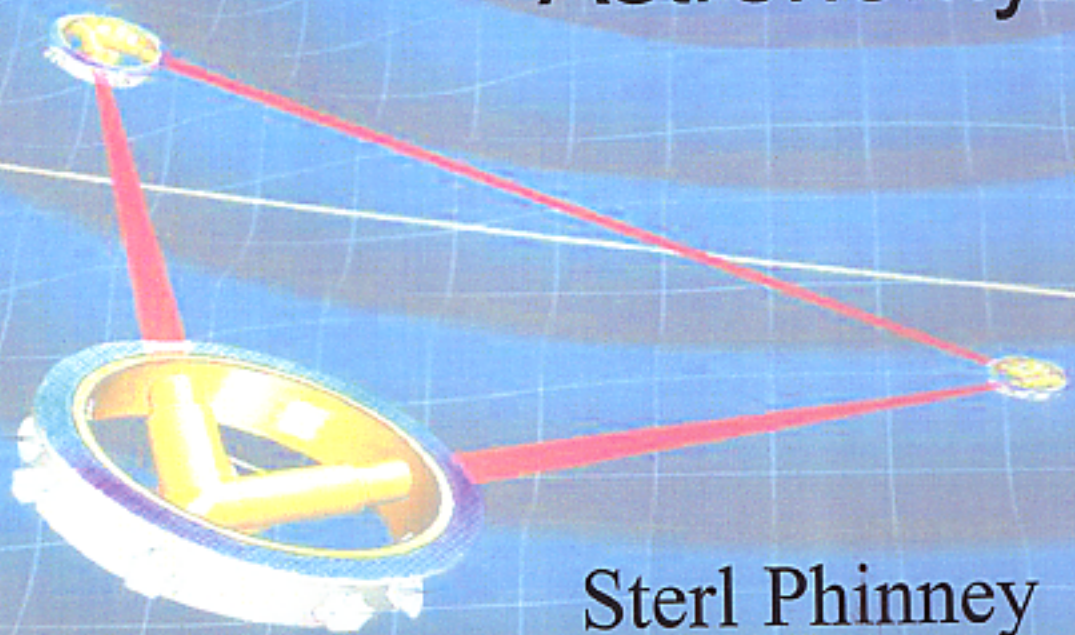
LISA and the Future of Gravitational Wave Physics and Astronomy



Sterl Phinney

Caltech

FNAL 5 Mar 2003

LISA timeline

- 2000: ESA approves LISA as 50/50 NASA share.
- 2001: ESA approves SMART-2 (LISA test package) for launch 2006. OMB funds $6M/y LISA technology development
- 2002: NASA approves $62M ST-7 as US LISA disturbance reduction tests on SMART-2.
- 2003: Bush puts full funding for LISA in NASA 2004 budget as part of *Beyond Einstein*.
- **2006: launch of SMART-2** LTP/ST-7/DRS LISA technology demonstration
- **2011: LISA launch**

2003 Events:

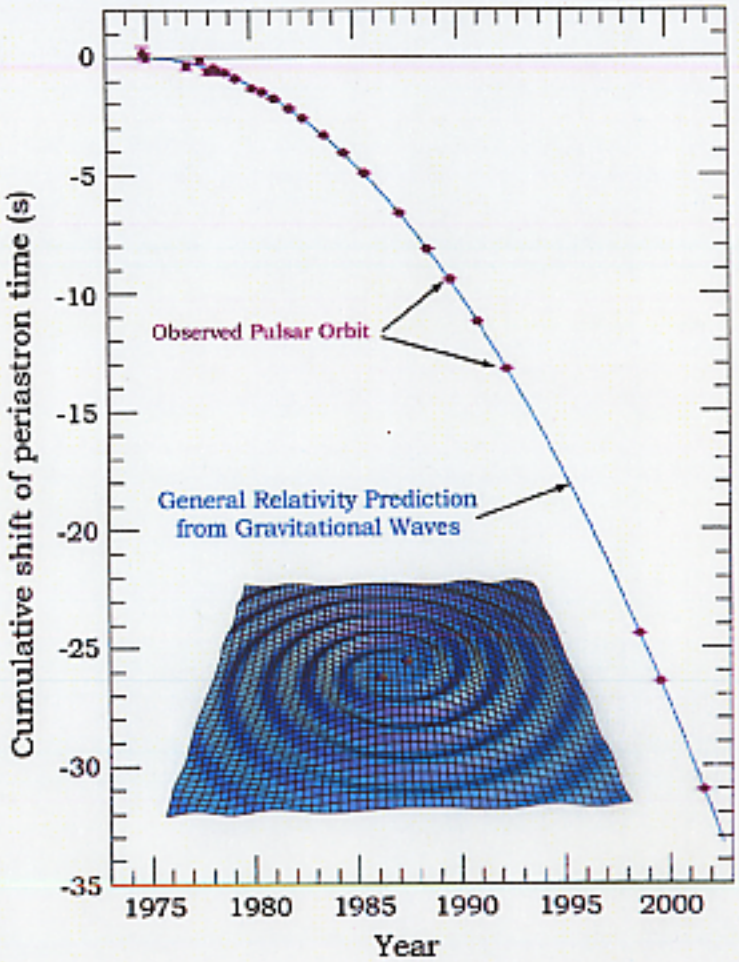
- ❑ March 4-7. TRIP review of LISA and Constellation-X: technology readiness, feasibility of schedule, cost.
- ❑ ESA awards SMART-2 industrial contracts.
- ❑NASA confirmation review of ST-7.

yr	02	03	04	05	06	07	08
$ M	6	7	35	69	89	120	147

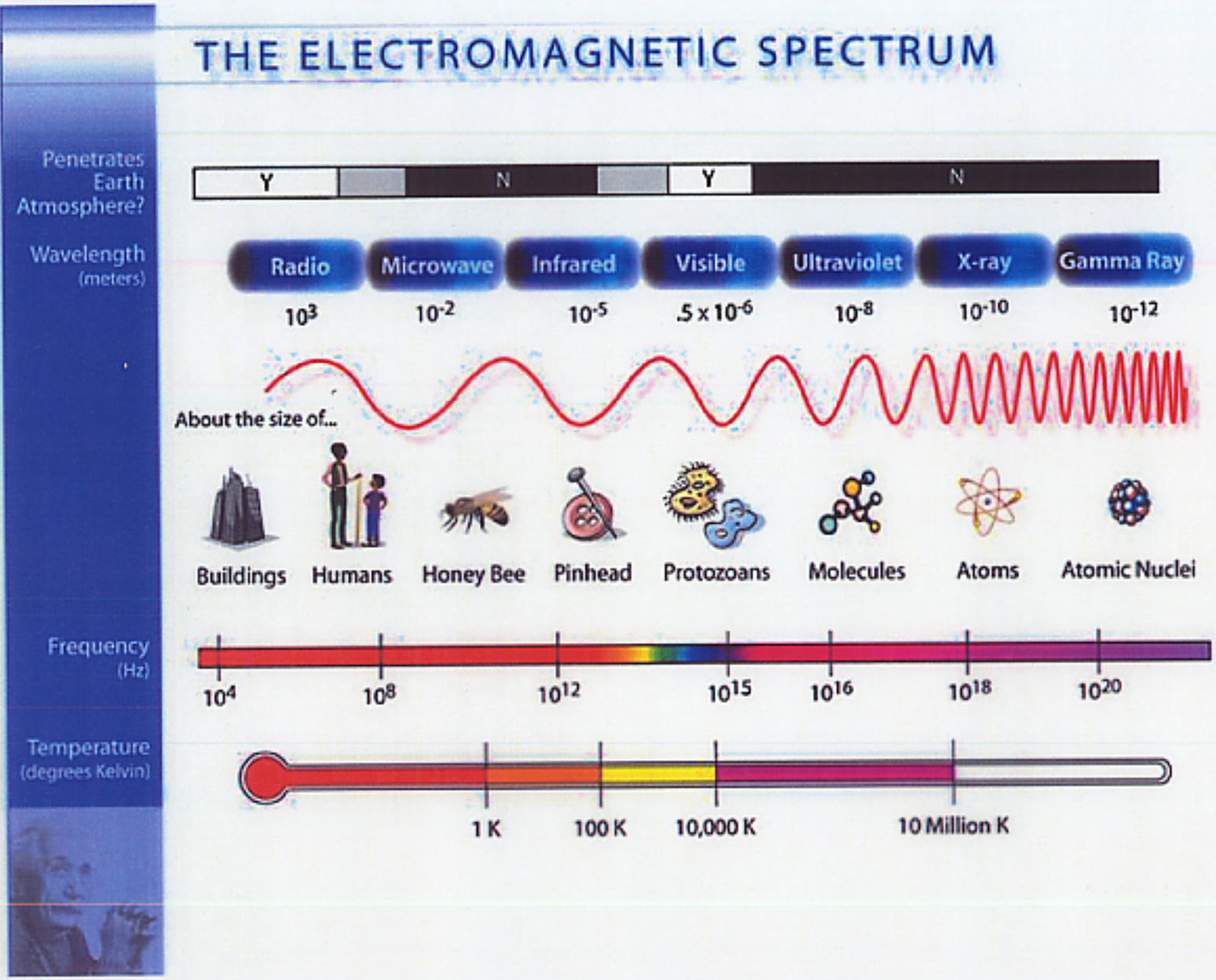
We have not yet detected gravitational waves. But we know they exist!

- jerking of masses makes ripples in spacetime
- The ripples carry of orbital energy
- So orbits shrink:
- cf. shrinking orbit of PSR 1913+16

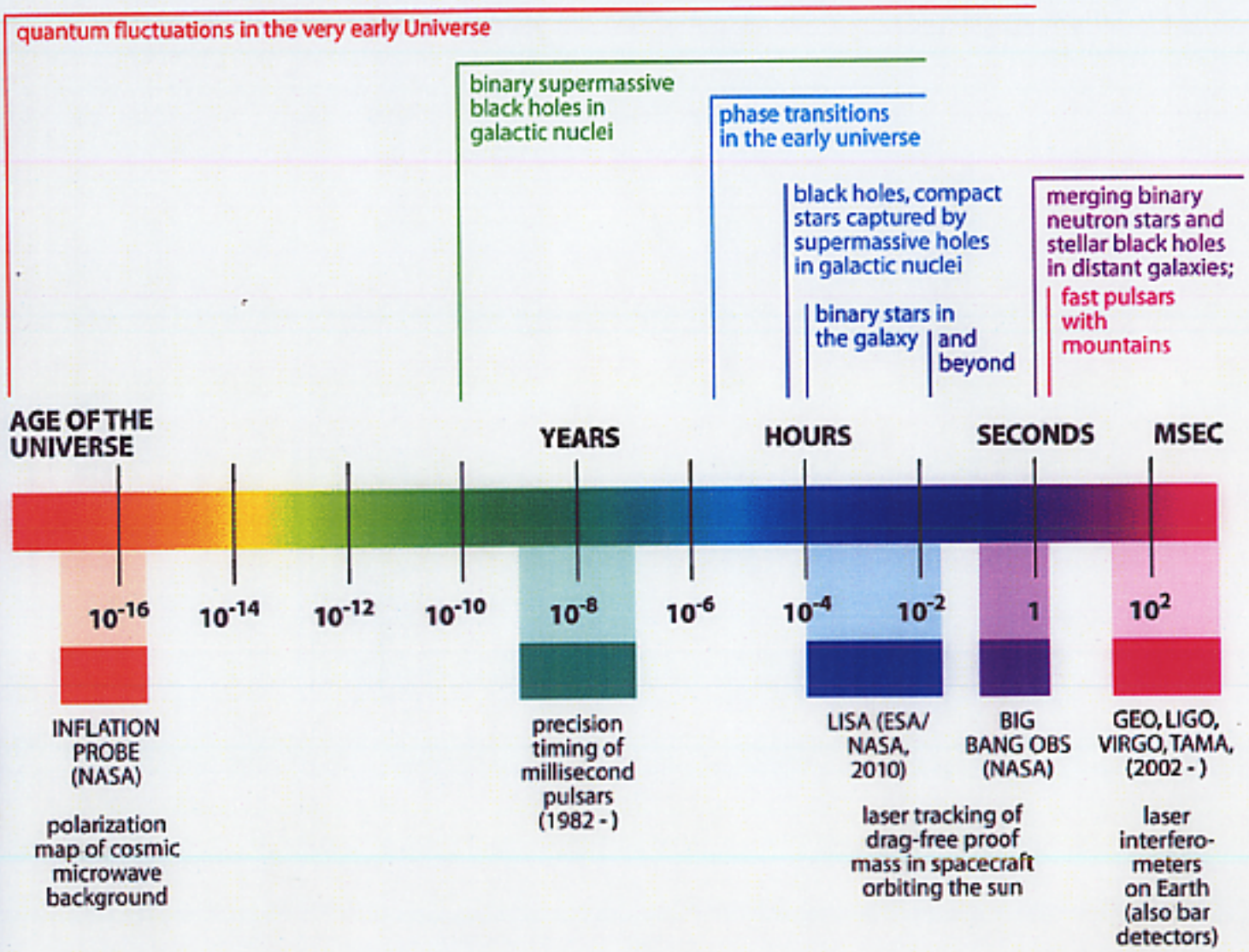
(Hulse-Taylor)



