ν Deeply Inelastic Scattering

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Lecture III - Life in the Real World: low Q^2, nuclear effects and more

- CJ Collaboration
- Leading twist versus power suppressed corrections
- Higher Twist contributions
- Target Mass Corrections
- Heavy Quarks
- Nuclear Effects
- Duality

Life in the Real World ...

- Large momentum transfer scales enable

    1. Use of perturbative techniques due to the small value of α_s
    2. Neglect of power suppressed contributions
    3. Neglect of nuclear effects since most collider data use protons or antiprotons

- Life is (relatively) simple

- Low momentum transfer scales require that attention be paid to these and other items

- The CJ Collaboration was formed to investigate a number of these issues

CJ Collaboration

- Cast of characters in the CTEQ-JLAB Collaboration - Alberto Accardi, Eric Christy, Cynthia Keppel, Simona Malace, Wally Melnitchouk, Peter Monaghan, Jorge Morfín, JFO, and Lingyan Zhu
- Formed to study the effects of decreasing the cuts on Q^2 and W^2 in DIS global fits, thereby allowing the large-x region to be probed
- Special motivation provided by Jorge Morfín precisely to be of service to the neutrino community
- I will use examples from the CJ fits to illustrate the effects of various power-suppressed contributions

- The formalism outlined in Lectures I and II pertains to the so-called leading twist contribution (an old bit of terminology that means the leading contribution in terms of powers of Q^2)
- There are other contributions which are suppressed by one or more powers of Q^2
- Examples most often encountered include
    1. Target mass corrections (TMC)
    2. Higher twist contributions (HT)
- Actually, both are examples of power suppressed corrections, so be careful when reading the literature
- Some call the TMCs "kinematic higher twist" and HT "dynamical higher twist"
- If Q^2 is not large on a scale of 1 GeV^2 or so, then these terms can be non-negligible

Nachtmann Variable

In Lecture I we saw that in lowest order we had a factor of $\delta[(p+q)^2]$. With a massless parton, neglecting the target mass and assuming $p = \xi P$ this lead to $-Q^2 + 2\xi P \cdot q = 0$ and, therefore

$$\xi = \frac{Q^2}{2P \cdot q} = x$$

Now, suppose we didn't neglect the target mass, *i.e.*, use $(\xi P)^2 = \xi^2 M^2$. Then we have

$$\xi^2 M^2 + 2\xi P \cdot q - Q^2 = \xi^2 M^2 + \xi \frac{Q^2}{x} - Q^2 = 0$$

This has the solution

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}}$$

Exercise: Show this.

ξ is called the Nachtmann scaling variable and replacing x by ξ is the first step in accounting for the effects of the target mass.

Georgi-Politzer TMCs

The Georgi-Politzer method for calculating target mass corrections is based on the Operator Product Expansion and is a scheme which is often used today. The corrections for F_2, for example, take the form

$$F_2^{TMC}(x,Q^2) = \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi,Q^2) + \frac{6M^2x^3}{Q^2r^4} h_2(\xi,Q^2) + \frac{12M^2x^4}{Q^4r^5} g_2(\xi,Q^2)$$

where

$$\begin{aligned} r &= \sqrt{1+\frac{4M^2x^2}{Q^2}} \\ h_2(\xi,Q^2) &= \int_\xi^1 du \frac{F_2^{(0)}(u,Q^2)}{u^2}, \\ g_2(\xi,Q^2) &= \int_\xi^1 dv(v-\xi)\frac{F_2^{(0)}(v,Q^2)}{v^2} \end{aligned}$$

$F_2^{(0)}$ is the uncorrected structure function. For additional details, see Schienbein *et al*, arXiv:0709.1775 [hep-ph].

- So, the TMC can be calculated in terms of two convolution integrals.
- One feature of this formalism is that when $x = 1$ one has $\xi < 1$.
- This gives a non-zero result for F_2 when $x = 1$
- There are other formalisms, *e.g.*, the "Collinear Factorization" formalism, where the convolution integrals cover the range $\xi \to \xi/x$ so that the integral vanishes at $x = 1$

Dynamical Higher Twist (HT)

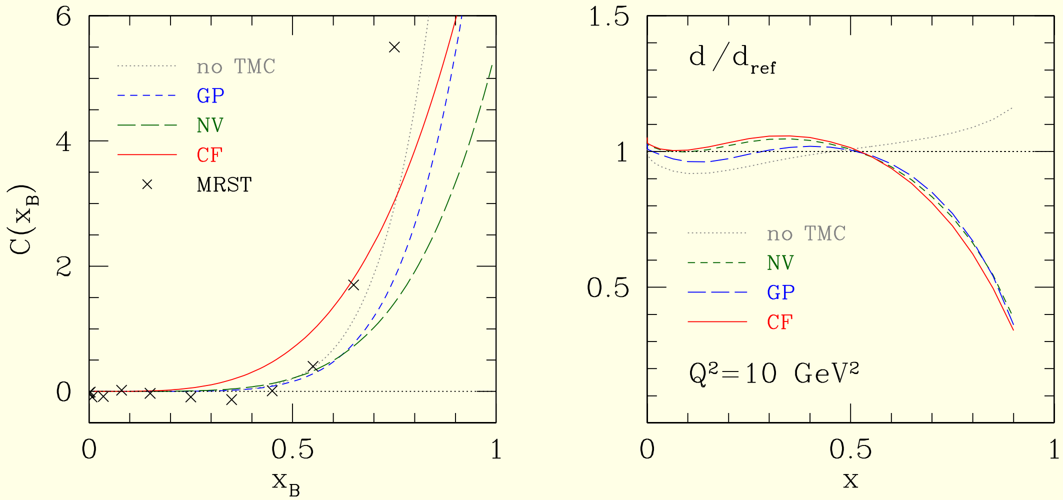
- The origin is matrix elements of operators that involve correlations between partons.
- Some work has been done on estimating these terms, but for the most part they have not been calculated
- HT contributions are typically parametrized and included in the global fits for PDFs
- Typical form is

$$F_2(data) = F_2(TMC)(1 + C(x)/Q^2)$$

where $C(x) = a\,x^b(1 + c\,x + d\,x^2)$

Comments:

- Parametrization is sufficiently flexible to give a good fit to the data
- Parameter d not really needed since for x near 1 there is not a lot of difference between x and x^2
- Differences in higher twist contributions for p or d can be included if/when required by data
- In principle, HT terms can be different for different structure functions
- Can be different for charged lepton DIS and neutrino DIS
- Extracted HT contributions depend on the order to which one is working in perturbation theory
- As one goes from LO $\rightarrow$ NLO $\rightarrow$ NNLO the size of the phenomenological HT terms decreases
- Leading twist PDFs, model-dependent TMCs, model-dependent HTs, and model-dependent nuclear corrections (yet to be discussed)
- How can one have a believable extraction of the PDFs and a believable model for the DIS cross sections?
- At least for the TMC/HT terms, the model-dependence cancels out



- Left-hand plot shows fitted HT terms corresponding to different choices of TMCs
- Right-hand plot shows the ratio of fitted d PDFs to a reference d PDF fitted to higher Q^2 data
- The leading twist d PDF is seen to be stable to the choice of TMC
- The HT parametrization compensates, leaving a unique PDF
- Similar results for other PDFs

Heavy Quarks

- The treatment of heavy quarks depends on whether or not you treat then as partons with their own distributions.
- Two types of schemes
  1. Zero Mass Variable Flavor scheme
  2. Fixed Flavor scheme
- For the ZMVF scheme the heavy quark H PDF is zero until one crosses the threshold $Q = M_H$ whereupon the H PDF is generated via the DGLAP Equations. As one crosses the threshold the number of active flavors increases by one. This threshold is the only place where then mass M_H explicitly enters. Terms of order $\log(M_H^2/Q^2)$ are resummed.
- For the Fixed Flavor scheme the heavy quarks are produced via the photon-gluon fusion mechanism $\gamma^* g \to H\bar{H}$ and the masses are explicitly retained.

Pros and Cons

- Near threshold the Fixed Flavor scheme is more precise
- In the ZMVF scheme, as Q^2 becomes much greater than M_H^2 the potential large logs are resummed, making this approach more precise
- One would like a scheme that interpolates between the two
- Examples of such schemes are the ACOT (Aivazis, Collins, Olness, and Tung) scheme and a simplified version called the S-ACOT scheme, although these are not the only ones
- Keeping track of the heavy quark mass effects near threshold is important because their contributions are suppressed by the mass terms, so the other PDFs have to change in order to fit the data
- This is most important at small values of x, so the sea quark terms are the most affected.

Example from Neutrino Scattering

- The lowest order processes to produce a charm quark would be $Ws \to c$ and $Wd \to c$
- Treating the threshold region for these subprocesses is important in order to get the s quark PDF correct.
- Some simple kinematics - keep the final state quark mass. Then the phase space constraint becomes

$$\delta[(p+q)^2 - M_c^2]$$

- Introducing $p = \xi P$ with $x = Q^2/2P \cdot q$ we find

$$\xi = x(1 + \frac{M_c^2}{Q^2})$$

  Exercise: Show this
- This is called slow rescaling. Near threshold the PDF is probed at $\xi > x$ thereby suppressing this subprocess

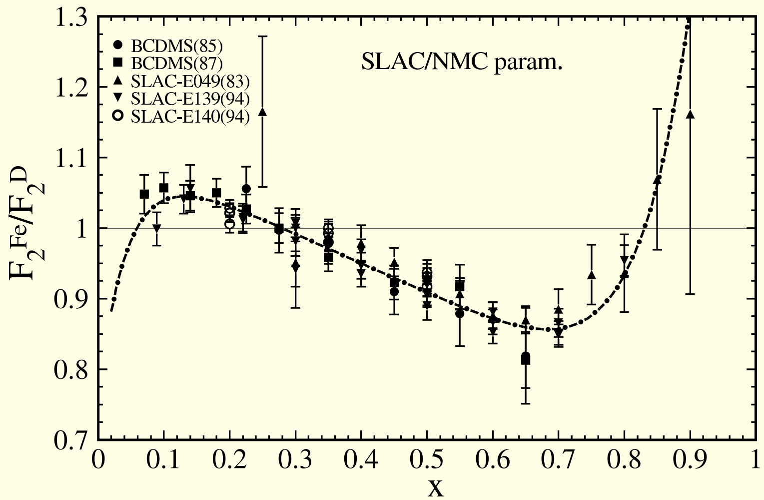
- In the full ACOT or S-ACOT schemes one also uses the correct kinematics near threshold for the higher order terms such as $Vg \to q\bar{q}'$
- Such considerations will be important for treating neutrino DIS processes in the few GeV to 10s of GeV region
- Charm production (observed via opposite sign dimuon events) is an important source of information on

