects (solve Dirac equation)
- $\mu^-$ can be absorbed by the nucleus
  \[(\mu^- A_Z)_{\text{bound}} \rightarrow \nu_\mu X \quad [\mu^- p \rightarrow n \nu_\mu]\]

\[\Gamma = -2 \, Im \, \Pi_{\mu^-}\]

Leptonic current

\[L(x) = \sqrt{2} G J^\mu(x) L_\mu^+(x)\]
\[\tilde{L}^\mu_+ \rightarrow \bar{u}_\nu(p_\nu) \gamma^\mu (1 - \gamma_5) u_\mu(p_\mu),\]
\[\tilde{J}^\mu \rightarrow \bar{u}_n(p_n) [g_V \gamma^\mu + i \frac{g_M}{2m_p} \sigma^{\mu\nu} q_\nu + g_A \gamma^\mu \gamma_5 + \frac{g_P}{m_\mu} q^\mu \gamma_5] u_p(p_p)\]

Hadron current

Juan Nieves, IFIC (CSIC & UV)
Lowest order contribution: imaginary part of the Lindhard function

\[ \Gamma = -2 \int \frac{d^3p_v}{(2\pi)^3} \frac{2m_v}{2E_v} \frac{2m_\mu}{2E_\mu} \frac{2m_n}{2E_n} \sum \sum |T|^2 \text{Im} \tilde{U}(p_v - p_\mu) \]

Strong renormalization effects:

\[ \mathbf{V}_l(q) + \mathbf{V}_t(q) \]

Finite nuclei: LDA (local density appx)

\[ \Gamma = \int d^3r |\Phi_{1s}(r)|^2 \tilde{\Gamma}(\rho_p(r), \rho_n(r)) \]

Juan Nieves, IFIC (CSIC & UV)
Shell model

(Capturing Rate vs Atomic Number)

(Nuclear Effects? SM vs FG
(Amaro, Nieves, Maieron and Valverde EPJA 24, 343)

Juan Nieves, IFIC (CSIC & UV)
Pionic Atoms

[J. Nieves et al., NPA 554 (1993) 509]

- Precise experimental measurements (spectroscopy techniques): shifts $\varepsilon = B_{\text{exp}} - B_{\text{em}}$ and widths $\Gamma \rightarrow$ information on the pion-nucleus interaction
- $\Pi(q^0, \bar{q}, \rho(r)) = 2q^0 V_{\text{opt}}(r)$, we solve the Klein-Gordon equation
- Impulse approximation

$$\Pi(q^0, \bar{q}) \approx T(q^0, \bar{q}) \rho \Rightarrow \Gamma = 0$$

- Widths come from absorption by two nucleons: $\pi^- NN \rightarrow NN$ that give rise to complex terms, proportional to $\rho^2$ in $\Pi(q^0, \bar{q}, \rho(r))$.
- There exists experimental information on 1S and 2P levels in light nuclei and 3D and 4F in heavy nuclei. Phenomenological potentials fail to describe these data: problem of the anomalies in pionic atoms.

Juan Nieves, IFIC (CSIC & UV)
Precisions of the order of 5% through the whole periodic table.

Range: eV to 20-30 KeV

Juan Nieves, IFIC (CSIC & UV)
Many body (density) expansion in the number of hole lines!

Juan Nieves, IFIC (CSIC & UV)
\[ 2 \omega V^{(s)}_1(r) = -4\pi [(1 + \epsilon)(b_0 + \Delta b_0(r))f(T)\rho + (1 + \epsilon)b_1(\rho_0 - \rho_p) \]
\[ + i(\text{Im} B_0(1+\frac{1}{2}\epsilon)2(\rho^2_p + \rho_p\rho_0) + \text{Im} B_0^\omega(T)(1+\frac{1}{2}\epsilon)\rho^2) ] \]

\[ 2 \omega V^{(p)}_{\mathrm{opt}}(r) = 4\pi \frac{M_N}{s} \left[ \nabla \frac{P(r)}{1 + 4\pi g' P(r)} \nabla - \frac{1}{2} \epsilon \Delta \left( \frac{P(r)}{1 + 4\pi g' P(r)} \right) \right] \]