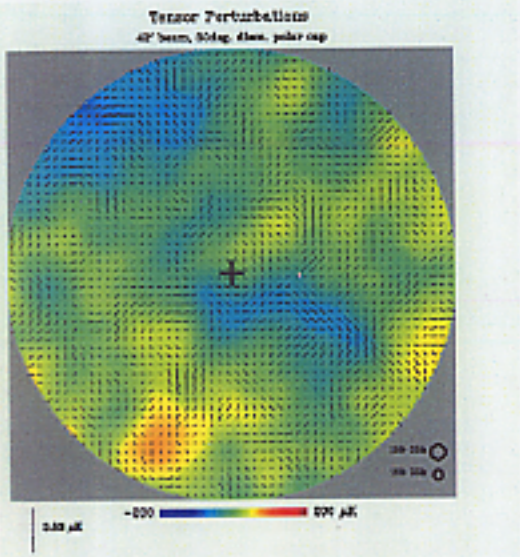
THE ELECTROMAGNETIC SPECTRUM



THE GRAVITATIONAL WAVE SPECTRUM

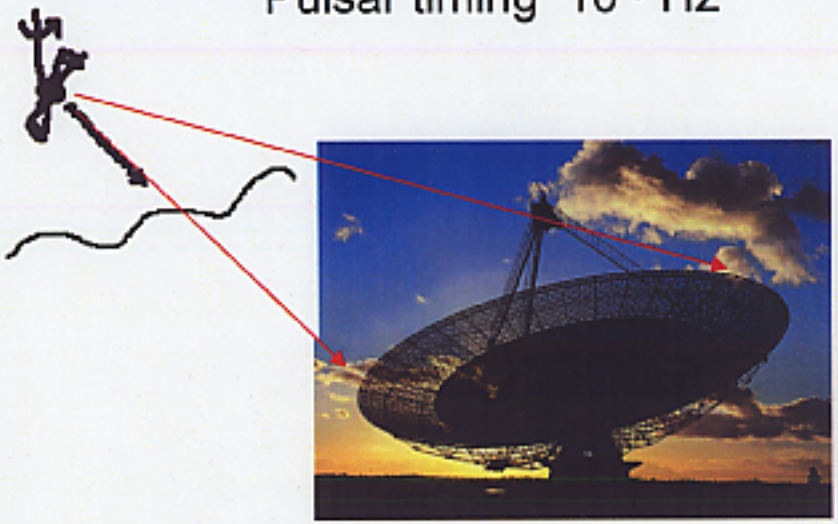


Four windows on the gravitational wave spectrum

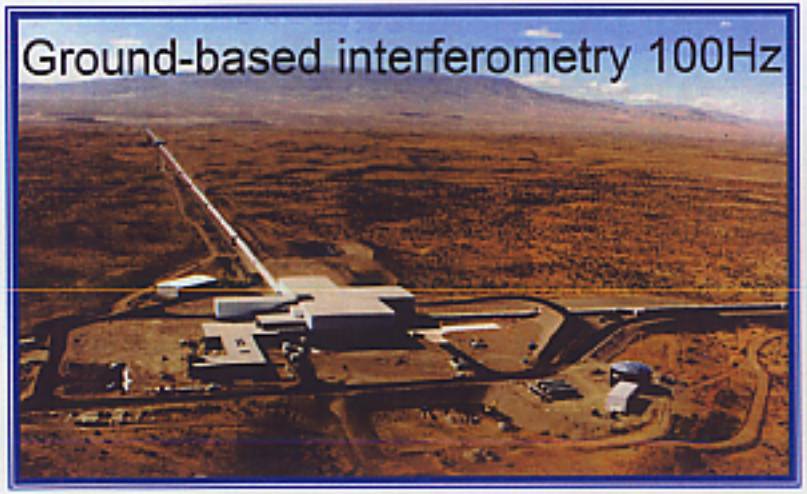


CMB B-mode polarization 10^{-16} Hz

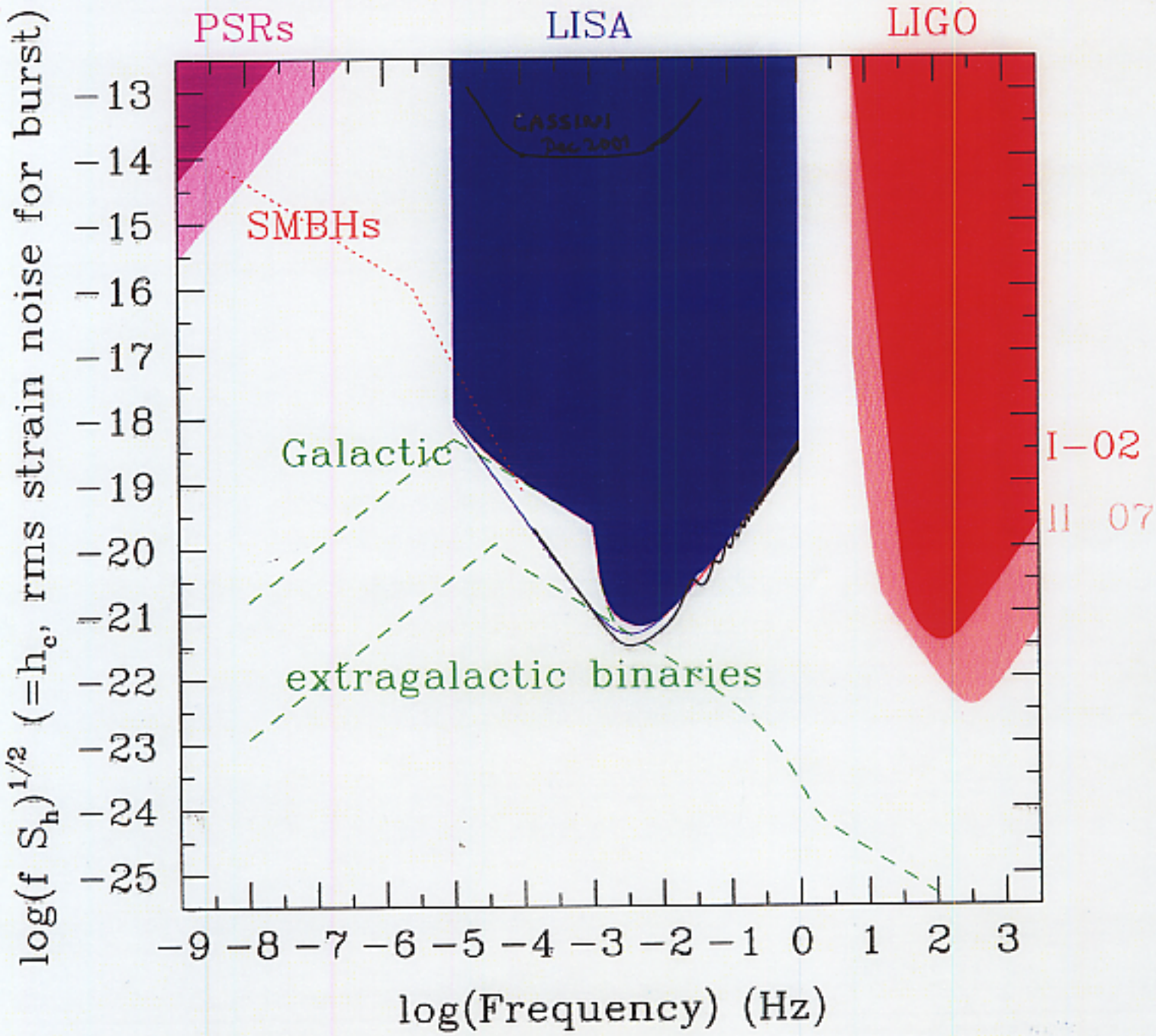
Pulsar timing 10^{-8} Hz



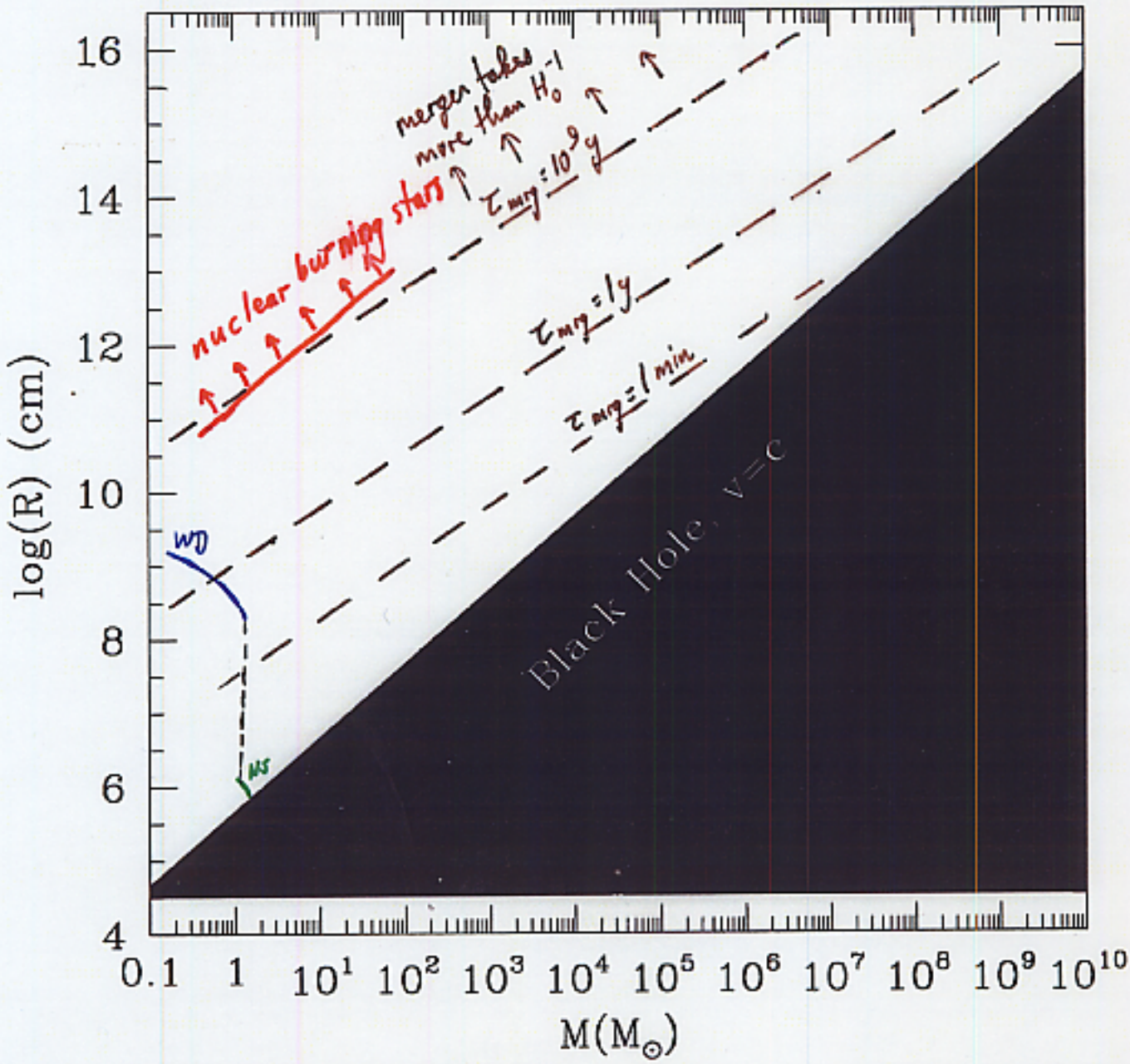
Spacecraft tracking
10^{-2} Hz



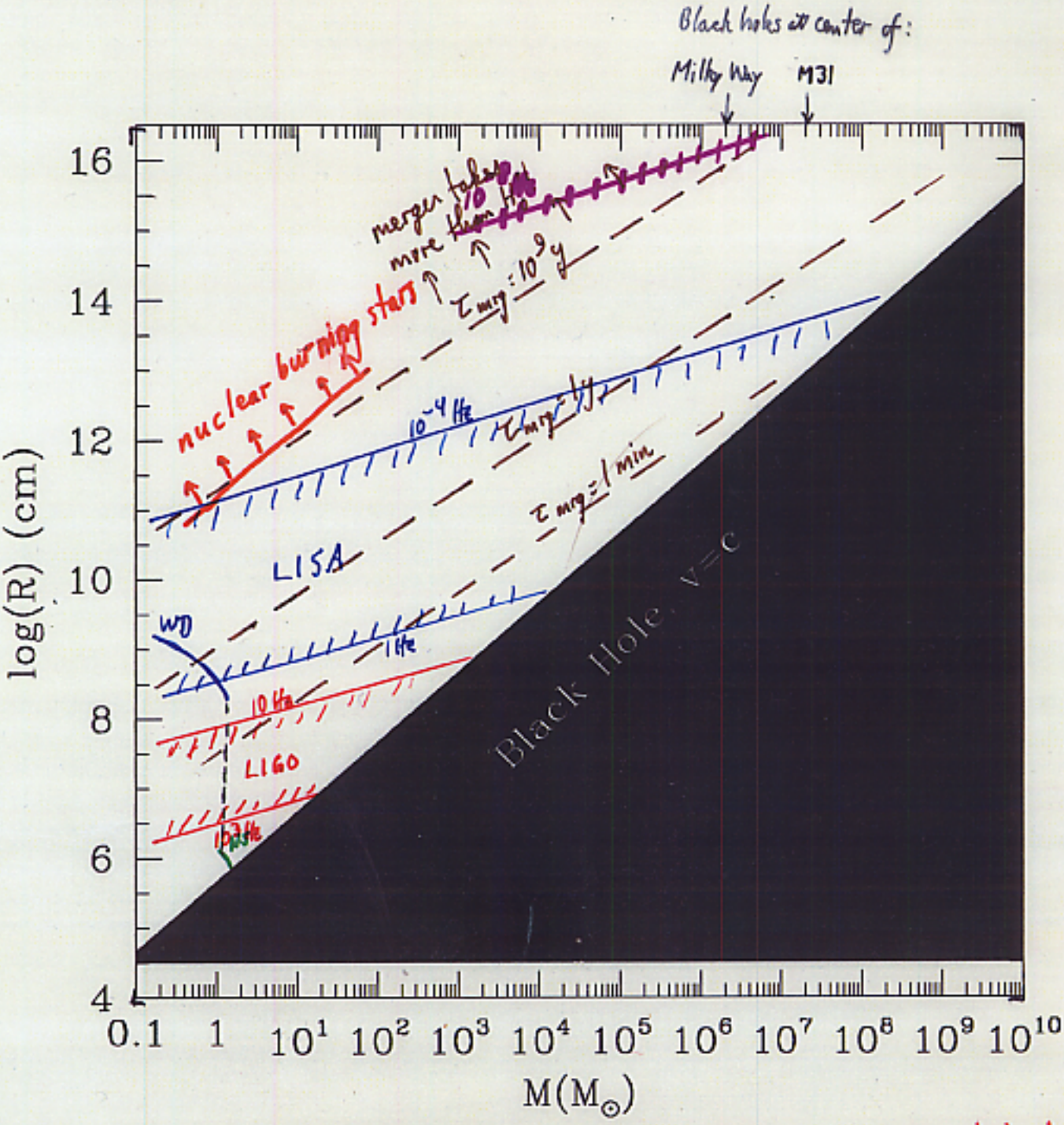
some agency should seriously support this!
cheap + potential payoff huge - $ 500k/y would enable. (10^{-2} LIGO, 10^{-2} LISA)



Equal mass binary assumed for lifetime



Equal mass binary assumed for lifetime



∴ LIGO only sees Neutron stars + 1-100 $M_\odot$ BH's — no white dwarfs
binary $\tau_{mrg} < 1$ min

LISA sees WD, NS, BH with $1y < \tau_{mrg} < 10^9 y$, supermassive black holes $10^5 < M < 10^8 M_\odot$, contact main sequence binaries.

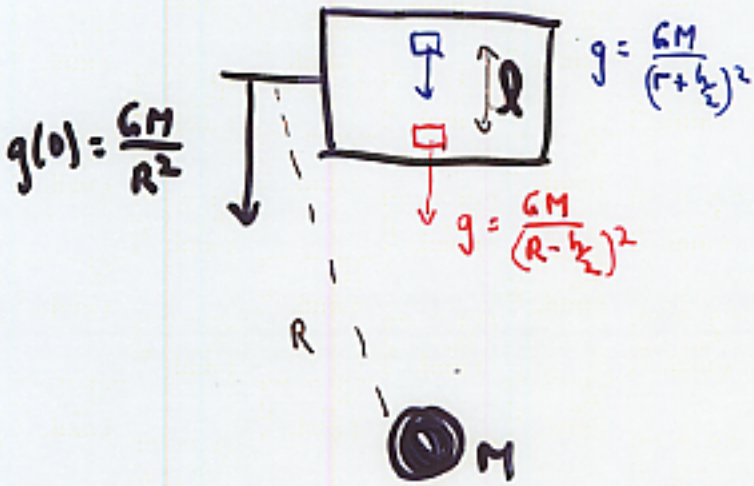
msec Pulsar timing sees supermassive black holes $M > 10^7 M_\odot$

Detecting gravity (static or waves)

Equivalence principle: in small lab, everything falls at same acceleration.

Gravity disappears if your lab is freely falling (of spaceshuttle!)

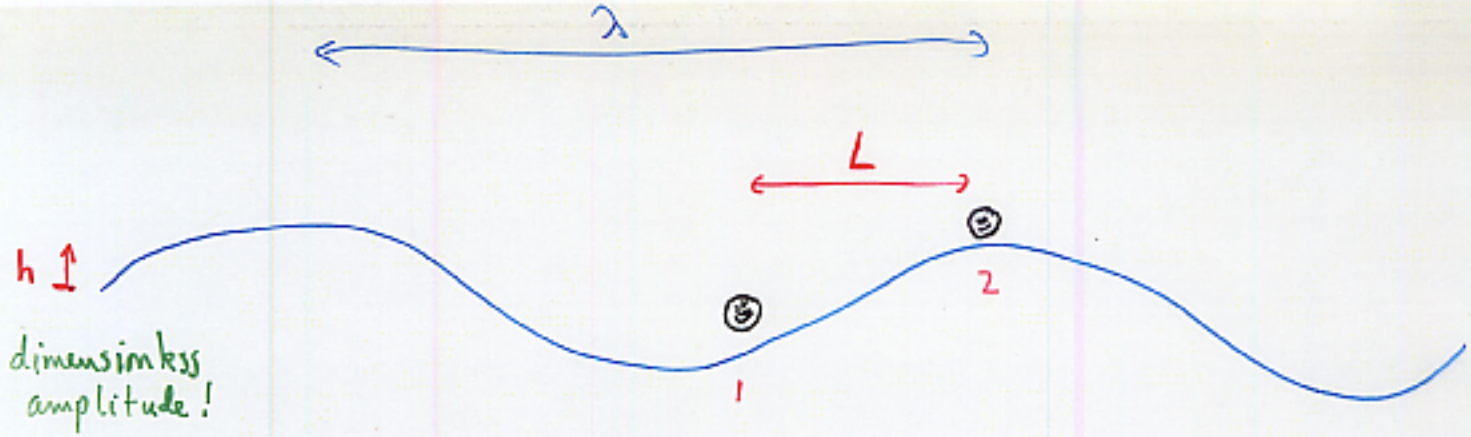
∴ Can detect only tidal effects of gravity



$$g - g = \frac{2GM}{R^2}\frac{\ell}{R} = \nabla\nabla\phi \cdot \ell$$

$c^2 \times$ metric

is only part of gravity measurable in lab.



Detection of gravitational waves:

acceleration due to wave rippling spacetime:

$$a = -c^2 \nabla h(\underset{\sim}{x}, t)$$

one mass in free fall -can't detect gravitational acceleration

(equivalence principle!)

Need two separated masses in free fall -measure *difference* in their gravitational accelerations (i.e. tidal acceleration, Riemann tensor)

LISA observable - change in fringe rate

crude: $\boxed{\ddot{L}} = a(2) - a(1) = L \cdot \nabla a$ for $L \ll \lambda$

$= -L c^2 \nabla\nabla h$ but $h = h(x - ct)$

so $\ddot{h} = c^2 \nabla\nabla h$

$= -L \ddot{h}$ $\dot{L} = -L \dot{h} + \Delta v_0$

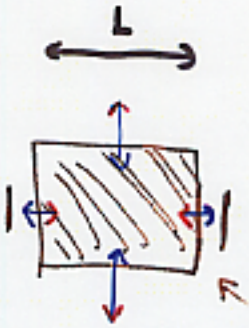
$$\boxed{\begin{aligned} \delta L &= -L\, h(t)/2 + \Delta v_0 t \\ \delta L^i &= -\tfrac{1}{2} h_{ij}(t) L^j + \Delta v_0 t \end{aligned}}$$ for $L \ll \lambda$

Exact

Passing wave produces change in separation of freely falling bodies

Detection methods for gravitational waves

Bar
(Weber ...)



