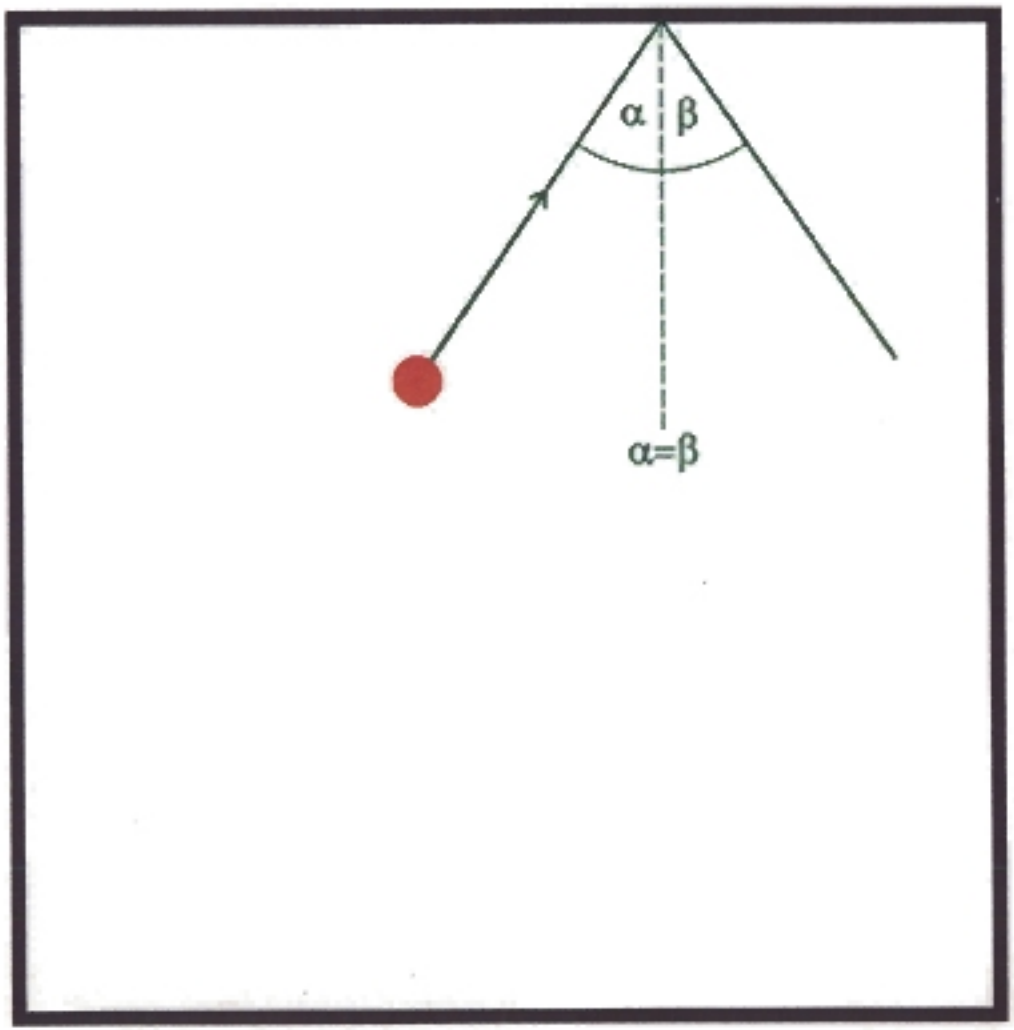
Playing Billiards with Microwaves: Quantum Manifestations of Classical Chaos

- Classical and quantum billiards
- Random Matrix Theory (Wigner 1951 – Dyson 1962)
- Spectral properties of billiards and nuclei
- Periodic Orbit Theory (Gutzwiller 1970)
- Generic and non-generic features of billiards and nuclei: the „scissors mode"
- Three-dimensional billiards
- Open billiard systems and wave functions

Supported by DFG within SFB 634

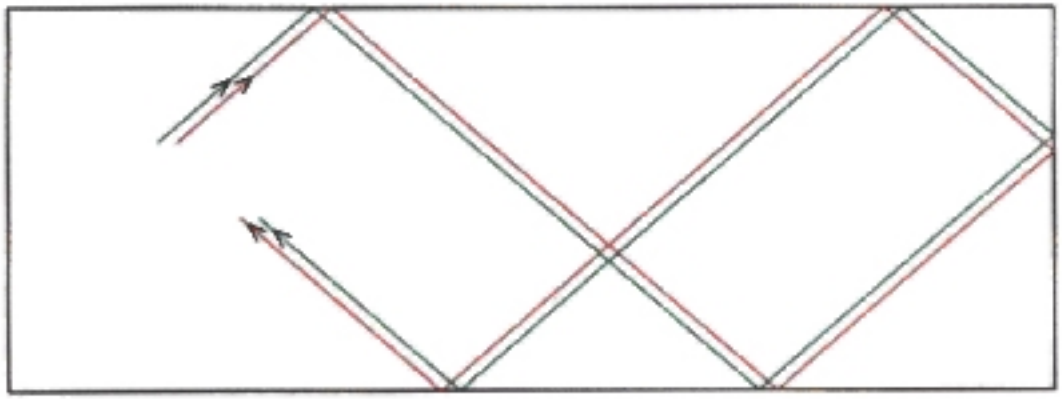
C. Dembowski, B. Dietz, T. Friedrich, H.-D. Gräf, T. Guhr, H.L. Harney, A. Heine, W.D. Heiss, M. Miski-Oglu, T. Papenbrock, A. R.

Classical Billiard

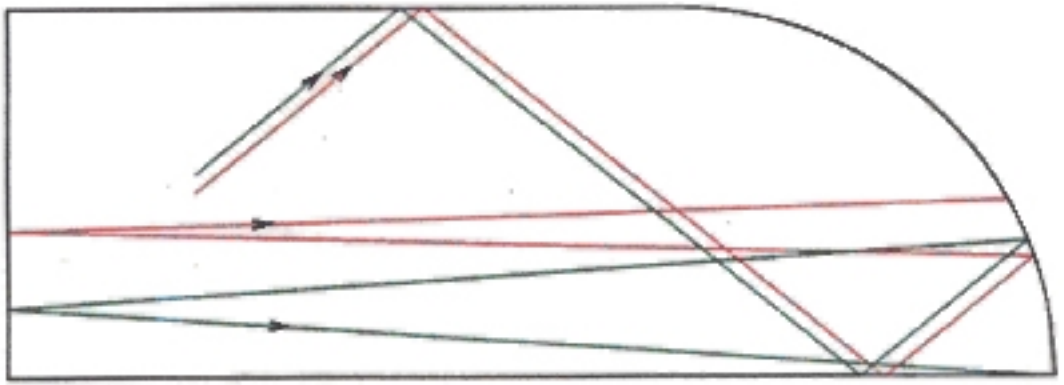


Regular and chaotic billiards

rectangular billiard (integrable)



Bunimovich stadium (chaotic)



Small changes → Large actions

- Sensitivity of the solutions of a deterministic problem with respect to small changes in the initial conditions is called Deterministic Chaos.

- Beyond a fixed, for the system characteristic time becomes every prediction impossible. The system behaves in such a way as if not determined by physical laws but randomness.

J.C. Maxwell (1873)

It is a metaphysical doctrine that from the same antecedents follow the same consequents. [...] But it is not of much use in a world like this, in which the same antecedents never again concur, and nothing ever happens twice. [...]

The physical axiom which has a somewhat similar aspect is „That from like antecedents follow like consequents." But here we have passed from sameness to likeness, from absolute accuracy to a more or less rough approximation. There are certain classes of phenomena [...], in which a small error in the data only introduces a small error in the result. [...] There are other classes of phenomena which are more complicated, and in which cases of instability may occur. [...] At these points, influences whose physical magnitude is too small to be taken account of by a finite being, may produce results of the greatest importance. [...]

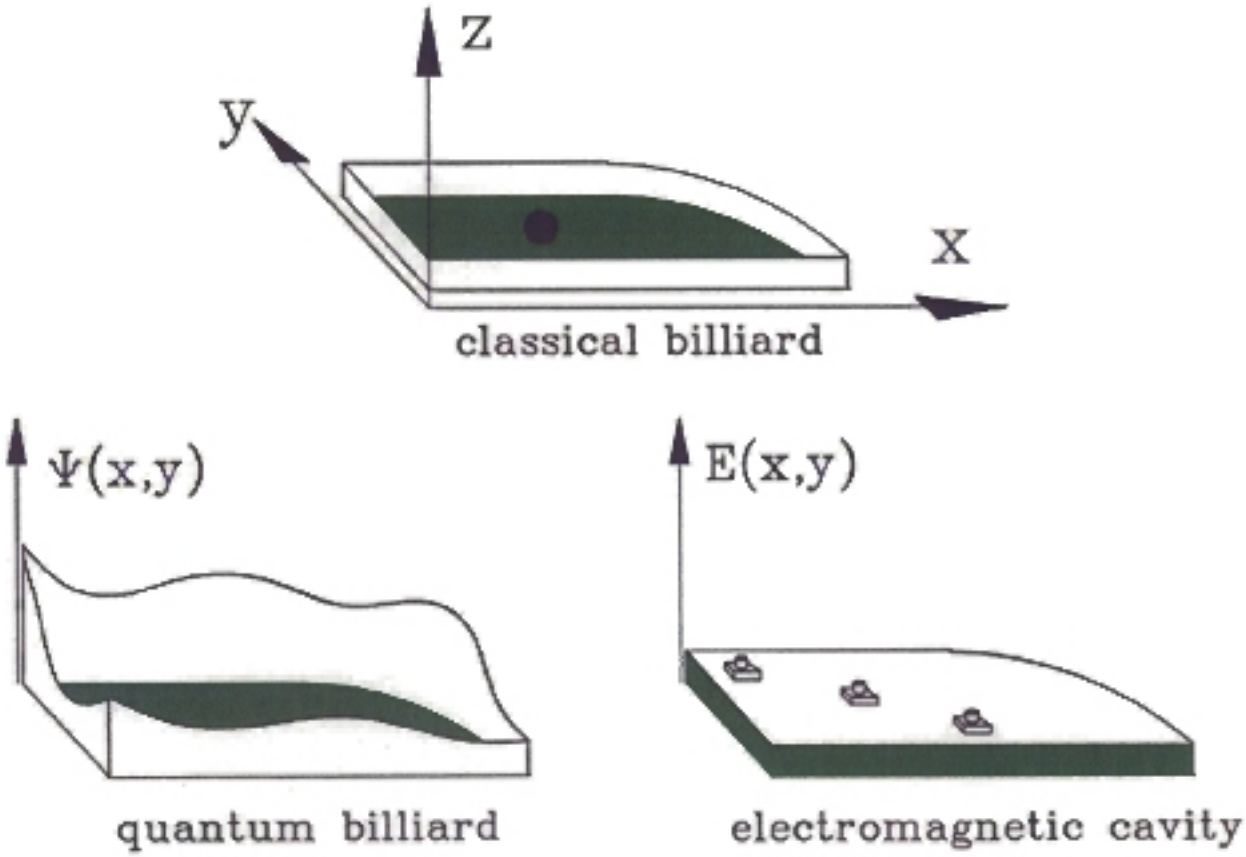
If, therefore, those [...] are led [...] to the study of singularities and instabilities, rather than continuities and stabilities of things, the promotion of natural knowledge may tend to remove that prejudice in favour of determinism which seems to arise from assuming that the physical science of the future is a mere magnified image of the past.

Our main interest

- How are these properties of classical systems transformed into corresponding quantum-mechanical systems?
  →Quantum chaos?

- What might we learn from generic features of billiards and nuclei?

The quantum billiard and its simulation



Schrödinger ↔ Helmholtz

quantum billiard

$$\left(\Delta + k^2\right)\Psi = 0$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

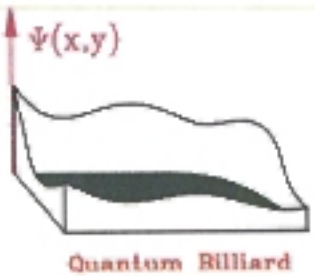
2D microwave cavity: $h_z < \lambda_{min}/2$

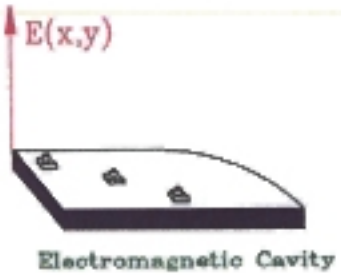
$$\left(\Delta + k^2\right)E_z = 0$$

$$k = \frac{2\pi f}{c}$$

Helmholtz equation and Schrödinger equation are equivalent in 2D. The motion of the quantum particle in its potential can be simulated by electromagnetic waves inside a two-dimensional microwave resonator.

Quantum billiards and microwave cavities



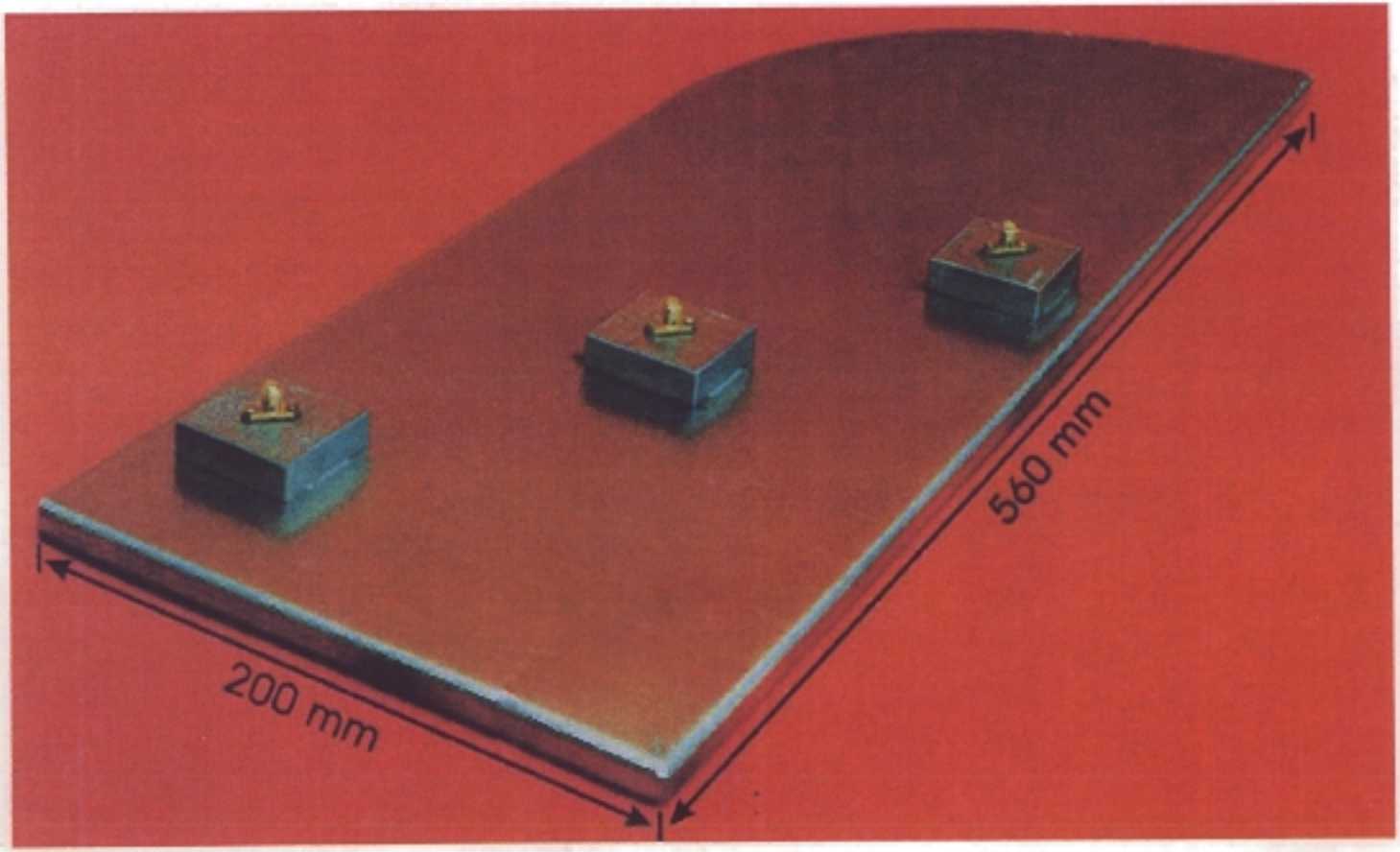


$$\left\{\begin{array}{l}\text{Re}\{\text{Eigenvalue}\}\\ \text{Im}\{\text{Eigenvalue}\}\end{array}\right\} \Leftrightarrow \left\{\begin{array}{c}\text{Resonance Frequency}\\ \text{Resonance Width}\end{array}\right\}$$