Juan Nieves, IFIC (CSIC & UV)
low densities

Fig. 8. Results for the shifts of different pionic states with different potentials. (a), (c), (e), (g): SM potential (solid line) of ref. 16 given by eqs. (57) and (58), and Lt potential (dashed line) of ref. 17 given by eqs. (59) and (60); (b), (d), (f), (h): Our potentials, theoretical potential (TH, solid line) and the semiphenomenological potential (THM, dashed line). The experimental values and the errors are statistical averages (see ref. 18) for detailed values of the experimental results. The lines between data points are just to guide the eye.
 medium densities

Juan Nieves, IFIC (CSIC & UV)

Theoretical potential
Theoretical solution to the problem of the anomalies in pionic atoms!

Juan Nieves, IFIC (CSIC & UV)
\[ \pi^\pm \rightarrow \text{nucleus reactions} \]

- \( \pi^\pm \rightarrow \text{nucleus reactions} \) [J. Nieves et al., NPA 554 (1993) 554]
  - \( \pi^\pm A_Z \rightarrow \pi^\pm A_Z \) [elastic]
  - \( \pi^\pm A_Z \rightarrow \pi' X \) [quasielastic]
  - \( \pi^\pm A_Z \rightarrow X \) (no pions) [absorption]

- Determination of neutron distributions from pionic atom data [C. García-Recio et al., NPA 547 (1992) 473]

- Radiative pion capture [H.C. Chiang et al., NPA 510 (1990) 573]
  \( (\pi^- A_Z)_{\text{bound}} \rightarrow \gamma X \)

- Chiral symmetry restoration [C. García-Recio et al., PLB 541 (2002) 64]
  \[ \frac{f_\pi(\rho)}{f_\pi} \rightarrow 0, \rho \gg 0 \]

Juan Nieves, IFIC (CSIC & UV)
Juan Nieves, IFIC (CSIC & UV)
Absorption + Quasielastic = Reaction

Q= pions which have changed either charge, energy or momentum

Juan Nieves, IFIC (CSIC & UV)
Inclusive electron-nucleus scattering

Juan Nieves, IFIC (CSIC & UV)

[A. Gil et al., NPA 627 (1997) 543; NPA 627 (1997) 599]
\[ e + A_Z \rightarrow e' X \]

\[
\frac{d^2 \sigma}{d \Omega_e' dE_e'} = \frac{\alpha^2 |k'|}{q^4 |k|} L^{\mu \nu} W_{\mu \nu}
\]

\[ L_{\mu \nu}(e, e') = 2 \left( k_{e \mu} k_{e \nu} + k_{e \nu} k_{e \mu} + \frac{q^2}{2} g_{\mu \nu} \right) \]

\[ W^{\mu \nu} = -\frac{1}{\pi e^2} \int d^3 r \frac{1}{2} (\text{Im} \Pi^{\mu \nu} + \text{Im} \Pi^{\nu \mu}) \]

virtual photon self-energy in the nuclear medium

\[ \Gamma(k) = -2 \frac{m_e}{E_e} \text{Im} \Sigma \]

\[ \Sigma_r(k) = ie^2 \int \frac{d^4 q}{(2\pi)^4} \bar{u}_r(k) \gamma^\mu \frac{(k' + m_e)}{k'^2 - m_e^2 + i\epsilon} \gamma^\nu u_r(k) \frac{\Pi_{\gamma \gamma}^{\mu \nu}(q)}{(q^2 + i\epsilon)^2} \]

\[ d\sigma = \Gamma(k) dt dS = -\frac{2m}{E_e} \text{Im} \Sigma dl dS = -\frac{2m}{|k|} \text{Im} \Sigma d^3 r \rightarrow \sigma = -\int d^3 r \frac{2m}{|k|} \text{Im} \Sigma(k, \rho(r)) \]