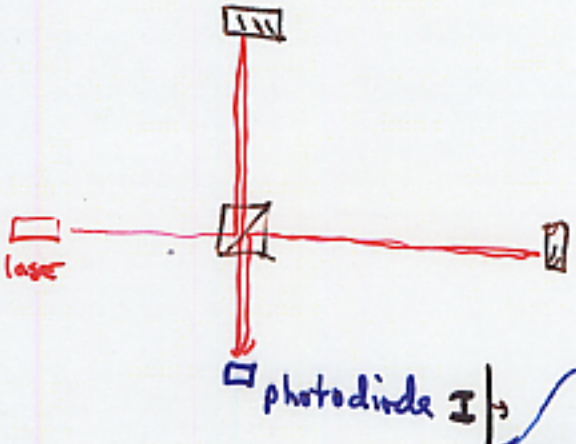
$f \sim \frac{c_s}{L} \sim \frac{2\,10^3\,m/s}{1\,m} \sim 2\,kHz$

capacitive sensor
detect expansion of bar
at resonant freqs of bar
- Narrow band due to sensor coupling.

Laser interferometer
(LIGO, Geo 600 ...)

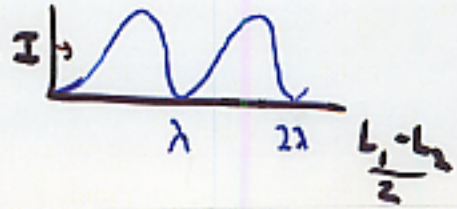
pendulum suspension
→ free fall for $\sim \frac{\text{pendulum period}}{2\pi}$
(in 1 dimension) $\sim 0.2\,s$
Seismic noise ⇒ $f > 20\,Hz$

to reduce effect of laser frequency drift, use Michelson interf.:



operate $L_1 = L_2 + \frac{\lambda}{4}$ (dark fringe)

GW shifts $\frac{2(\delta L_1 - \delta L_2)}{\lambda} = \frac{h}{\lambda}$

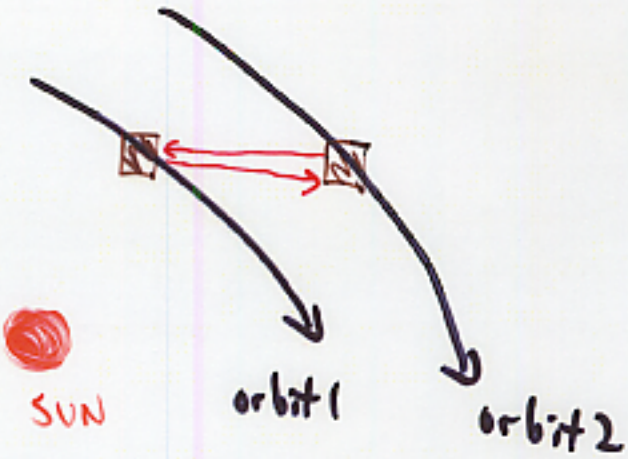


Detection methods cont'd

Spacecraft tracking

CASSINI (radio, 1 baseline)

LISA (laser, 3 baseline)



Freely falling spacecraft : const relative velocity (w/ 1yr drift)

$\Rightarrow$ constant fringe rate (L.O. vs signal from distant osc.)

$$\dot{L}/\lambda = \frac{15\,m/s}{\lambda} = 15\,MHz$$

LISA

Gravitational wave modulates fringe rate:

$$\ddot{L} = (2\pi f)^2 h L \qquad \text{for } \lambda = \frac{c}{f} > 2L$$

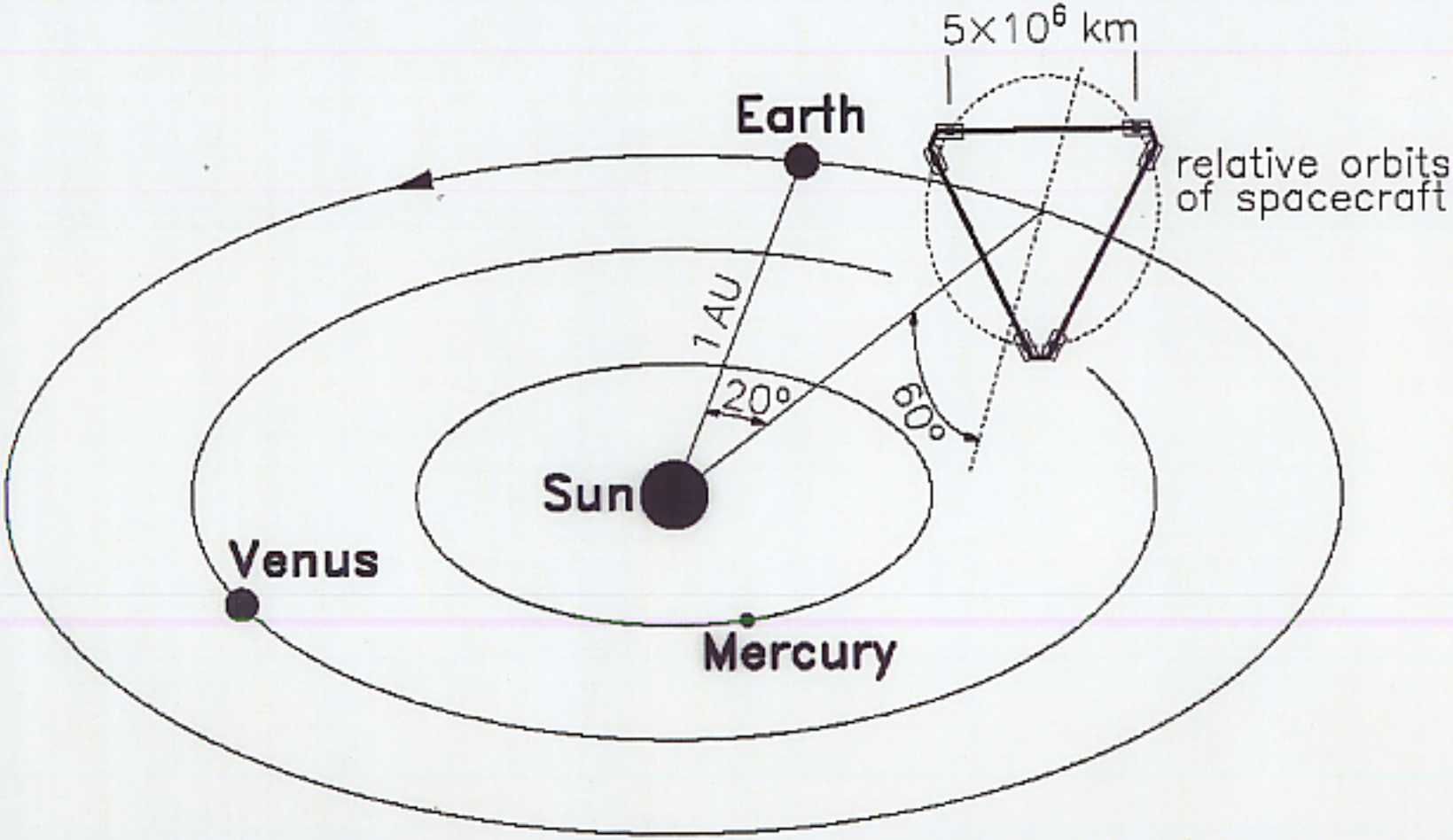
To remove laser frequency drift, need 3 spacecraft (2 arms) - of Michaelson

$L_1 \neq L_2 \Rightarrow$ must time shift by $\frac{2(L_1 - L_2)}{c}$ to remove effect of freq. drift

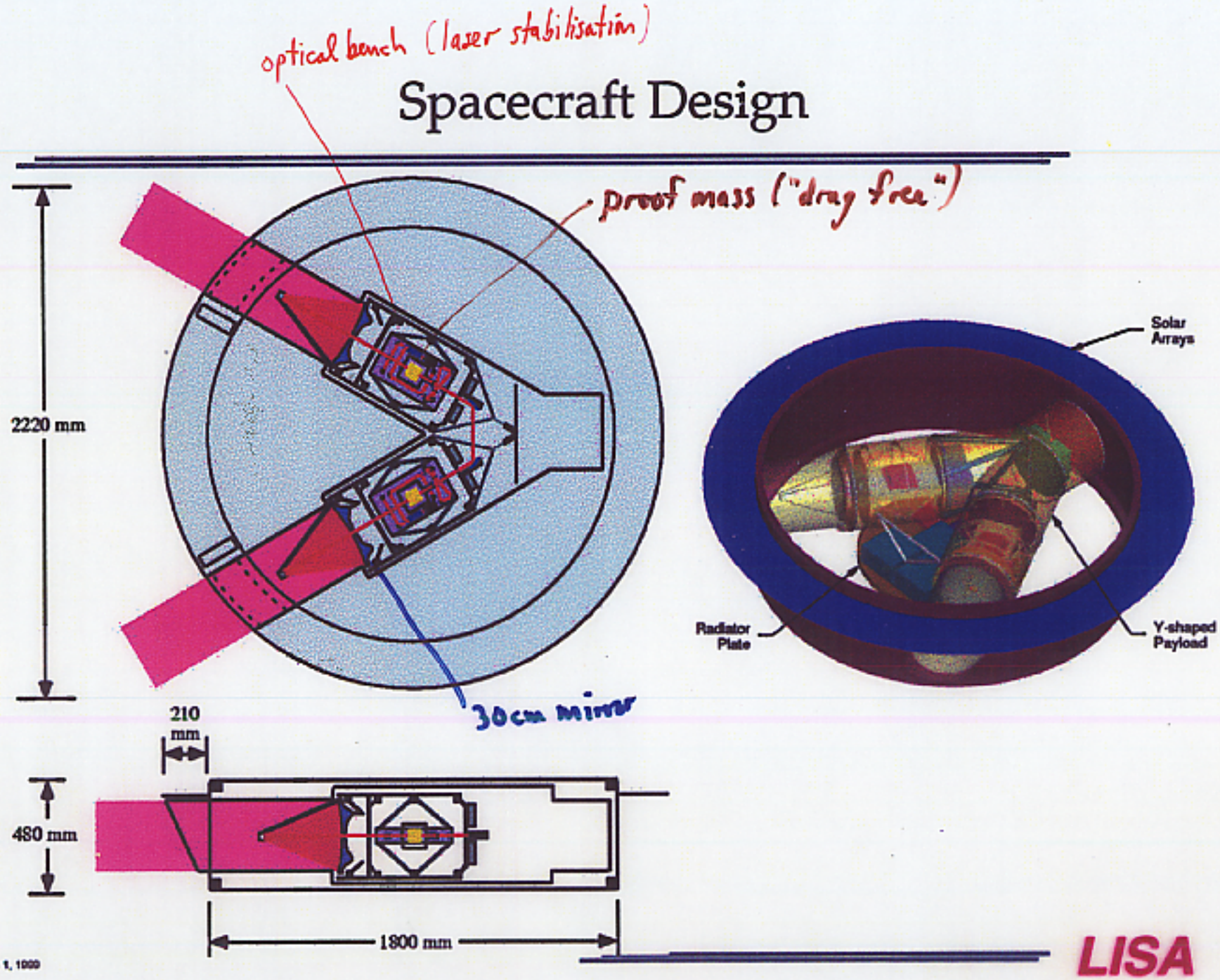
"time delay interferometry

LISA

5×10^6 km=0.03AU=17 light seconds



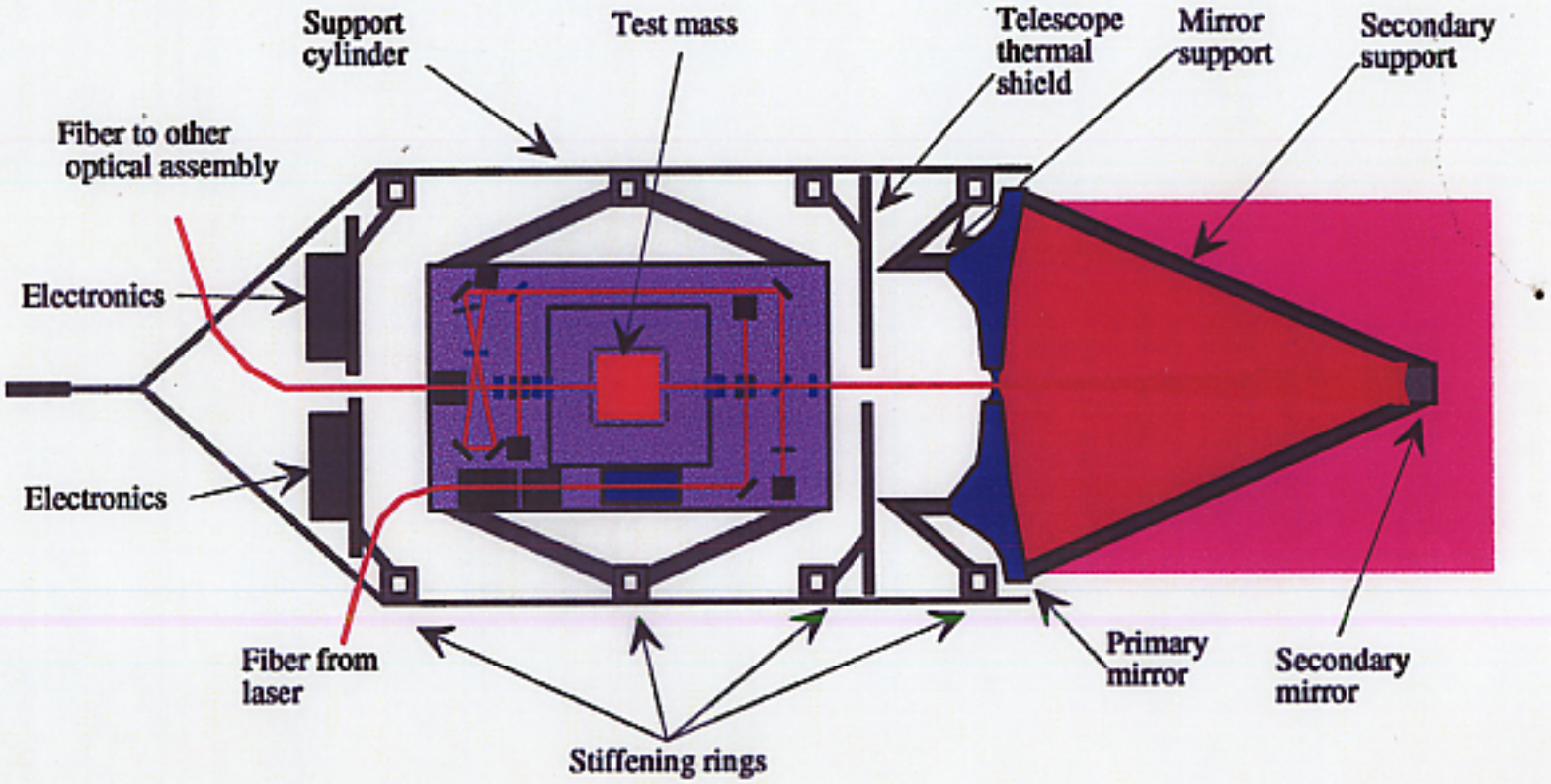
Spacecraft Design

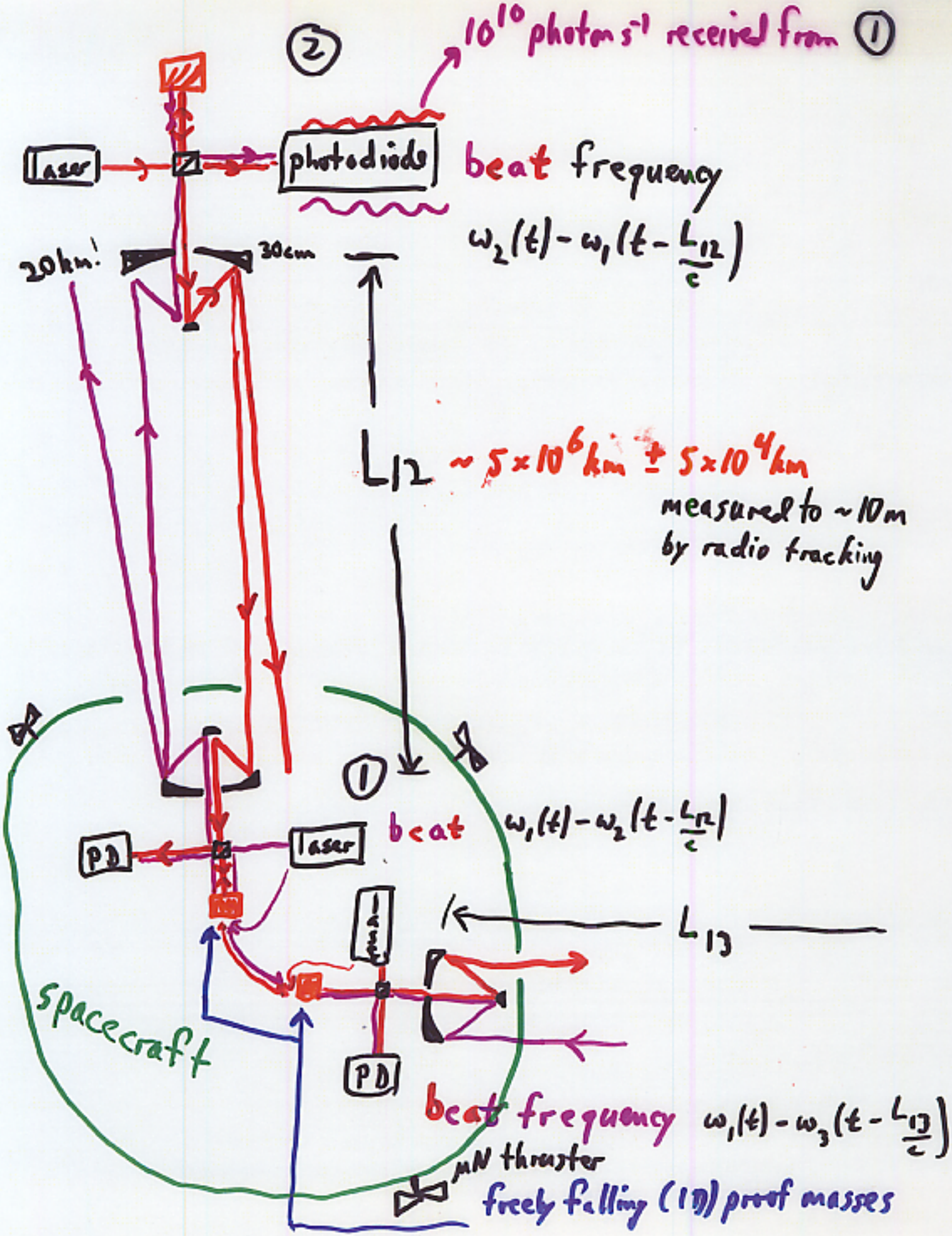


March 1, 1999

LISA

Optical System




2
10^{10} photons s^{-1} received from 1
laser
photodiode
beat frequency
$\omega_2(t) - \omega_1(t - \frac{L_{12}}{c})$
20km!
30cm
$L_{12} \sim 5\times10^6$ km $\pm 5\times10^4$ km
measured to ~10m
by radio tracking
1
PD
laser
beat
$\omega_1(t) - \omega_2(t - \frac{L_{12}}{c})$
laser
L_{13}
spacecraft
PD
beat frequency $\omega_1(t) - \omega_3(t - \frac{L_{13}}{c})$
μN thruster
freely falling (1D) proof masses

