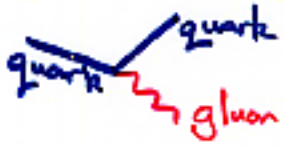
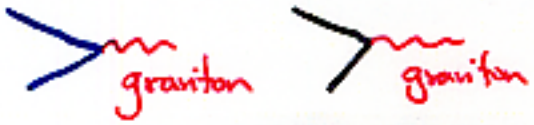
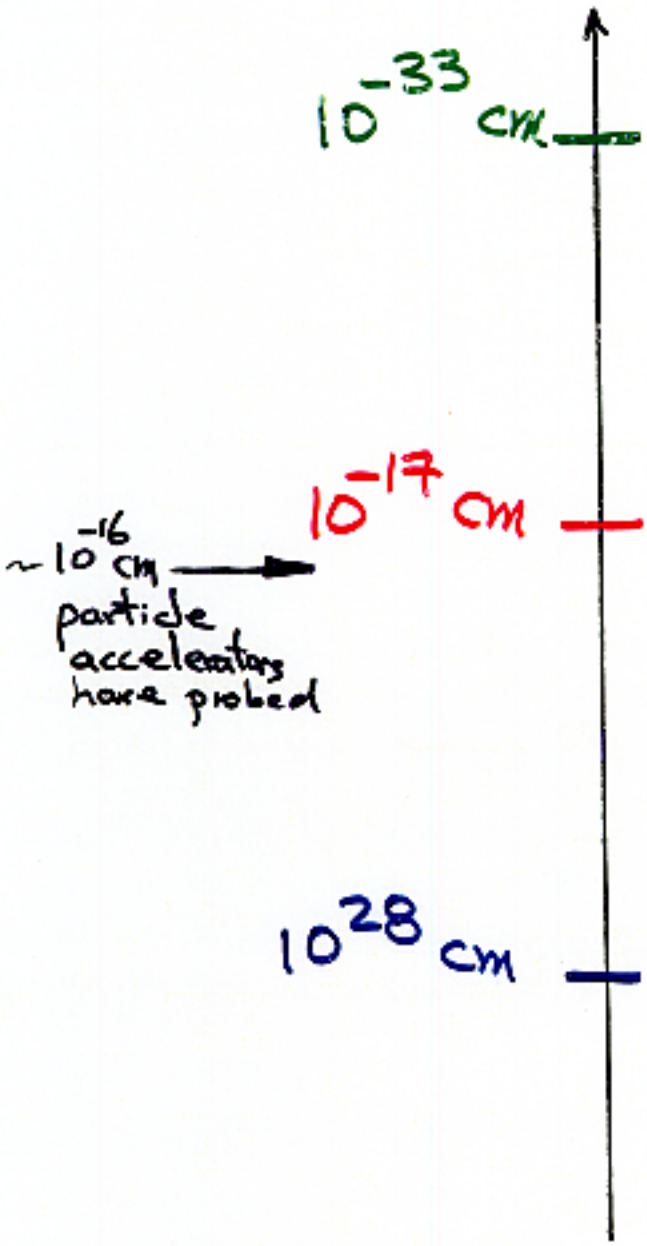
Current understanding of basic forces in Nature:

Force		Interaction strength
Strong, Weak, EM	Described by Standard Model	 g_s
		g_w
		e
Gravity	Described by G.R.	 $G_N \sim (10^{-33}\ \text{cm})^2$ [units where $\hbar = c = 1$]

g_s, g_w, e dimensionless, $\sim O(1)$ [$\alpha = e^2/4\pi \sim 1/137$]

Is in excellent agreement with experiment.

Some important length scales:



The "Planck" scale ℓ_{Pl}

$G_N \sim \ell_{Pl}^2$

"Electroweak" scale ℓ_{EW}

~ range of weak force

~ scale where mass is generated

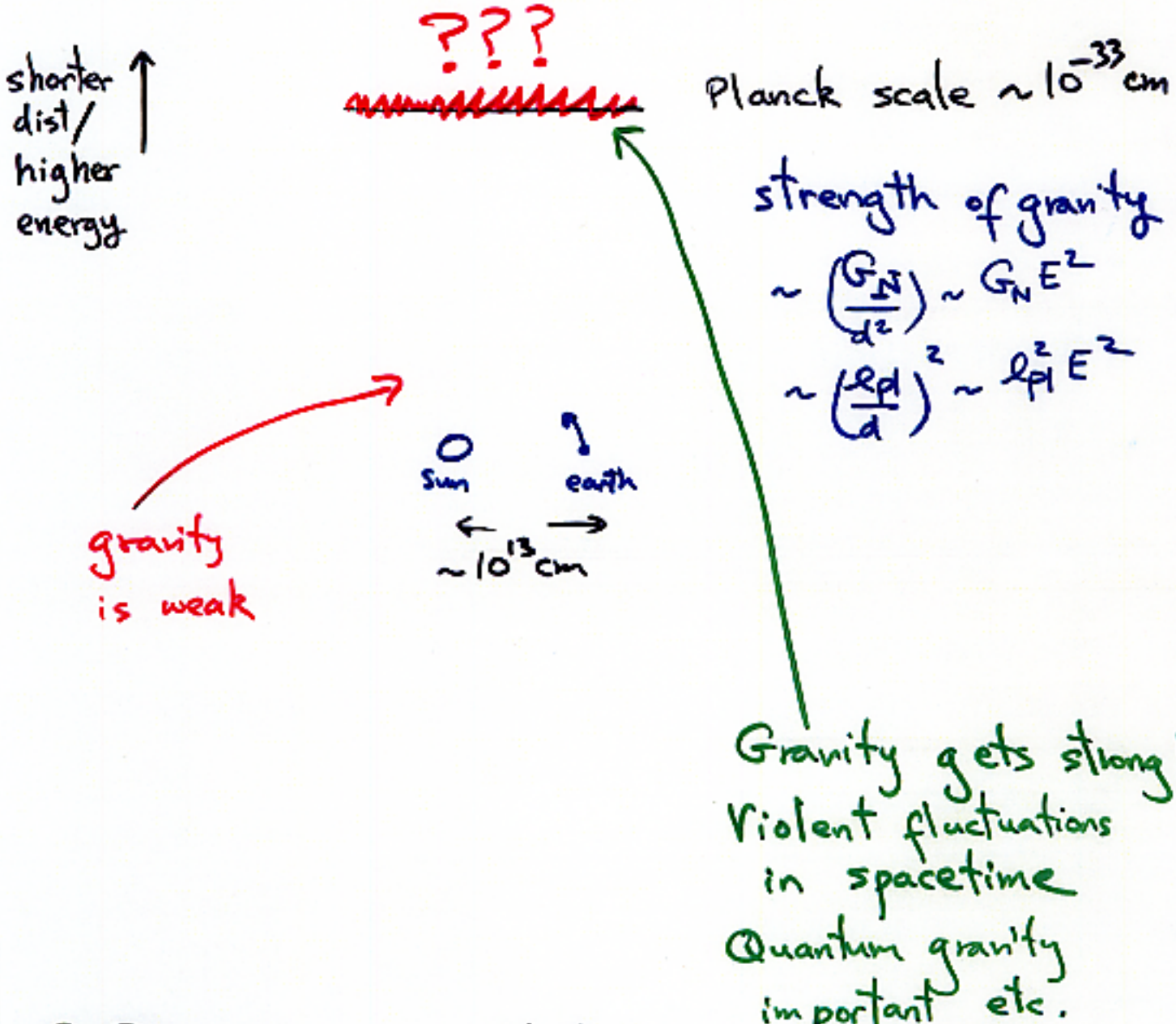
~ scale where strong, weak, em become "comparable" in strength

"Hubble" scale

Size of universe ℓ_u

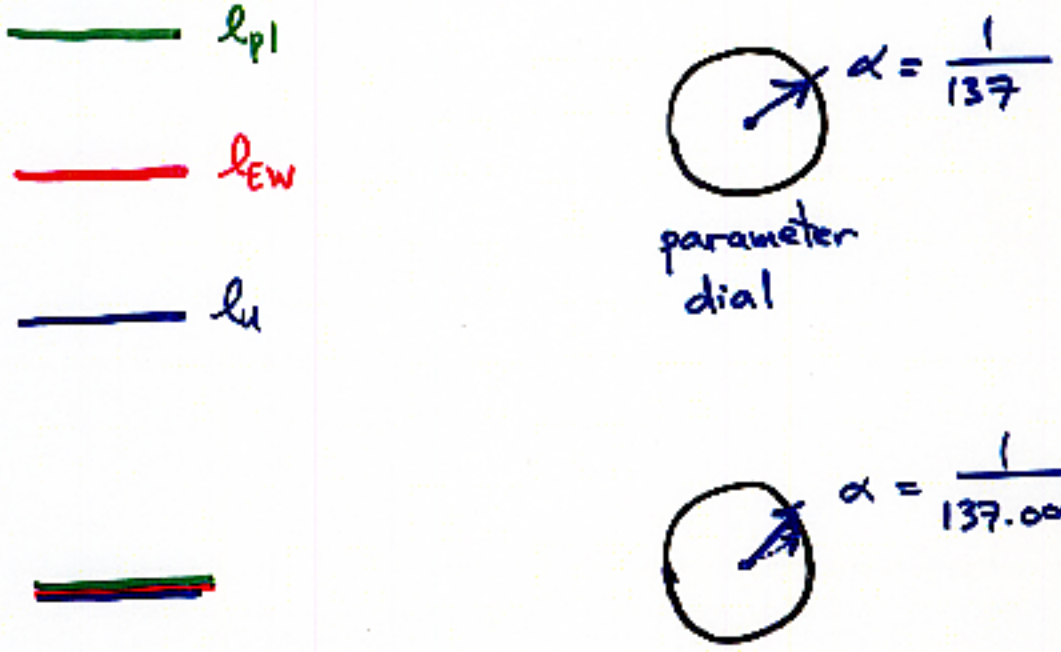
But all is not well: theoretical crises on many fronts.