- Random Matrix Theory (RMT)
- Periodic Orbit Theory (POT)

$$\text{Eigenfunction} \Leftrightarrow \text{Field Distribution}$$

Superconducting Niobium microwave resonator



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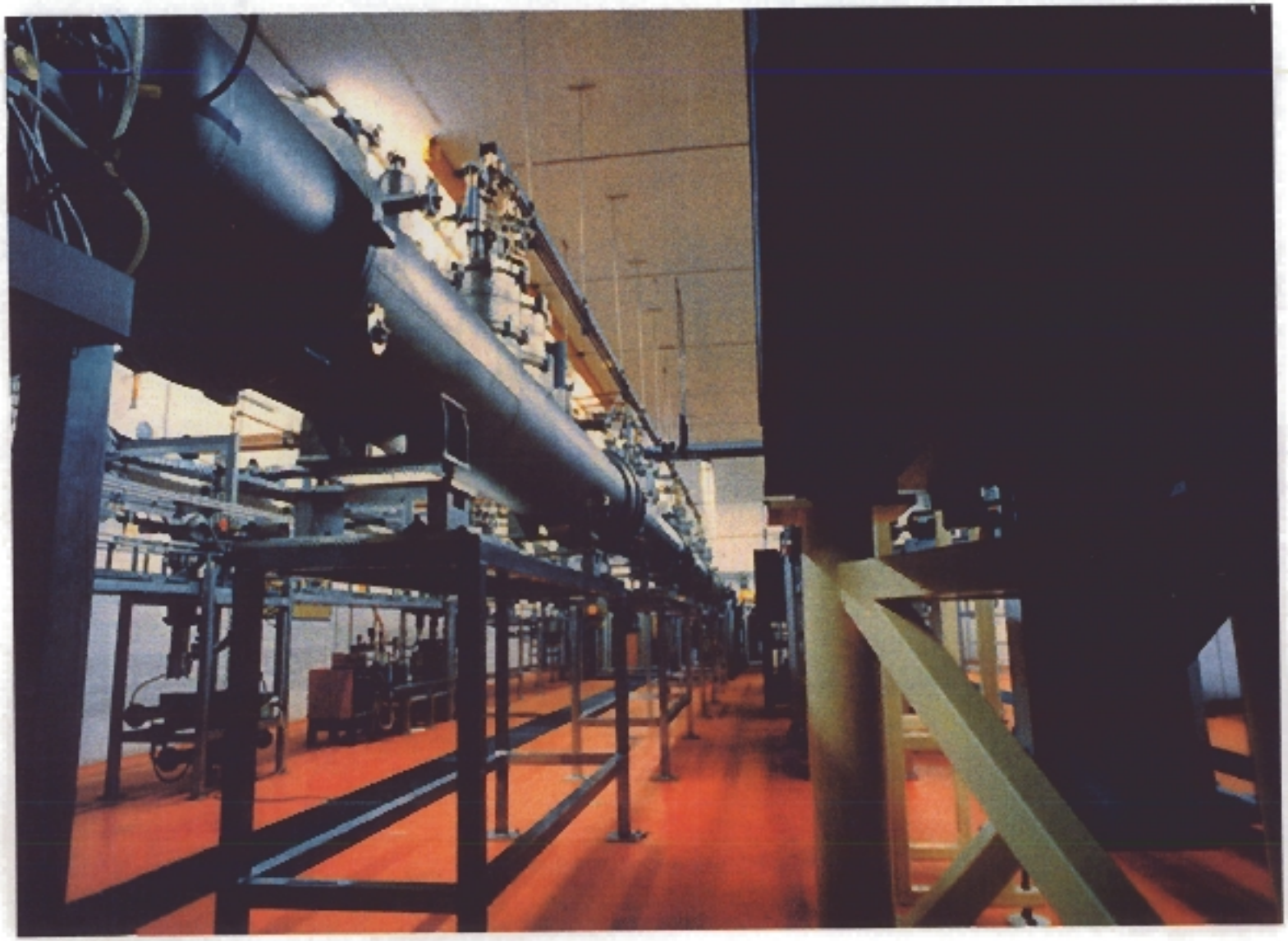
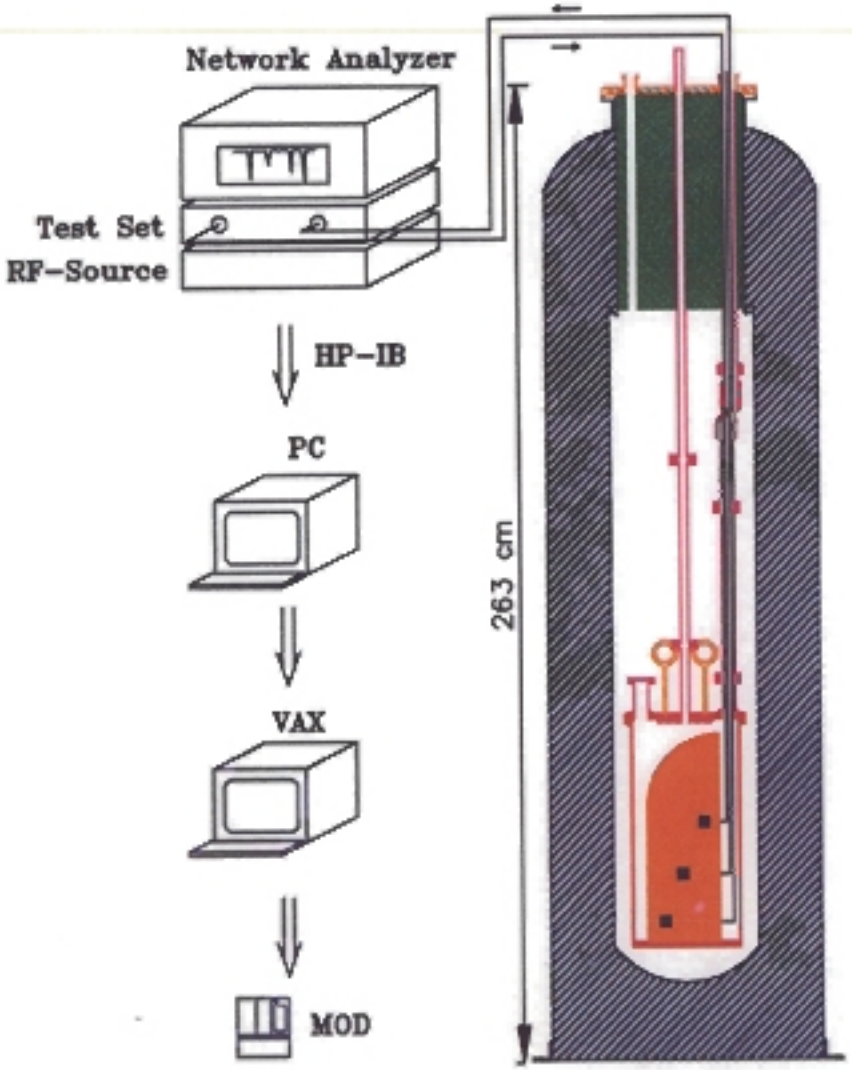
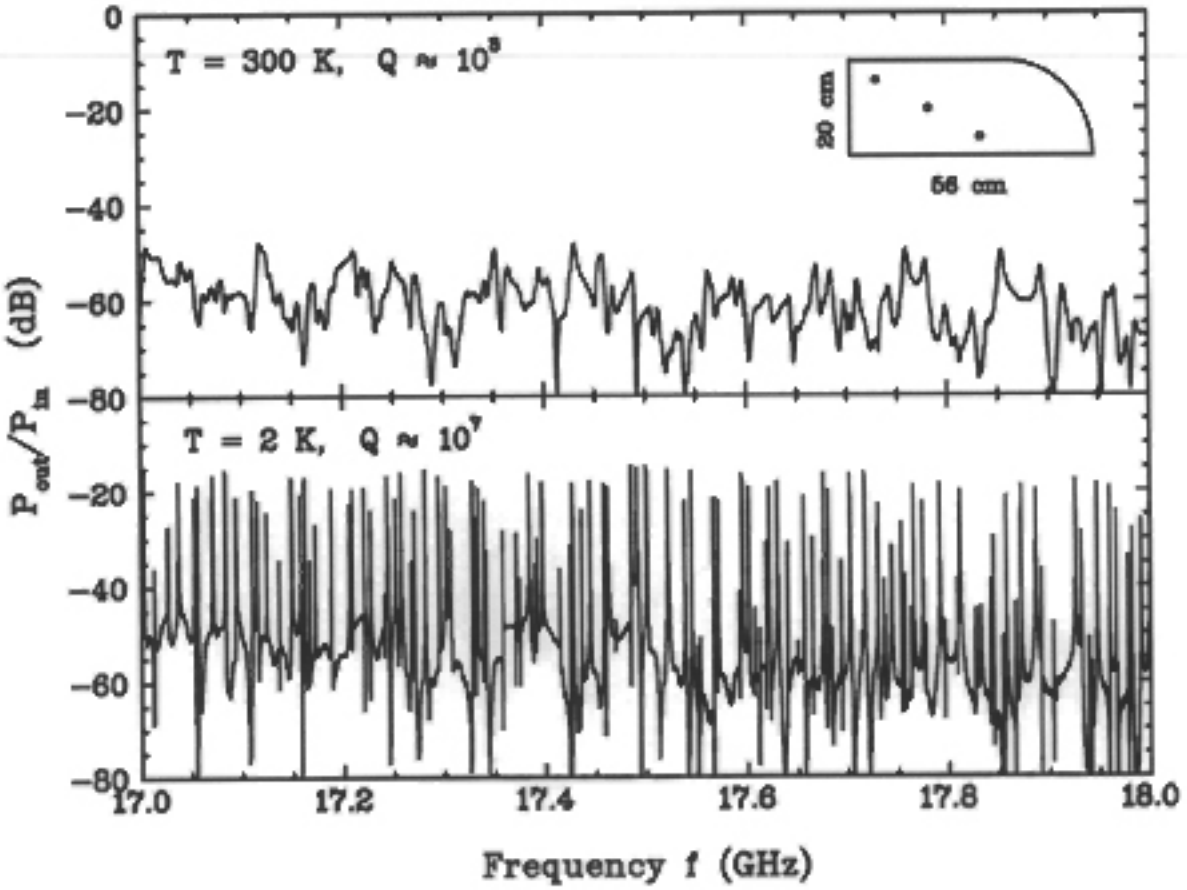


Experimental setup



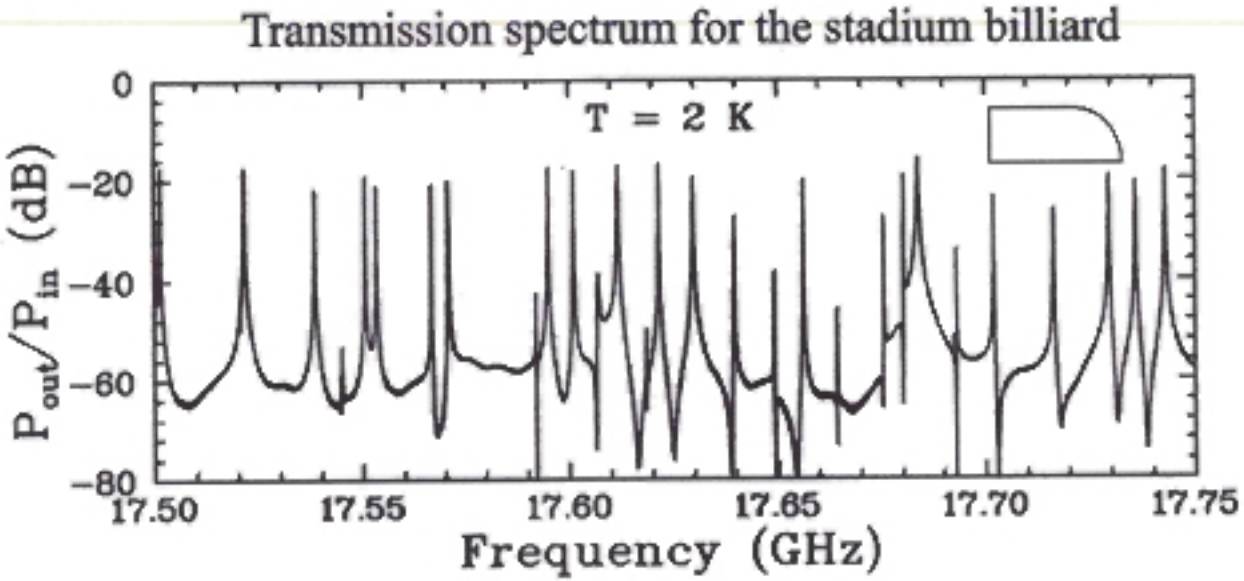
- Superconducting cavities
- LHe (T = 4.2 K)
- f = 45 MHz ... 50 GHz
- $10^3 ... 10^4$ eigenfrequencies
- $Q = f/\Delta f \approx 10^6$

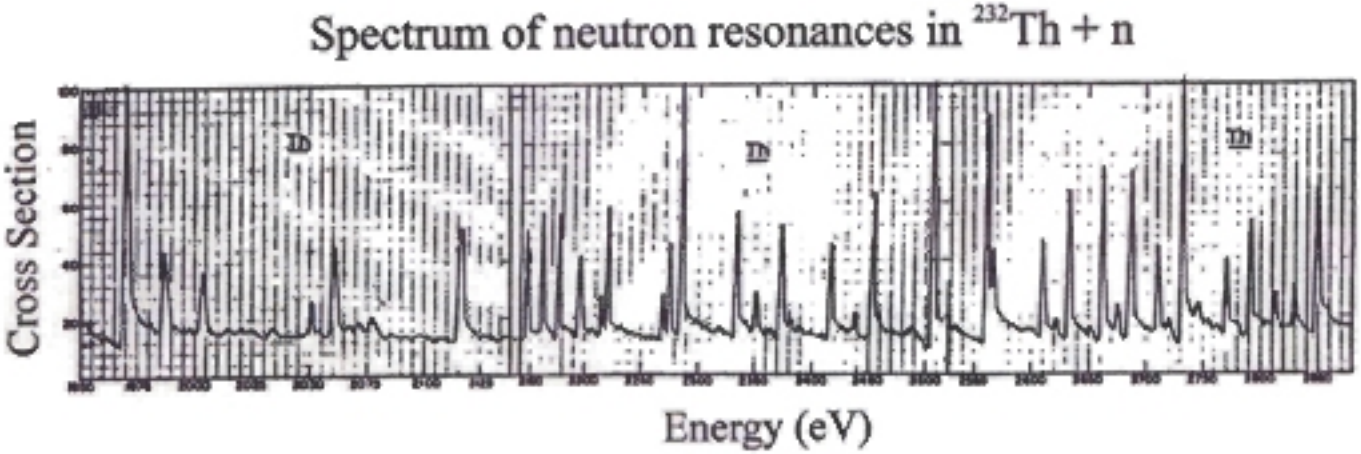
Spectra



- Complete spectrum of eigenfrequencies can be measured in superconducting billiard cavities.