1 W lasers at 1 μm $\times \varepsilon$ $= 2\,10^{18}$ photons/s emitted

receive fraction $\dfrac{\pi D^2}{\pi\left(\frac{\lambda}{D}L\right)^2} = \left(\dfrac{30\text{ cm}}{20\text{ km}}\right)^2 = 2\,10^{-8} \times \varepsilon$

$\Rightarrow$ receive $\sim 10^{10}$ photons s^{-1} to define fringe rate changes.

shot noise

in 100 s $\sim \dfrac{1}{f} \sim \dfrac{1}{\sqrt{10^{10}\text{ ph/s} \times 10^2\text{ s}}} \sim 10^{-6}$ fringe $\sim 10^{-12}$ m

$\Rightarrow h_{rms}$ sensitivity $\sim \dfrac{\delta L}{L} \sim \dfrac{10^{-12}\text{ m}}{5\times10^{3}\text{ m}} \sim \boxed{2\,10^{-22} \sim h_{rms}}$ 100 s

in 3 years $\sim T_{mission}$. $\boxed{h_{rms} \sim 2\,10^{-25}}$ 3 years

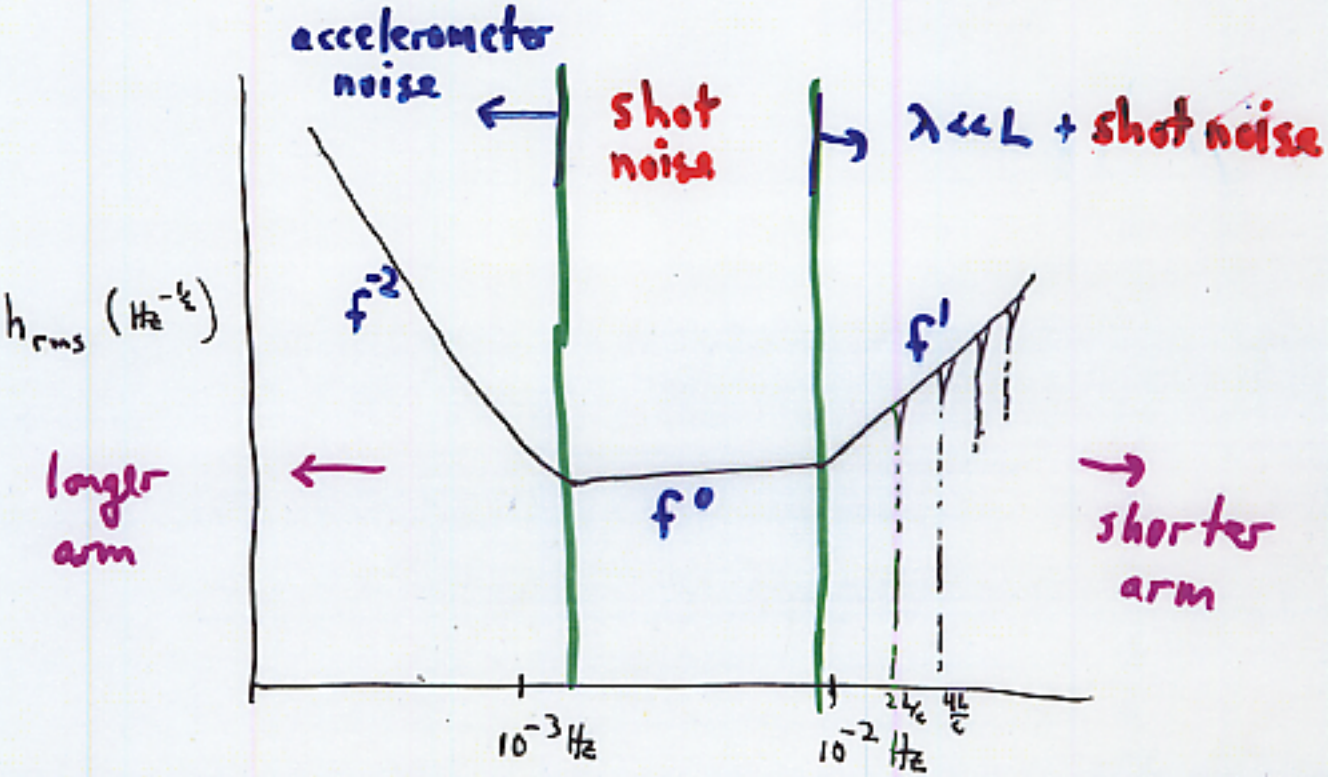
Design: keep acceleration noise on proof masses + laser frequency/phase noise below shot noise at freqs of interest $< \dfrac{c}{2L} = 0.03$ Hz

3. Gravitational wave sensitivity

for $\lambda > 2\pi L$, $h_{rms} = \frac{\delta L_{rms}}{L}$ $f < 10^{-2}$ Hz

$\lambda < 2\pi L$, cancellation $h_{rms} = \frac{\delta L_{rms}}{(\lambda/2\pi)}$ $f > 10^{-2}$ Hz

resonances at $\frac{1}{f} = n\frac{2L}{c}$ $n = 1, 2, 3, 4 \ldots$



Example: periodic source at $f = 3 \times 10^{-3}$ Hz → shot

$T = 3$ year mission

$$\text{detect} > h_{1\sigma} = \frac{\delta L_{rms}}{L\sqrt{T}} = 10^{-25}$$

Example sources:

1. Two $0.3\,M_\odot$ white dwarfs with $P = 700\,s \Rightarrow f_{gw} = \frac{2}{P} = 3\,10^{-3}\,Hz$

at 10 kpc across Milky Way $\qquad \tau_{gw} \sim 10^6\,y$

$$h \sim \frac{GM}{c^2 r}\,\frac{r_g}{a}\,\frac{m}{M} \sim 10^{-23} \quad \Rightarrow \frac{S}{N} \sim 100\,!$$

vs $h_{1\sigma} = 10^{-25}$

2. WD 0957-666 $\quad f_{gw} = 3.8\,10^{-4}\,Hz \quad h = 4\,10^{-22}$

KPD 0422+4521 $\quad f_{gw} = 2.6 \times 10^{-4}\,Hz \quad h = 5\,10^{-22}$ $\quad \Rightarrow \frac{S}{N} \sim 30^*$

$\tau_{gw} \sim 2\,10^8\,y$

* but of WD background...

3. $10\,M_\odot + 10^6\,M_\odot$ with $f_{gw}(z=0) = 1.5 \times 10^{-3}\,Hz$, 2 years before merger

@ $z=1$ $\quad h \sim 4\,10^{-23} \quad \rightarrow \frac{S}{N} \sim 400^*$

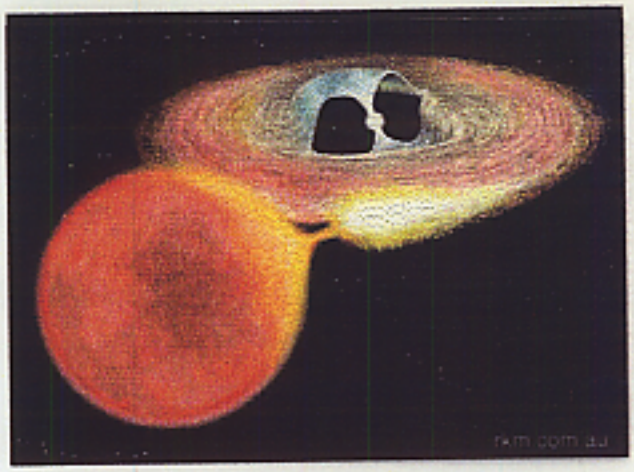
* if can coherently integrate w/ optimal signal processing

4. $10^6 + 10^6\,M_\odot$ @ $z=2$ at $r = 6M \rightarrow f_{gw} = 7\,10^{-4}\,Hz$ at $z=0$

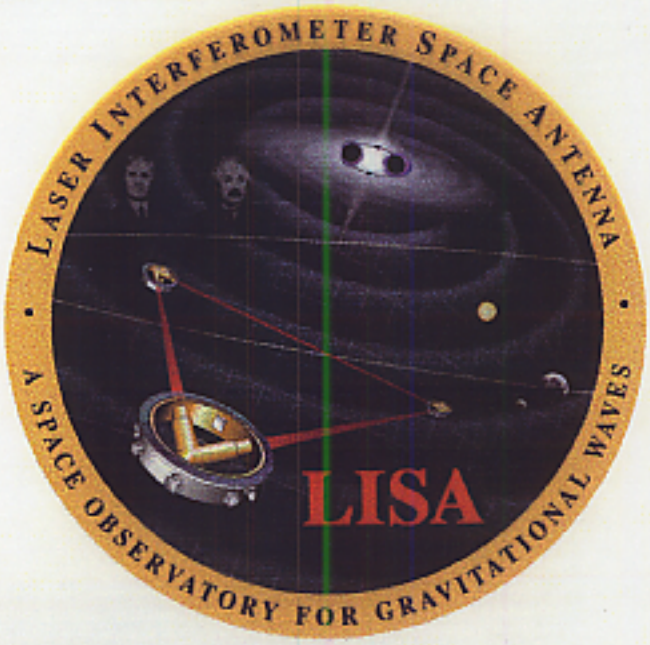
$\tau_{gw} \sim 3000\,s$ = T not 3y!

$h \sim 10^{-18} \quad \rightarrow \frac{S}{N} \sim 10^4\,!$

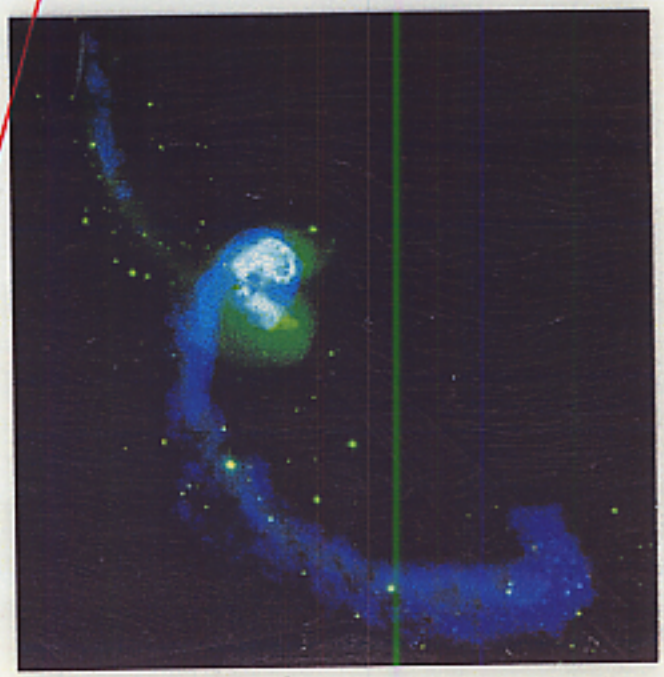
$h_{1\sigma}(3000\,s) = 2\,10^{-23}$



Detect every binary star in the Milky way with orbital period <~750s (ca. 30,000 sources)



Detect every compact object captured by supermassive black holes (10^5 – 3×10^6 solar masses) in galactic nuclei at z<1 (ca. 500–5000 sources)



Detect every merging pair of supermassive black holes with $M=3\times10^{6+/-1}/(1+z)$ in merging galaxies to z<100 (ca. 1–500 sources)

LISA - can measure <u>both</u> polarisations of gravitational waves*

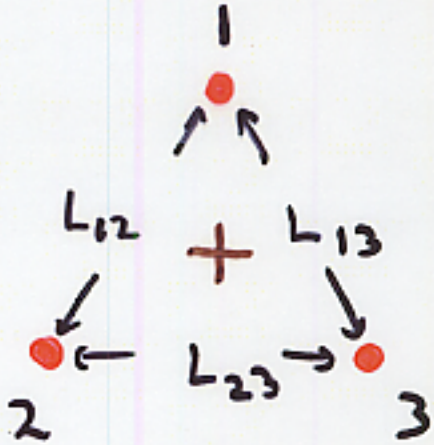
$h_+ , h_\times$

Consider $\lambda \gg L$

($f \ll 0.03$ Hz)

wave incident from above

$\otimes$



$$\begin{cases} \delta L_{12} = \left(\frac{1}{4} h_+ - \frac{\sqrt{3}}{4} h_\times\right) L \\ \delta L_{23} = -\frac{h_+}{2} L \\ \delta L_{31} = \left(\frac{1}{4} h_+ + \frac{\sqrt{3}}{4} h_\times\right) L \end{cases}$$

$$\begin{cases} \delta L_{31} - \delta L_{12} = L \frac{\sqrt{3}}{2} \boxed{h_\times} \\ \delta L_{12} + \delta L_{31} - \delta L_{23} = L \boxed{h_+} \\ \text{and } \delta L_{12} + \delta L_{23} + \delta L_{31} = 0 \end{cases}$$

(max signal - or $\delta L_{31} + \delta L_{21} = -\delta L_{23} = \frac{L}{2} h_+$)

Sagnac - measure of laser noise - no sensitivity to GWs for $\lambda \gg L$

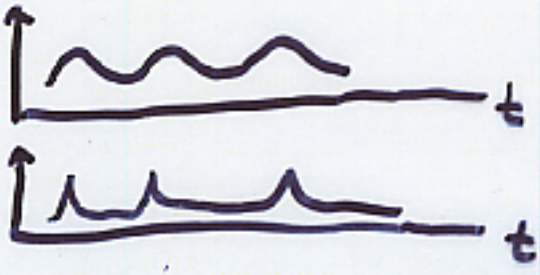
(Estabrook, Tinto + Armstrong 2000)

LIGO alone can't

LISA data:

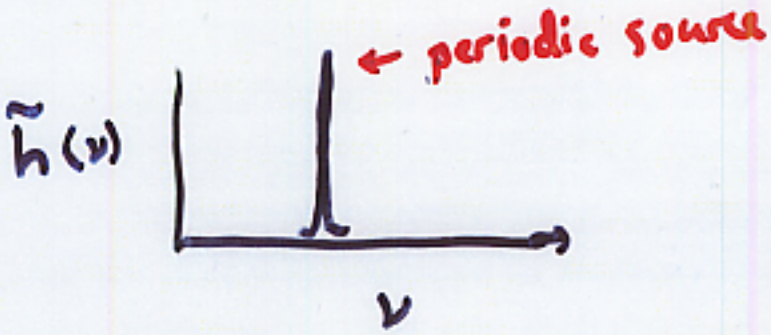
$h_+(t)$

$h_\times(t)$



3×10^{-3} Hz $\times$ 3 years $= 3\times10^{5}$ wave cycles!

usually present |FFT| of h



(not how will be analysed!)