- **Crisis with Gravity:**

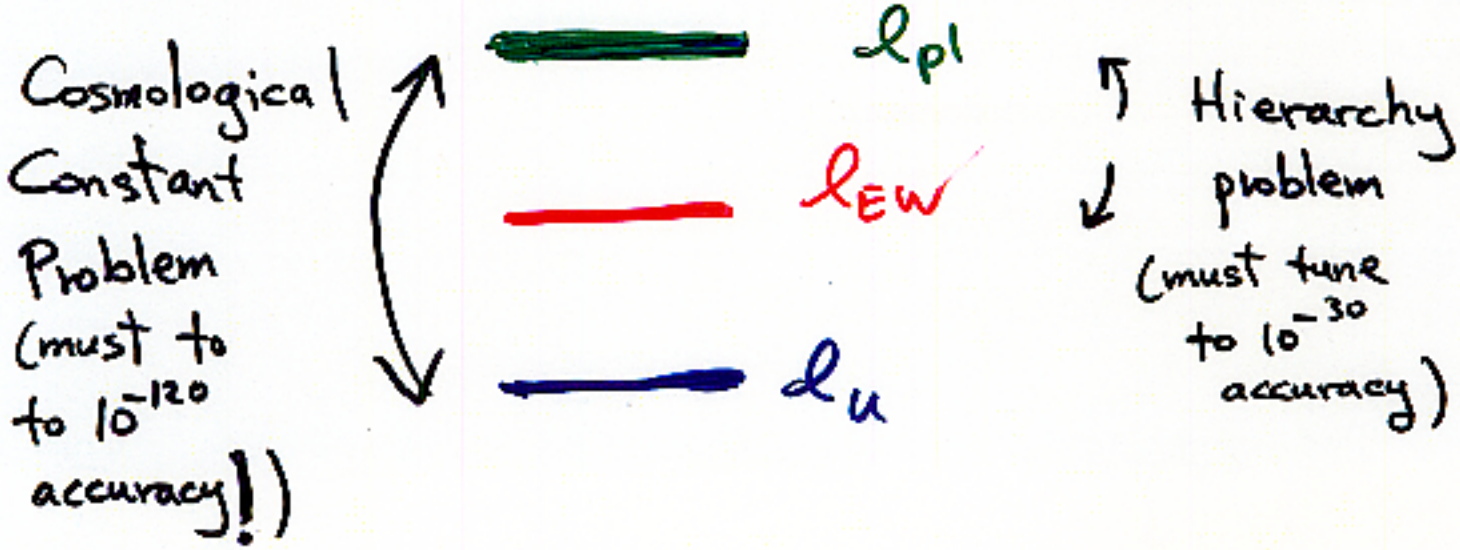


GR is incomplete: must emerge from a deeper theory.

• Stability Crises:



⇒ Enormous hierarchy of scales arranged by enormous tuning of parameters!



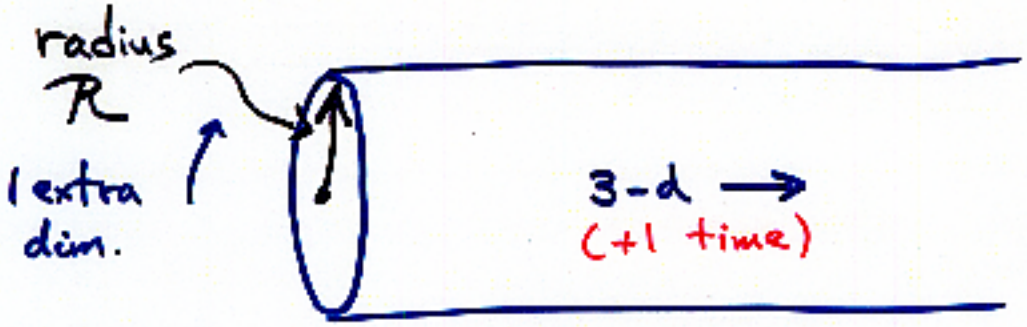
. Our picture of physics must be fundamentally incomplete ! We need new physics beyond the Standard Model + GR, to resolve these mysteries.

$\Longrightarrow$ new degrees of freedom.

In order to go beyond the Standard Model + address some of its fundamental mysteries, we need to add degrees of freedom to our description of nature.

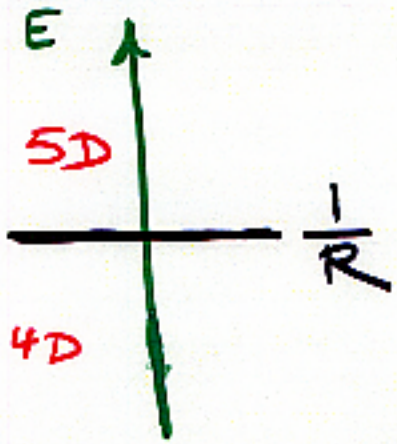
. One possibility for these new degrees of freedom involve

new dimensions of space:



At distances $\gg R$, can't see the extra dim.

At distance $\ll R$, they become apparent



• Higher-dimensional theories have similar problems as gravity in 4D:

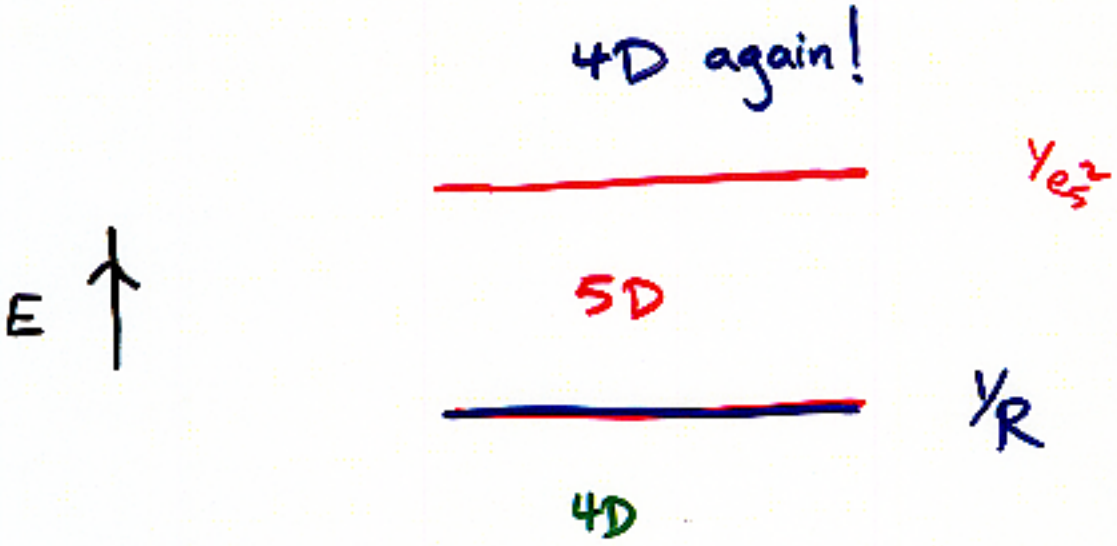
e_4^2 dimensionless $\left[V(r) \sim \frac{e_4^2}{r}\right]$

e_5^2 dimension of length! $\left[V(r) \sim \frac{e_5^2}{r^2}\right]$

⇒ 5D physics has to emerge from a more fundamental theory.

[Combine w/ gravity? Could do, but it's overkill.
⇒ SHUT OFF Gravity for now]

We will construct such a "short-distance completion" of 5D theories, and find



- Fundamentally theory is 4D, 5'th dimensions emerges dynamically at low energies / large distances.

Dimensional Deconstruction

Idea: we will build a 5-D theory out of a bunch of 4D theories.

Much like we build a 1-D rope from balls + springs in Physics 15C!

Infinite chain

or

finite circular chain.

At distances larger than the inter-ball spacing,

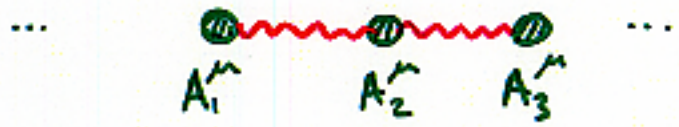
Infinite cont. rope

rope on a circle

Allows 1D wave propagation

$$E = \sum_i \frac{1}{2} m \dot{x}_i^2 + \frac{1}{2} k (x_i - x_{i+1})^2$$

We want a 5D theory where the Standard Model particles can wander in the 5th d. Say, photons / gluons / W, Z.
"gauge fields"



Different o-d ball positions X_i $\longrightarrow$ Different 4D gauge fields

We want to somehow get

$$E = \sum_i \frac{1}{2}\dot{A}_i^2 + \frac{m^2}{2}(A_i - A_{i+1})^2$$

$\frac{1}{2}\dot{A}_i^2$: kinetic term

$\frac{m^2}{2}(A_i - A_{i+1})^2$: "hopping" term.

(More precisely:

$$\mathcal{L} = \sum_i -\frac{1}{4}(F_{\mu\nu\, i})^2 + \frac{m^2}{2}(A_{\mu i} - A_{\mu i+1})^2)$$

Why not just use this Lagrangian?

$$\mathcal{L} = \sum_i -\frac{1}{4}(F_{\mu\nu\,i})^2 + \frac{1}{2}m^2(A_{\mu i} - A_{\mu i+1})^2$$

(Exactly this is why it was hard to get massive W/Z bosons in Standard Model)

This is a mass term, which is not so innocent for gauge fields!

$m = 0$: spin 1 particle has 2 polarizations

$m \neq 0$: spin 1 particle has 3 polarizations

Interactions of longitudinal components of massive gauge field $\sim E/m$ at high energies, and blow up!

becomes strongly interacting for $E \gg m$!

What to do?

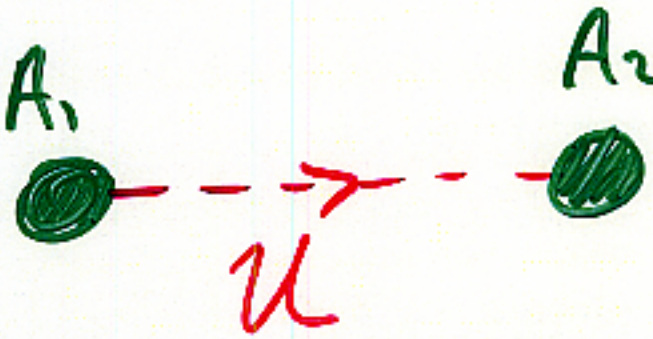
We want to have "springs" @ low energies, so we can build the 5'th dim. But these same springs destroy the theory right where things start getting interesting.

$\Rightarrow$ We need to build the springs, so that they appear at low energies but disappear at high energies.

$\Rightarrow$ We need emergent springs.

Familiar problem!

Higgs Mech.



$U = e^{i\pi}$

U's are goldstone bosons eaten to get $(A_1 - A_2)$ massive.