Stadium billiard ↔ n + ^{232}Th





News and Views

Neutron Capture and Nuclear Constitution

THE new views of nuclear structure and the processes involved in neutron capture, presented by Prof. Niels Bohr in an address which appears elsewhere in this issue, were expounded by him in a lecture to the Chemical and Physical Society of University College, London, on February 11 and were illustrated by two pictures here reproduced. The first of these is intended to convey an idea of events arising out of a collision between a neutron and the nucleus. Imagine a shallow basin with a number of billiard balls in it as shown in the accompanying figure. If the basin were empty, then upon striking a ball from the outside, it would go down one slope and pass out on the opposite side with its original velocity. But with other balls in the basin, there would not be a free passage of this kind. The struck ball would divide its energy first with one of the balls in the basin, these two would similarly

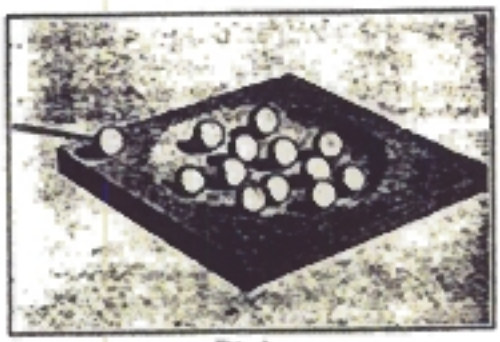


FIG. 1.

share their energies with others, and so on until the original kinetic energy was divided among all the balls. If the basin and the balls are regarded as perfectly smooth and elastic, the collisions would continue until the kinetic energy happens again to be concentrated upon a ball close to the edge. This ball would then escape from the basin and the remainder of the balls would be left with insufficient total energy for any of them to climb the slope. The picture illustrates, therefore, "that the excess energy of the incident neutron will be rapidly divided among all the nuclear particles with the result that for some time afterwards no single particle will possess sufficient kinetic energy to leave the nucleus".

Nuclear Energy Levels

THE second figure illustrates the character of the distribution of energy levels for a nucleus of not too small atomic weight. The lowest lines represent the levels with an excitation of the same order of magnitude as ordinary excited γ-ray states. According to the views developed in Prof. Bohr's address, the levels will for increasing excitation rapidly become closer to one another and will, for an excitation of about 15 million electron volts, corresponding to a collision between a nucleus and a high-speed neutron, be continuously distributed, whereas in the region of small excess energy of about 10 million volts excitation they will still be sharply separated. This is illustrated by the two lenses of high magnification placed over the level-diagram in the two above-mentioned regions. The dotted line in the middle of the field of the lower magnifying glass represents zero excess energy, and the fact that one of the levels

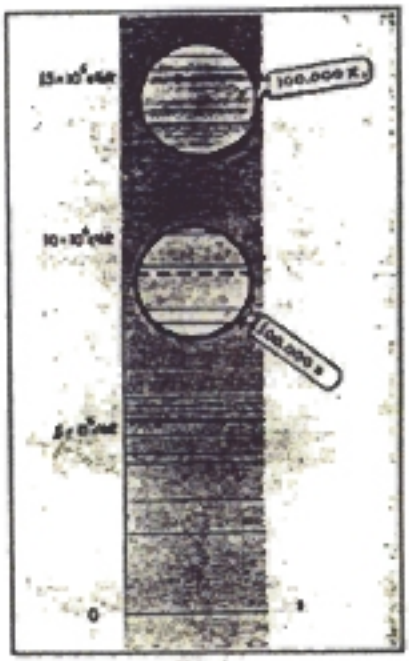
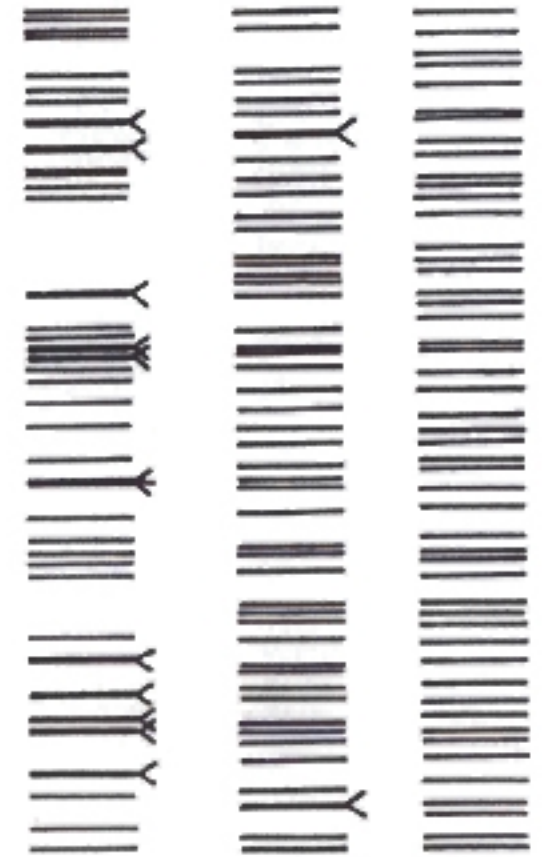


FIG. 2.

is very close to this line (about $\frac{1}{4}$ volt distant) corresponds to the possibility of selective capture for very slow neutrons. The average distance between the neighbouring levels will in this energy region be about ten volts as estimated from the statistics for the occurrence of selective capture. The diagram shows no upper limit to the levels, and these actually extend to very high energy values. If it were possible to experiment with neutrons or protons of energies above a hundred million volts, several charged or uncharged particles would eventually leave the nucleus as a result of the encounter; and, adds Prof. Bohr, "with particles of energies of about a thousand million volts, we must even be prepared for the collision to lead to an explosion of the whole nucleus".

Typical Sequences of Eigenvalues

Level Schemes



Poisson GOE GUE

Random Matrices ↔ Level Schemes

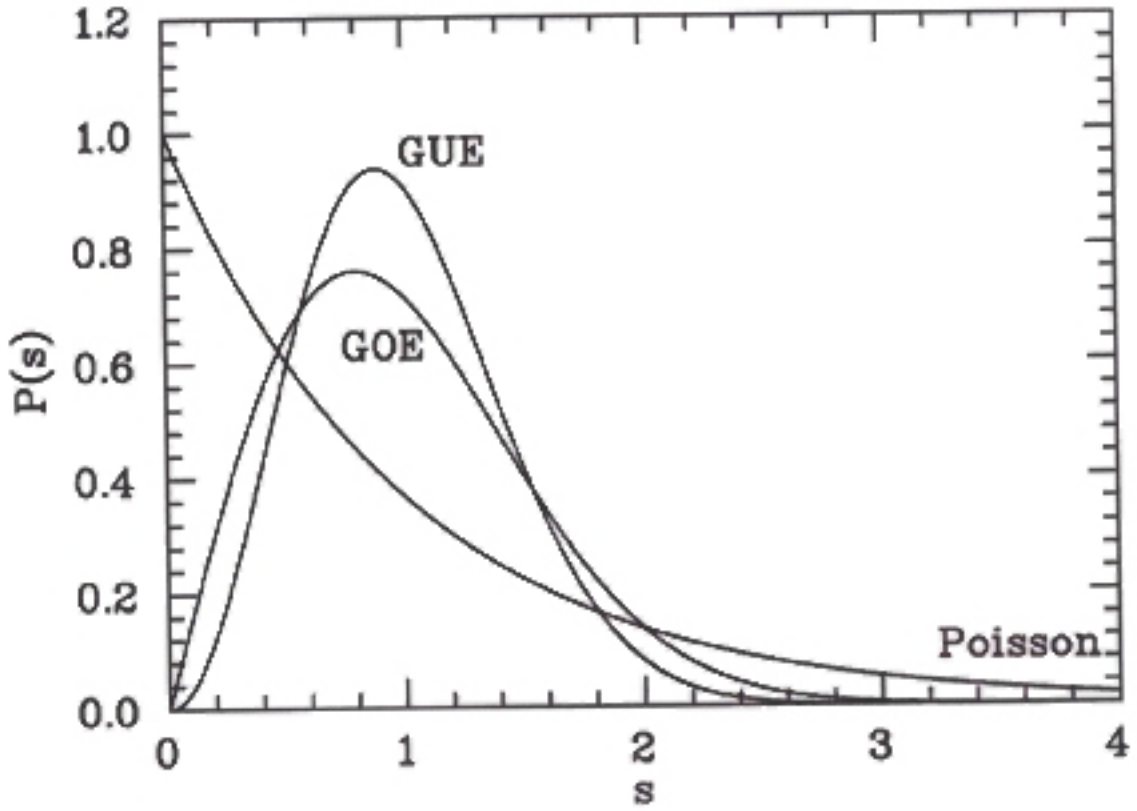
Random Matrix

$$H = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

Eigenvalues

$$H\phi_n = E_n\phi_n$$

Nearest neighbor spacings distribution

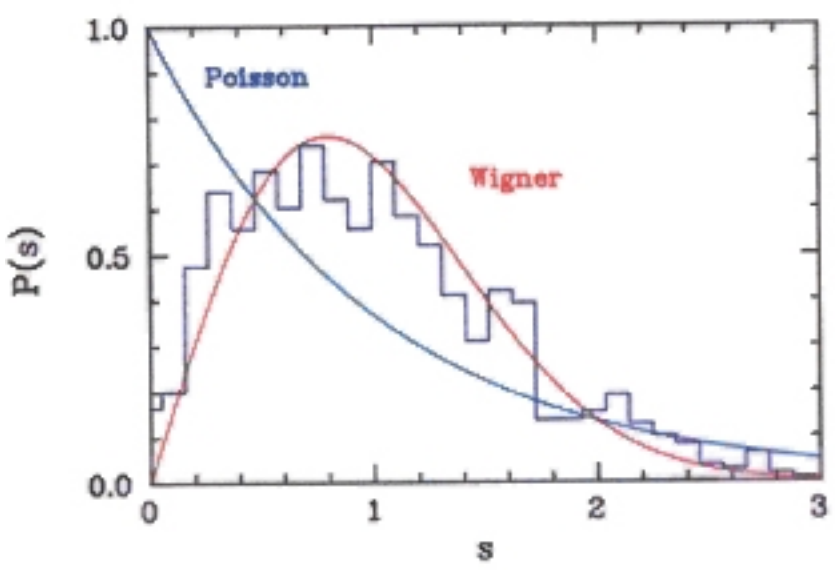


GOE and GUE ↔ "Level Repulsion"

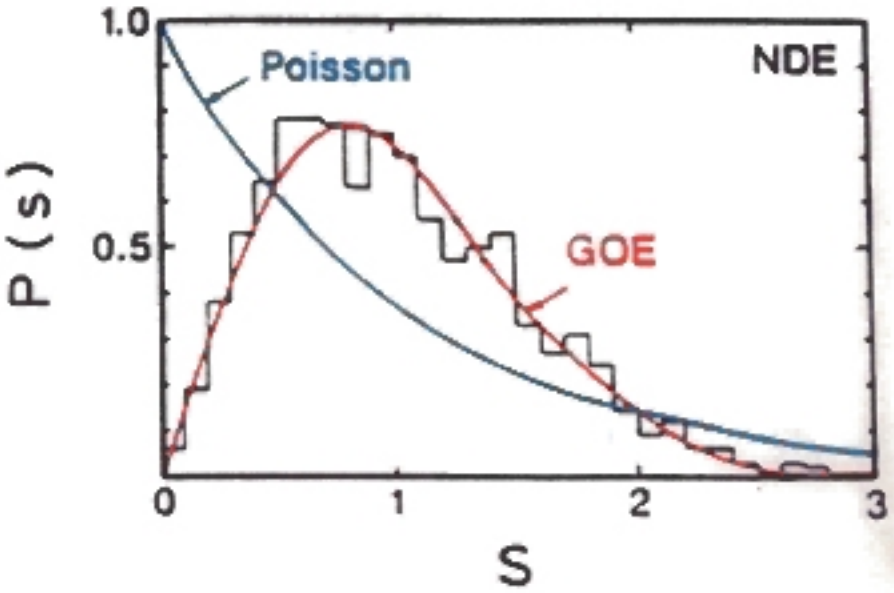
Poissonian Random Numbers ↔ "Level Clustering"

Nearest Neighbor Spacings Distributions

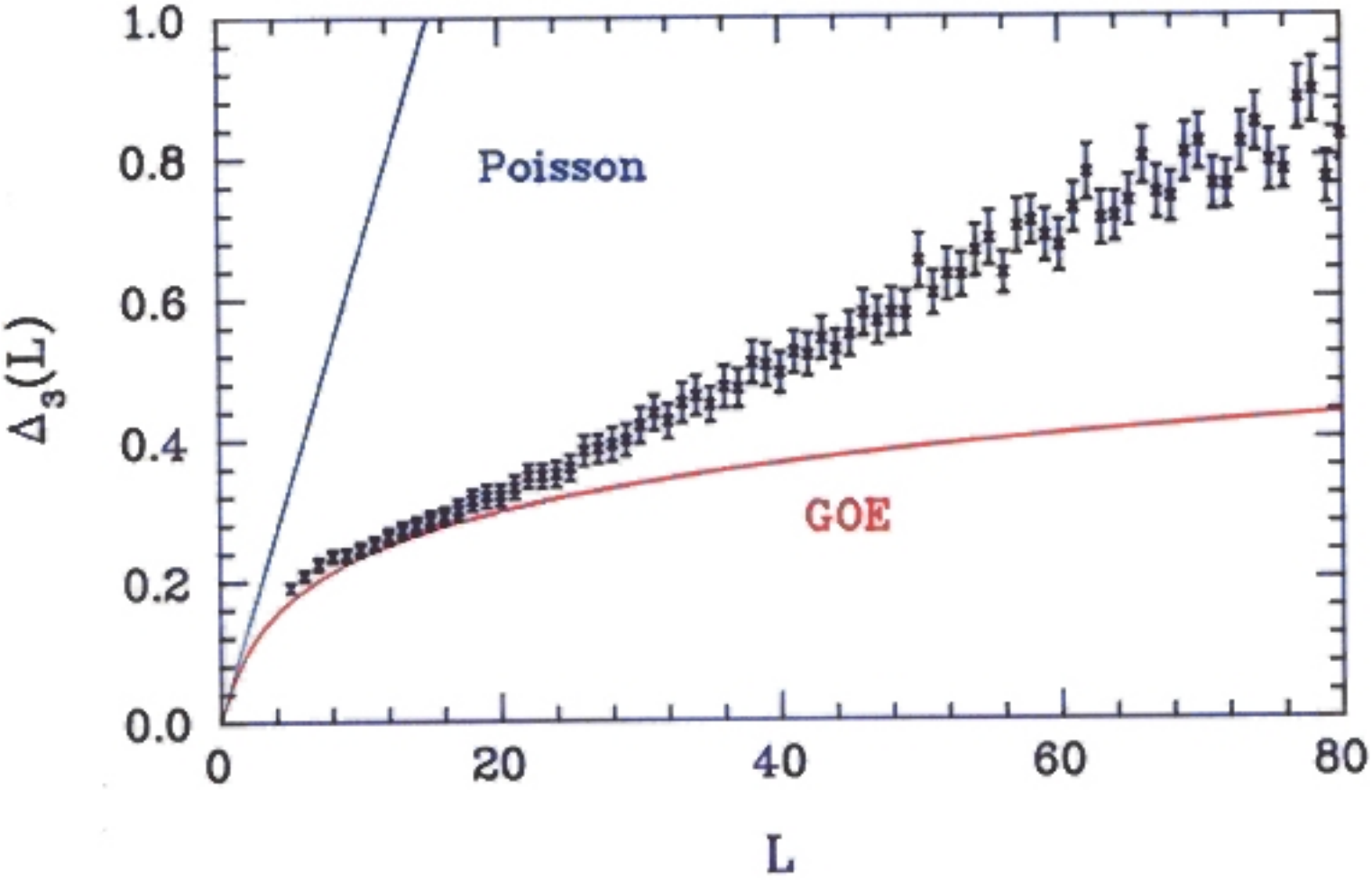
stadium billiard



nuclear data ensemble



Dyson-Mehta statistics for the stadium: something non-generic seems to happen



Periodic orbit theory (POT)