  1. The average strange PDF $\frac{s+\bar{s}}{2}$
  2. The difference of the PDFs $\frac{s-\bar{s}}{2}$

- For this reason it is important to get the kinematics right near threshold

Nuclear Corrections

- Often use nuclear targets
    1. Increased event rate (relevant for neutrinos)
    2. To probe different PDFs, *e.g.*, use of deuterium in DIS as a way to scatter off neutrons
- PDFs in nuclei are in principle different from PDFs on a free nucleon target
- Several sources
    1. Fermi motion
    2. Binding/off-shell effects
    3. Screening
    4. Collective effects (scattering coherently from more than one nucleon)



See three distinct regions in the ratio F_2^{Fe}/F_2^d

- Screening at low values of x
- enhancement followed by suppression in the mid-x region
- Enhancement from Fermi motion at high values of x

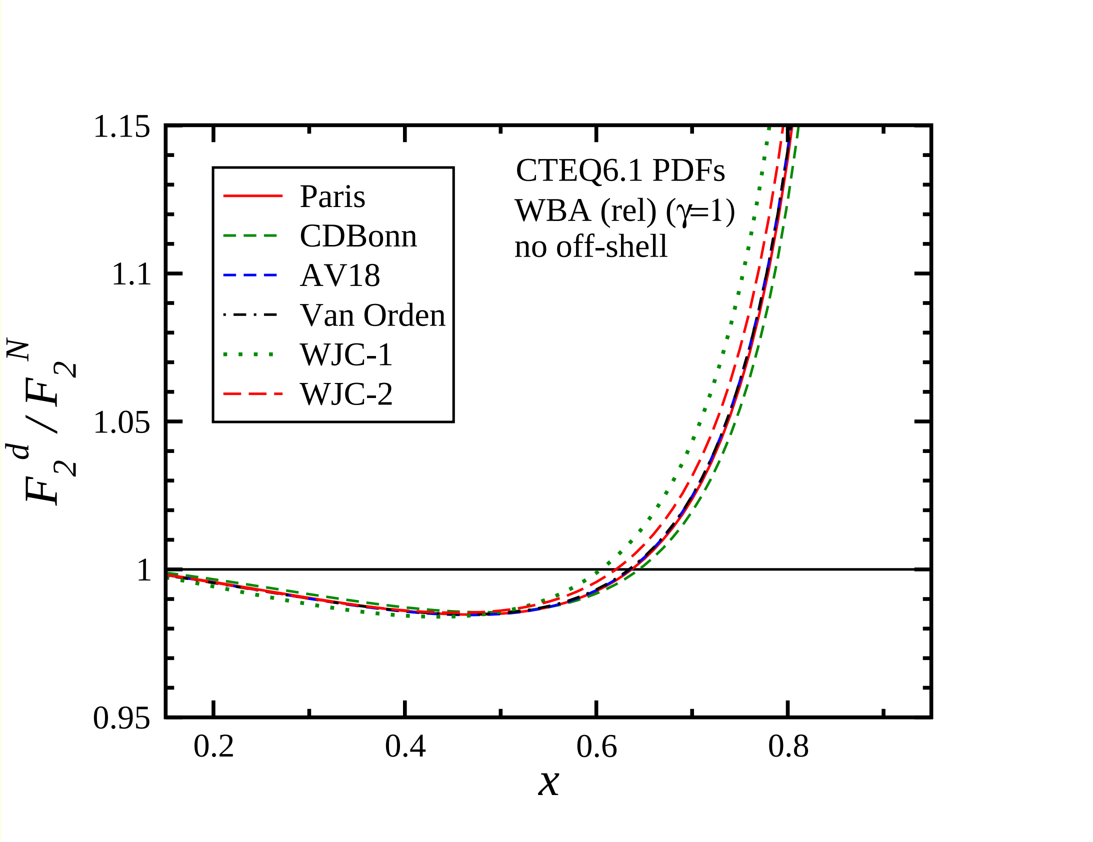
Fermi Motion

- Motion of the individual nucleons causes a smearing of the x distribution (lab frame is not the nucleon rest frame)
- Effect is largest where the cross section is steeply falling (large-x region)
- Calculate as a convolution

$$F_2^A(x, Q^2) \approx \sum_{N=p,n} \int_x^{M_A/M} dy \; f_{N/A}(y, \gamma) \; F_2^N\left(\frac{x}{y}, Q^2\right) .$$

where γ is a kinematic factor given by $\gamma = \sqrt{1 + 4x^2 M^2/Q^2}$.

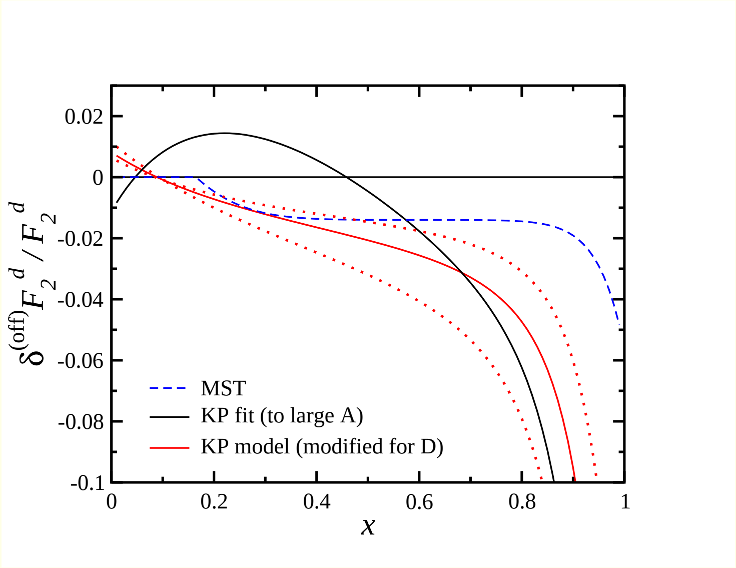
- Requires knowledge of the wavefunction of a nucleon in the nucleus A
- For heavy nuclei often use a parametrization of the ratio F_2^A/F_2^d, for example
- For deuterium we have a selection of wavefunctions to choose from