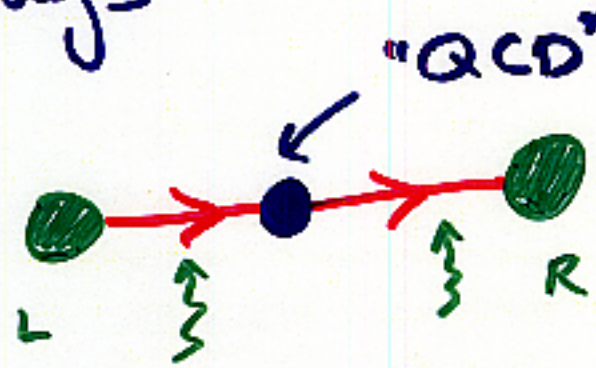
$$U \to g_1 U g_2^{-1}$$

$$D_\mu U = \partial_\mu U + igA_1 U - iUgA_2$$

"Unitary gauge" $U = 1$

$$v^2 |D_\mu U|^2 \to g^2 v^2 (A_1 - A_2)^2$$

Where can the goldstone bosons from? Again familiar ways



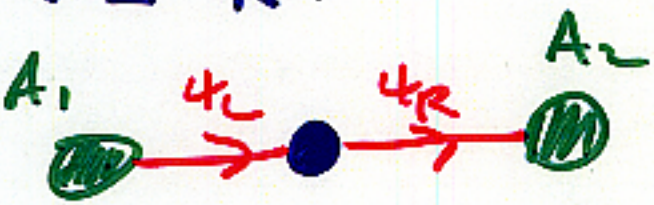
"left-handed quarks" ψ_L

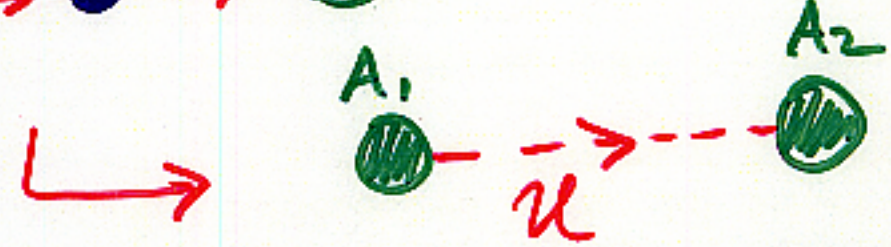
"Right-handed quarks" ψ_R

Confinement + Chiral Symmetry breaking

$\mathcal{U}$, $U = e^{i\pi}$ pions

$\langle \psi_L \psi_R \rangle \sim \Lambda_{QCD}^3$

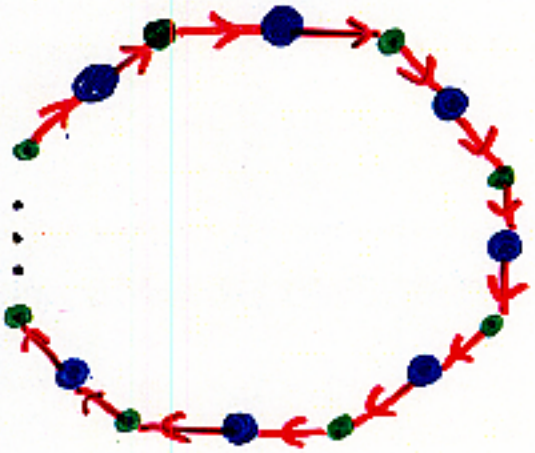




$\sim g^2 \Lambda_{QCD}^2 (A_1 - A_2)^2$.

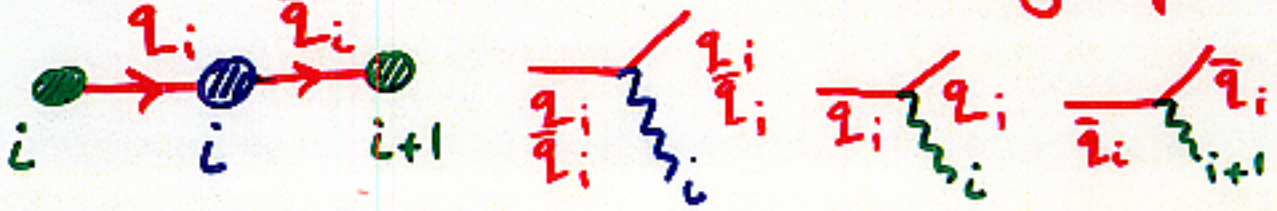
EMERGENT SPRING!

Now that we have balls + springs, we can make our 5'th dimension



N-sided polygon.

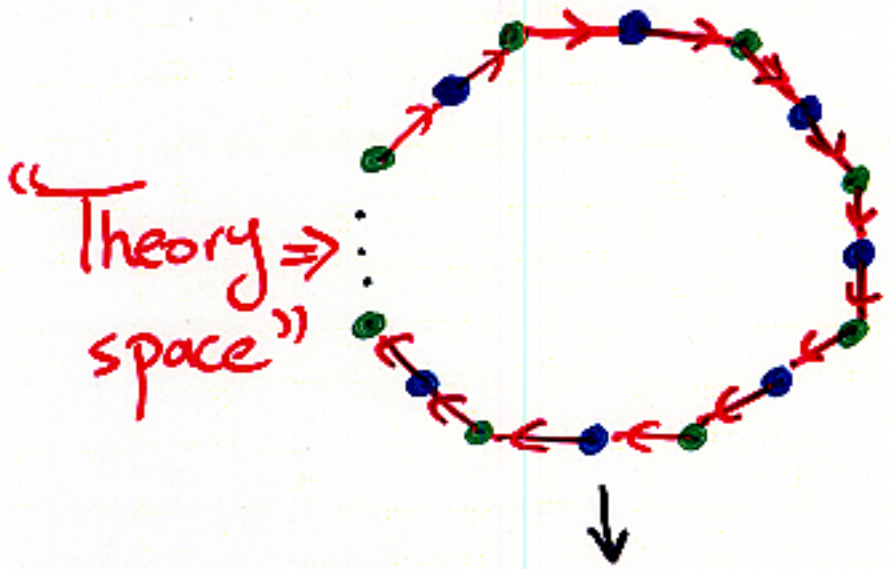
Again: this diagram is just a mnemonic for the particles + interactions of a 4D theory ["Moose" diagram, "Quiver" diagram, "Theory space"] that makes sense to arb. high energies.

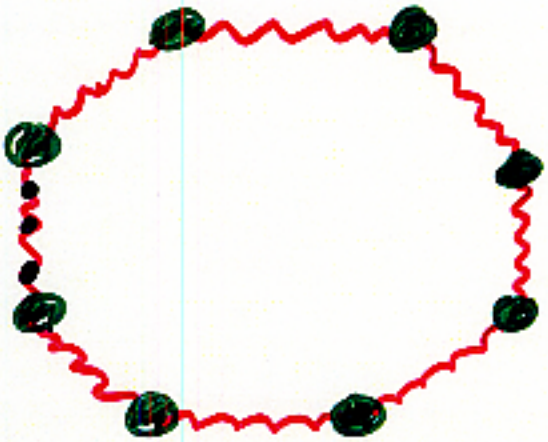


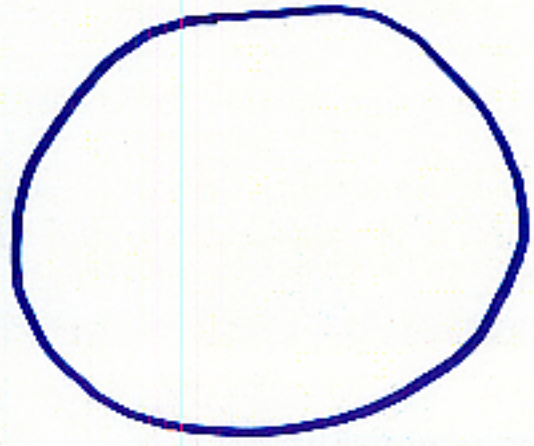
. At high energies, we have free "quarks" and "gluons", 4D theory.

But when interactions become strong at energy $\Lambda_{\bullet}$, the springs are created + 5'th dimension is born:

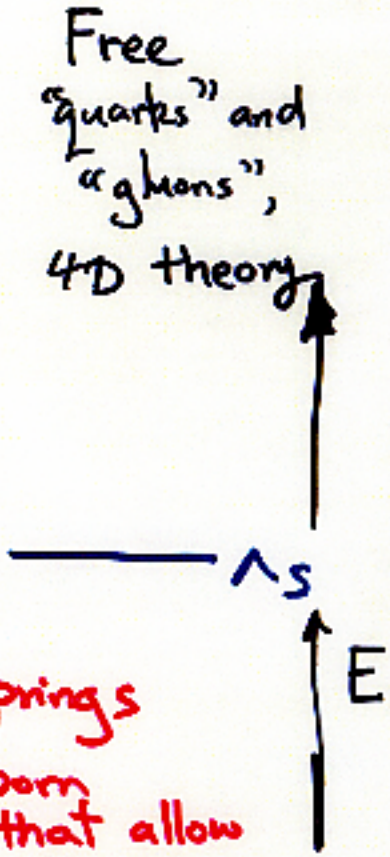
the Theory Space goes from being a mnemonic for the particles and interactions of the 4D theory, into a picture of the Extra dimension.







transmuted to



Springs born that allow "hopping". Latticized 5'th dim

At distances large compared to the "lattice" spacing, looks like a continuous 5'th dimension

• The parameters of the effective 5D theory are determined by the fund. 4D theory [which are $\Lambda_\bullet$, e, N]

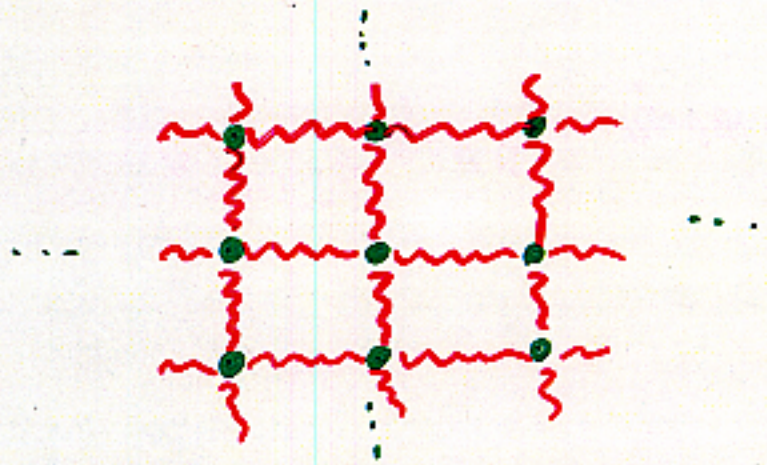
"lattice spacing" $a = \frac{1}{e}\frac{1}{\Lambda_\bullet}$

"Radius" $R = \frac{Na}{2\pi} = \frac{1}{2\pi}\frac{N}{e\Lambda_\bullet}$

"5D gauge coupling" $g_5^2 = \frac{e}{\Lambda_\bullet}$

So: we have constructed a 5'th dimension dynamically, from 4D theories that make sense to arb. short distances. (5'th dim melts away @ high energies).