Monitor all sky

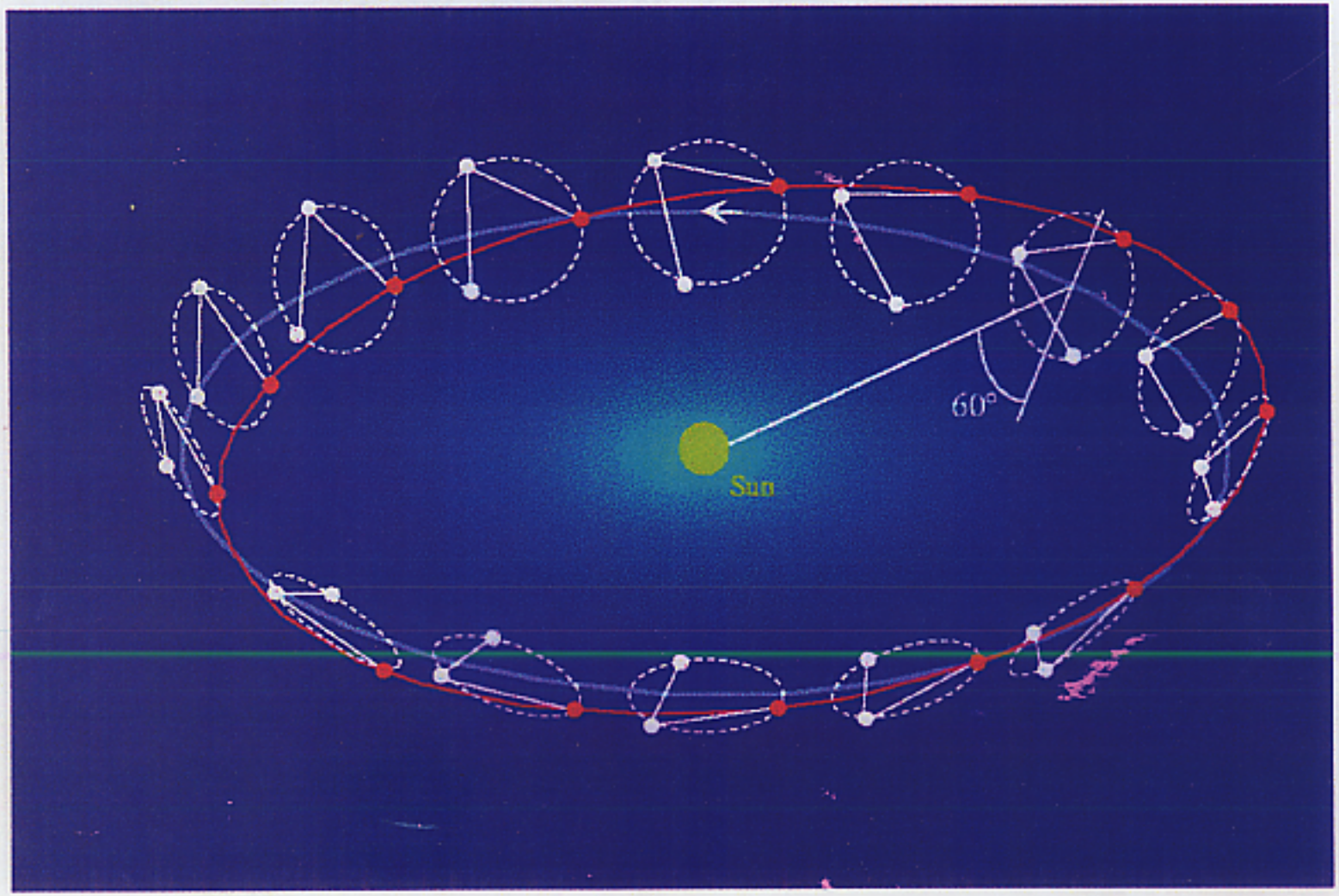
separate sources by
- frequency (FFT = template for NBs
- interferometer orbital modulation → position on sky
- template matching searches

Spacecraft Orbits

5×10^6 km

$= \frac{1}{30}$ AU

arms

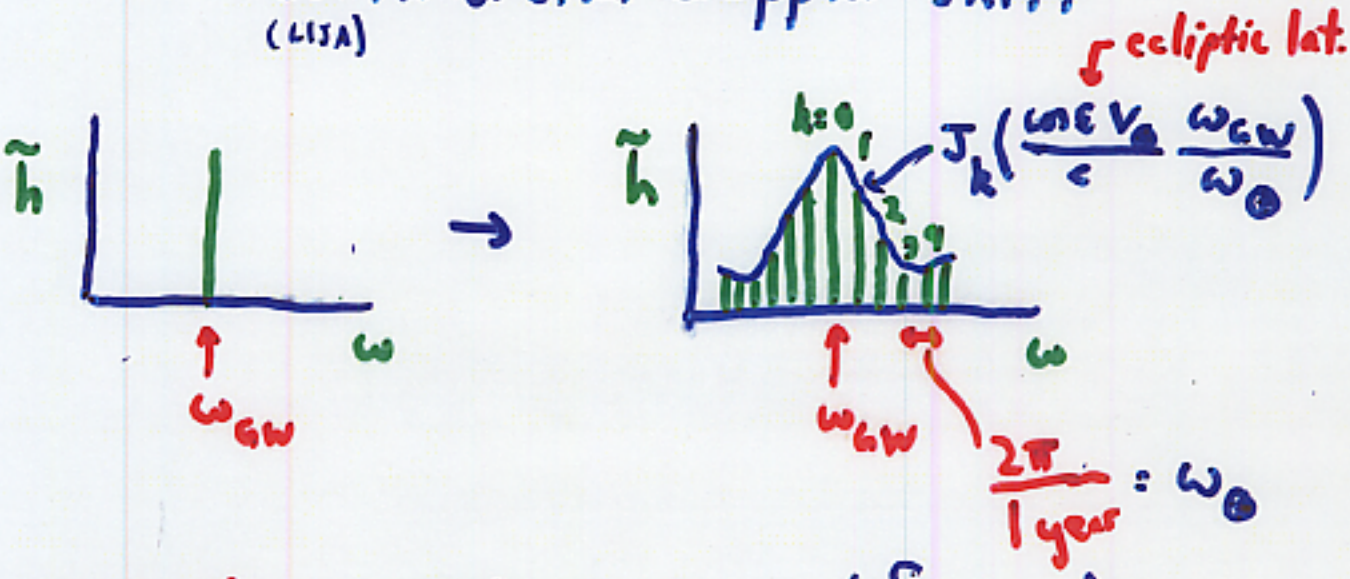


Determining Source Positions with LISA

- Most sources detected for > 1 year
- Method similar to pulsar timing positions (high freq)
- antenna pointing (low freq)

Two effects:

1. **FM** - modulation of GW frequency by "earth" orbital doppler shift (LISA)



No of strong side bands $\sim \cos\varepsilon\left(\frac{f_{GW}}{3\,10^{-4}\,\text{Hz}}\right)$

char freq $= \frac{c}{v_\oplus}\frac{\omega_\oplus}{2\pi} = \frac{c}{v_\oplus}\frac{1}{1y}$

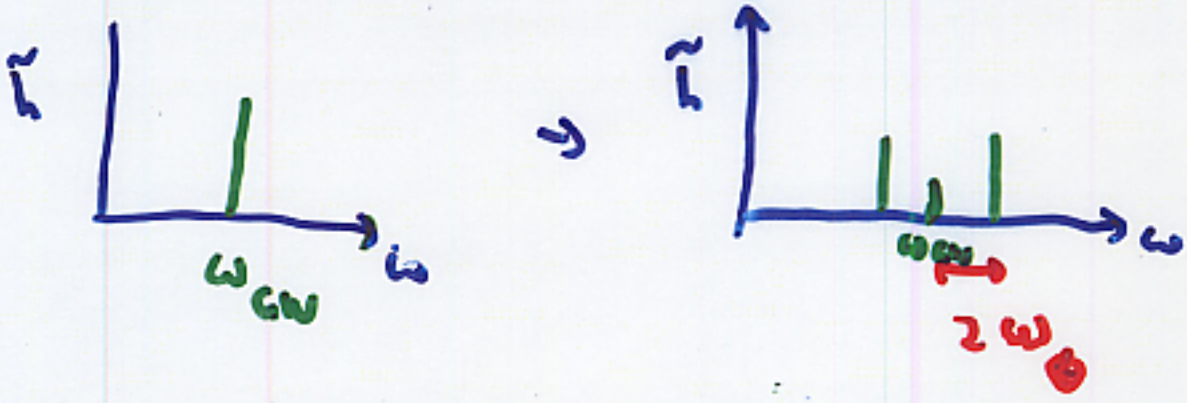
dominates for $f_{GW} > 10^{-3}$ Hz

Angular resolution $\sim 1^\circ$ for $f = 10^{-2}$ Hz $S/N = 10$

$\sim 8^\circ$ for $f = 10^{-3}$ Hz $S/N = 10$

$\sim 1^\circ$ $f = 10^{-3}$ Hz $S/N = 10^3$

2. AM - modulation of signal amplitude due to annual change in orientation of interferometer arms

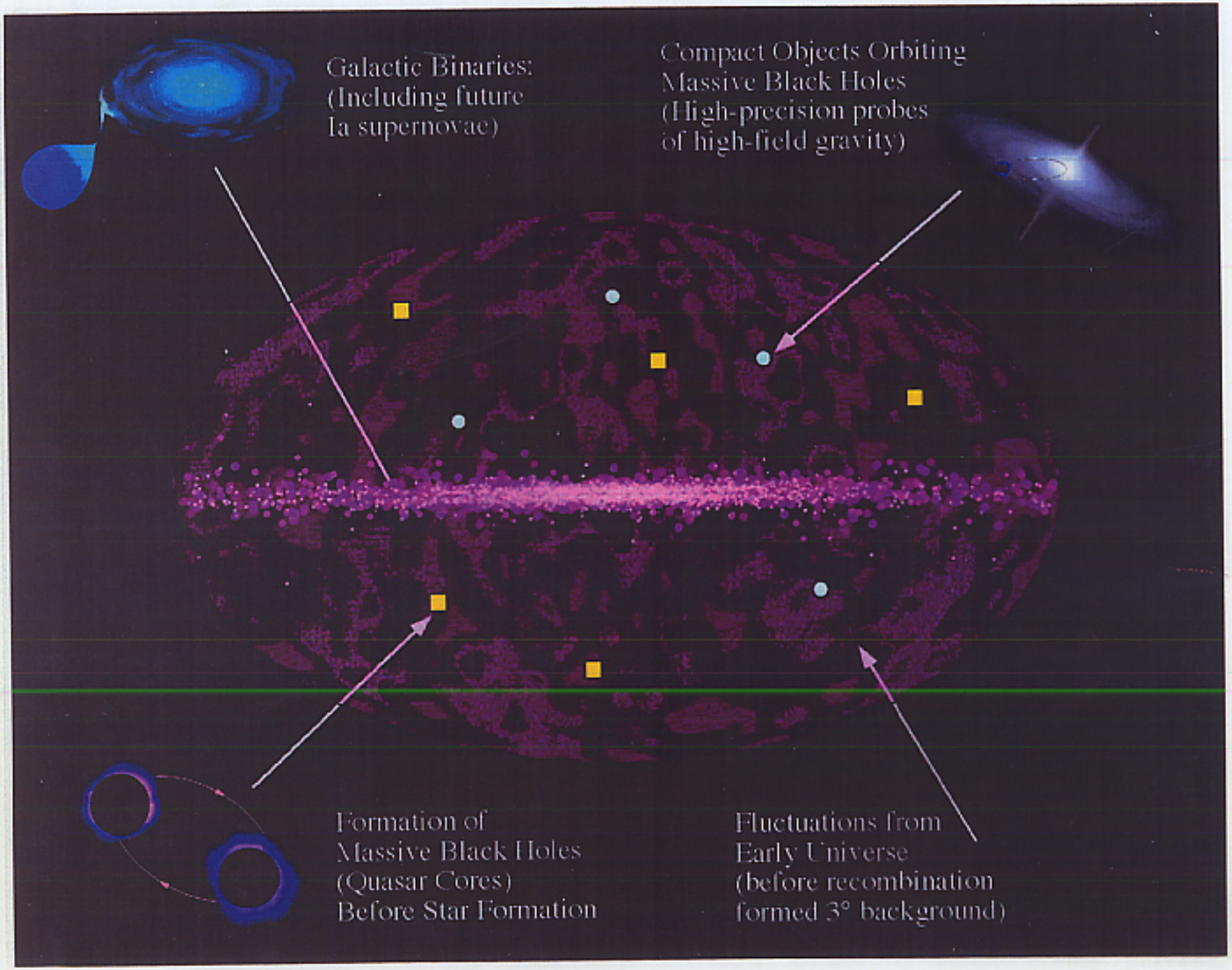


dominates for $f_{GW} < 10^{-3}$ Hz

Angular resolution for $f < 10^{-3}$ Hz :

$\sim 10^{\circ}$ $\quad S/N = 10$

$\sim 1^{\circ}$ $\quad S/N = 10^{3}$


Galactic Binaries:
(Including future
Ia supernovae)
Compact Objects Orbiting
Massive Black Holes
(High-precision probes
of high-field gravity)
Formation of
Massive Black Holes
(Quasar Cores)
Before Star Formation
Fluctuations from
Early Universe
(before recombination
formed 3° background)

class	source	dist pc	$f = 2/P_b$ mHz	M_1 $M_\odot$	M_2 $M_\odot$	τ_{mrg} 10^8y	h
WD+WD	WD 0957-666	100	0.38	0.37	0.32	2	4×10^{-22}
	WD 1101+364	100	0.16	0.31	0.36	20	2×10^{-22}
	WD 1704+481	100	0.16	0.39	0.56	13	4×10^{-22}
	WD 2331+290	100	0.14	0.39	> 0.32	< 30	$> 2 \times 10^{-22}$
WD+sdB	KPD 0422+4521	100	0.26	0.51	0.53	3	6×10^{-22}
	KPD 1930+2752	100	0.24	0.5	0.97	2	1×10^{-21}
AM CVn	RXJ0806.3+1527	300	6.2	0.4	0.12	–	4×10^{-22}
	RXJ1914+245	100	3.5?	0.6	0.07	–	6×10^{-22}
	KUV05184-0939	1000	3.2	0.7	0.092	–	9×10^{-23}
	AM CVn	100	1.94	0.5	0.033	–	2×10^{-22}
	HP Lib	100	1.79	0.6	0.03	–	2×10^{-22}
	CR Boo	100	1.36	0.6	0.02	–	1×10^{-22}
	V803 Cen	100	1.24	0.6	0.02	–	1×10^{-22}
	CP Eri	200	1.16	0.6	0.02	–	4×10^{-23}
	GP Com	200	0.72	0.5	0.02	–	3×10^{-23}
LMXB	4U1820-30	8100	3.0	1.4	< 0.1	–	2×10^{-23}
	4U1626-67	3-8000	0.79	1.4	< 0.03	–	6×10^{-24}
W UMa	CC Com	90	0.105	0.7	0.7	–	6×10^{-22}

The population of Galactic binary stars in LISA frequency band ($\gtrsim 10^{-4}$ Hz $\Rightarrow P_{orb} < 5.5$ hours)

Mostly double degenerates - created by spiral-in at $P_{orb} < 10h$ $\quad \dot{N} \simeq \frac{1}{20\text{-}100\,y}$

Then evolve by gravitational radiation losses

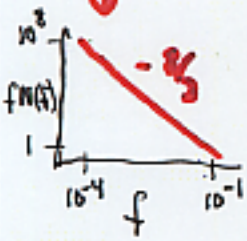
$$t_{decay} \sim 6\times10^{5}\,y \left(\frac{f}{3\times10^{-3}\,\text{Hz}}\right)^{-8/3} \qquad f = \frac{2}{P_{orb}} \text{ quadrupo}$$