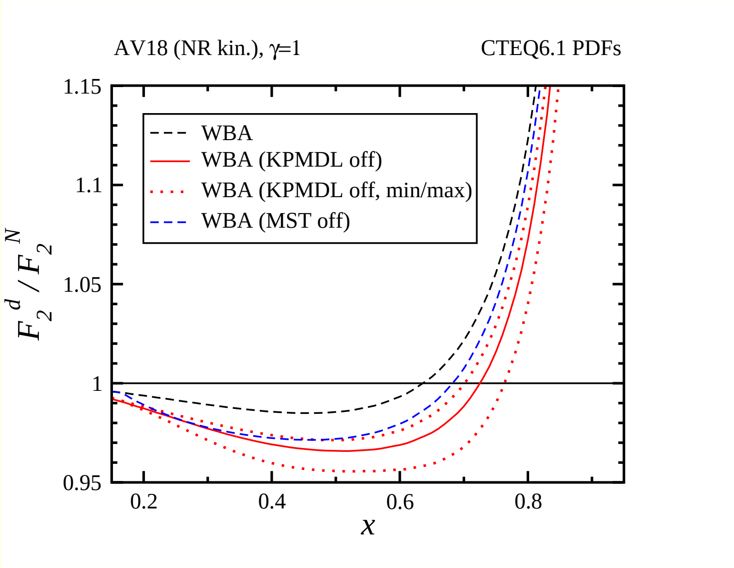


- Various wavefunctions seem to yield rather similar results
- Deceptive - the curves are rapidly rising, so look closely at the vertical displacement at fixed x
- Theme I will return to later - collider data (needing no nuclear corrections) can help select the correct model(s) for nuclear corrections since the extracted PDFs will depend on the nuclear corrections

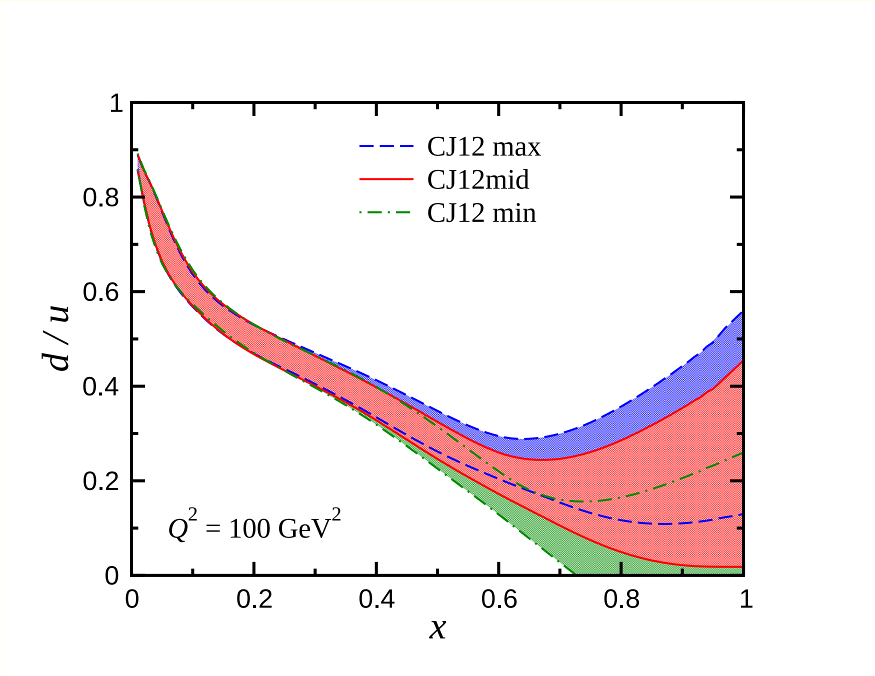
Off-Shell Corrections

- Start with a parametrization due to Kulagin and Petti which is fitted to data for a range of heavy nuclei
- Parameters were adjusted (Wally Melnitchouk) to provide a range of corrections representative of the average offshellness of nucleons in a deuteron





- Easy way to think about the effects of the nuclear corrections on the PDFs
- The deuterium data are divided by this ratio, yielding effectively the sum of neutron and proton data
- When the ratio is less than one the data are enhanced and the d PDF will increase
- Conversely, the d PDF will be reduced when the ratio is greater than one
- So why am I talking about deuterium so much?
    1. Simplest isoscalar nucleus
    2. Cross section and structure function nuclear results are usually expressed as ratios to deuterium
    3. Deuterium is as close to a neutron target as we can get
    4. Important to understand deuterium before proceeding to other heavier targets
    5. Provides important information on the d PDF at large x - u PDF already well constrained by proton data



- Three nuclear models employed - give large variations in the d/u ratio
- Comparison to various types of collider data can choose which PDF set does best and thereby constrain the nuclear model(s)

Strategies for Heavier Nuclei

Different strategies address different physics issues

- What are the PDFs like in a particular nucleus? - fit data on one type of nucleus and compare to proton (or deuterium) results
- How do the PDFs depend on A? - fit data from a range of targets with an A-dependent parametrization
- Are the nuclear corrections the same or different for charged lepton and neutrino DIS?
- How do I "correct" data taken on a heavy target so that I effectively have the result I would have measured on a proton or deuteron target?
- What is the best way to calculate results for data taken on heavy targets?