- It's straightforward to extend to more extra dimensions



. We can now investigate higher-dim physics on a firm footing, without worrying about what happens at high energies.

We have provided a "short-distanc completion" of higher-dimensional field theories.

But much more interestingly:

we have learned that extra dimensions aren't "special". They are just the tip of the iceberg.

$\Rightarrow$ Dynamics of 4D theories with the "site" - "link" structure

— characterized by a Theory Space —

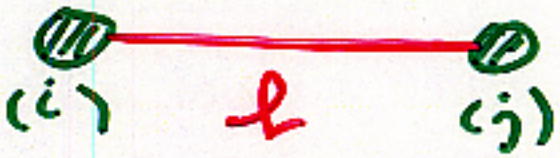
is surprisingly rich, offering a new set of ideas beyond extra dimensions, with more interesting physics.

Theory Space Rules

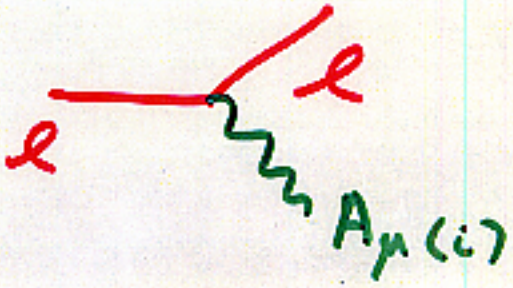
Sites

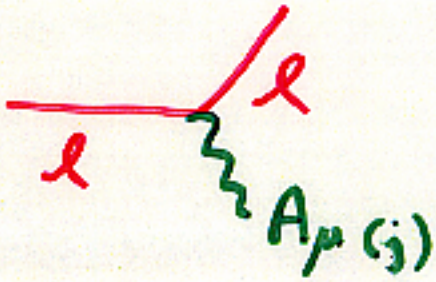
Specifies gauge symmetry + associated "photons" $A_{\mu(i)}$

Links

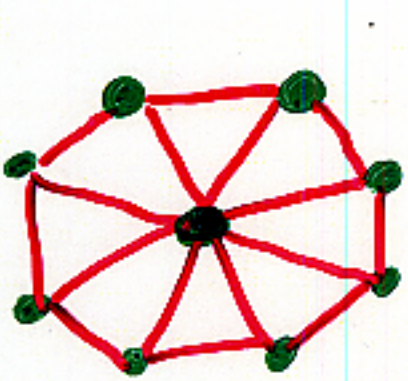


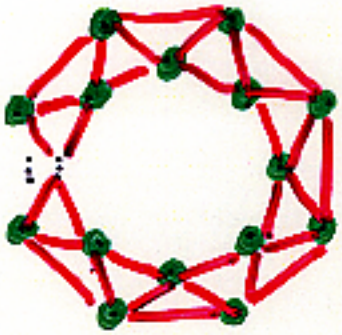
ℓ charged under (i), (j), neutral under everything else

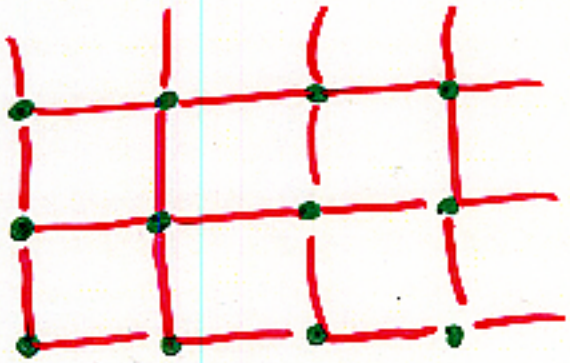


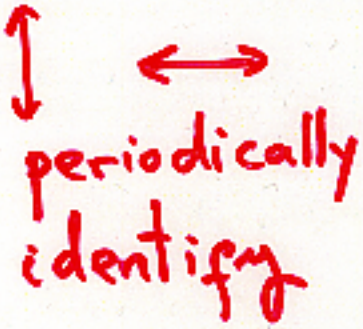


Some theory spaces:









All of these theory spaces specify some 4D theory. In some phases of the theory, extra dimensions emerge – in others, other surprising things happen. (Topology is significant).

This has opened up two different directions to explore:

Deconstructing Dimensions

"Field-theoretic xtra-dim suck"

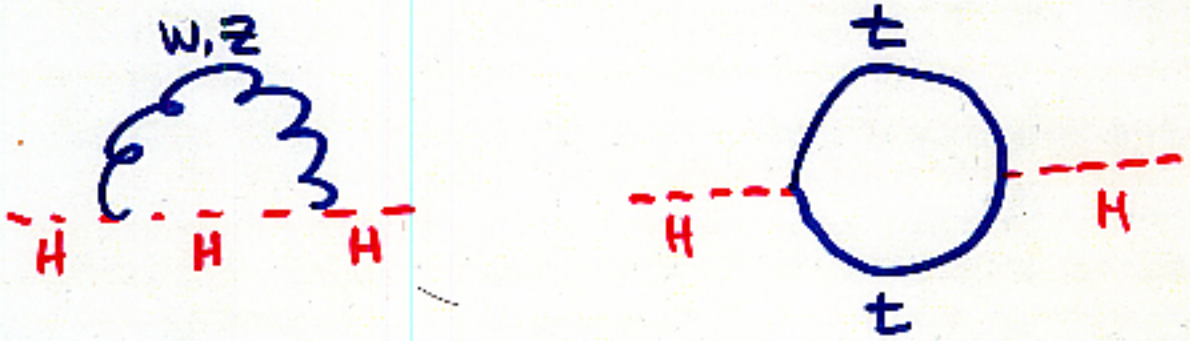
"F-T xtra dim are boring"

- EWSB from theory space
- Accelerated Unification
- "Twisted" SUSY

- Deconstructing string theory

Crisis in the Origin of Mass a.k.a Hierarchy Problem

. The reason we have a crisis is that the very interactions that give mass <u>dramatically destabilize</u> m^2_{Higgs}

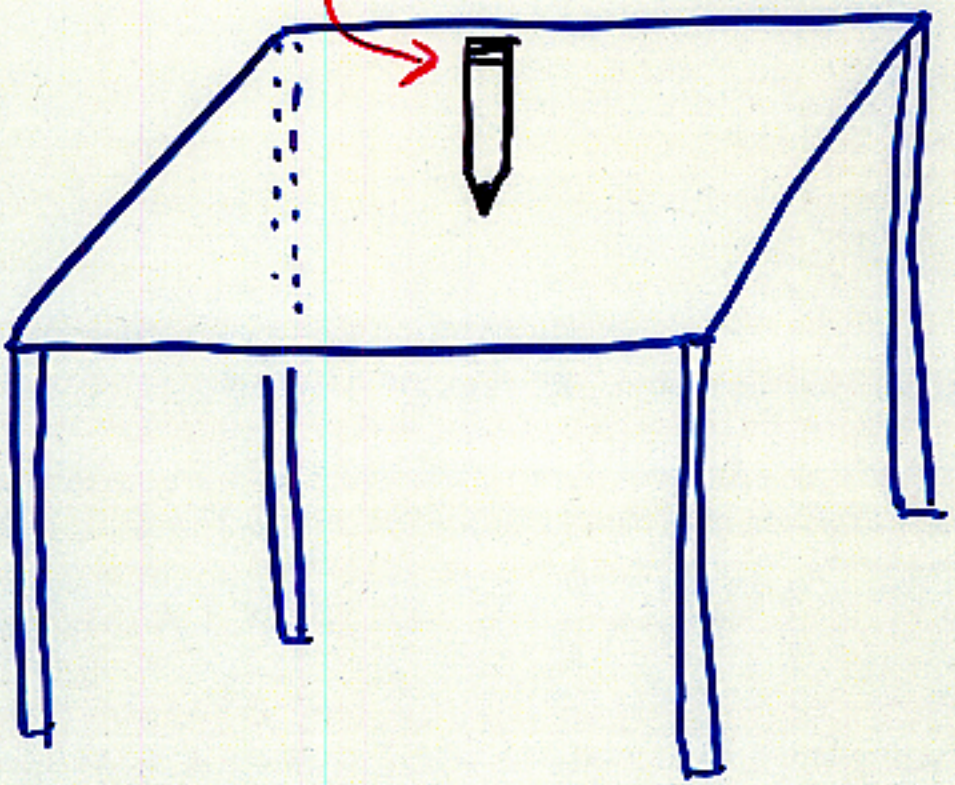


$\Delta m_H^2 \sim \Lambda^2_{cutoff}$!

If its just SM + gravity, $\Lambda_{cutoff} \sim M_{Pl}$, so naturally $\langle H \rangle \sim M_{Pl}$. Disaster.

. Can only avoid by tuning m_{Cl}^2 against Δm^2 to accuracy $\frac{(300\,GeV)^2}{M_{Pl}^2} \sim 10^{-34}$!

Pencil standing on its tip, within 10^{-34}° of vertical.



Not inconsistent, but very very strange!

Makes us suspect we are missing something big.

To date, there are two main approaches to tackling this problem

(I) "Compositeness"

$\leftarrow 10^{-17}\,\text{cm} \rightarrow$

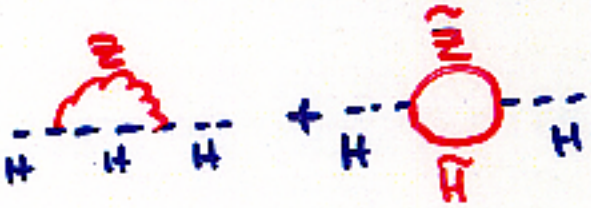
Higgs

has some sort of "size" $\sim 10^{-17}$ cm

STRONG DYNAMICS AT 10^{-17} cm

(II) Supersymmetry

A new symmetry is revealed @ 10^{-19} cm



$\sim (10^{-17}\,\text{cm})^{-2}$

WEAKLY INTERAC. DYNAMICS AT 10^{-17} cm.

Are there any qualitatively new possibilities for physics at the TeV scale?

Electroweak Symmetry Breaking from Theory Space.

We find option (III):

"Whatever we can do in "real" space we can also do in theory space"

e.g. Compositeness in space

⇓

Higgs as extended object in <u>theory space</u> (but point-like in 4D).