Motivation: description of quantum spectra in terms of classical closed orbits (Gutzwiller's trace formula).

spectrum

$$\{f_i\}$$

$$\downarrow$$

spectral density

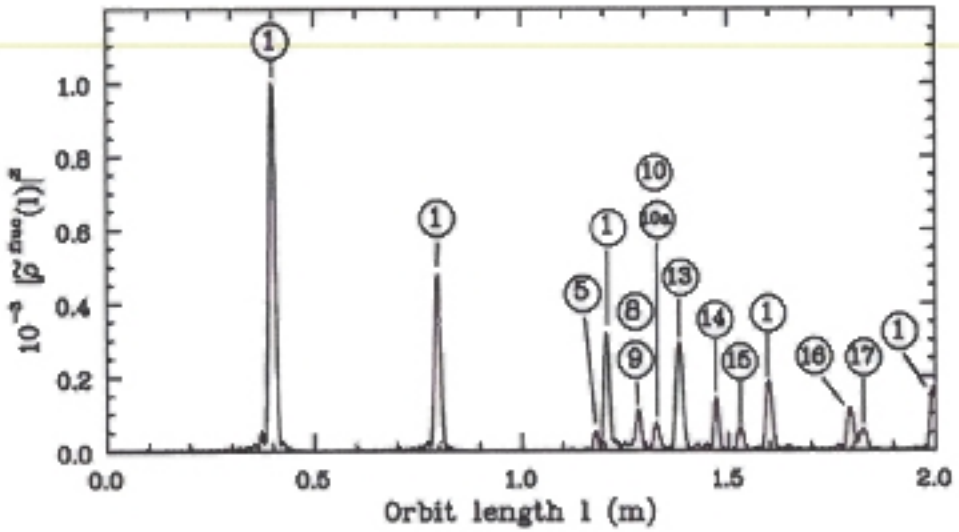
$$\rho(f) = \sum_i \delta(f - f_i)$$

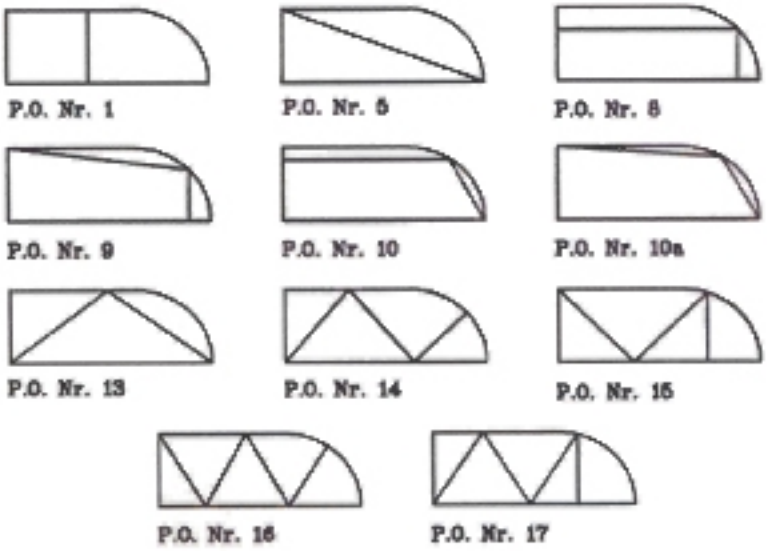
$$\downarrow$$

length spectrum

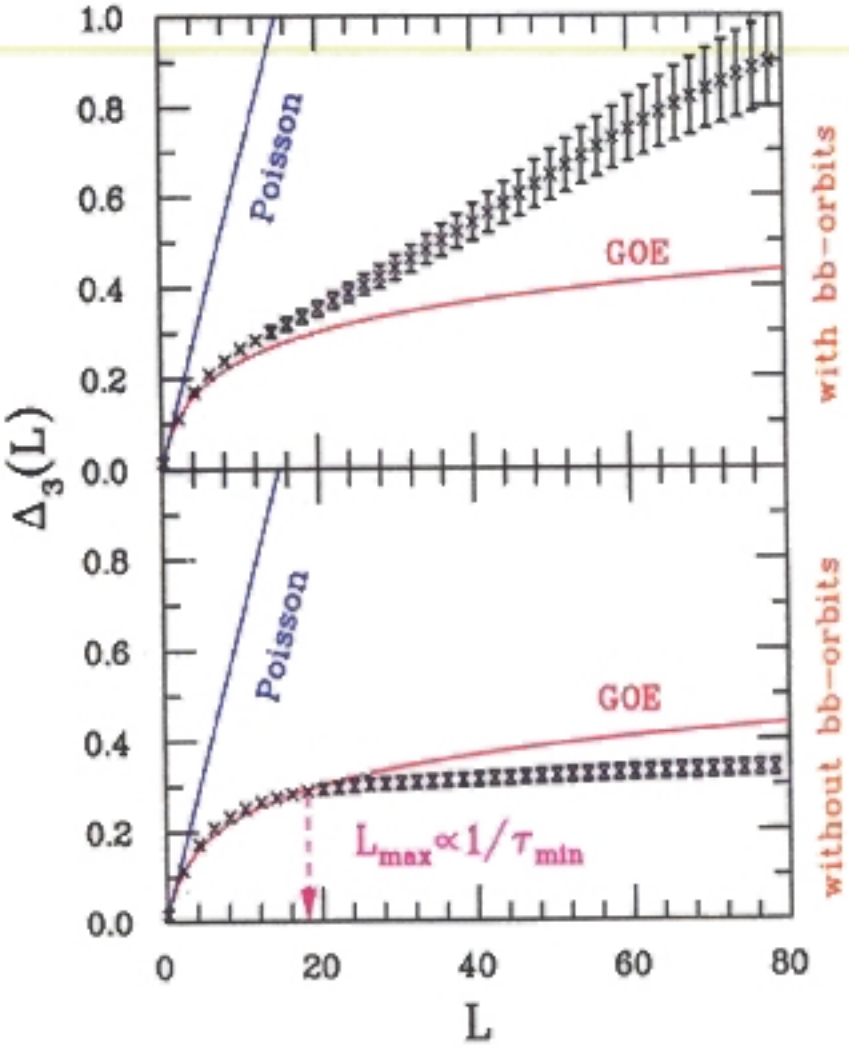
$$\tilde{\rho}(x) = \int_0^{f_{\max}} \rho(f) e^{i\frac{2\pi}{c}xf}\, df$$

Periodic orbits in the stadium





Extraction of the bouncing ball orbits



How is the behavior of the classical system transferred to the quantum system?

- There is a one-to-one correspondence between the two systems. → Bohigas' conjecture

- After extraction of the (non-generic) bouncing ball orbits of the classical system the quantum counterpart behaves like a typical chaotic system.

Definition: `generic´

1. 2nd Concise Edition of Webster's New World Dictionary of the American language (1975):

   `refering to a whole kind, class, or group´
   `something inclusive or general´

2. Oriol Bohigas:

   `opposite of specific´
   `non-particular´
   `common to all members of a large class´

3. Thomas Seligman:

   `structurally stable against small perturbations

Generic and non-generic features of billiards and nuclei

Generic

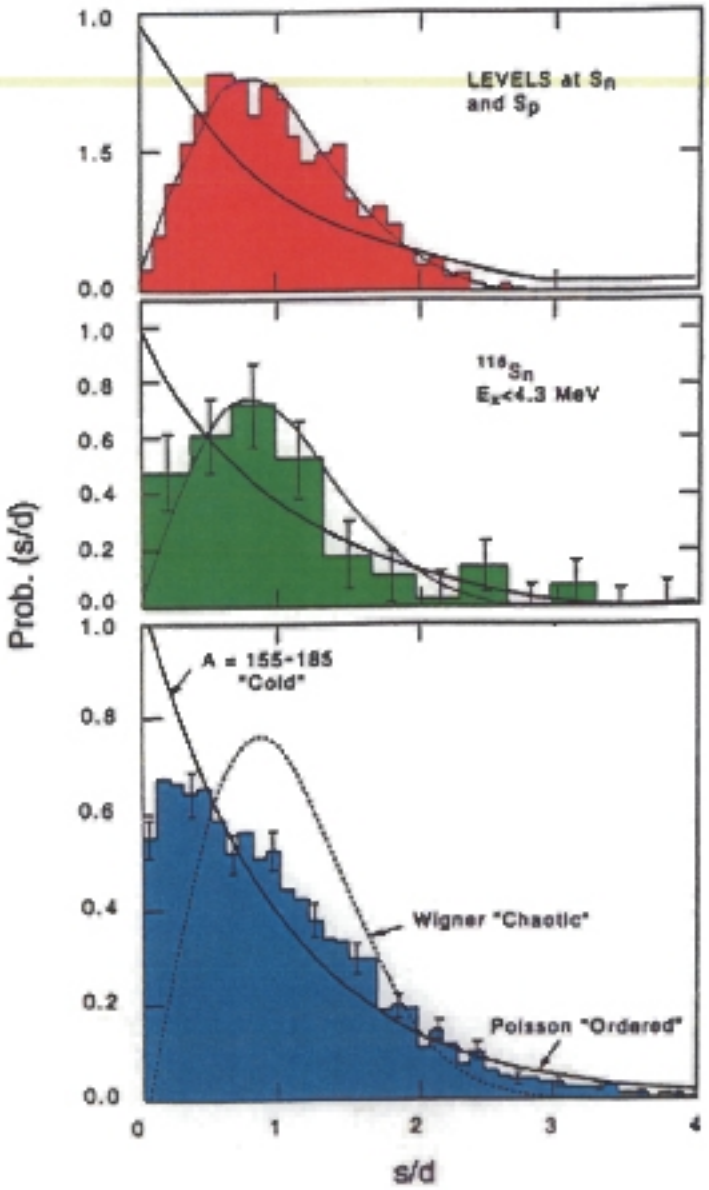
- Level statistics
- Width distributions

Non-generic

- Certain POs (BBOs)
- Collective rotations and vibrations

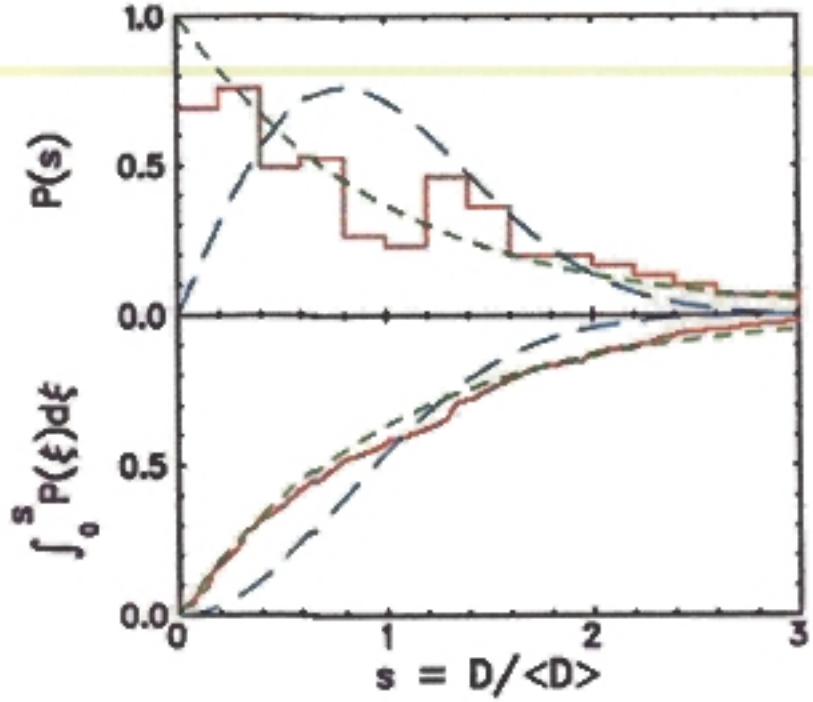
→ The Scissors mode DALINAC 1984

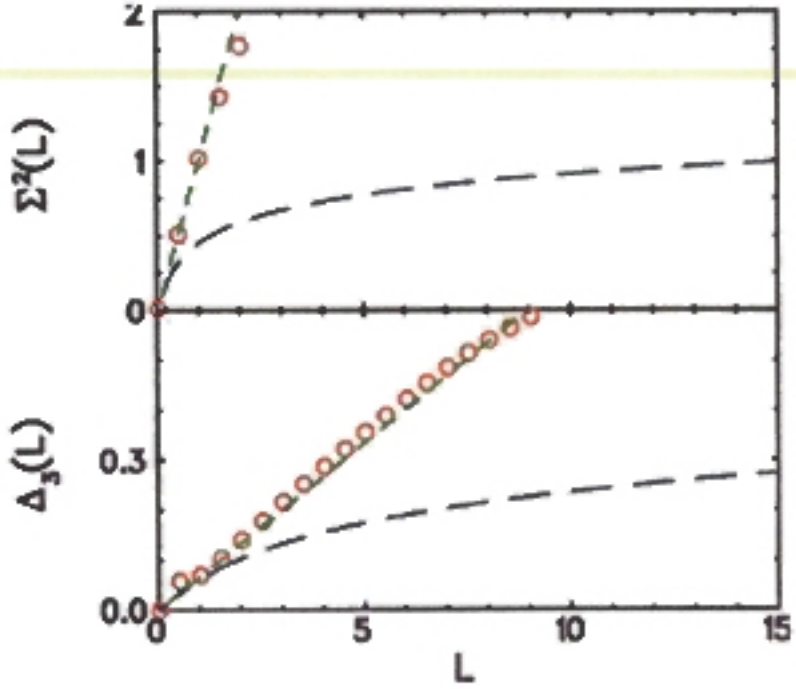
Example: Cold and Hot Nuclei



J.D. Garrett, J.Q. Robinson, A.J. Foglia, H.-Q. Jin, PLB **392**, 24 (1997).