Juan Nieves, IFIC (CSIC & UV)
Technically Cutkowsky’s rules are used to obtain the imaginary part of the many body Feynman diagrams

\[ \Sigma(k) \to 2i \text{Im} \Sigma(k) \Theta(k^0), \]
\[ \Xi(k') \to 2i \text{Im} \Xi(k') \Theta(k'^0), \]
\[ \Pi^{\mu\nu}(q) \to 2i \text{Im} \Pi^{\mu\nu}(q) \Theta(q^0), \]

where

\[ \Xi(k') = \frac{1}{k'^2 - m^2 + i\epsilon} \]
Quasielastic peak

(lowest order: imaginary part of the Lindhard function)

\[
\text{Im } \Pi^{00} = \frac{1}{2} \text{Im } \hat{U}(q, \rho) \langle \text{Tr}(V^0 V^\dagger) \rangle, \\
\text{Im } \Pi^{xx} = \frac{1}{2} \text{Im } \hat{U}(q, \rho) \langle \text{Tr}(V^x V^\dagger^x) \rangle.
\]

\[
p^2 = (p + q)^2 = M^2 \rightarrow q^2 = -2pq \rightarrow q^0 = - \frac{q^2}{2M}
\]

Cutkowsky’s rules

spectral function?

Juan Nieves, IFIC (CSIC & UV)
Spectral Function (SF) + Final State Interaction (FSI): dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the $ph$ excitation

- Change of nucleon dispersion relation:
  * hole $\Rightarrow$ Interacting Fermi sea (SF)
  * particle $\Rightarrow$ Interaction of the ejected nucleon with the final nuclear state (FSI)

\[
G(p) \rightarrow \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \vec{p}')}{p^0 - \omega - i\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega, \vec{p}')}{p^0 - \omega + i\epsilon}
\]

The hole and particle spectral functions are related to nucleon self-energy $\Sigma$ in the medium,

Juan Nieves, IFIC (CSIC & UV)
\begin{align*}
S_{p,h}(\omega, \vec{p}) &= \mp \frac{1}{\pi} \frac{\text{Im}\Sigma(\omega, \vec{p})}{[\omega^2 - \vec{p}^2 - M^2 - \text{Re}\Sigma(\omega, \vec{p})]^2 + [\text{Im}\Sigma(\omega, \vec{p})]^2} \\
\text{with } \omega \geq \mu \text{ or } \omega \leq \mu \text{ for } S_p \text{ and } S_h, \text{ respectively (}\mu \text{ is the chemical potential).}
\end{align*}

To take into account SF+FSI \rightarrow \text{replace Im}\bar{U}_R^N(q) \text{ by the response function:}

\begin{align*}
- \frac{1}{2\pi} \int_0^{+\infty} dp p^2 \int_{-1}^{+1} dx \int_{\mu - q^0}^{\mu} d\omega S_h(\omega, \vec{p}) S_p(q^0 + \omega, t)
\end{align*}

with \( t^2 = \vec{p}^2 + \vec{q}^2 + 2|\vec{p}||\vec{q}|x. \)

This nuclear effect is additional to those due to RPA (long range) correlations!!
\[ S_{p,h}(\omega, \vec{p}) = \mp \frac{1}{\pi} \frac{\text{Im} \Sigma(\omega, \vec{p})}{[\omega^2 - \vec{p}^2 - M^2 - \text{Re} \Sigma(\omega, \vec{p})]^2 + [\text{Im} \Sigma(\omega, \vec{p})]^2} \]

with \(\omega \geq \mu\) or \(\omega \leq \mu\) for \(S_p\) and \(S_h\), respectively (\(\mu\) is the chemical potential).

---

For non interacting fermions [\(\Sigma = 0\)],

\[ S_p(\omega, \vec{p}) = \frac{\theta(|\vec{p}| - k_F)}{2E(\vec{p})} \delta(\omega - E(\vec{p})) \]

\[ S_h(\omega, \vec{p}) = \frac{\theta(k_F - |\vec{p}|)}{2E(\vec{p})} \delta(\omega - E(\vec{p})) \]

and only Pauli blocking is incorporated!!
The simplest description ⇒ relativistic Fermi Gas with non interacting fermions \( \Sigma = 0 \).

\[
S_p(\omega, \vec{p}) = \frac{\theta(|\vec{p}| - k_F)}{2E(\vec{p})} \delta(\omega - E(\vec{p}))
\]

\[
S_h(\omega, \vec{p}) = \frac{\theta(k_F - |\vec{p}|)}{2E(\vec{p})} \delta(\omega - E(\vec{p}))
\]

and only Pauli blocking is incorporated!!