- Weakly coupled @ TeV scale, calculable.

- Un constrained by precision EW data, so viable.

- Rich new spectrum of particles from 100's GeV – multi TeV scale.

• Consider an SU(k) pure gauge theory in 5D, compactified on a circle of radius R

• Low-E theory is 4D,

$A_M \xrightarrow{\text{zero-modes}} A_\mu^{(0)}$ 4D SU(k) gauge bosons

$A_M \xrightarrow{\text{zero-modes}} A_5^{(0)} \equiv \phi$ Real scalar in adjoint rep.

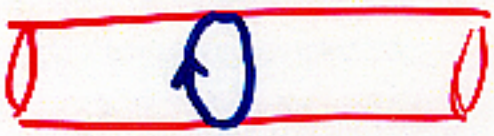
$\phi \leftrightarrow$ Wilson loop $W = \mathrm{P} e^{i \oint A_5 \, dx_5}$

• Classically, ϕ massless. But in 4D LE theory,

quad. div. mass!

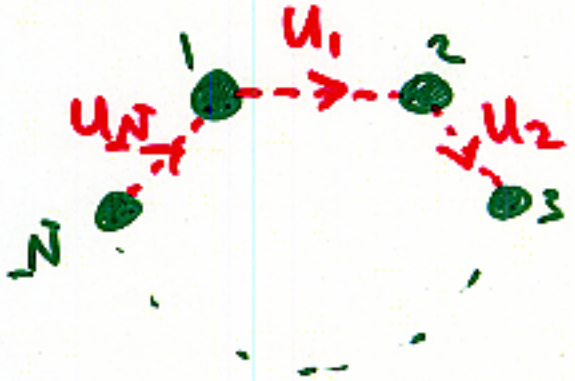
... However, this div. must be cut-off at the scale $1/R$. At dist. smaller than R, it becomes manifest that $\phi \leftrightarrow A_5$, and 5D sorgauge invariance prevents div.

How can any mass be generated at all?

Wilson loop $W = Pe^{i\oint dx_5 A_5}$ 

is gauge covariant, nothing prevents $|\mathrm{tr}\, W|^2$ from being generated... But this is a non-local operator; can't be generated with UV div coefficient!

• Return to toy 5D model + deconstruct. Sufficient to look @ low-E theory → "condensed moose"



$$U_i = e^{i\pi_i^a T^a/f}$$

• We can <u>almost</u> go to a unitary gauge where $U_1, \ldots, U_N = 1$. But not quite:

$$(U_1 \cdots U_N) \to g_1 (U_1 \cdots U_N) g_1^{-1}$$

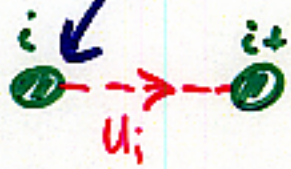
can't be gauged to 1. [⇔ Wilson loop in contin.].

$\phi = \frac{(\pi_1 + \cdots + \pi_N)}{\sqrt{N}}$ is classically massless [⇔ zero mode of A_5 in contin.]

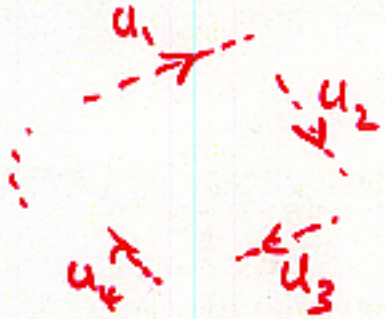
It is useful to look at symmetries.

Turn off gauge couplings. Then, there is chiral symmetry.

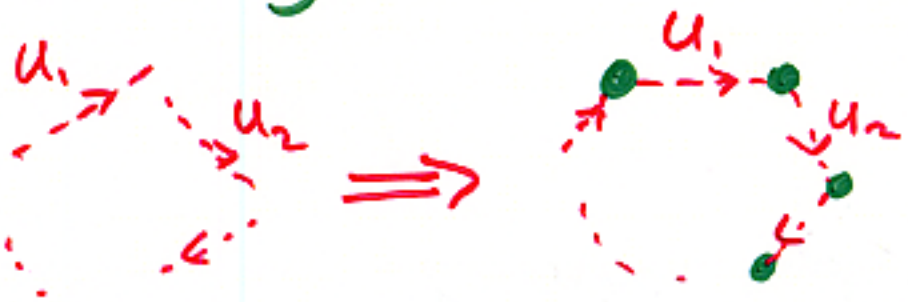
$$U_i \to L_i U_i R_{i+1}^\dagger$$



Moose falls apart:



Only gauge couplings glue Moose together:



So, to make a closed chain of U's, every coupling is needed

$\Rightarrow$ coefficient of $|\mathrm{tr}\, U_1 \ldots U_N|^2$

is $\propto \left(\frac{g_1^2}{16\pi^2}\right)\left(\frac{g_2^2}{16\pi^2}\right)\cdots\left(\frac{g_N^2}{16\pi^2}\right)$!

$N=1$ quad. div @ 1-loop

$N=2$ log div @ 1-loop (quad @ 2-loops)

$N=3$ no div @ 1-loop, log at 2, quad @ 3 ....

$N=1$, $m_\varphi^2 \sim \Lambda_{UV}^2$

(analogous to $\pi^\pm$ mass in QCD)

$N=2$, only log div.

$N=3$, finite!

[Closely related model: Final exam problem in Coleman QFT course...]

At large N, looks like a 5thD,

but $N=2,3$ don't have any

extra-d interpretation!

Let's verify this analysis by computing 1-loop potential for ϕ, using Coleman-Weinberg.

The background is

$$U_i = e^{i\hat{\phi}}, \quad \hat{\phi} = \frac{\phi}{\sqrt{N} f}.$$

Take SU(k=2) example, and choose

$$\hat{\phi} = \theta \sigma^3.$$

CW potential is

$$V(\theta) = \frac{3\Lambda^2}{32\pi^2} \mathrm{tr} M^2(\theta) + \frac{3}{64\pi^2} \mathrm{tr}(M^2(\theta))^2 \log \frac{M^2(\phi)}{\Lambda^2}$$

where $M^2(\theta)$ is gauge boson mass matrix.

Easy to diagonalize. Neutral gauge boson m^2 are θ-indep; charged:

$$m_n^2(\theta) = 4g^2f^2\sin^2\left(\frac{n\pi}{N}+\theta\right) \quad ; \frac{N}{2} < n \leq N/2.$$

N=1: ——— $m^2(\theta) = c\sin^2\theta$

$\mathrm{tr}\, m^2(\theta) \neq 0.$

$\Rightarrow$ quad. div.

N=2

——— $\leftarrow c\sin^2(\pi/2+\theta) = c\cos^2\theta$

——— $\leftarrow c\sin^2\theta$

$\mathrm{tr}\, m^2(\theta) = c\sin^2\theta + c\cos^2\theta = c.$

θ indep!

$\Rightarrow$ <u>NO</u> quad. div.

$\mathrm{tr}\, m^4(\theta) \propto \sin^4\theta + \cos^4\theta$

is θ dep $\Rightarrow$ log div.

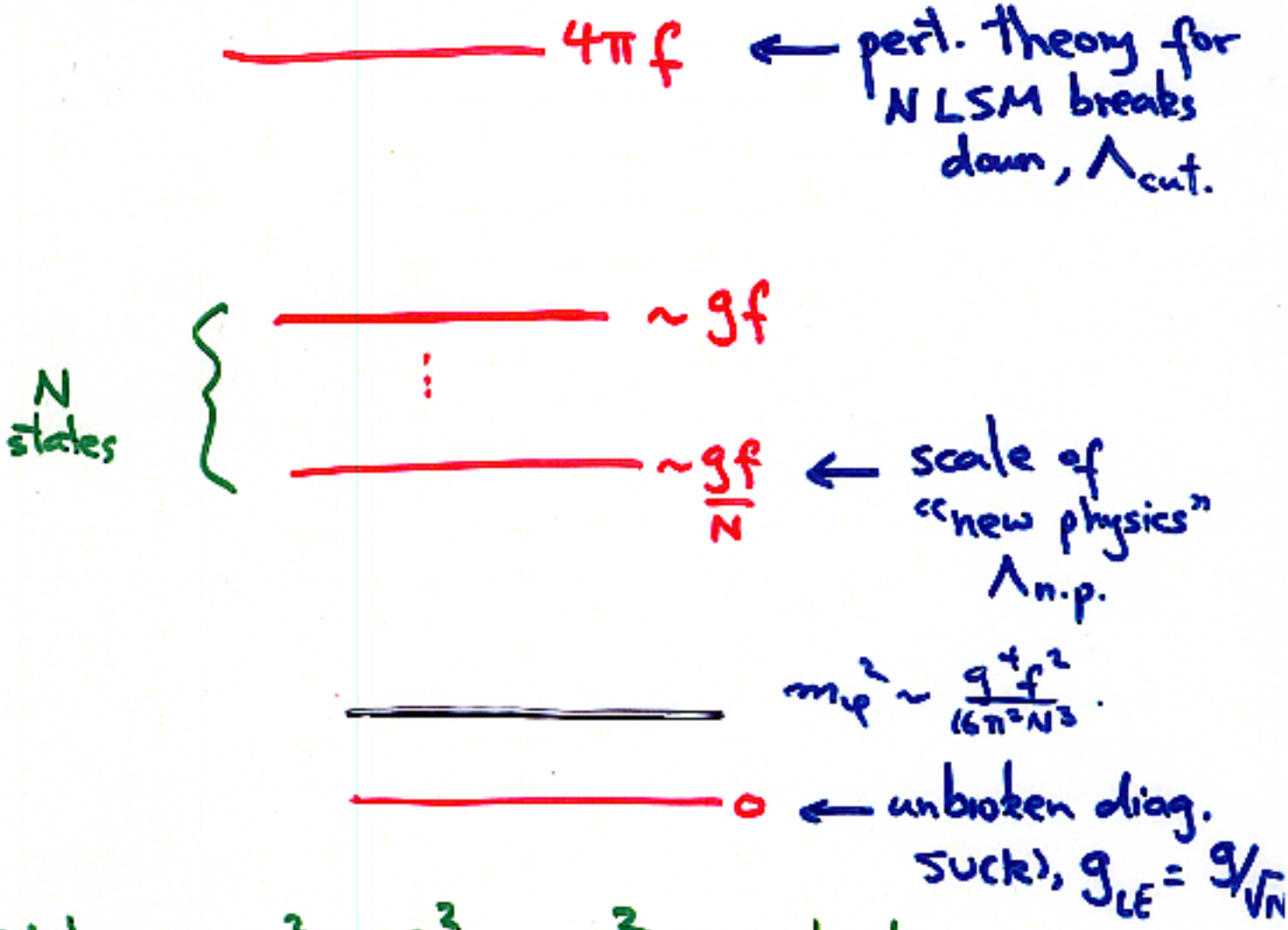
$N > 2$: Both $\mathrm{tr}\, m^2(\theta)$, $\mathrm{tr}\, m^4(\theta)$ are θ-indep. Can perform CW sum exactly, and we find finite result:

$$V(\theta) = \frac{8 g^4 f^4}{(N^2-1)(N^2-4)} \sum_{n=1}^{\infty} \frac{\cos n\theta}{n^5} + \text{const.}$$

$$\left[\Rightarrow m_\varphi^2 \sim \frac{g^4 f^2}{16\pi^2 N^3} \right]$$

. We have seen cancellation <u>without SUSY</u>. Amusing that contr. from each eigenstate is div; but spectrum + couplings "conspire" to give finite answer [As guaranteed by our symmetry considerations]

Scales:



Note: $m_\varphi^2 \sim \frac{g_{LE}^2}{16\pi^2} \Lambda_{n.p.}^2$, but

$\Lambda_{n.p.} \ll \Lambda_{cut}$ parametrically. Pert. new phys.