All in galaxy detectable in h

but LISA frequency resolution $\sim \frac{1}{T}$ ← mission duration 1-10y

GW

$\frac{d}{df}(\dot{f}N(f)) = 0$



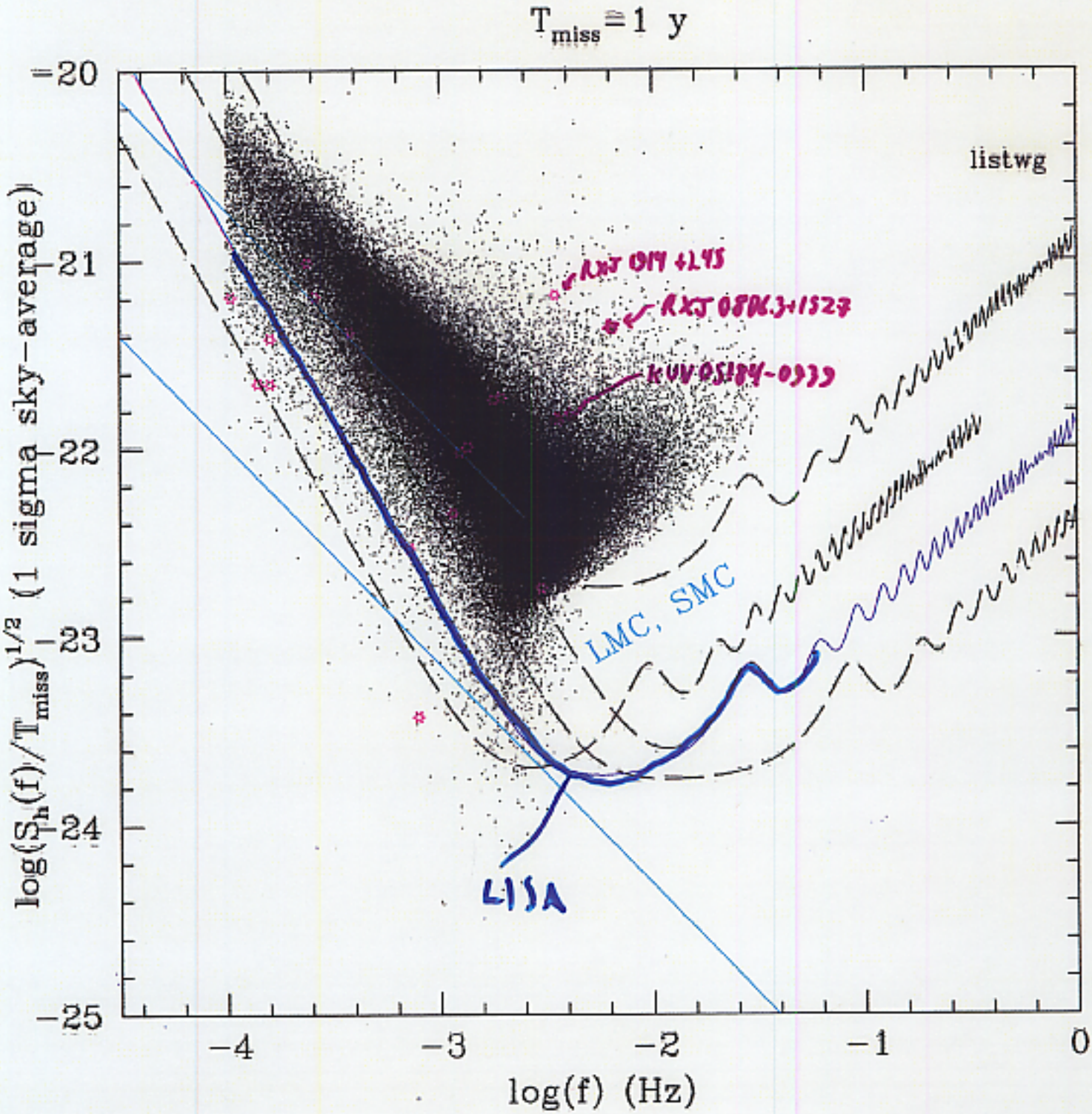
$\Rightarrow$ Confusion (>1 binary per frequency bin) if

$$\underbrace{N(>f)}_{\#f\to 2f} \simeq \dot{N}\, t_{decay} > \underbrace{fT}_{\#\text{bins } f\to 2f} \quad \Leftrightarrow \quad \frac{dN}{df}\cdot\overbrace{\frac{1}{T}}^{\Delta f} > 1$$

$\Rightarrow$ no confusion for

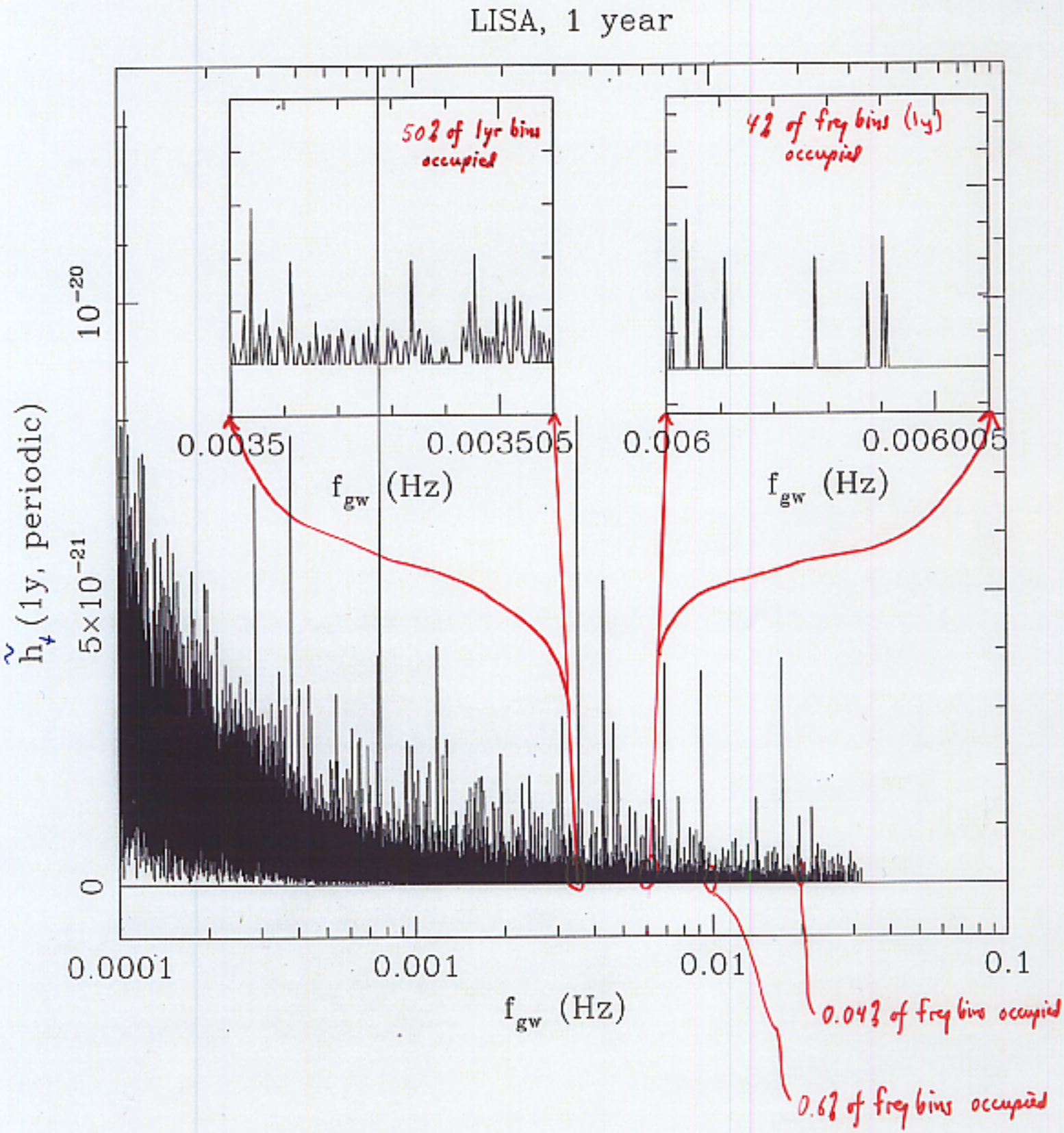
$$f \gtrsim f_{conf} \sim 2\,10^{-3}\,(T/\text{year})^{-\frac{3}{11}}\left(\frac{\dot{N}}{1/30\,y}\right)^{3/11}\ \text{Hz}$$

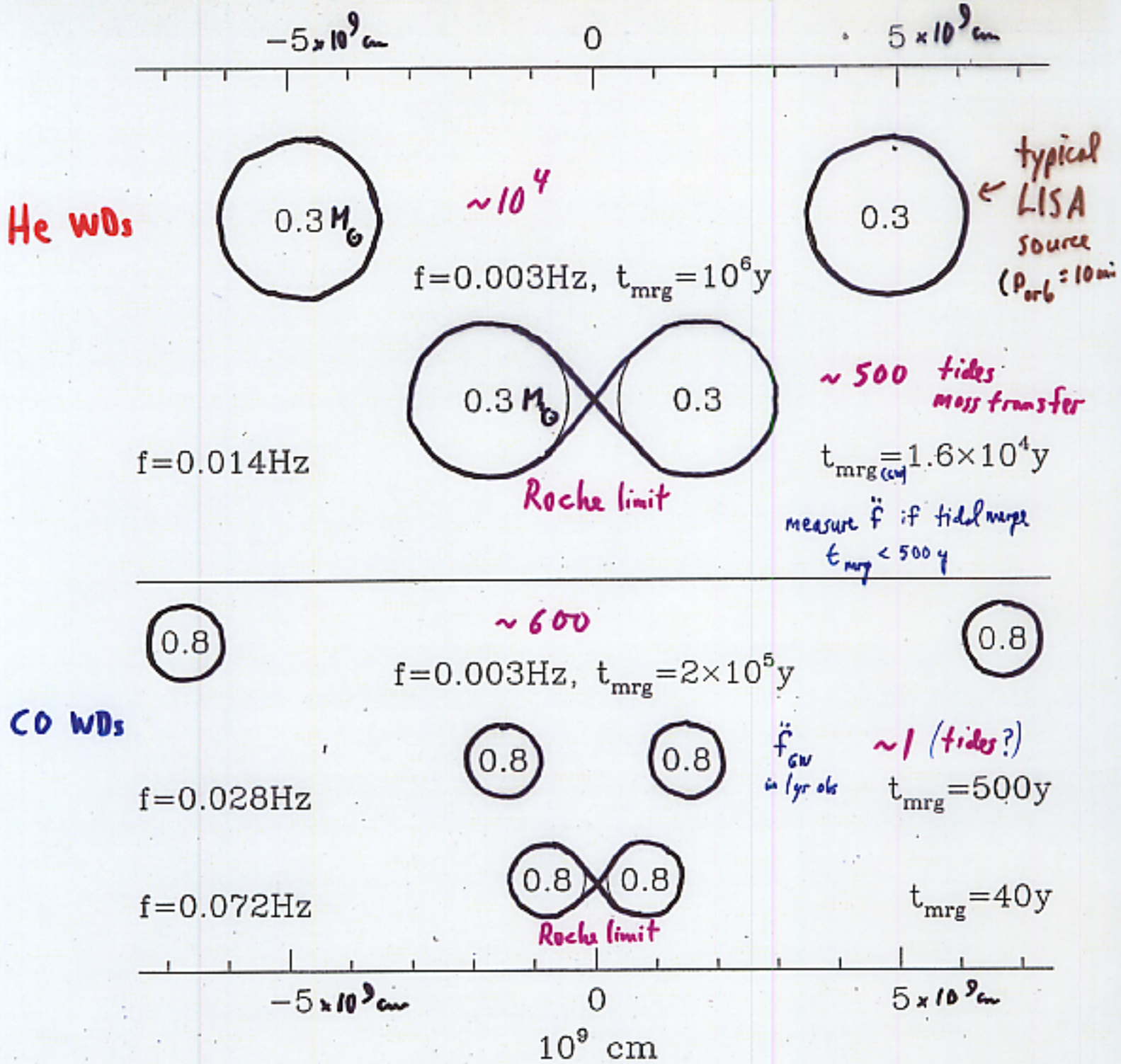
for $f < f_{conf}$ - only nearby strong sources above the din identifiable



$T_{miss} = 1$ y
listwg
RXJ 1914+245
RXJ 0806.3+1527
KUV 05184-0939
LMC, SMC
LISA
$\log(S_h(f)/T_{miss})^{1/2}$ (1 sigma sky-average)
log(f) (Hz)
−20
−21
−22
−23
−24
−25
−4
−3
−2
−1
0

FFT of LISA signal (+pol^n) - Galactic binaries only





LISA can measure $\ddot{f}_{GW}$ for $f > 0.027$ Hz - is for a couple of CO WDs approaching Roche limit - test for tidal heating

- if pure GW loss $\ddot{f} f \dot{f}^{-2} = 5/3$

Dominant Galactic LISA sources

IN - **Double degenerates** (WD+WD) dominate power

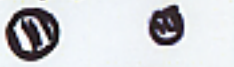
from Double Common envelope evolution of wide binaries (30-1000 $R_\odot$)

2-4 $M_\odot$ primaries dom, but 1-10 contribute

and Roche lobe overflow in Hertzsprung gap ($\lesssim 10 R_\odot$) followed by common envelope in 2nd mass transfer.

Mix of He-He, He-CO, CO-CO (dominate $L_{galaxy,gw}$)
CO-ONe, ONe-ONe less important, rare

5 mHz, $\tau \sim 10^5$ y
$h(1\,kpc) \sim 4\,10^{-22}$



0.4 0.6

Chirp masses $\mathcal{M}$ up to $\sim 1 M_\odot$
$\mathcal{M} = 0.3 - 0.5\, M_\odot$ typical

OUT - **AM CVn** (WD + Roche filling WD He) dominate numbers

Roche lobe filling
- backing out if conservative (J) transfer ??