Local vs Global Fermi Gas ?

\[
k_F^{p,n}(r) = \left[\frac{3\pi^2}{2}\rho^{p,n}(r)\right]^{1/3} \quad \text{vs} \quad k_F = \text{cte} ?
\]

Convolution approach: C. Ciofi degli Atti, S. Liuti, and S. Simula, PRC 53, 1689 (1996), provide realistic distribution due to short-range correlations!
Polarization effects (RPA) at QE peak

\[ c_0\{f_0(\rho) + f'_0(\rho)\vec{\tau}_1\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2 \} + \sum_{i,j=1}^{3} \sigma_i^j \sigma_j^i V_{ij} \]

Fig. 37. Polarization (RPA) effect in the evaluation of \( R_L \).
RPA vs SF effects: Differential cross sections for the CCQE reaction on $^{12}$C averaged over the MiniBooNE flux
(Alvarez-Ruso L et al., 2009 AIP Conf. Proc. 1189 151)

It depends on the specific kinematics and observable!

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$R_L$ and $R_T$ QE response functions for $e + {}^{40}\text{Ca} \rightarrow e' + X$
Pion production

one of the terms generates the $\Delta$ contribution

Juan Nieves, IFIC (CSIC & UV)
RPA corrections to the dominant $\Delta h$ term

Juan Nieves, IFIC (CSIC & UV)
• Δ dominant component of the pion production contribution
• Missing strength both at the dip region and the Δ peak
2p2h (two body absorption) contributions

RPA corrections to 2p2h contributions

Two cuts: \( \gamma^* NN \rightarrow NN \)

\( \gamma^* N \rightarrow N\pi \) (dressed)

Juan Nieves, IFIC (CSIC & UV)
Fig. 44. Two-body photon absorption (solid line) versus pion production (dotted line) for $^{12}$C.

Juan Nieves, IFIC (CSIC & UV)
Additional contributions generated from the channel

$$\gamma^* N \rightarrow N\pi\pi$$

Fig. 23. Relevant Feynman diagrams that enter in the evaluation of the $$\gamma^* N \rightarrow N2\pi$$ cross section.
\[ e + {}^{12}\text{C} \rightarrow e' + X \]
and by means of a Monte Carlo simulation we obtain cross sections for the processes \((e, e'N), (e, e'NN), (e, e'\pi), \ldots\)
Important difference:

\[ p^2 = (p + q)^2 \]

is forbidden!

Real Photon Results

Same formalism applied to the study of the interaction of Real Photons with Nuclei at Intermediate Energies: Total Photo-absorption cross section \( \gamma A_2 \rightarrow X \) [Carrasco + Oset, NPA 536 (1992) 445] and Inclusive \((\gamma,\pi), (\gamma,N), (\gamma,NN)\) and \((\gamma,N\pi)\) reactions [Carrasco + Oset + Salcedo NPA 541 (1992) 585 and Carrasco+Vicente-Vacas+ Oset NPA 570 (1994) 701]

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does not have solution when all particles are in the mass shell, i.e.

\[ p^2 = (p + q)^2 = M^2 \text{ and } q^2 = 0. \]
\[ \gamma + A \rightarrow X \]

Fig. 45. Results for \( \sigma_e/A \) as a function of the photon energy for \(^{12}\text{C}\). Experiment from ref. \(^{1}\). The lower curve is the result for direct photon absorption.

Fig. 46. Results for \( \sigma_e/A \) as a function of the photon energy for \(^{16}\text{O}\). Experiment from ref. \(^{1}\).

Fig. 47. Continuous line: results for \( \sigma_e/A \) as a function of the photon energy for \(^{208}\text{Pb}\). The dashed line shows the impulse approximation result \( i \Delta \sigma_{e}\) for comparison. The dotted line is the result for direct photon absorption. Experimental data: dark dots from ref. \(^{1}\), while dots from ref. \(^{2}\).

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