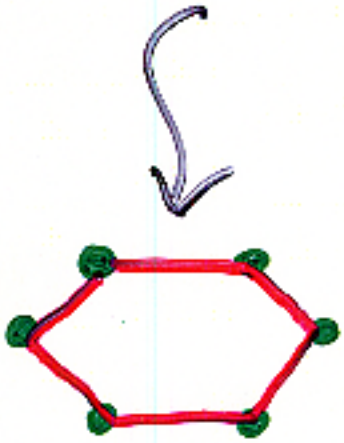
[In our realistic examples,

$m_H \sim 100$ GeV, $\Lambda_{n.p.} \sim 1$ TeV,

$\Lambda_{cut} \sim 10 - 100$ TeV]

What was essential here?

$\phi \leftrightarrow$ Non-contractible chain in theory space



. If interactions are "local" in theory space, then a "non-local" (and high # of links) operator can't be generated with div coefficient @ low loop order)

"Higgs" mass is soft because it is an extended object in theory space.

Realistic Theories

We need

(1) Higgs to be a doublet under $SU(2)_L$, not adjoint.

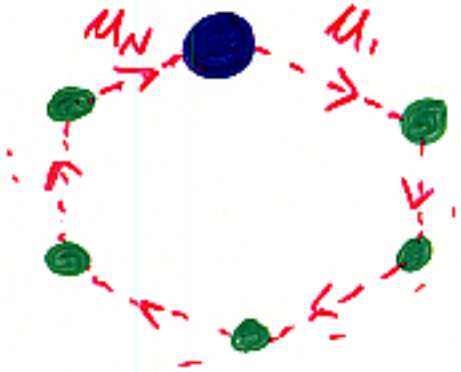
(2) $m_H^2 < 0$, but $\lambda_t \sim O(1)$

(3) $\lambda_{top} \sim O(1)$

... We'll now work directly with 4D moose picture... indeed our ultimate models will have no extra-d interp. at all ...

(1) Getting Higgs out of adjoint

Many ways; here is one:



● SU(3)

● SU(2)×U(1) ⊂ SU(3)

$\left[\begin{pmatrix} SU(2) & \\ & 0 \end{pmatrix}, \begin{pmatrix} 1 & \\ & -2 \end{pmatrix}\right]$

LE gauge group is $SU(2)\times U(1)$.

$$\varphi \sim \left(\begin{array}{c|c} \chi+\eta & h \\ \hline h^\dagger & -2\eta \end{array}\right) \qquad \chi \sim 3_0,\ \eta \sim 1_0$$

$h \sim 2_{1/2}$ ← quant# of SM Higgs

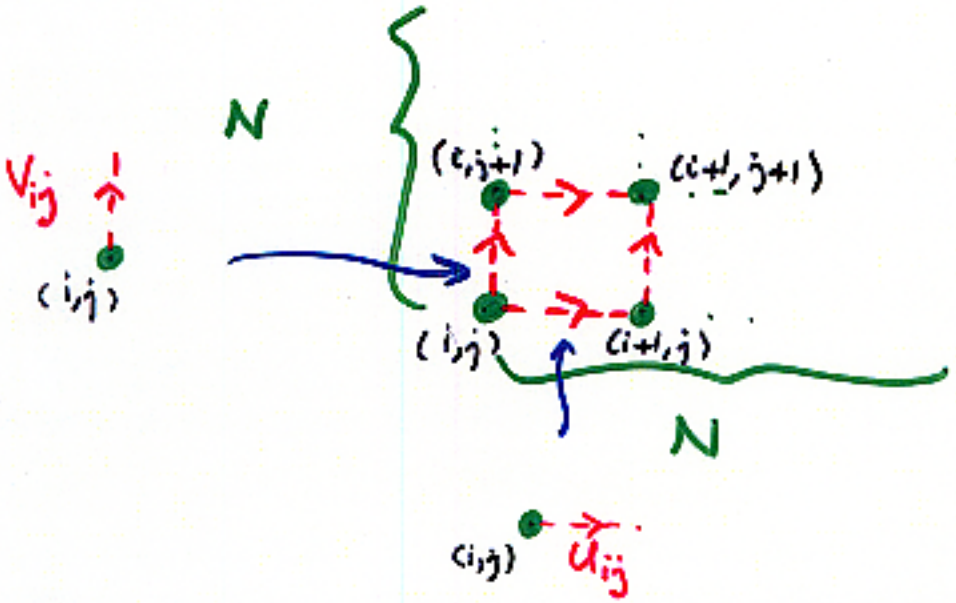
Gauge symmetry no longer relates χ, η, h. They can get different m^2's.

(2) We need $\mathcal{O}(1)$ quartic potential.

Inspiration from higher-d: in e.g. 6D, action contains $\mathrm{tr}\, F_{56}^2$

$\rightarrow$ $\mathrm{tr}\,[A_5^{(0)}, A_6^{(0)}]^2$ for zero modes.

Deconstruct:



N×N "toms"

$i \sim i+N$

$j \sim j+N$

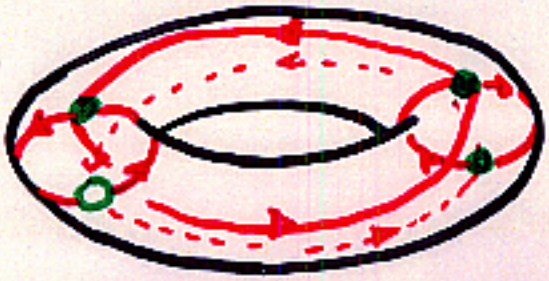
Add "plaquette" operators. $V \sim \sum_{i,j} \lambda_{ij}\, \mathrm{tr}(U_{ij} V_{i,j+1} U^\dagger_{i+1,j} V^\dagger_{ij}$ + h.c.

"Chain" scalars $U_{ij} = e^{iu}$, $V_{ij} = e^{iv}$

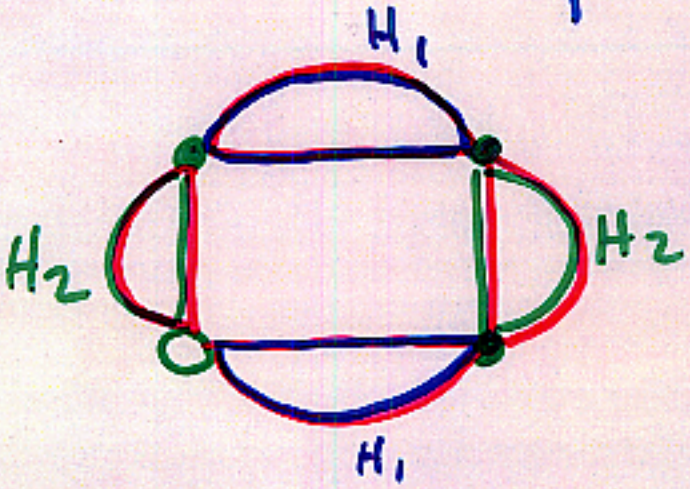
Plaquette action $\Rightarrow$ const + $\lambda\, \mathrm{tr}\,[u,v]^2$; $\lambda \equiv \sum_{ij} \lambda_{ij}$

Quartic potential

Realistic models



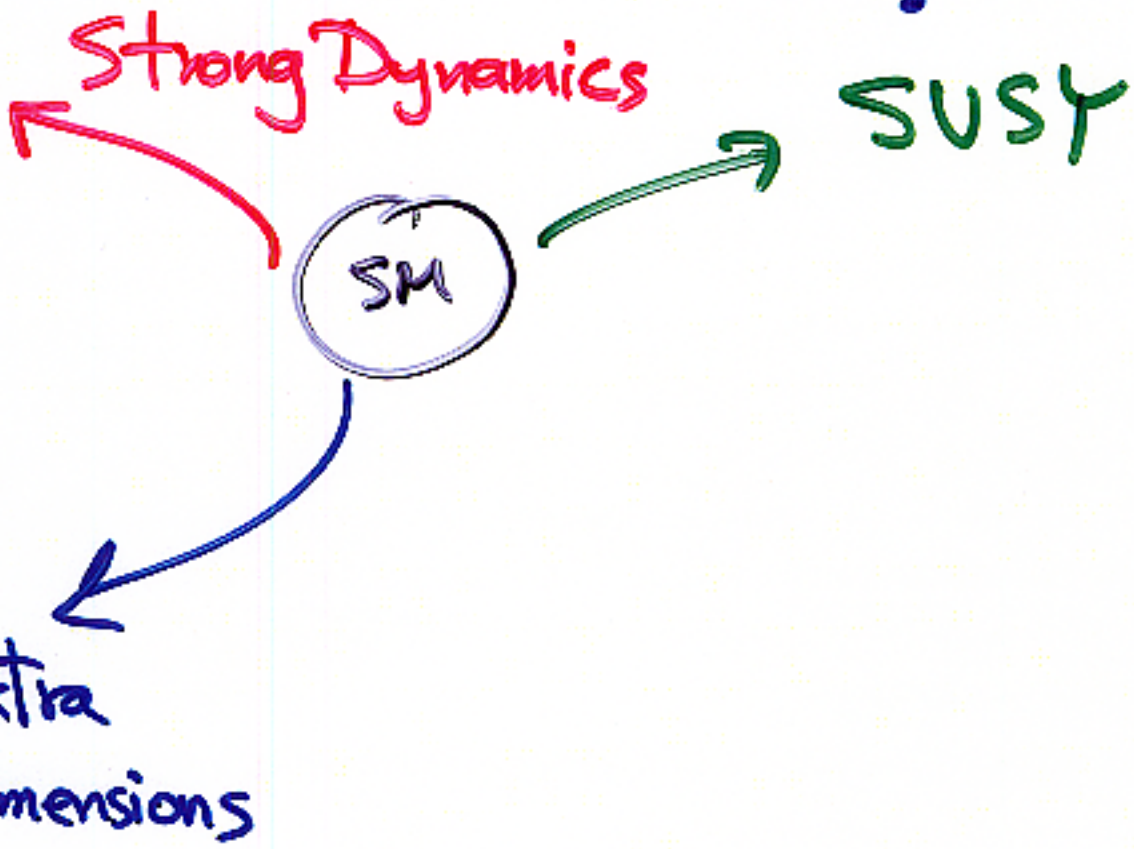
Theory space with topology of torus. Higgs fields $\leftrightarrow$ non-contractible loops in theory space.