Scissors Mode





- Non-generic behavior according to Poisson
- Proof of a simple collective excitation

Temperature Dependence of Damping and Frequency Shifts of the Scissors Mode of a Trapped Bose-Einstein Condensate

Onofrio Maragò, Gerald Hechenblaikner, Eleanor Hodby, and Christopher Foot

Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford, OX1 3PU, United Kingdom

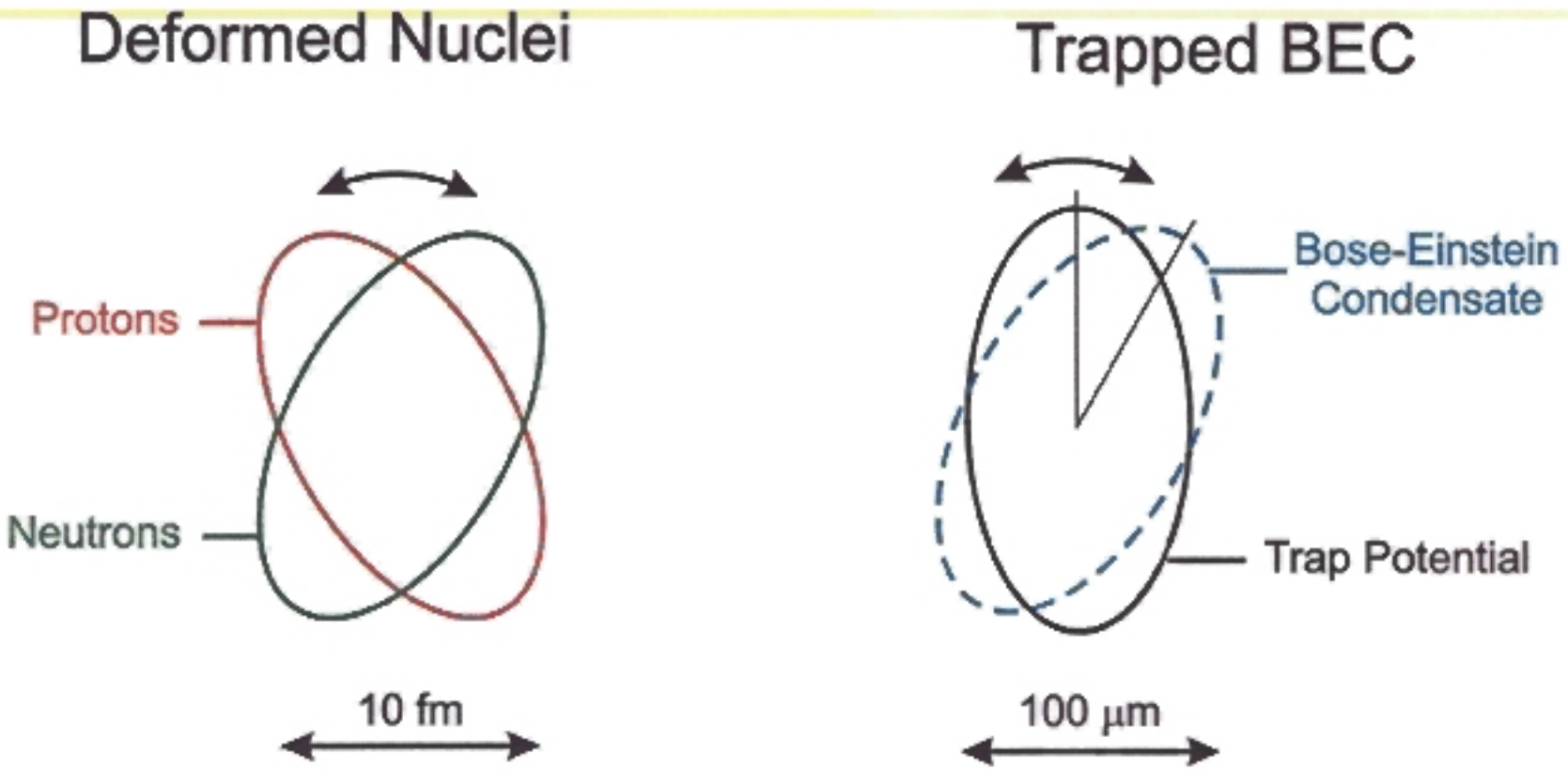
The experimental discovery of the scissors mode [1], first predicted in a geometrical model [2], has been one of the most exciting findings in nuclear physics during the past two decades (see [3] for a review). According to the geometrical picture, such a mode arises from a counter-rotational oscillation of the deformed proton and neutron fluids. Extensive studies of this mode in the past two decades investigated the dependence of its strength on the nuclear deformation and the relationship with the quadrupole collective mode and with the superfluid moment of inertia of the nucleus [3].

Recently it has been possible to study the scissors mode in Bose-Einstein condensates (BEC) of dilute gases [4]. This transverse mode of excitation was used to demonstrate that the condensate can flow only in an irrotational

In this paper we report a systematic study of the temperature dependent damping and frequency shifts of the scissors mode excitation in a trapped condensed gas. After a brief description of the experimental procedure, we present the scissors mode data for the frequency shifts and damping rates. We then compare our results with the available theoretical calculations for other collective modes. Finally, we show how the scissors mode frequency shifts are related to quenching of the moment of inertia of the boson gas.

We excite the scissors mode by using the technique described in our previous paper [6]. In summary we prepare atoms at the desired temperature T in an untilted time-averaged orbiting potential (TOP) trap ($\omega_x = \omega_y = 126$ Hz, $\omega_z = \sqrt{8}\,\omega_x$). We then adiabatically tilt the con-