Case Study - nCTEQ

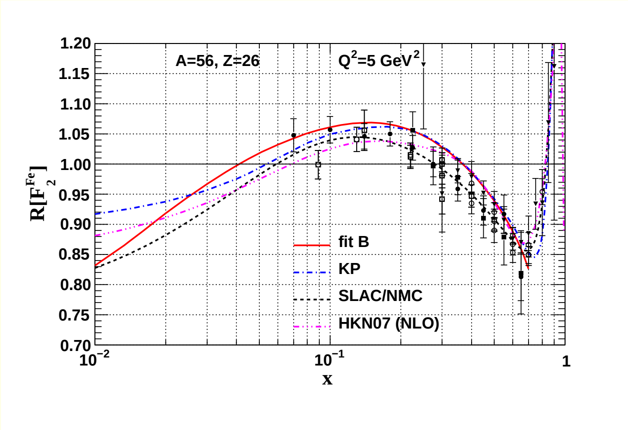
1. arXiv: 0710.4897[hep-ph] Phys. Rev. D77, 054013
2. arXiv:0907.2357[hep-ph] Phys. Rev. D80, 094004
3. arXiv:1012.1178[hep-ph] Phys. Rev. Lett. 106(2011)122301

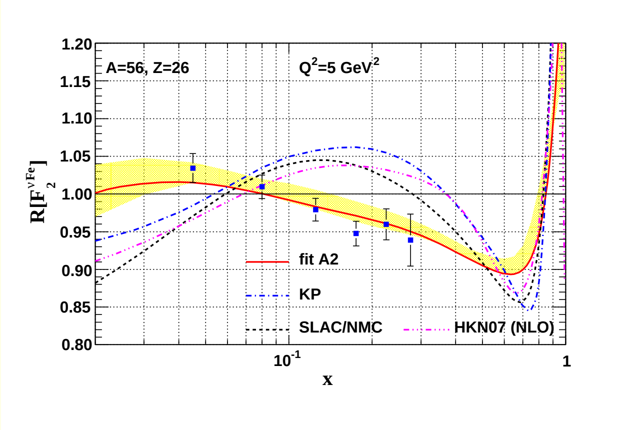
- First step - fit NuTeV data using a parametrization similar to that used in CTEQ6
- Caveats
  - Can not constrain all the PDFs, *e.g.,* the gluon, so some parameters are frozen
  - Related - some parameters have essentially flat regions in parameter space, so these were frozen, too.
  - Only fitting data on Iron so essentially one is getting Iron PDFs
- NuTeV data represent the highest statistics available with the smallest correlated systematic errors (more on this later)

Second step - fit charged lepton DIS and lepton pair production data using an A-dependent parametrization

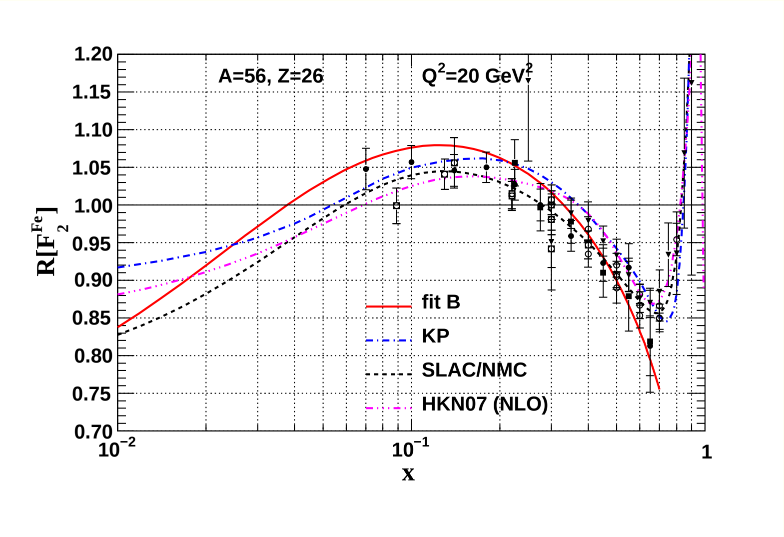
$$
\begin{aligned}
x f_k(x, Q_0^2) &= c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5} \\
\bar{d}/\bar{u}(x, Q_0^2) &= c_0 x^{c_1}(1-x)^{c_2} + (1+c_3 x)^{c_4} \\
c_k &\rightarrow c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}}) \\
f_i(x, Q^2) &= \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}(x, Q^2)
\end{aligned}
$$