- ● SU(3)
- ○ SU(2)×U(1)

+ Plaquette terms

What new principles govern physics beyond the Standard Model?



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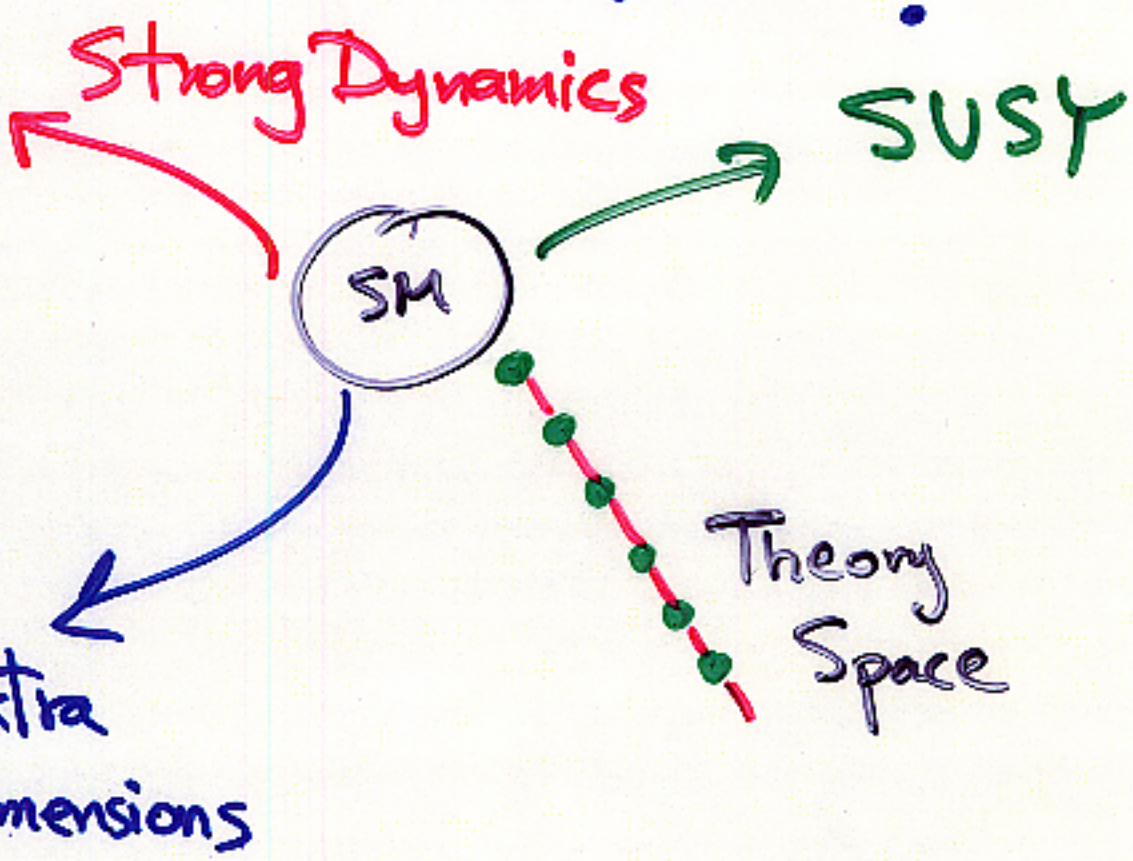

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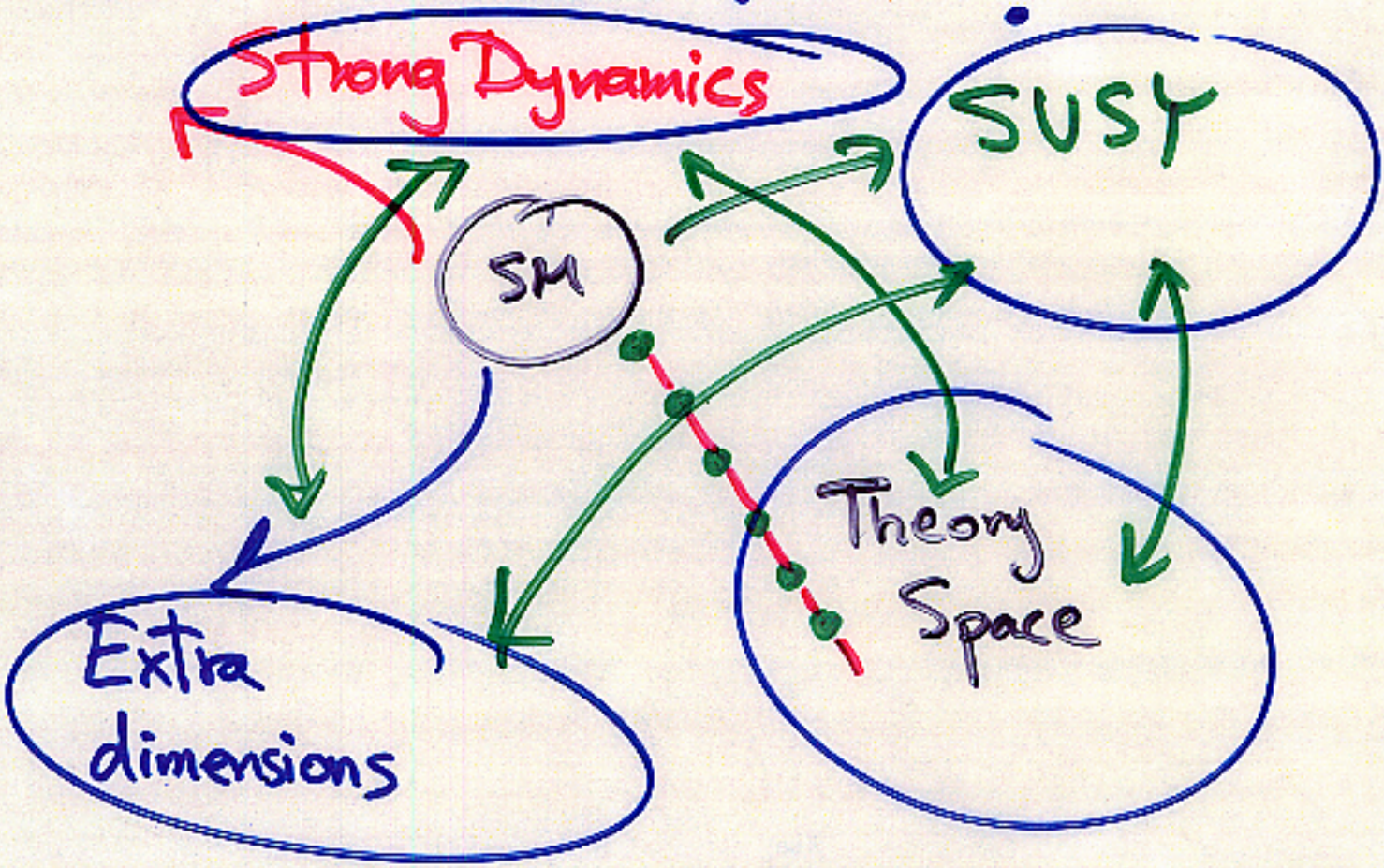


Perturbative,

Calculable, Weakly-coupled

EWSB with a Light

Higgs $\Longrightarrow$ SUSY

Perturbative,

Calculable, Weakly-coupled

EWSB with a Light

Higgs $\not\Rightarrow$ SUSY

Can be EWSB from

theory space instead.

Low energy theory scalar potential

$$V \sim m_{h_1}^2 |h_1|^2 + m_{h_2}^2 |h_2|^2 + m_\phi^2 |\phi|^2 + m_\eta^2 |\eta|^2$$

$$\longrightarrow + 3\lambda \left(|h_1|^2 - |h_2|^2\right)^2 + \text{terms with } \phi, \eta$$

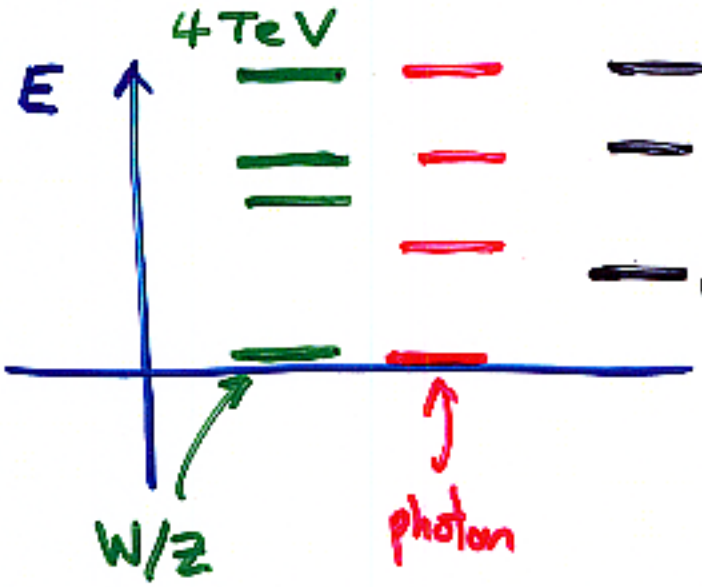
Quartic pot. from plaquettes

Wide range of parameters where

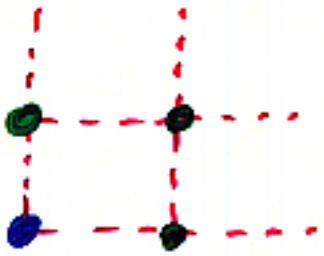
$m_{h_1}^2 < 0$, $m_{H_2, \phi, \eta}^2 > 0$ (Can be triggered by Heavy Top Quark)

$\Rightarrow$ Correct pattern of $SU(2) \times U(1)$ breaking!