Scissors Mode

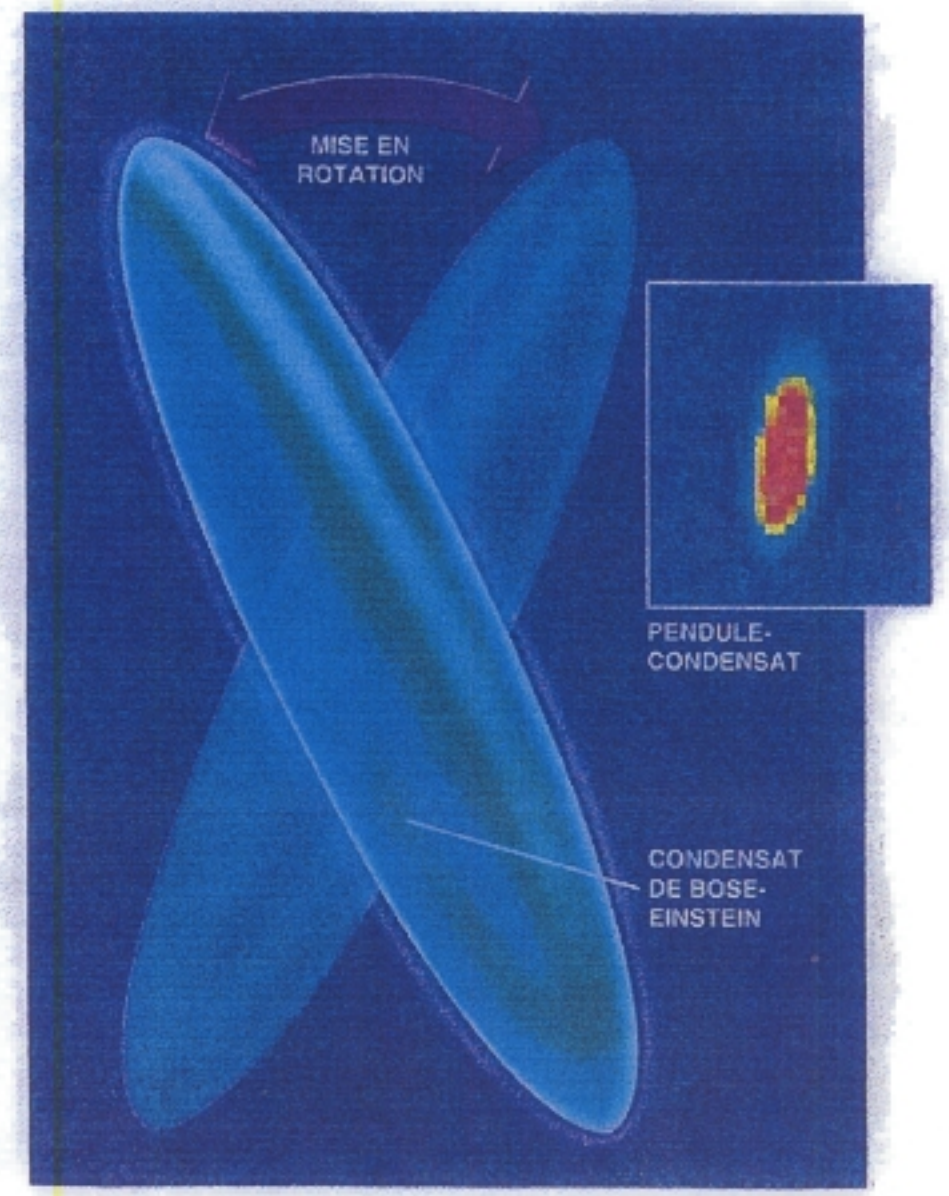


- Superfluidity?
- Moments of inertia

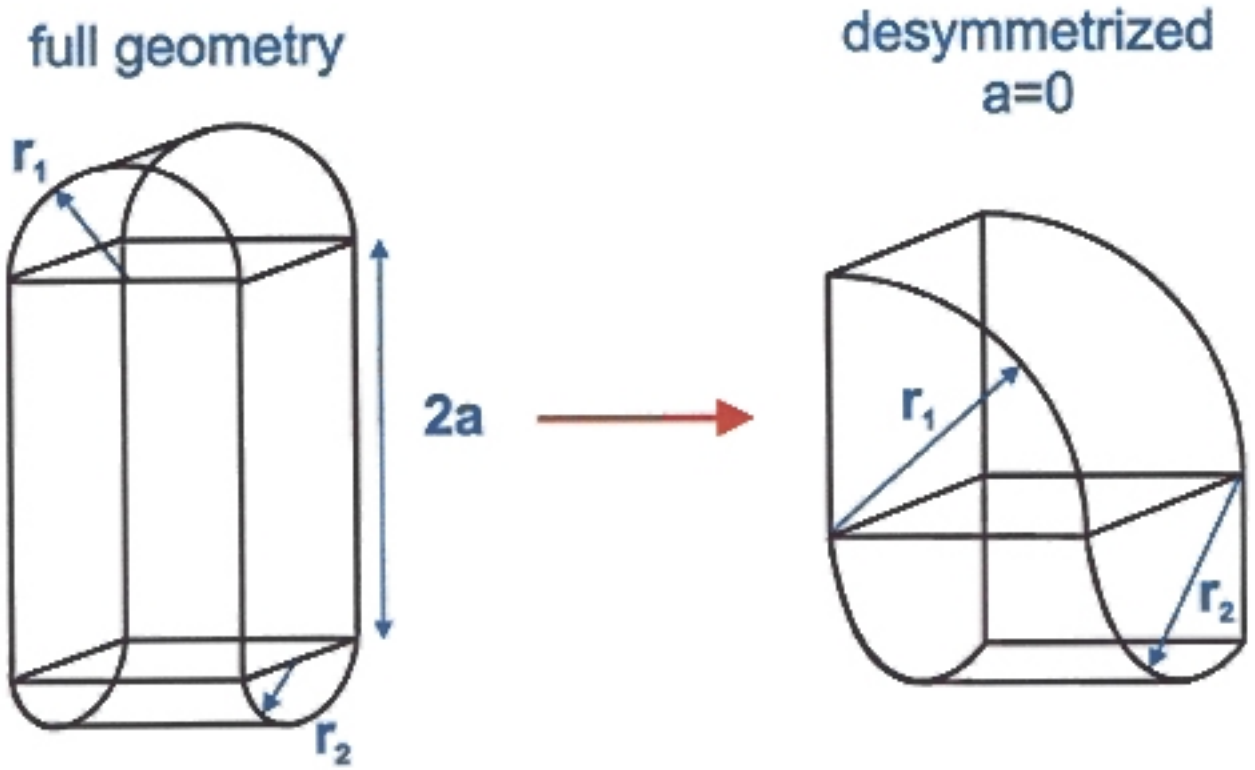
La superfluidité des condensats

DAVID GUÉRY-ODELIN

POUR LA SCIENCE - N° 296 JUIN 2002

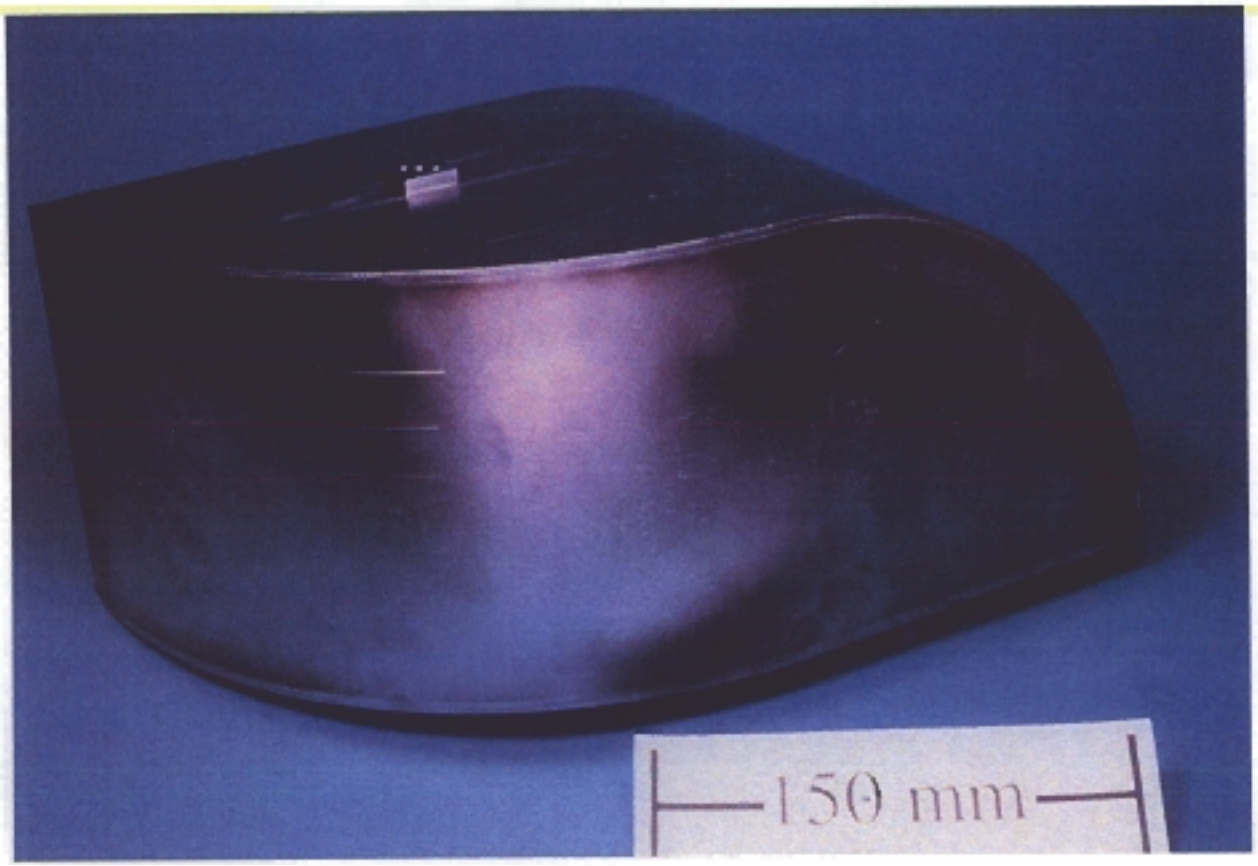


3D stadium billiard

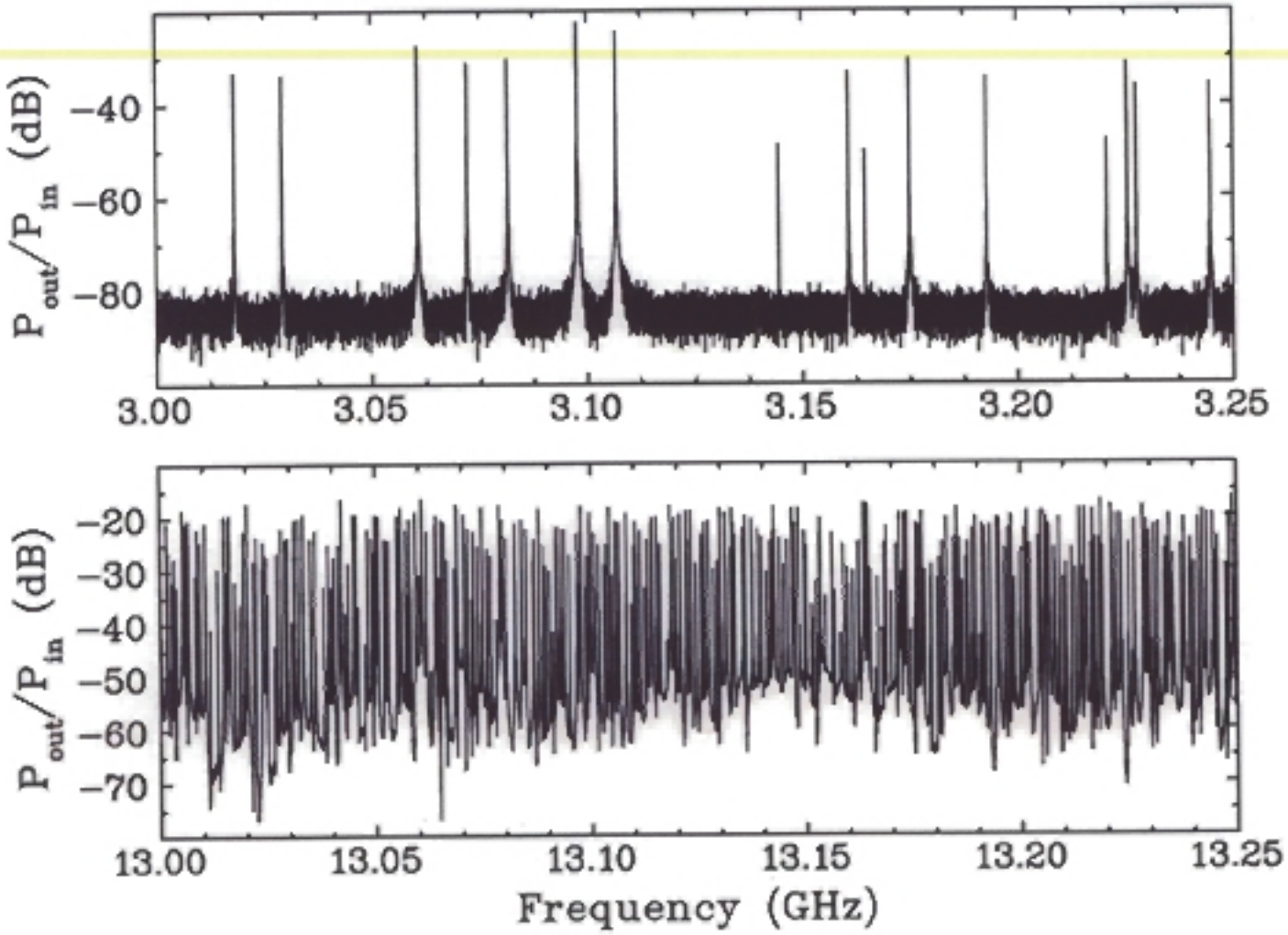


T. Papenbrock, Phys. Rev. E **61** (2000) 4626

3D Niobium resonator



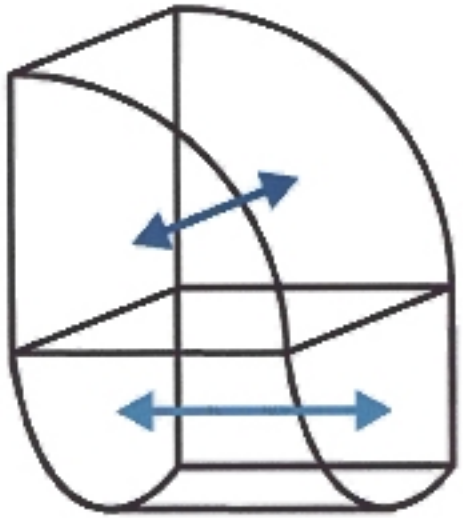
Transmission spectra



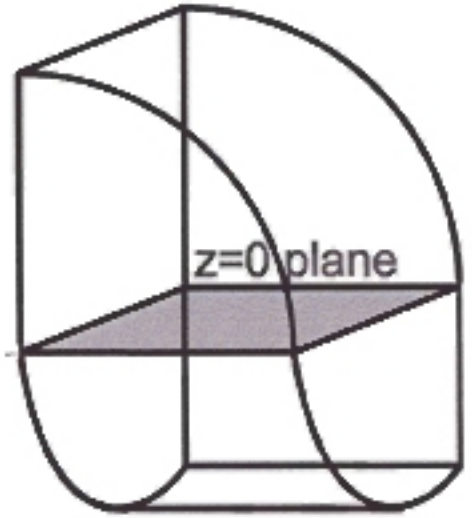
18764 resonances up to f=20 GHz

Non-generic periodic orbits

3D bouncing ball orbits



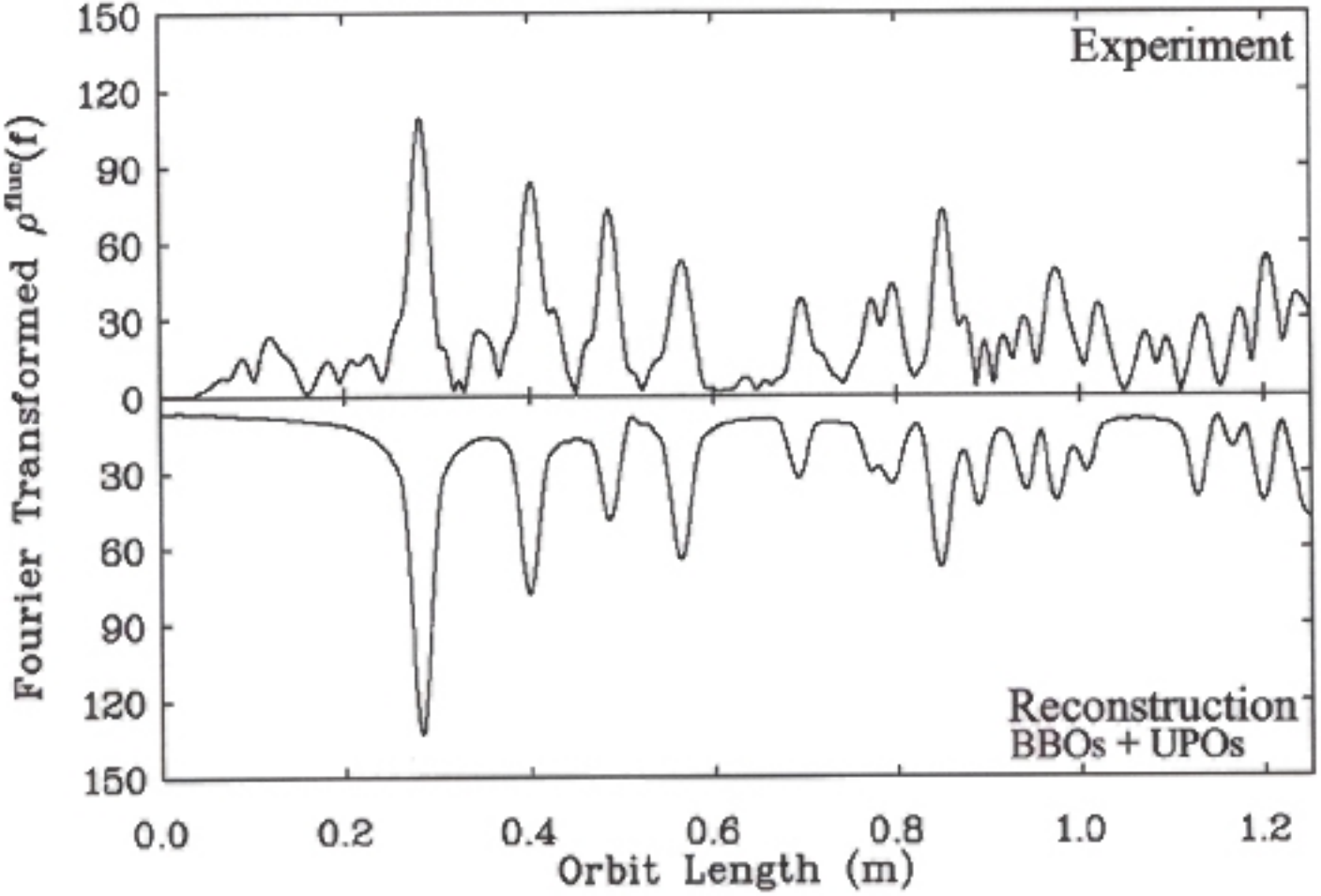
2D bouncing ball orbits



- There are two families of three dimensional bouncing ball orbits. They evolve parallel to the axis of the lower and the upper quarter-cylinder, respectively.

- There are trajectories in the plane z=0, that are linearily stable with respect to deviations parallel to this plane and unstable with respect to perpendicular deviations.

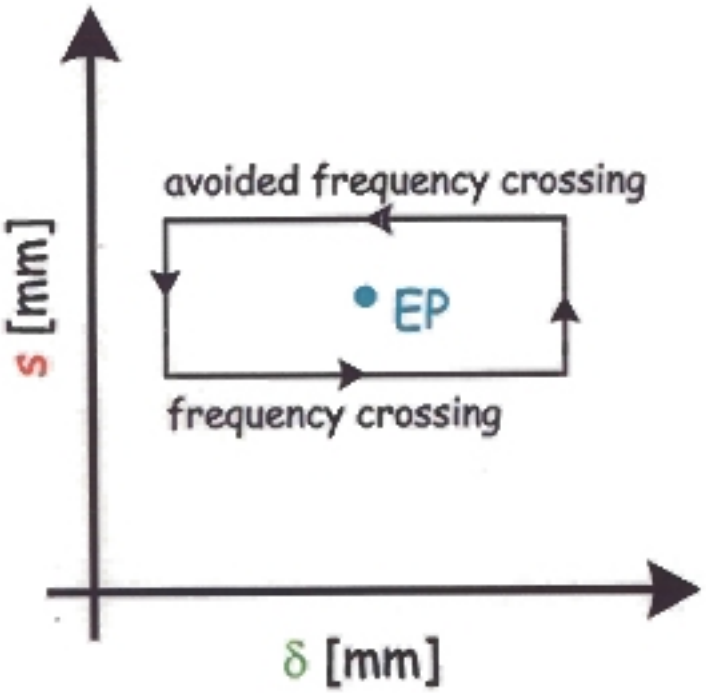
Comparison of length spectra



Frequencies, Widths and Wave Functions in Open Billards

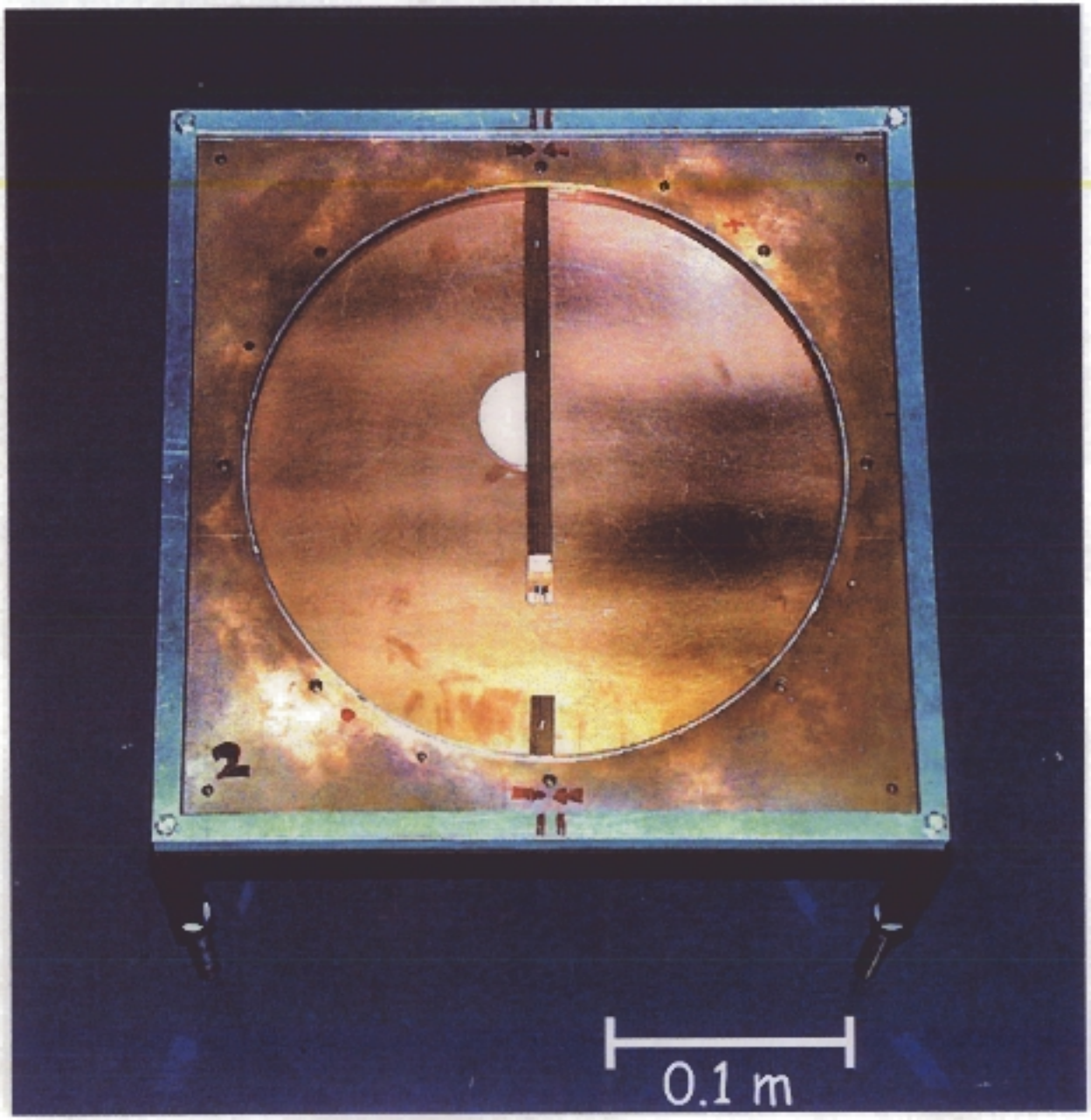
couple two slightly different microwave resonators

$$H(\delta, s) = \begin{pmatrix} \delta \cdot E_1 + i \cdot \gamma_1 & s \\ s & E_2 + i \cdot \gamma_2 \end{pmatrix}$$