0.12 0.6

direct impact
tides in small disks

5 mHz
$h(1\,kpc) \sim 10^{-22}$
$\tau_{GW} \sim 4\,10^5$ y
$\tau \sim ?$

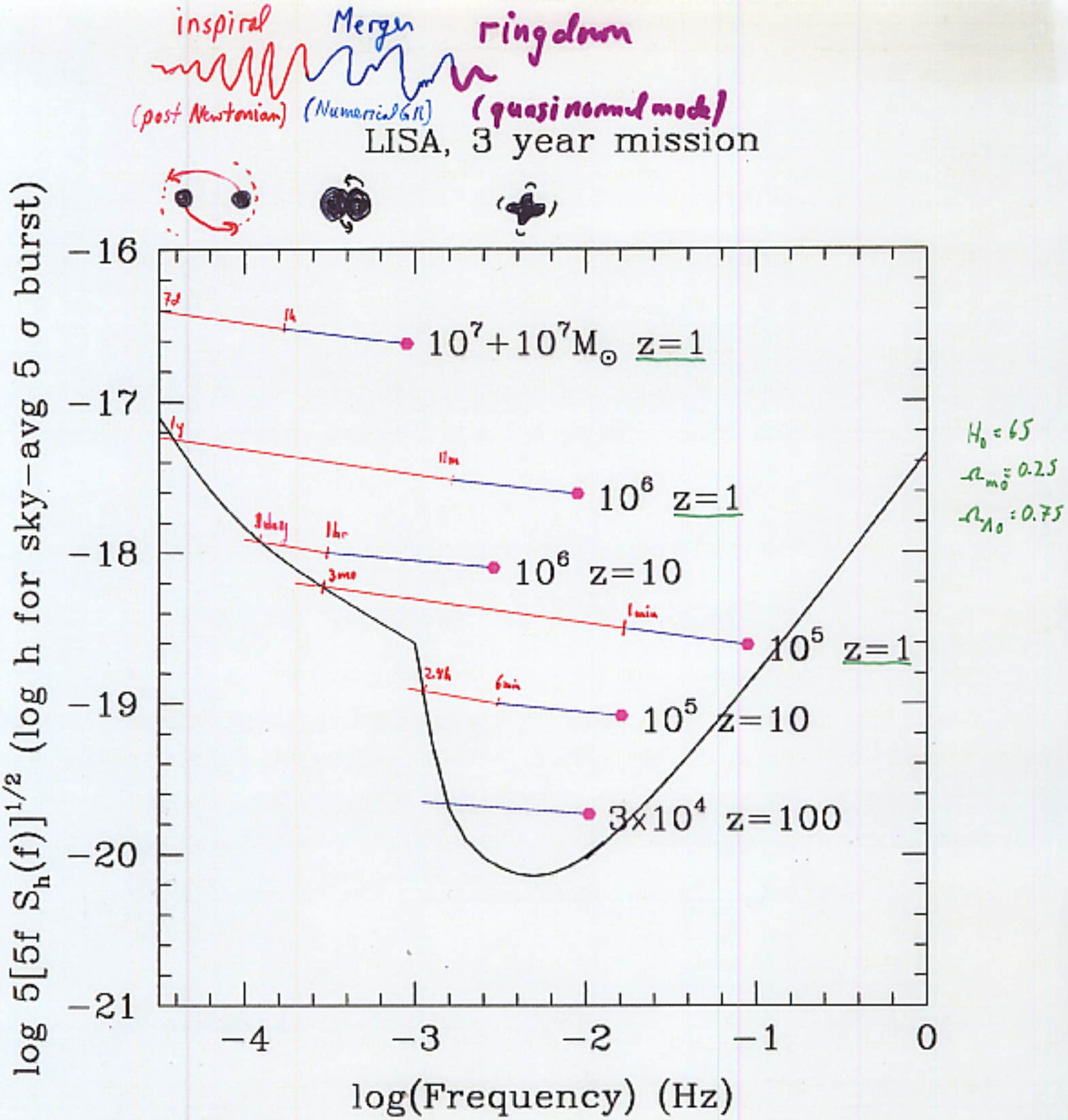
$h \propto m$

$L_{gw} \propto m^2$

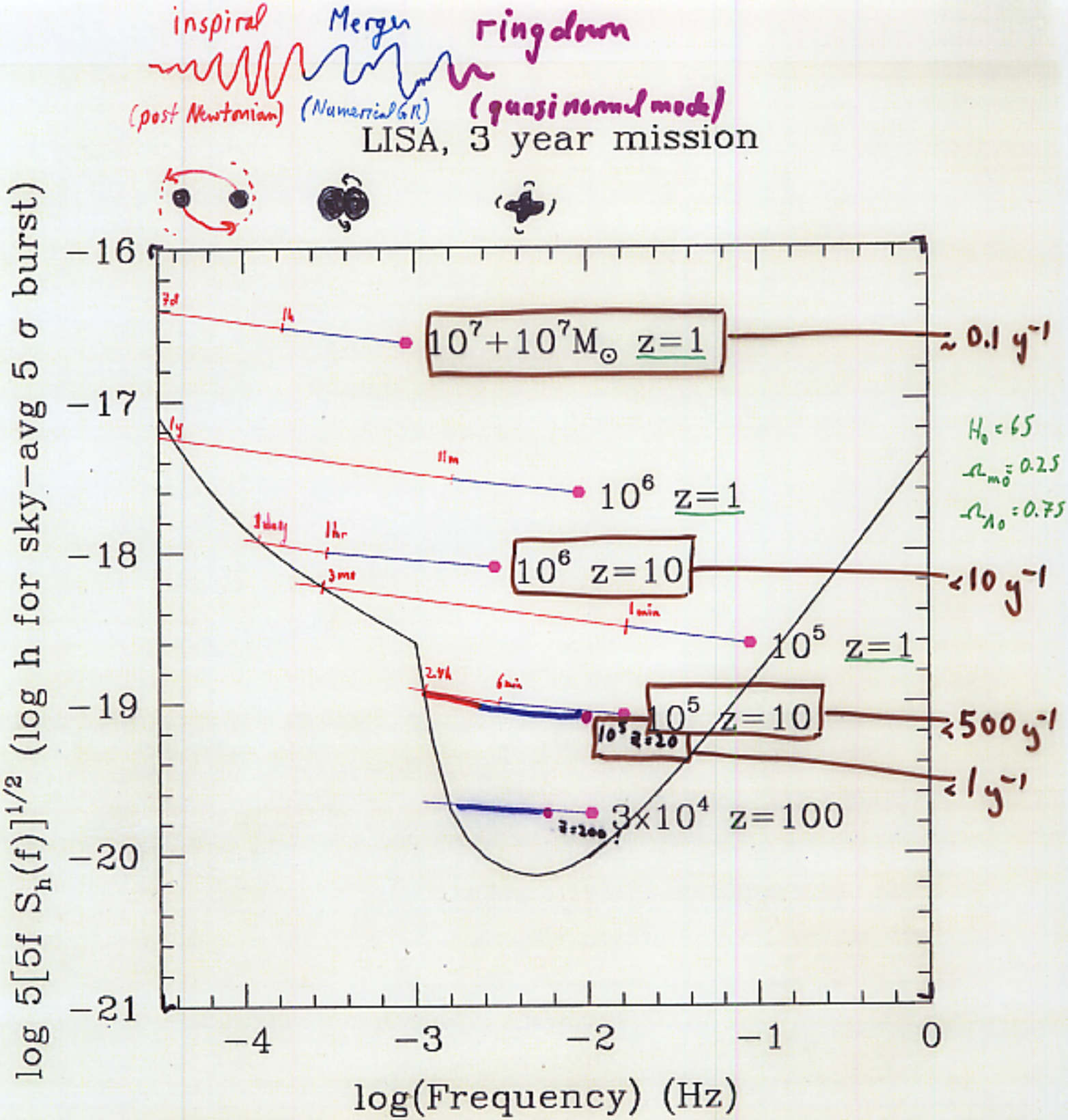
$\tau_{gw} \propto m^{-1}$

if back out on τ_{GW} $\Rightarrow$ $N(f)_{out} \lesssim \underbrace{\frac{m_{in}}{m_{out}}}_{\sim 4} N(f)_{in}$

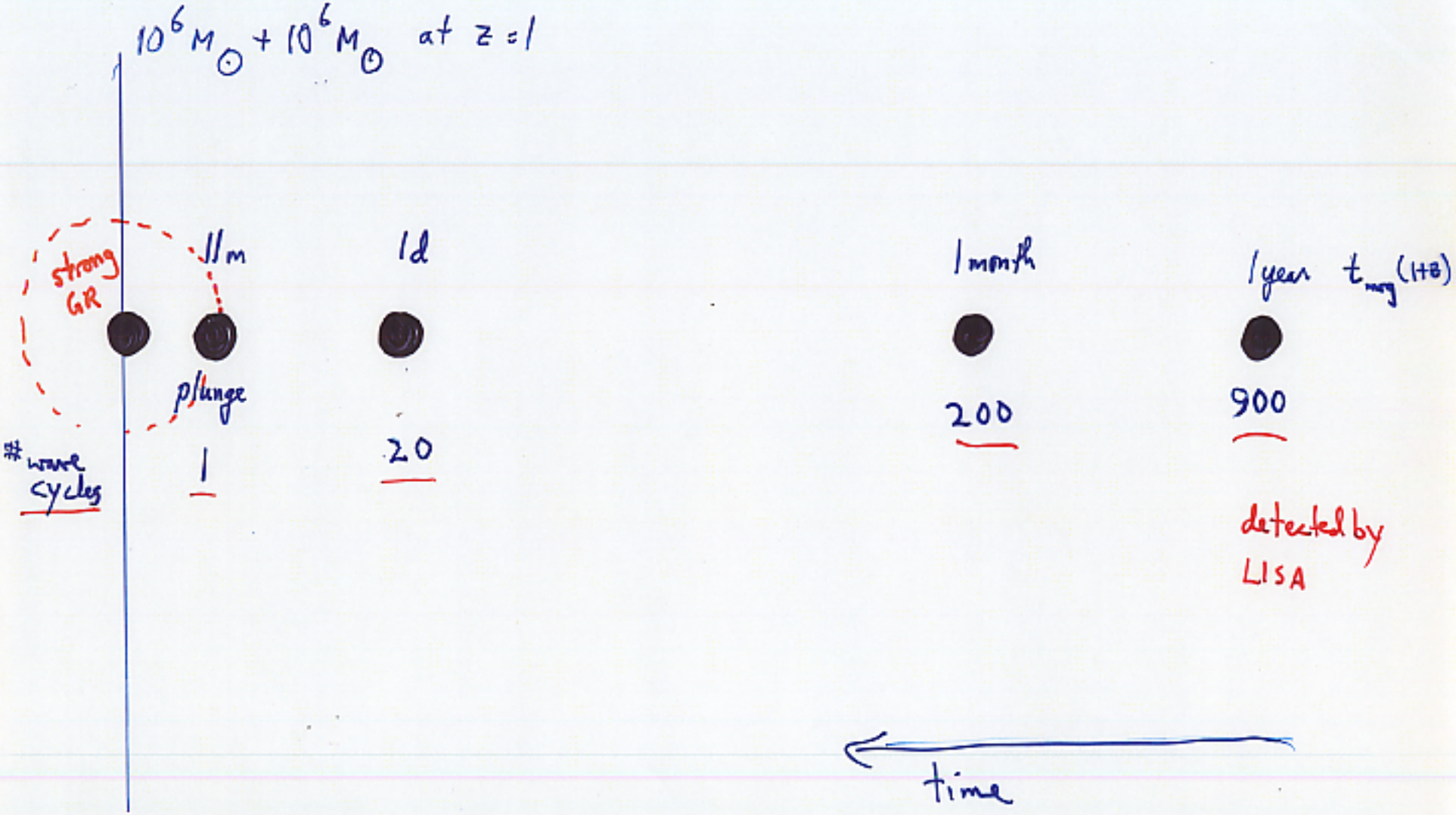
power at f $= \underbrace{\frac{M_{out}}{M_{in}}}_{\sim 0.25} \times$ (DD Power)



h_c plotted - source height above 5σ noise curve × 5 = $\frac{\text{sky-avg SNR}}{d\ln f}$

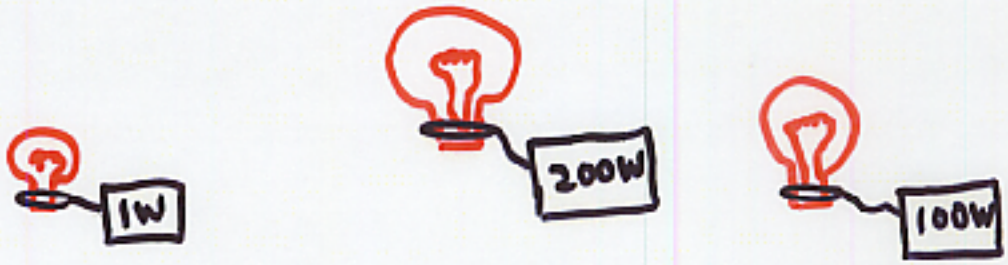


h_c plotted - source height above 5σ noise curve × 5 = $\frac{\text{sky-avg SNR}}{d\ln f}$


$10^6 M_\odot + 10^6 M_\odot$ at $z=1$
strong GR
11m
1d
1month
1year
$t_{merg}(1+z)$
plunge
wave cycles
1
20
200
900
detected by LISA
time

Gravitational waves from binaries:

The self-calibrating standard candles



Measurable gravitational wave amplitudes

$$(1) \qquad \begin{matrix} h_+ \\ h_\times \end{matrix} = \left\{ \begin{matrix} \frac{1+\cos^2 i}{2} \\ 4\cos i \end{matrix} \right\} \frac{\mathcal{M}_z}{D_L} \left(\pi \mathcal{M} f_r\right)^{2/3} \cos\left(\int 2\pi f(t)\,dt + \psi_0\right)$$

f_r: freq at earth

D_L = luminosity distance

$\mathcal{M}_z = (1+z)\left(\frac{M_1^3 M_2^3}{M_1+M_2}\right)^{1/5}$ chirp mass

Measurable frequency derivative due to orbit decay by gravitational wave loss:

$$(2) \qquad \frac{df}{dt} = \frac{96}{5\pi} \frac{(\pi \mathcal{M}_z f_r)^{11/3}}{\mathcal{M}_z^2}$$

→ solve for $\mathcal{M}$

→ insert in (1); get $D_L \times Q(i)$

→ $h_+/h_\times$ → i → $\boxed{\text{Get } D_L.}$ (in cm)

Hughes MNRAS 331, 805 (2002)

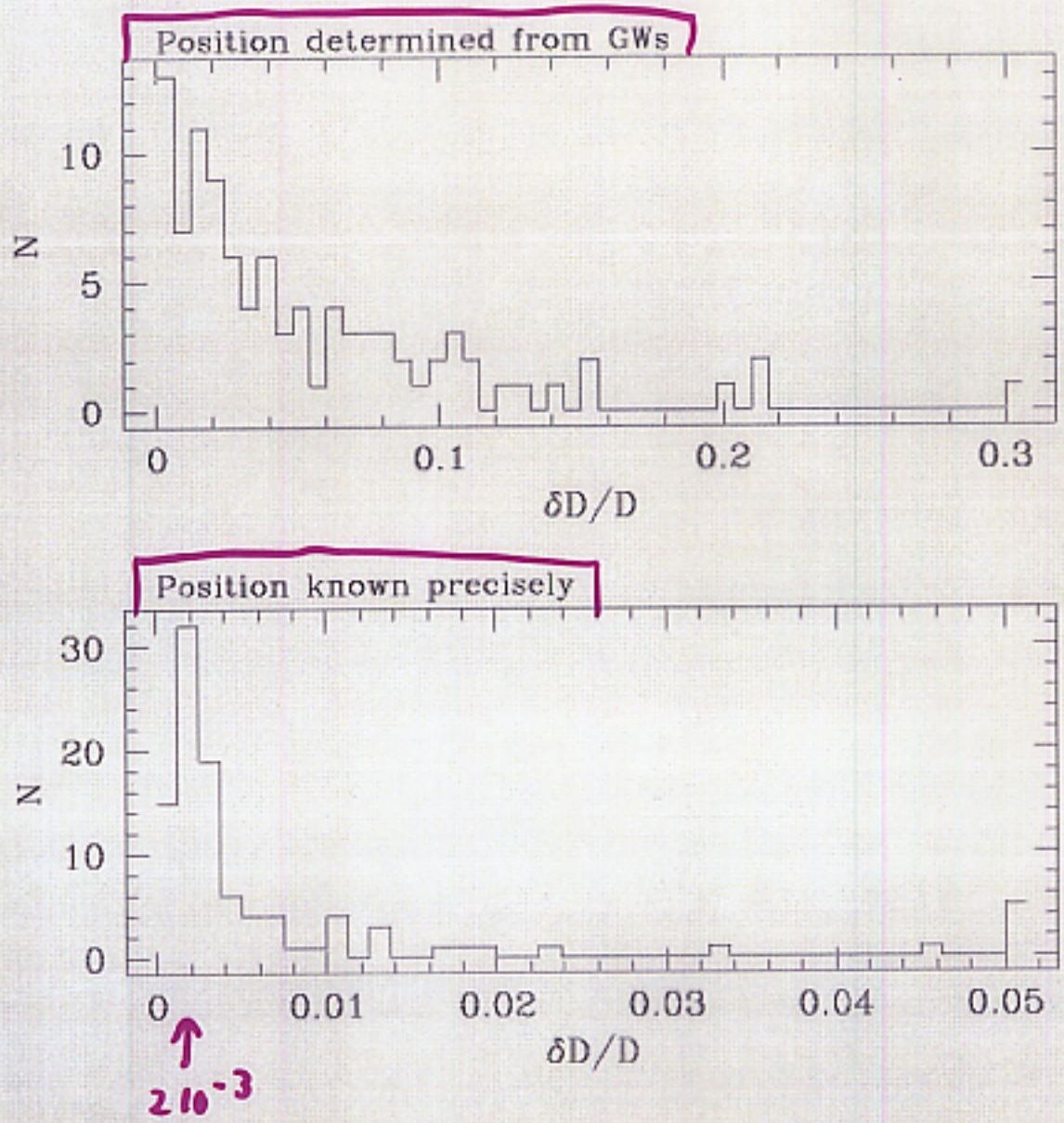


Figure 3. Comparison of measurement errors in luminosity distance for *LISA* measurement of a binary with $m_1 = m_2 = 10^5\ M_\odot$ at $z = 1$. The top panel shows the error distribution when the sky position is determined using gravitational waves; the bottom panel shows the distribution assuming the sky position is measured with no error. Because of strong correlations between the luminosity distance and the sky position angles, improving the accuracy with which the position is measured has a big impact on the distance determination. In this case, D is measured with roughly an order of magnitude less error.

If electromagnetic signal accompanies gravitational wave signal ($L_{EM} = 10^{-20} L_{GW}$ → 20th mag!)

- to find in <1° error circle with 6 months notice of date!

→ precise position

→ redshift

⇒ $D_L(z)$ with individual measurement errors of few $\times 10^{-3}$ for $z = 0-10$.

1–100 sources in 3 yr operation

Independent of and possibly more precise than other proposed methods (e.g. SNAP).

w, w' ?

precision limited by gravitational lensing
⇒ line of sight variations in D_L (8 level)

Compact Stars captured from stellar cusps around nuclear black holes:
(WD, NS, BH, He stars)

rate ~ $\frac{1}{5\,10^5\,y}$ Milky Way

⇒ LISA see ~ 2/year at S/N > 50 volume

2 stars pass close
- deflect one to pass close to BH
- if $\left.\frac{a}{\dot{a}}\right|_{GW}$ < time to scatter to larger pericenter

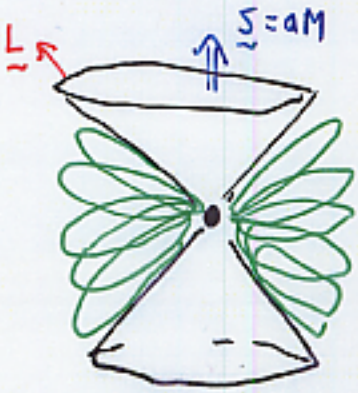
then star will be captured

orbit grind down by gravitational radiation

- still eccentric when period enters LISA band ⇒ many harmonics of $1/P_{orb}$
- Generally not in plane ⊥ BH spin axis ⇒ plane precesses
- have pericenter < $8\frac{GM}{c^2}$; inspiral lifetime < 10y
- Number of orbits required to disappear into horizon $\gtrsim \frac{M_\bullet}{m_*}$ $\gtrsim 10^5$!!

Kepler

Schwarzschild



Kerr

Lense-Thirring precession of orbit plane

Read off the moments of expansion of spacetime in tensor spherical harmonics. All moments should be determined by first two (M, S)
test of No hair theorem!

Cusps of stars around supermassive black holes

Black hole dominates dynamics for

$$r < r_{gr} \equiv \frac{G M_\bullet}{\sigma_*^2} \simeq 10.8\, M_8^{1/2} \text{ pc}$$

use $\sigma_* \simeq 200 \text{ km s}^{-1}\, M_8^{1/4}$ $\qquad M_8 = \frac{M_\bullet}{10^8 M_\odot}$

<u>Observe</u>:

in massive galaxies (with $M_\bullet \geq 10^8 M_\odot$).