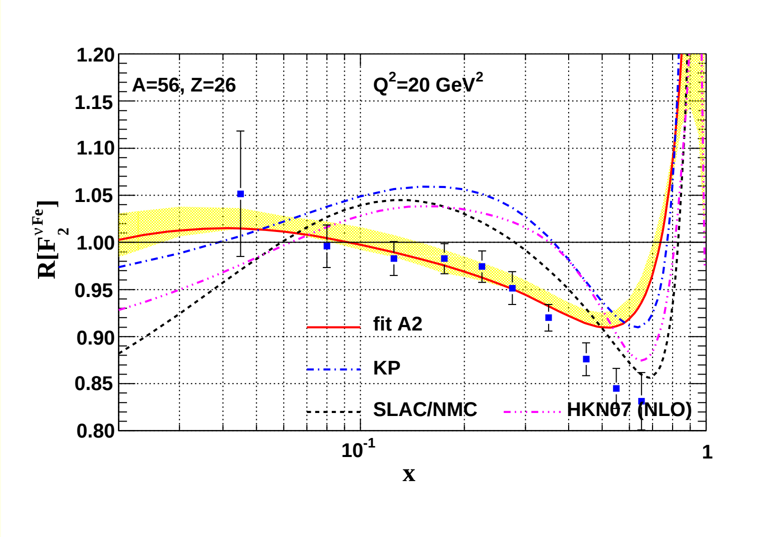
- Vary initial PDF parameters at Q_0^2 as well as the A-dependent ones
- Note that the ansatz insures that the $A \rightarrow 1$ limit reduces to the proton PDFs
- Isospin is used to relate the proton PDFs to the neutron PDFs
- Data fit were of the form $F_2^A/F_2^d, F_2^A/F_2^{A'}$ and $\sigma_{lpp}^{pA}/\sigma_{lpp}^{pA'}$
- End result is a set of A-dependent PDFS from which one can calculate, for example, F_2^{Fe}/F_2^d which can then be compared to the same quantity calculated wit the Iron PDFs found from the NuTeV data in step one





- Left figure shows F_2^{Fe}/F_2^d from the charged lepton DIS fits with several others shown for comparison at $Q^2 = 5\ \mathrm{GeV}^2$
- Right figure shows the same ratio but with the neutrino Fe PDF result shown as the yellow band with some representative data
- The neutrino results show a reduced nuclear correction in the x region from about 0.5 to 0.7, a ratio closer to one in the x region between about 0.1 to 0.2, and a lack of screening at low values of x
- So, it would seem that the nuclear corrections might be different for neutrino and charged lepton DIS





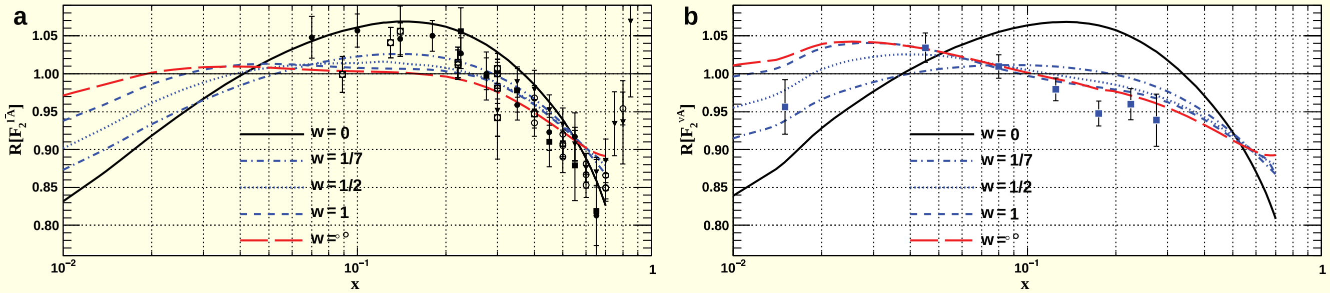
- These plots show the same quantities at $Q^2 = 20$ GeV^2 with the trends being the same.

- So, is the situation as these results suggest or is it possible to get simultaneous fits with good chi square values for both charged lepton and neutrino DIS experiments?

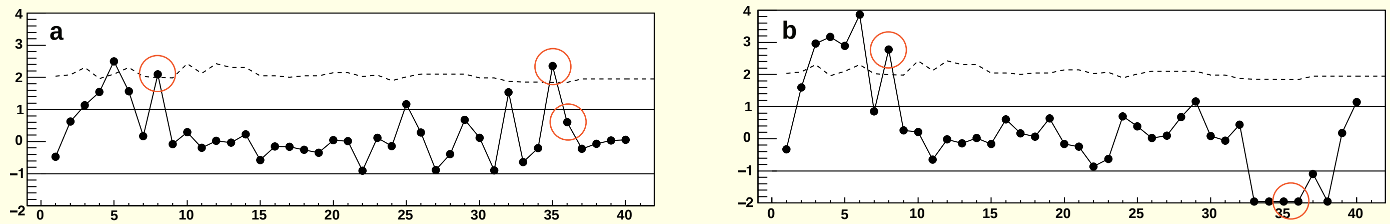
- That takes us to step 3...

Step 3 - Joint Charged Lepton and Neutrino Fits

- Is it possible to get simultaneous fits to charged lepton and neutrino data simultaneously?
- Weight the neutrino data sets (NuTeV, CHORUS, CCFR (dimuon only)) with a weight w such that $w = 0$ corresponds to fitting only the charged lepton and lepton pair production data (DY) while $w = \infty$ corresponds to fitting the neutrino data only



- Can successfully fit charged lepton data or neutrino data, but not both
- Left (right) plot shows charged lepton (neutrino) DIS data