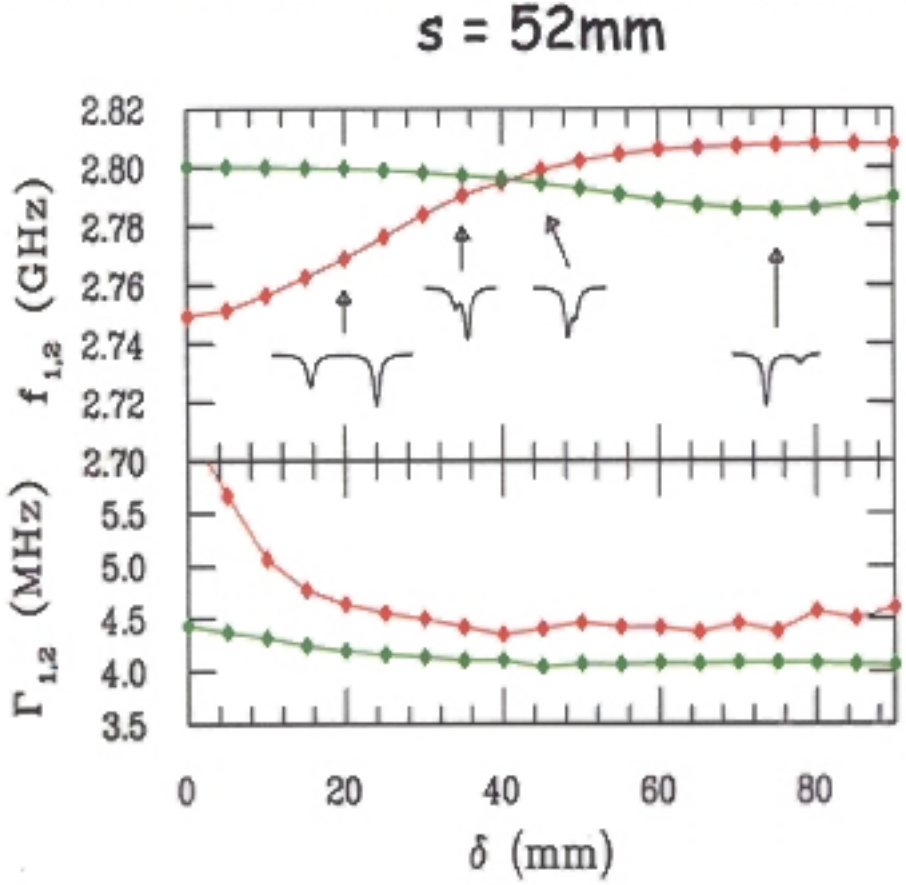


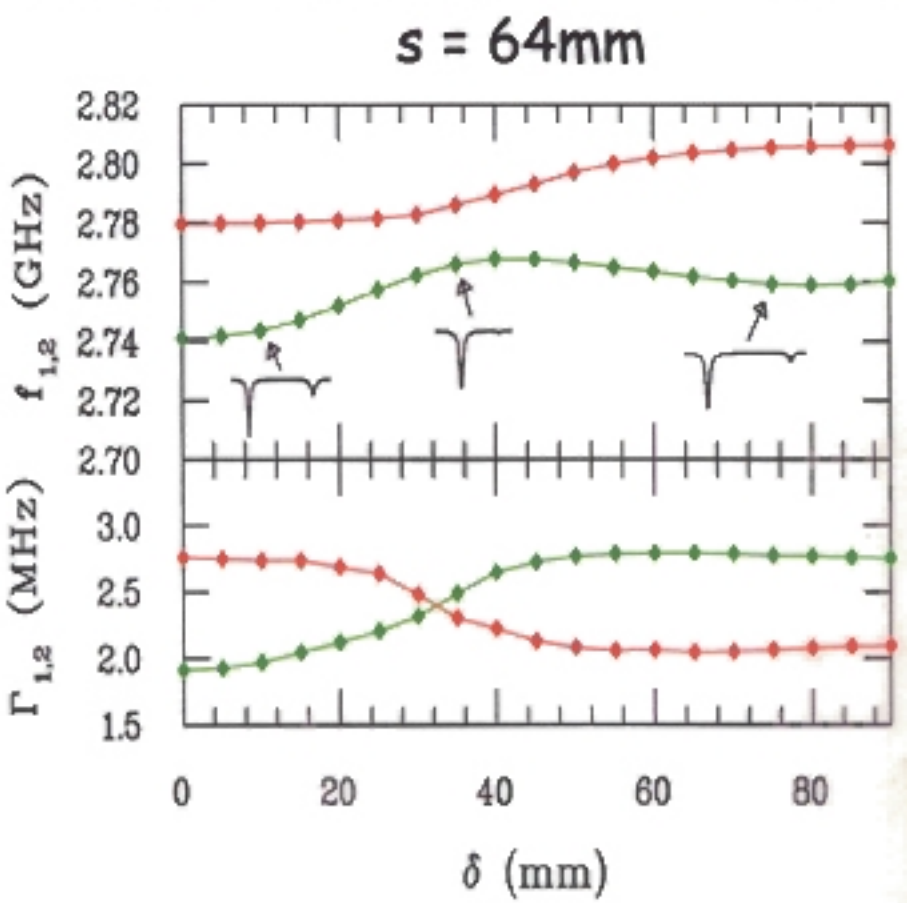
and encircle the EP in the parameter space