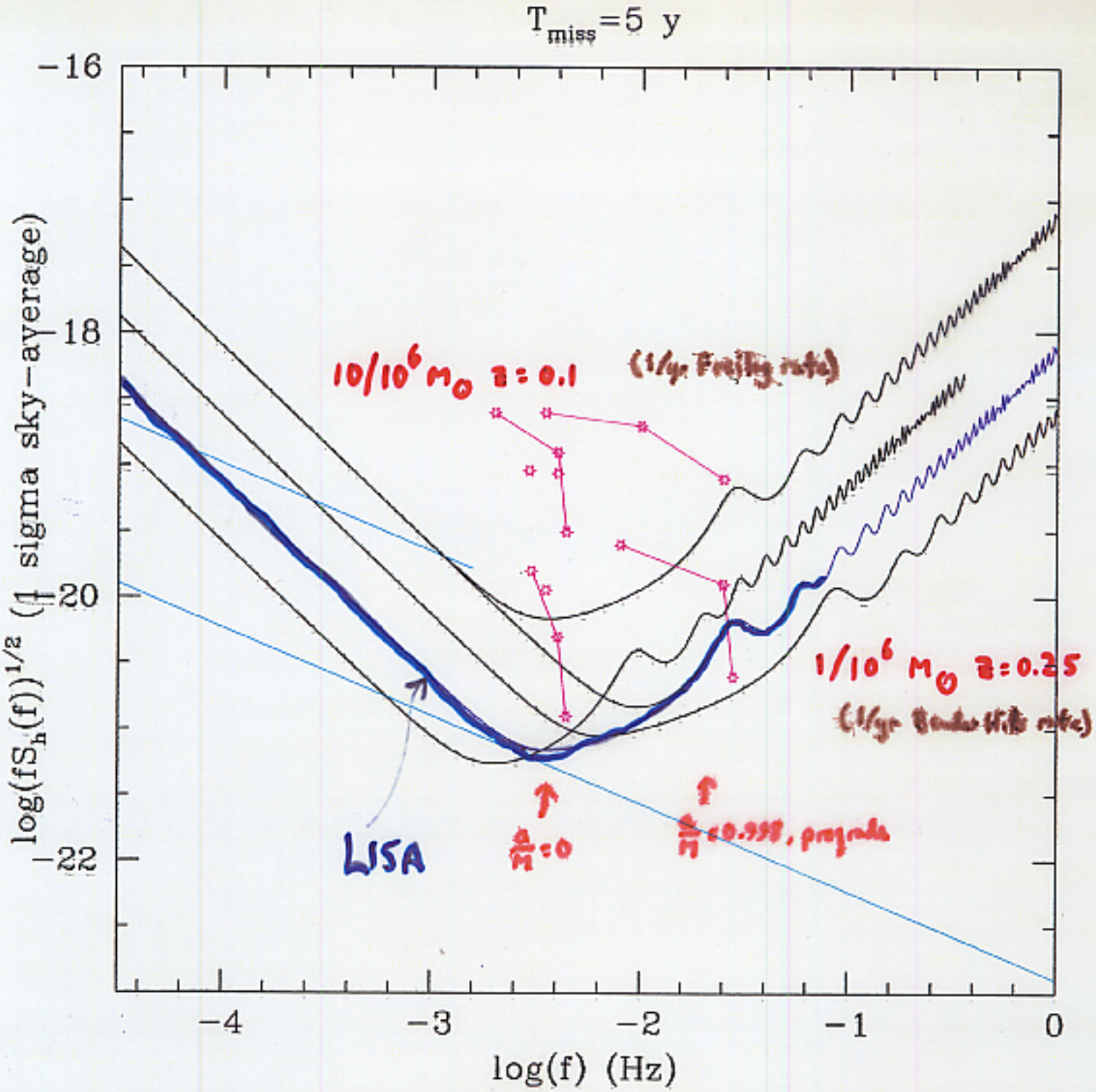
cusp with $\rho_* \underset{\sim}{\propto} r^{-1}$ for $r < r_{gr}$, steeper beyond

cf M87 "core"

in low mass galaxies (with $M_\bullet \lesssim 10^8 M_\odot$)

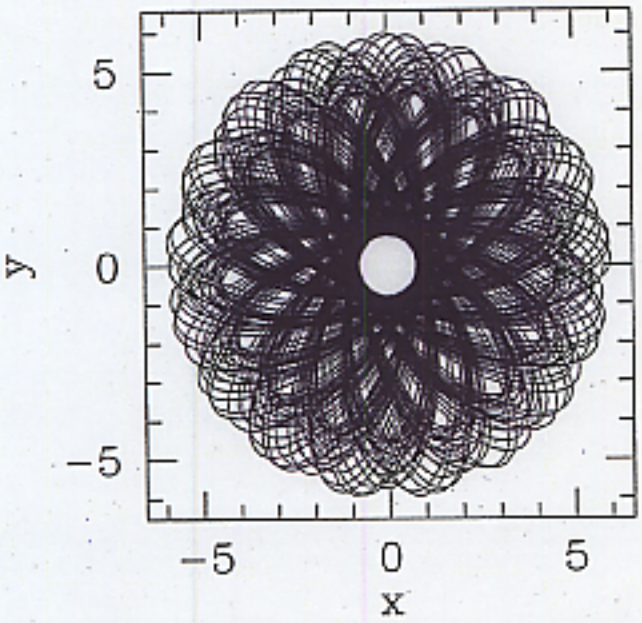
cusps with $\rho_* \underset{\sim}{\propto} r^{-2}$ for $r < r_{gr}$, similar or steeper beyond

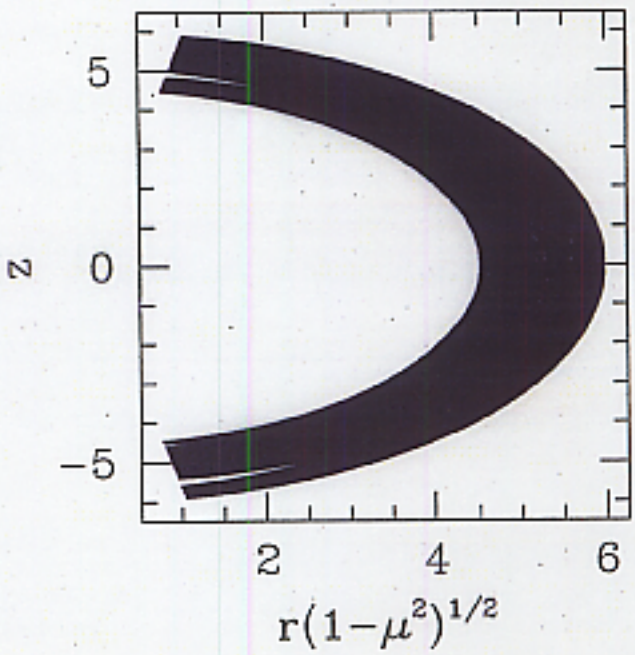
cf Milky Way M32 "powerlaw"

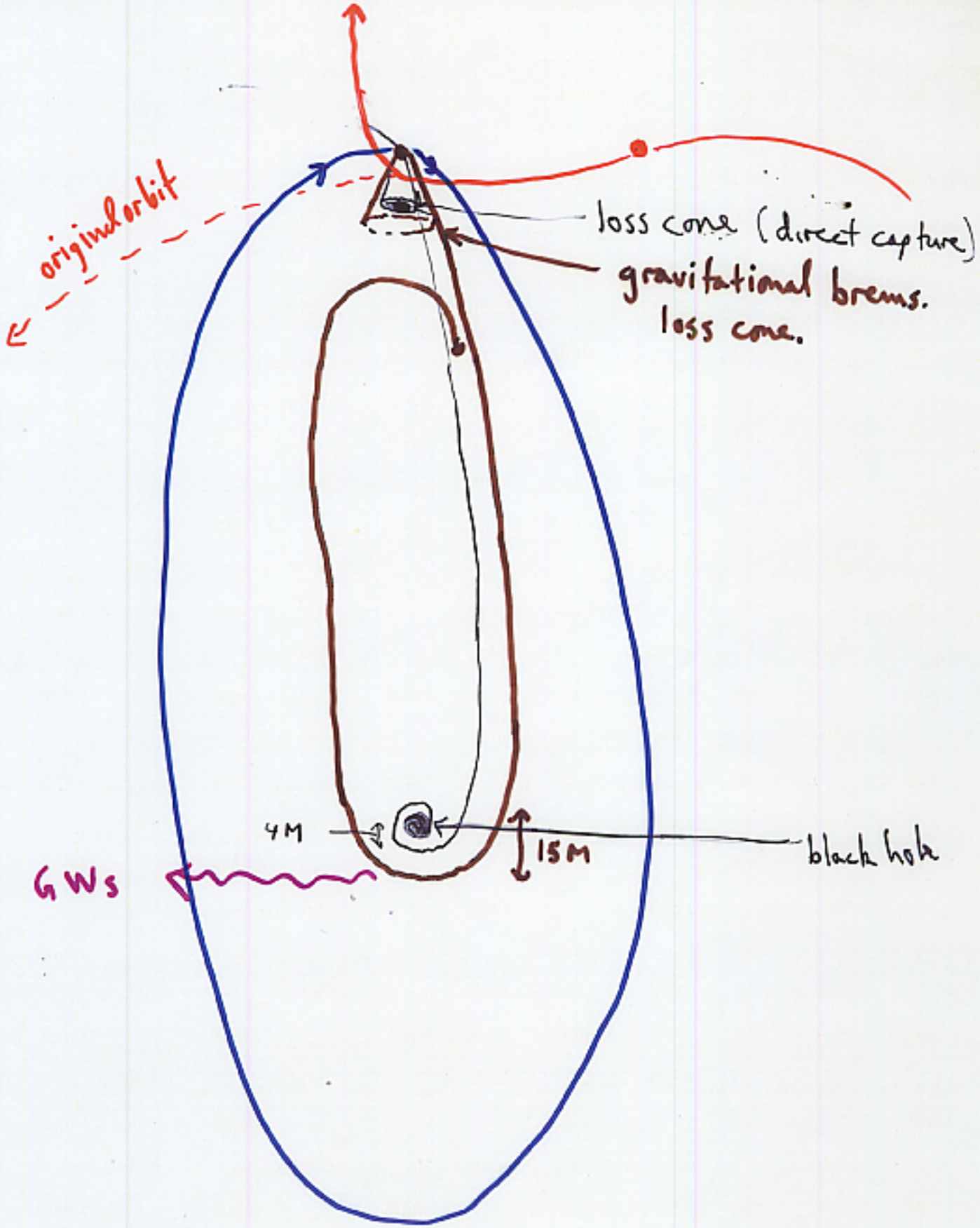


Points: 1 yr
1 month
1 day
before end.

$\frac{a}{M} = 0.95 \quad r_a = 6 \quad e = 0.2 \quad i = 80°$






original orbit
loss cone (direct capture)
gravitational brems. loss cone.
4M
15M
black hole
GWs

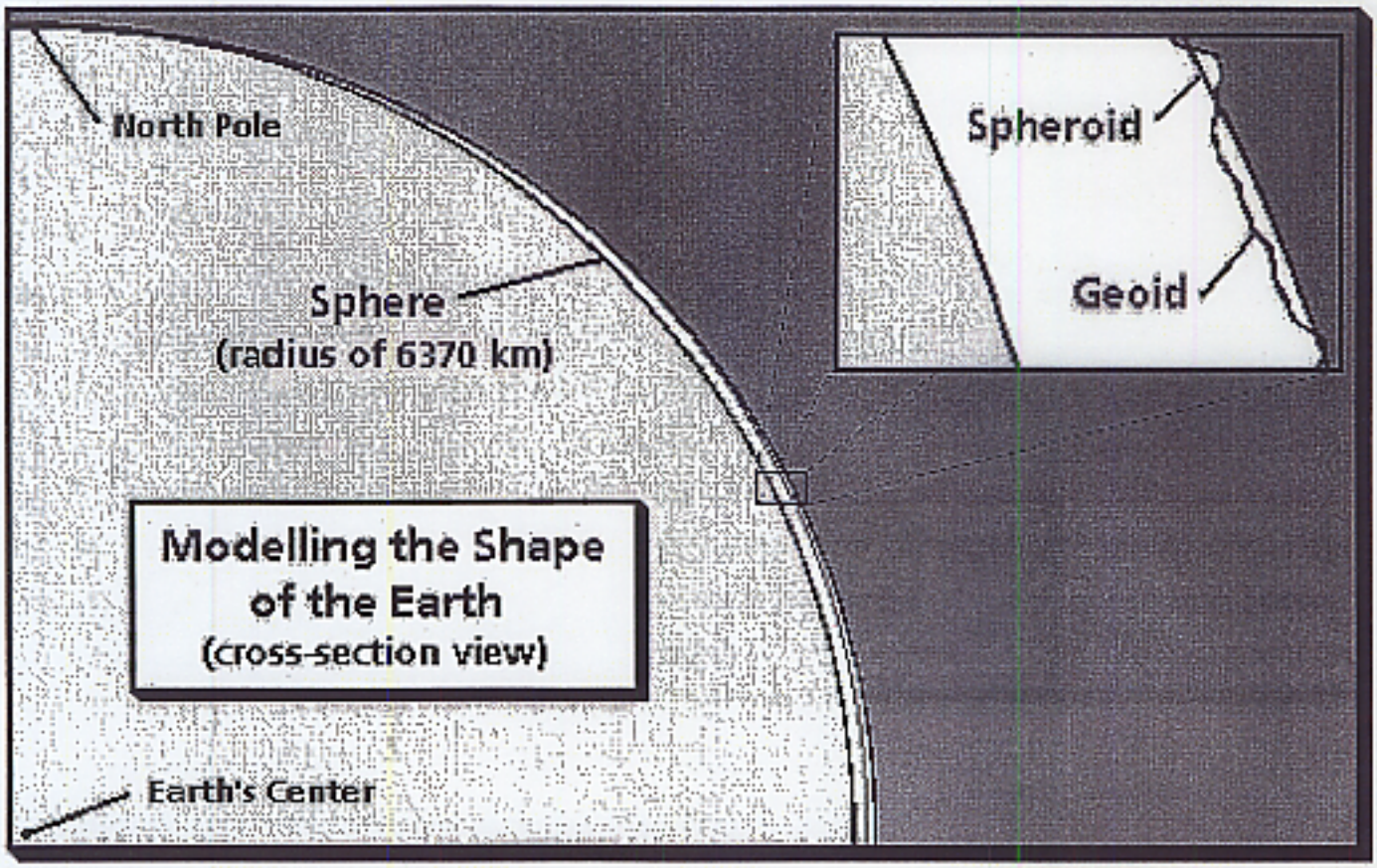
Gravitational radiative capture if

$$\left.\frac{a}{\dot{a}}\right|_{GW} < \text{time to scatter to larger pericenter}$$

$$\uparrow$$

$$\propto r_p^{3.5}\, a^{0.5}$$

Geodesy



Measuring the geoid:

Outside earth, $\nabla^2 \phi = 0$ ⇒ vacuum spherical harmonics.

$$\underset{\sim}{g}(\underset{\sim}{r}) = -\nabla \phi$$

$$\phi(\underset{\sim}{r}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \boxed{B_{\ell m}} \underbrace{Y_{\ell m}(\theta,\varphi)}_{r^{-(\ell+1)}}$$

track satellite w/ drag-free test mass (CHAMP, GRACE, GOCE..)

$$\frac{d^2 \underset{\sim}{r}}{dt^2} = \underset{\sim}{g}(\underset{\sim}{r})$$

Solve for $B_{\ell m}$

diagnostic of interior structure by matching to interior sol$^{\underline{n}}$ $\nabla^2 \phi = 4\pi \rho(\underset{\sim}{r})$

Bothrodesy

in GR: Mass, current multipoles (Geroch, Hansen)

assume axisymmetry $B_{\ell m} = B_\ell$

$$M_\ell = \int d^3r\, r^\ell \rho P_\ell(\cos\theta) \qquad = MR^\ell J_\ell \text{ in geophysics}$$

dimensionless

$$S_\ell = \int d^3r\, r^{\ell-1} (\underset{\sim}{r} \times \underset{\sim}{v} \rho) P_\ell$$

reflection symmetry about equatorial plane: $M_{3,5,7,9\ldots} = 0$

$S_{2,4,6\ldots} = 0$

[but of earth $J_3 = -0.254 \times 10^{-5}$ "pear shape"]

M_0 mass

M_2 mass quadrupole

$\vdots$

S_1 spin angular momentum

S_3 current octupole

$\vdots$

Uniform density sphere $\quad M_2 = \frac{25}{8}\left(\frac{R}{GM}\right) a^2 M \qquad a = \frac{J}{M}$

ang mom/mass

Black hole: no hair theorem $\quad M_\ell + i S_\ell = M(ia)^\ell$

i.e. $\quad M_2 = Ma^2$

Science return:

Mino 2003
Barack + Ori 2002

once we figure out how to compute radiation reaction force in general...

Precision measurements of strong field space-time, indep of state of 4-D numerical GR.

1. Is it a GR black hole? M, J, no hair
   vs cubical soliton stars held up by Higgs fields
2. Does gravitational radiation scatter superradiantly from the horizon ($J \neq 0$)
   Scott Hughes! huge effect on orbit
3. Distribution of $\frac{J}{M^2}$ → Astrophysics
   - accretion
   - Energy extraction
4. If see many events
   ~1% perturbed by accretion disks in AGN

   ~2-10%? white dwarfs, He stars tidally disrupted in late stages → electromagnetic signal (≃ supernova or GRB)
   identify galaxy, get z and D_M, m
   Cosmography, astrophysics