The spectrum of this theory has a distinctive 15c structure: e.g. vector bosons

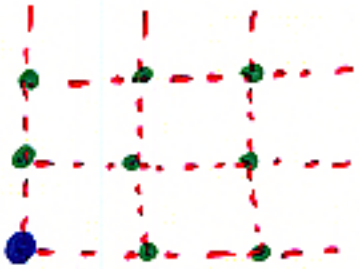


- In addition to the normal light Higgs ~ 100 GeV, there are additional neutral particles φ, η ~ 100 GeV, that are stable. Dark matter



- SU(3)
- SU(2)×U(1)

N=2 Only log div.



N=3 Finite

No extra-d interp.

. Main ingredients for successful models:

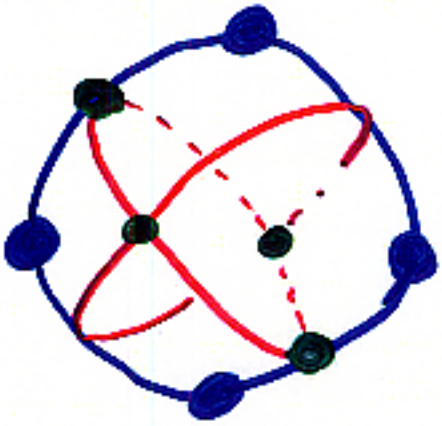
- Non-contractible loops in theory space $\Longleftrightarrow$ Higgs.

- "Plaquette" type action to get quartic couplings.

- "Local" interactions in theory space.

Many models can be constructed along these lines...

For instance



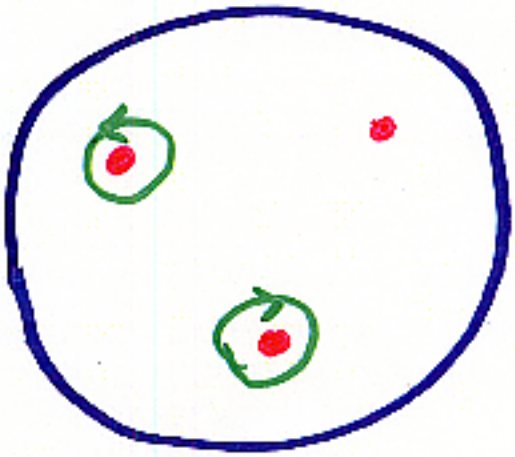
"Moosified"

[Only light scalars are $h_{1,2}$]

Punctured Sphere

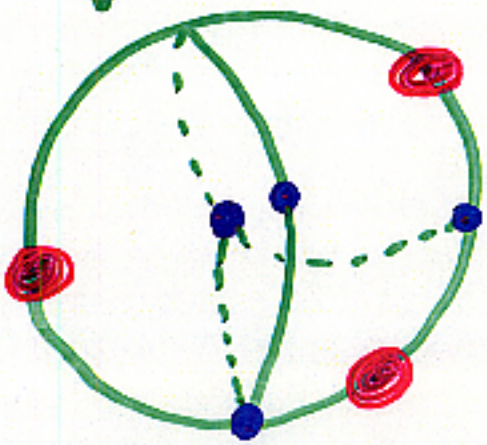
SU(2)×U(1)
SU(3)

No non-contractible loops ⇒ No Higgs



Two Higgses w/ quartic potential.

Again, only Higgs doublets light

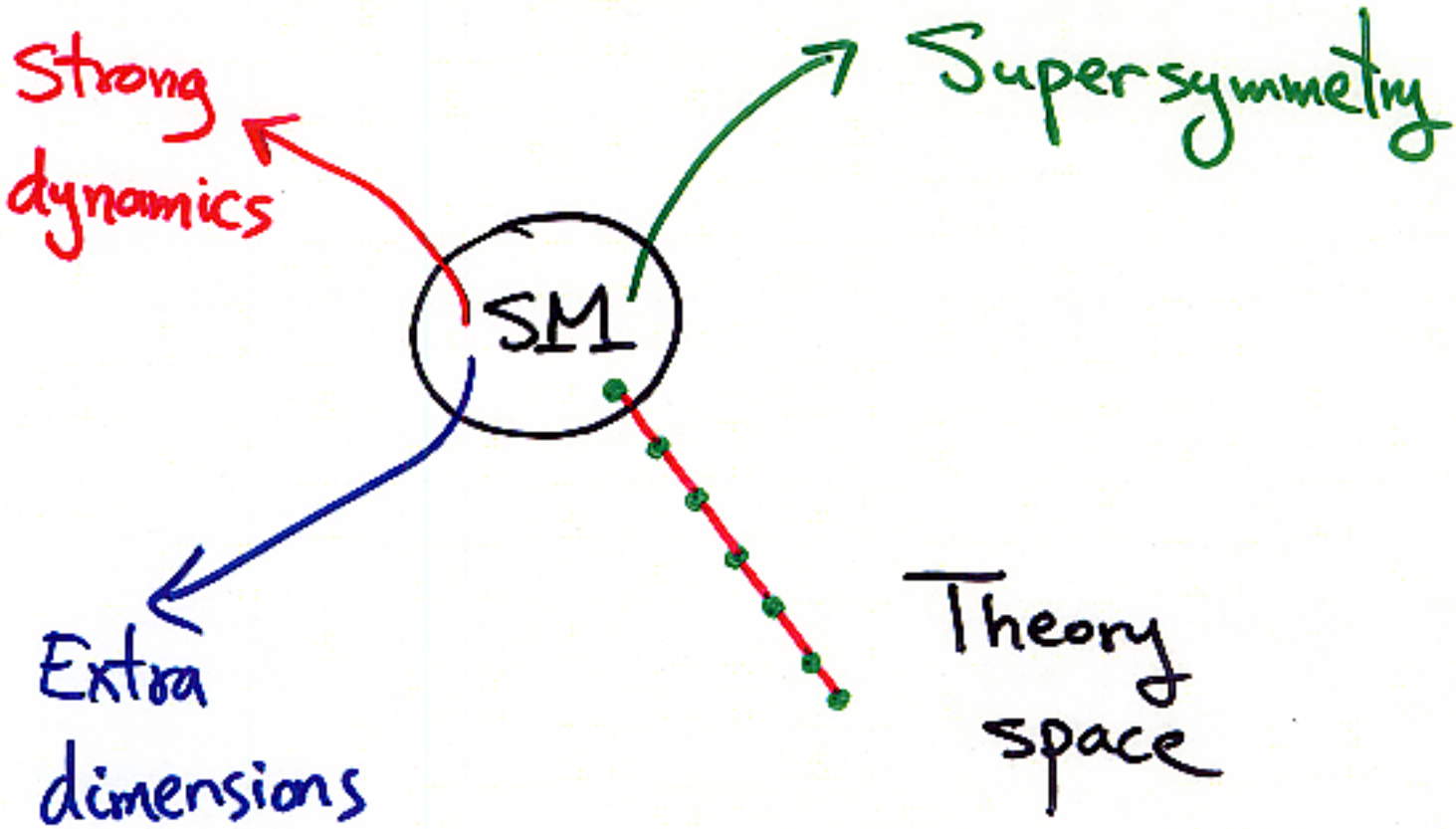


Conclusions

- We have seen that we can induce natural EWSB, with a light Higgs, if the Higgs descends from a "link-chain" in theory space.

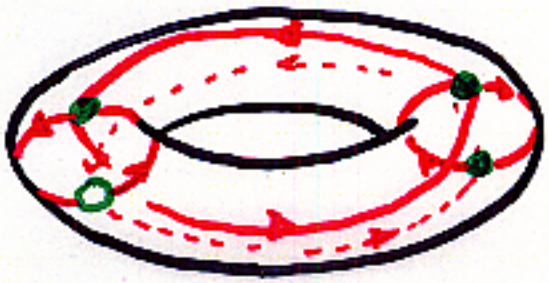
- Rich new phenomenology @ TeV scale:

Organizing principles for physics beyond the Standard Model

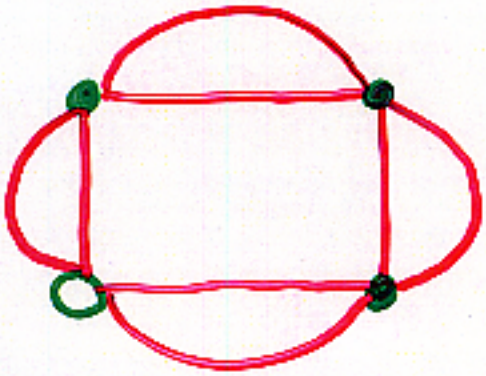


Which (if any) of these ideas are relevant to describing the real world?

Realistic models



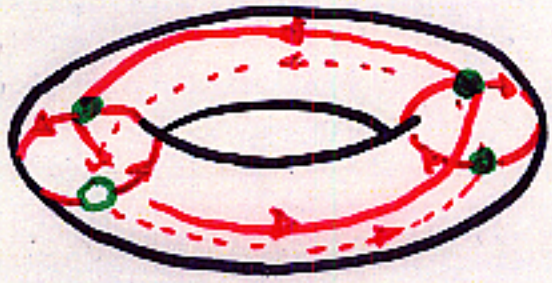
Theory space with topology of torus. Higgs fields $\leftrightarrow$ non-contractible loops in theory space.



- ● SU(3)
- ○ SU(2)×U(1)

+ Plaquette terms

Realistic models



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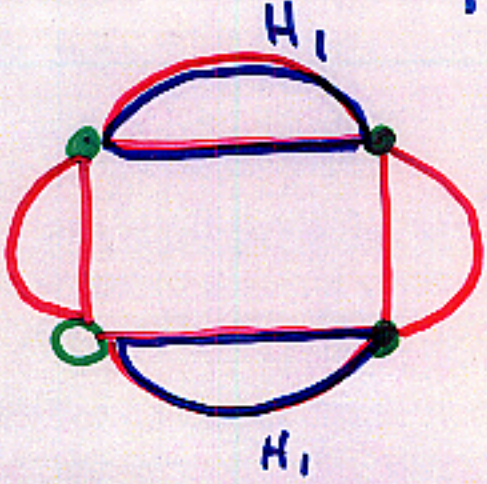

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