Experimental setup

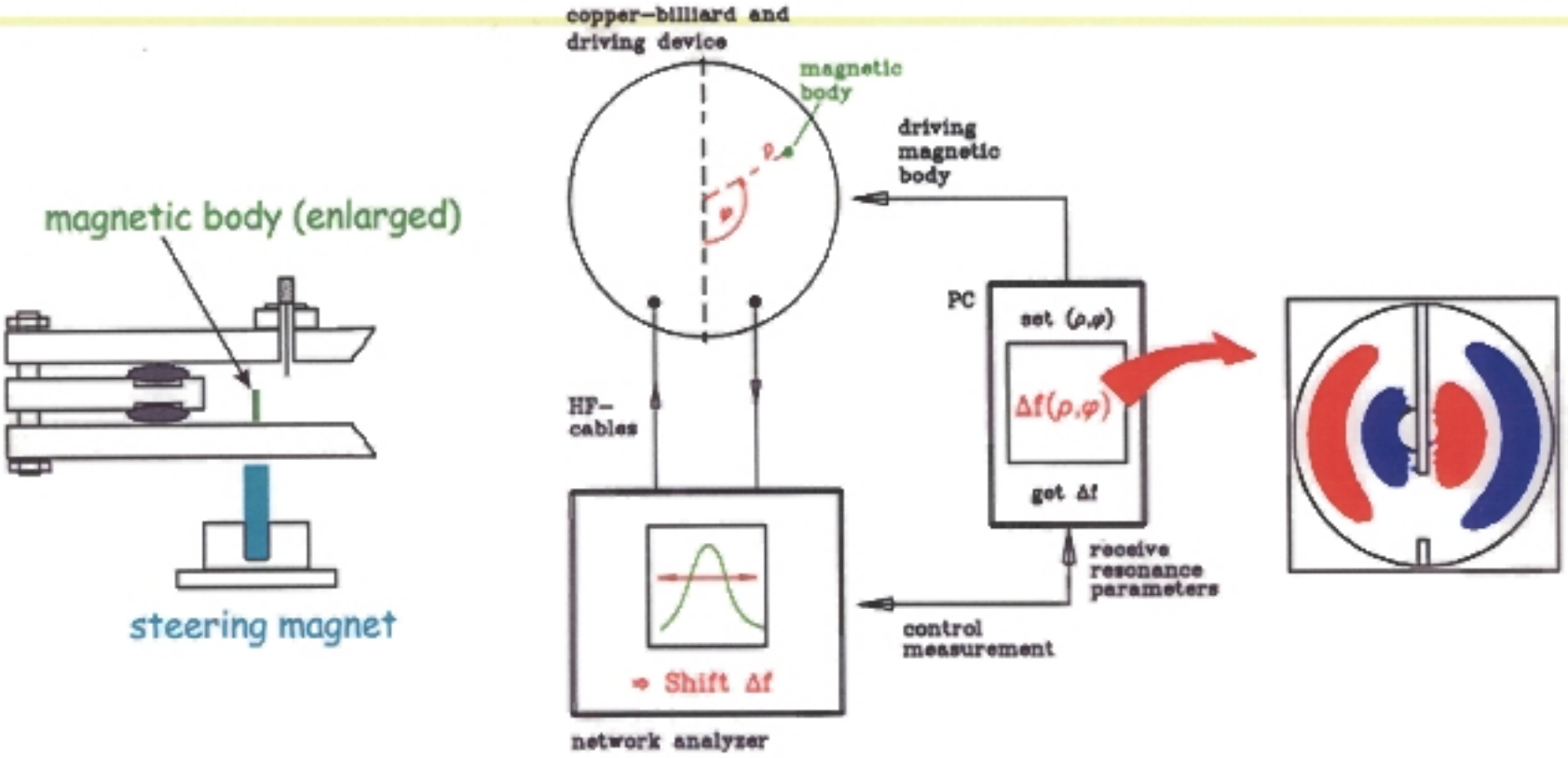


Frequencies and Widths

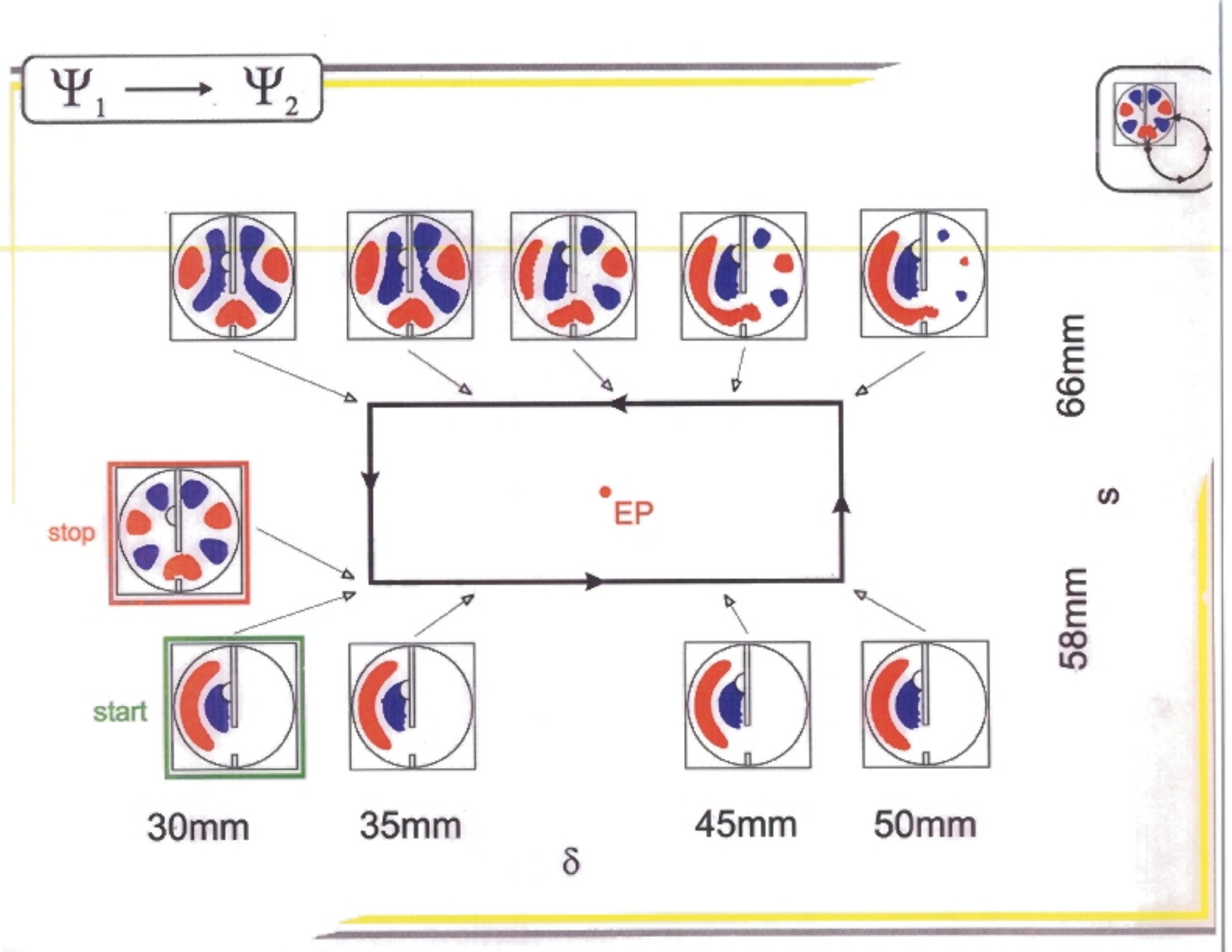




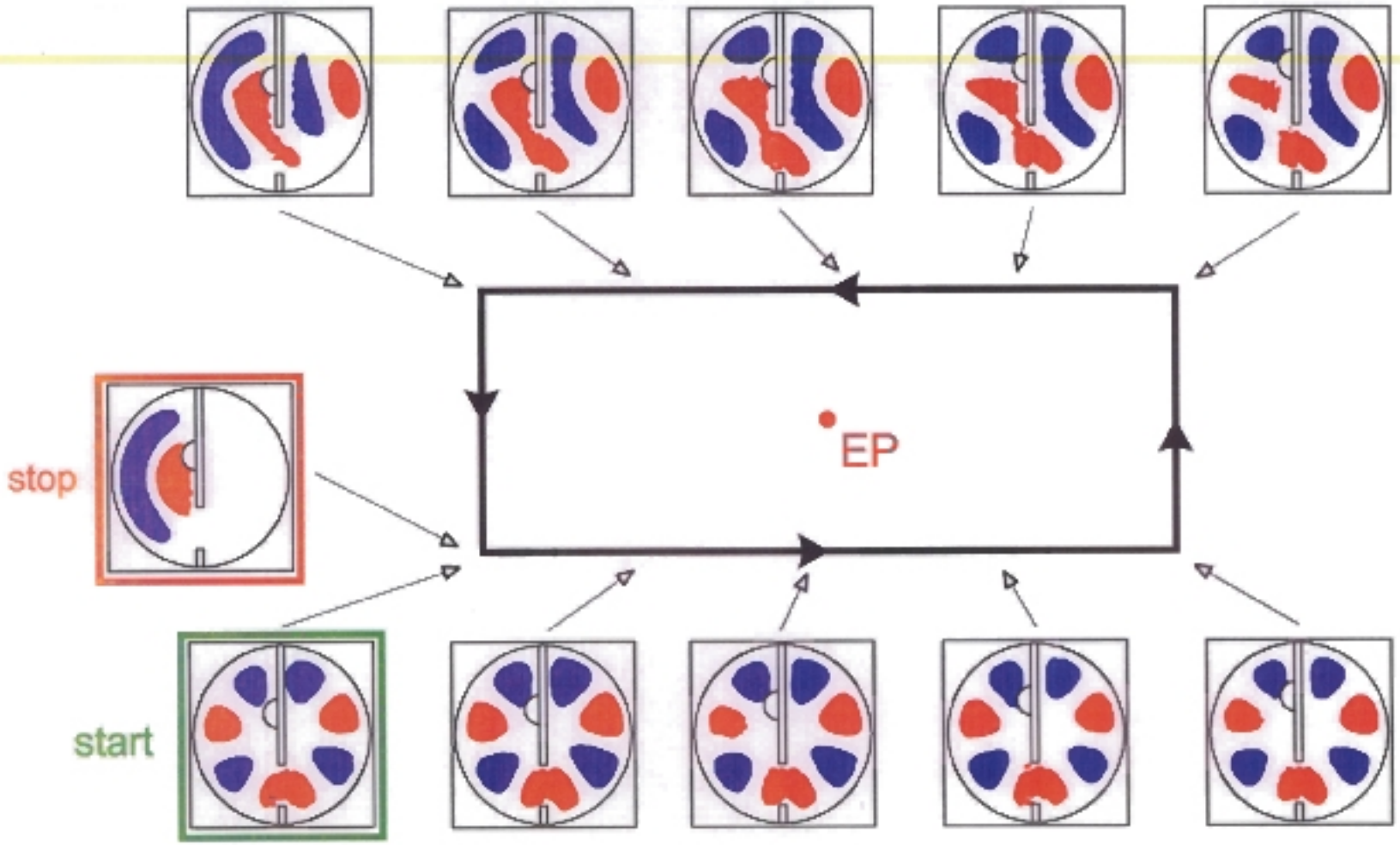
Measuring Field Distributions



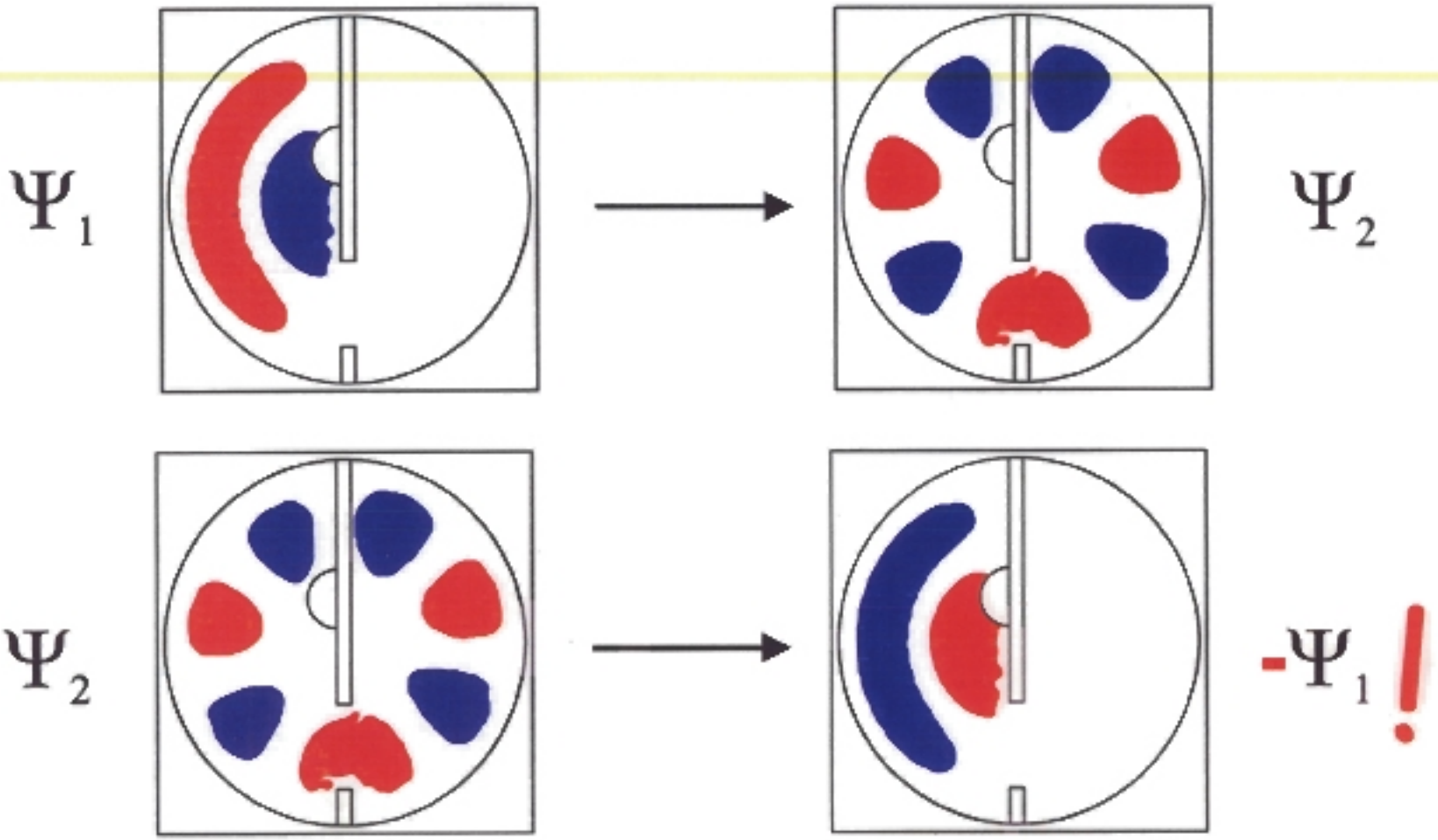
for a special geometry: $\Delta f \propto \vec{E}^2(\vec{r})$ (Slater theorem)


$\Psi_1 \longrightarrow \Psi_2$
66mm
s
58mm
EP
stop
start
30mm
35mm
45mm
50mm
δ

$\Psi_2 \longrightarrow -\Psi_1$



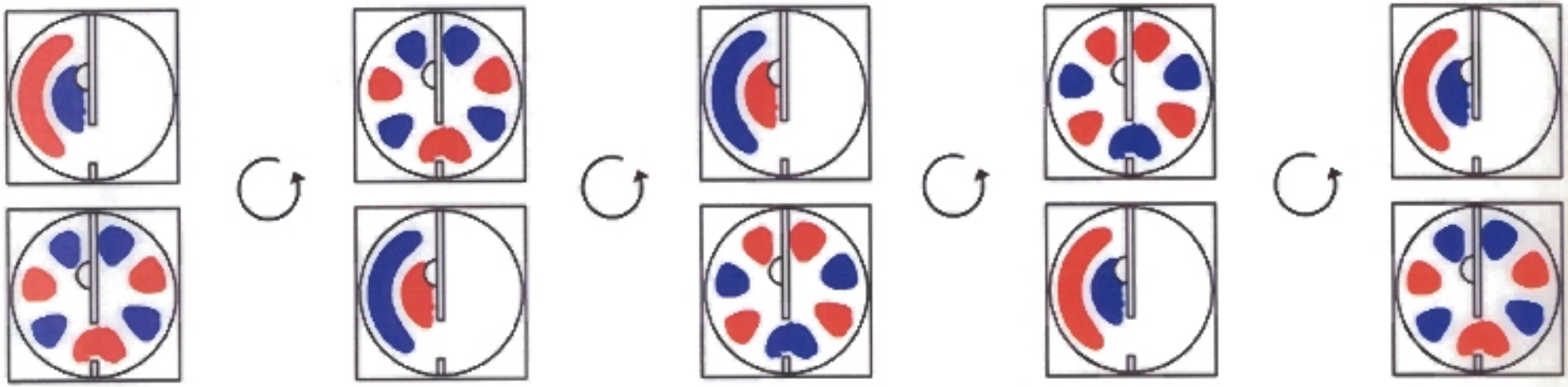
Chirality



Ψ_1 picks up a geometric phase of 2π and Ψ_2 of only π

Surrounding the EP Four Times

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \circlearrowright \begin{pmatrix}\psi_2\\ -\psi_1\end{pmatrix} \circlearrowright \begin{pmatrix}-\psi_1\\ -\psi_2\end{pmatrix} \circlearrowright \begin{pmatrix}-\psi_2\\ \psi_1\end{pmatrix} \circlearrowright \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$



Surrounding the EP four times restores the original situation!

Chirality II

ψ_1

ψ_2

right hand

ψ_2

$-\psi_1$

ψ_1

ψ_2

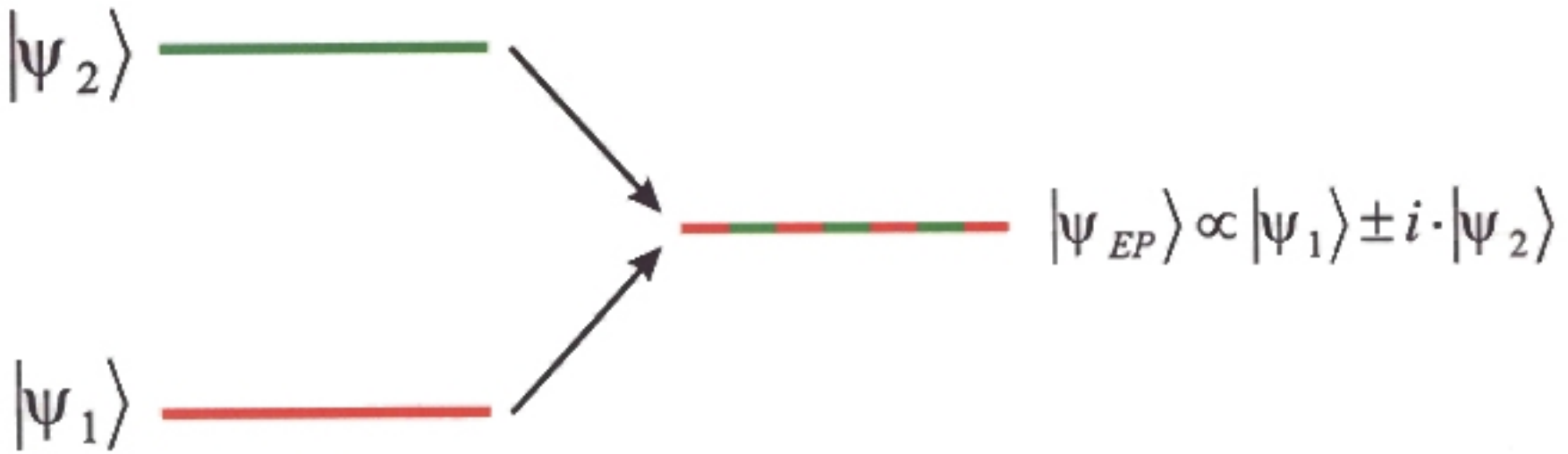
left hand

$-\psi_2$

ψ_1

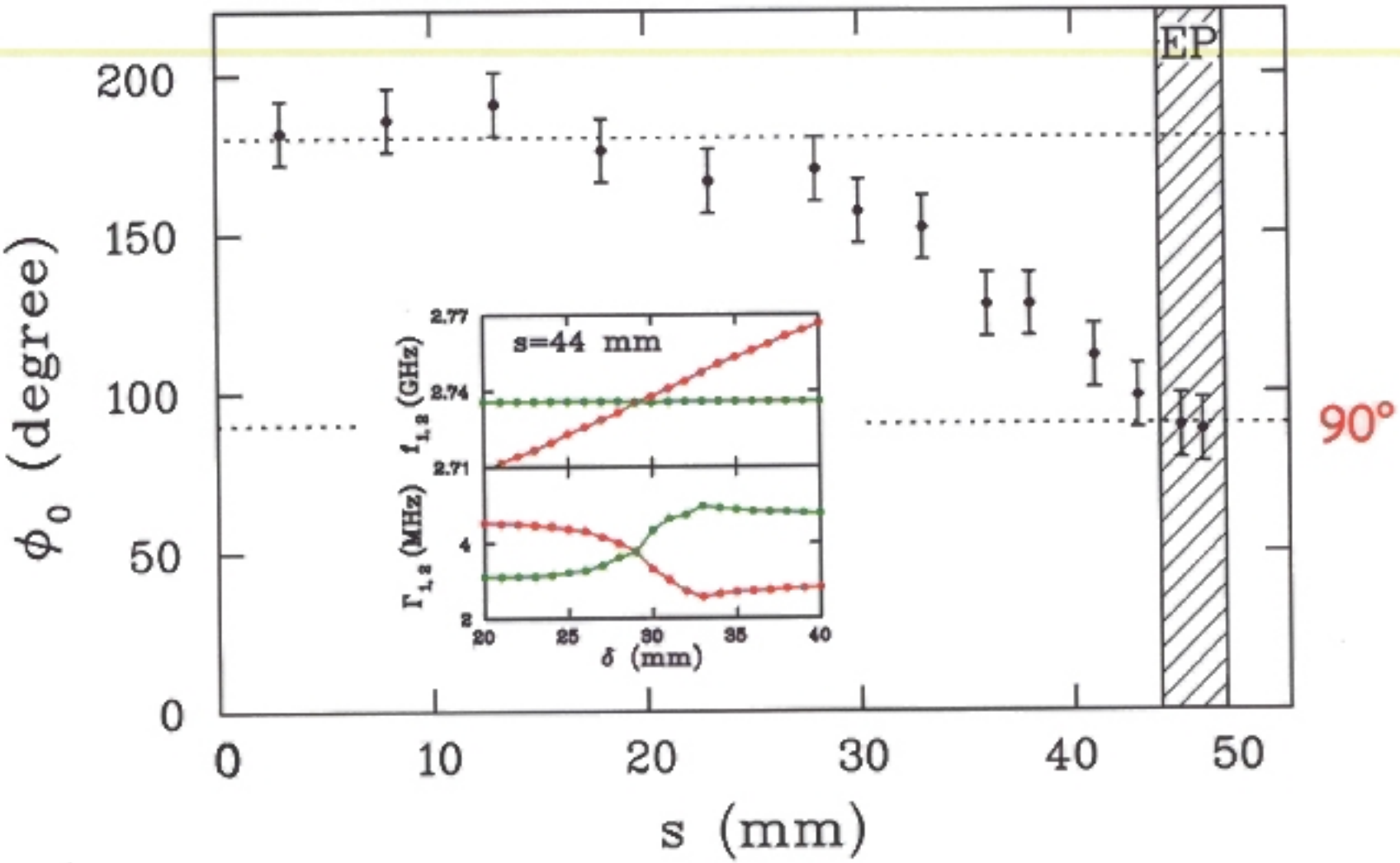
What Happens at the EP?

For $s \to s^{EP}$ the two modes $|\psi_1\rangle$ and $|\psi_2\rangle$ coalesce to $|\psi_{EP}\rangle$



can the factor $\pm i$ be measured as a phase difference of $\pm 90°$?

Experimental Data



Summary

- Quantum chaos manifests itself in the universal (generic) behavior of spectral properties and wave functions.

- Analogue experiments with microwave resonators provide interesting model systems for real quantum systems (atoms, molecules, nuclei,...).

„Wo das Chaos auf die Ordnung trifft,
gewinnt meist das Chaos,
weil es besser organisiert ist."

Friedrich Nietzsche (1844-1900)