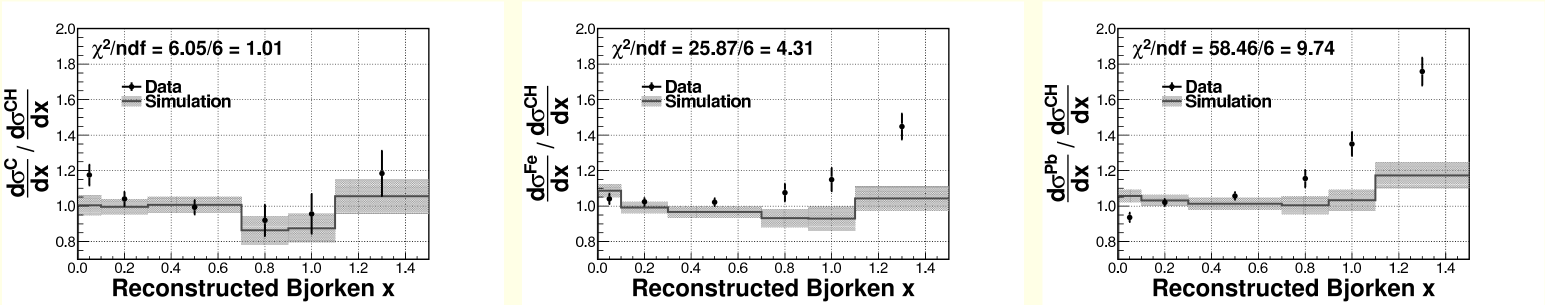
- χ-square confidence levels for each data set (charged lepton towards left end, neutrino towards right end)
- Dashed line - 99% confidence level, Solid line - 90% condidence level
- Left plot has $w = 1/2$ right plot has $w = 1$
- Plots illustrate that one can not simultaneously get acceptable fits for both the charged lepton and neutrino data
- Contradicting point of view: H. Paukkunen and C. Salgado, arXiv:1302.2001[hep-ph], PRL110(2013)212301 - allow normalization of NuTeV to be adjusted in each energy bin. Then the problem goes away and there is no conflict.

- They add the statistical and systematic errors in quadrature while the previous analysis used the full correlated systematic error matrix

- Adding in quadrature overestimates the errors and makes the corresponding χ-square smaller

- What about the CHORUS data? Statistical and systematic errors are such that the NuTeV data dominate

- Data from MINERvA



- See no screening for the Fe data, some for the Pb data
- Available simulations do not adequately describe the data
- Caveat - the data are at relatively low values of Q^2
- The jury is still out on whether nuclear corrections are different in charged lepton and neutrino DIS

Modeling ν DIS I

Until recently, most PDF determinations came from global fits that had rather high cuts on Q^2 and W^2. How, then, could one use these as the basis for modeling ν DIS on nuclear targets?

A. Bodek and U.K. Yang, arXiv:hep-ex/0203009

- Based on GRV 94 LO PDFs
- Increase the d/u ratio at high x in order to better fit DIS data
- Replace x With $x_w = \frac{Q^2+B}{2M\nu+A}$
  - Parameter A is provides an approximate way of including TMC and HT
  - Parameter B allows the fit to be extended to low values of Q^2 (near the photoproduction limit $Q^2 = 0$)

- Multiply all PDFs by $\frac{Q^2}{Q^2+C}$ - helps with the low-Q^2 limit
  - $\sigma_{\gamma p} \propto \frac{F_2}{Q^2}$
  - Finite $\sigma_{\gamma p}$ at $Q^2 = 0$ requires F_2 to be proportional to Q^2
- Freeze evolution of α_s at $Q^2 = 0.24$ GeV^2 - avoids the Landau pole at $Q^2 = \Lambda^2$
- Correct for nuclear effects using a Q^2-independent parametrization of F_2^{Fe}/F_2^d - model is only applied to ν scattering on iron
- End result is a good phenomenological description of existing DIS data - but it needs to be tested and extended to more nuclei

Based on what we've learned in these three lectures, can we do any better than this?

Modeling ν DIS II

- Can use modern PDFs describing DIS processes in NLO or NNLO
- Can calculate TMCs explicitly
- Can fit HTs using a simple parametrization
- Still need some modifications if one wants to really go to low values of Q^2
  - Transitioning from 5 to 4 to 3 flavors as Q^2 decreases
  - At some point perturbative QCD will not be adequate
  - Still have to do something about α_s and about the $Q^2 \rightarrow 0$ limit of the PDFs and structure functions
- So, there are areas of the modeling that can be updated, but there is still a need for parameters that go beyond the reach of perturbative QCD

A Few Words About Duality

Duality - more precisely "semilocal hadron duality" refers to the idea that there are dual descriptions of structure functions in the resonance region

- Recall that the squared hadronic mass is given by $W^2 = M^2 + Q^2(\frac{1}{x} - 1)$ so that as $x \to 1, W^2 \to M^2$
- Resonances appear as peaks in the cross section or structure functions
- As Q^2 increases the resonances move to higher values of x since their masses are fixed
- Duality suggests that a description of the structure functions in terms of PDFs should *on average* reproduce the resonance behavior
- Where does this come from?

Superconvergence Relations, Finite Energy Sum Rules, and Regge Theory

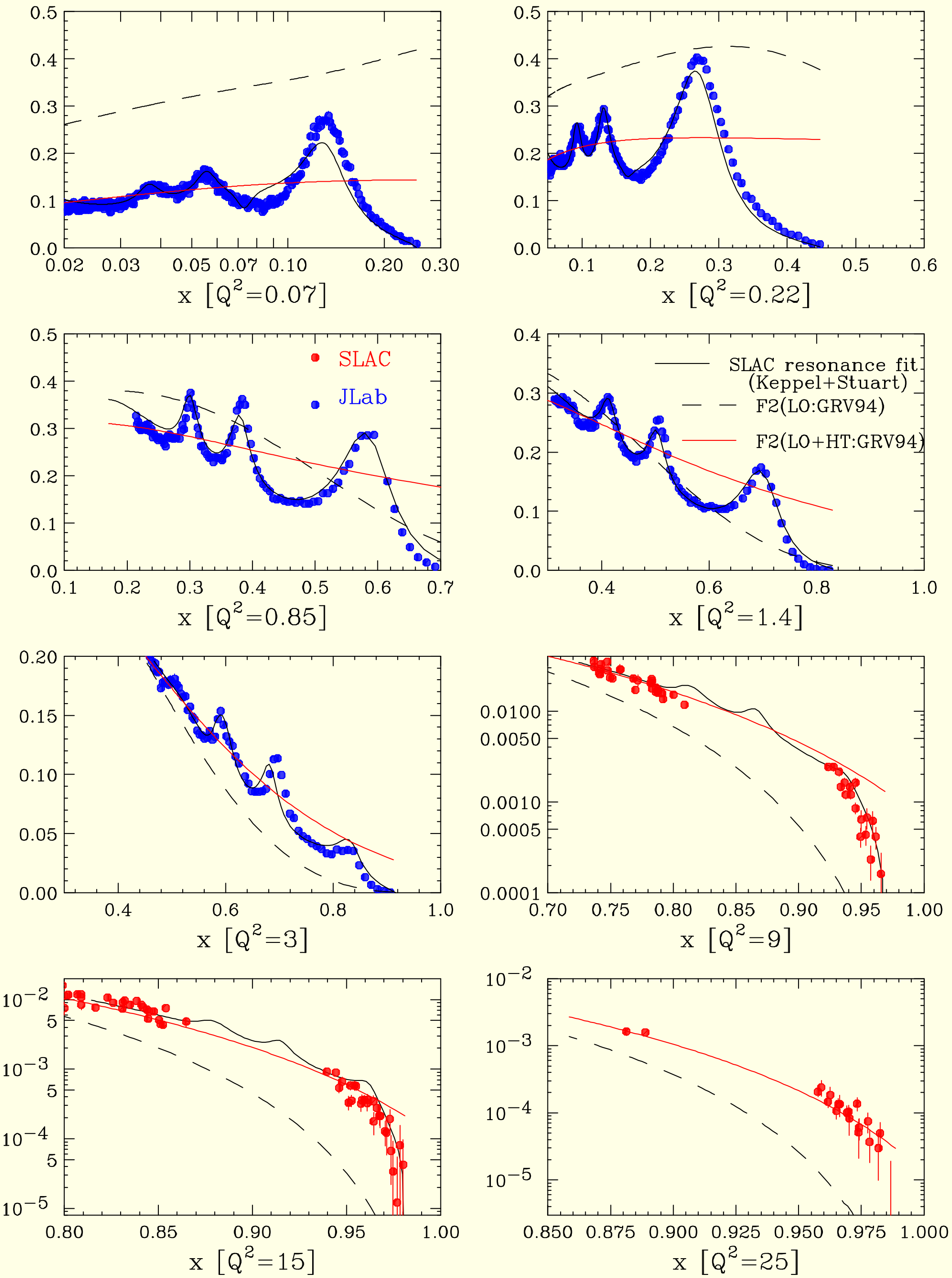
Consider an amplitude $A(s,t)$ for some two-body scattering process.

- At low energies there may be resonant structure, *e.g.*, the $\Delta(1236)$ in $\pi^+ p$ elastic scattering
- At high energies the amplitude could be expanded in terms of Regge pole exchanges giving a dependence on s going as $s^{\alpha(t)}$ where $\alpha(t)$ is a Regge trajectory
- Depending on the energy dependence of the process one can write a contour integral such that

$$\int_{s_0}^{\infty} Im A(s,t) ds = \text{constant}$$

- By suitable subtractions and adjustments one could arrange for the constant to be zero (superconvergence relation)
- Next, truncate the integration at some finite value of s which was large enough to enable a Regge description of the amplitude.
- The resulting expression basically says that the integral over the low energy region builds up a description in terms of Regge poles at some higher energy. (Finite Energy Sum Rule = FESR)
- Now lower the the upper limit into the resonance region. This says that averaging in a semilocal way over the resonances gives a description dual to that provided by the Regge poles.
- Applied to DIS structure functions, think of W^2 as playing the role of s in virtual photon proton Compton scattering

- At high W^2 (think low values of x) one can have a Regge description of the structure functions (that's why the valence PDFs go as $x^{-1/2}$ while the sea quark PDFs and gluons go as x^{-1}, but that's another story...)
- Going to large values of x puts one in the resonance region and the same idea of duality now says that the PDFs should *on average* reproduce the resonance description of the structure functions
- This is useful if one wants to make use of data that is a very low values of Q^2 and high values of x
- In the following plot one can see the ideas discussed above in action
  - Watch the resonance peaks move to higher values of x as Q^2 increases
  - The dahed lines are for a standard (high Q^2) LO PDF fit
  - The solid red curves are modified using the Bodek-Yang model
  - The solid black curve is a resonance-based description
- One can see the dual descriptions in action



x [Q^2=0.07]
x [Q^2=0.22]
SLAC
JLab
x [Q^2=0.85]
SLAC resonance fit
(Keppel+Stuart)
F2(LO:GRV94)
F2(LO+HT:GRV94)
x [Q^2=1.4]
x [Q^2=3]
x [Q^2=9]
x [Q^2=15]
x [Q^2=25]

Conclusions

- In these three lectures I have tried to give both an introduction to and an overview of DIS
- Parton model = lowest order QCD
- Higher order corrections in α_s
- Power supressed terms necessary for low values of Q^2
- Nuclear PDFs
- Duality