HOLOGRAPHIC COSMOLOGY

(OR THE REMARKABLE $P=\rho$ FLUID)

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THE REMARKABLE $p = \rho$ FLUID

(? HOLOGRAPHIC COSMOLOGY)

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A REMARKABLE FLUID

$$p = \rho \quad ?$$

$$ds^2 = dt^2 - a^2(t)\, d\bar{x}^2$$

$$\left(\frac{\dot{a}}{a}\right)^2 \sim \rho(a)$$

EQN. OF STATE

$$p = K\rho$$

$$\rightarrow \boxed{\rho(a) \sim (\text{VOLUME})^{-(1+K)}}$$

$$\text{VOLUME} \sim a^{D-1}$$

D : SPACE-TIME DIMENSION

NON REL. MATTER $\qquad p = 0$

$$\rho \sim \frac{1}{V^{R}} \sim \frac{1}{V^{3}}$$

RADIATION $\qquad p = \frac{1}{D-1}\rho$

$$\rho \sim \frac{1}{V^{4/3}} \sim \frac{1}{V^{D/D-1}}$$

$K=1 \qquad p = \rho$

$$\rho \sim \frac{1}{V^{2}}$$

VOLUME $\nearrow$ $\qquad \rho \searrow$ FASTEST $(K=1)$

$\updownarrow$

VOLUME $\searrow$ $\qquad \rho \nearrow$ FASTEST $(K=1)$

(ATTRACTOR WHEN VOLUME SHRINKS)

1) FASTEST — BLUESHIFT / REDSHIFT

$K = 1$

MOST INCOMPRESSIBLE FLUID

SATISFYING CAUSALITY

SPEED OF SOUND = SPEED OF LIGHT

$$c_s^2 = \frac{P}{\rho} = K = 1$$

2) MOST EFFICIENT IN

PACKING ENTROPY

STATIC STAR LIKE CONFIGURATION

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2 d\Omega^2$$

OPPENHEIMER - TOLMAN - VOLKOV EQS

$$\begin{cases} \dfrac{D-2}{2r}\,\dfrac{\partial_r (AB)}{AB^2} = 8\pi(\rho + p) \\[2ex] \dfrac{D-2}{r^2}\left[D-3 - \partial_r\left(\dfrac{r}{B}\right) + \dfrac{4-D}{B}\right] = -16\pi p \\[2ex] \partial_r p = -(\rho + p)\,\dfrac{\partial_r A}{A} \end{cases}$$

$$r \to \lambda r$$

INVARIANT:

$$\begin{Bmatrix} p \\ \rho \end{Bmatrix} \to \lambda^{-2} \begin{Bmatrix} p \\ \rho \end{Bmatrix}$$

B FIXED

AN ASIDE

$$dE = -p\,dV + T\,dS$$

$$V\,dp + p\,dV = -p\,dV + Ts\,dV + TV\,ds$$

$$\downarrow$$

$$p(r) + \rho(r) = Ts(r)$$

$$dp = T\,ds$$

$$\downarrow$$

$p = k\rho$

$$s \sim T^{1/k}$$

$$\rho \sim T^{1+1/k}$$

$k=1$ IN ALL DIMENSIONS

$$s \sim T \qquad \rho \sim T^2$$

! RELATIVISTIC GAS IN 1+1 DIM.!

$$\boxed{s \sim \sqrt{\rho}}$$

$$M(R) \sim \int_0^R dr \; r^{D-2} \rho(r)$$

$$M(R) \sim R^{D-3}$$

$$S(R) \sim \int_0^R dr \; r^{D-2} \underbrace{\sqrt{B(r)}}_{\sqrt{g_{rr}}} \; \underbrace{s(r)}_{\rho^{\frac{1}{1+k}}}$$

$$S(R) \sim R^{D-1-\frac{2}{1+k}}$$

$$k=1 \longrightarrow \boxed{\begin{aligned} S(R) &\sim R^{D-2} \\ M(R) &\sim R^{D-3} \end{aligned}}$$

(SAME SCALING AS BLACK HOLES)

$k<1$ PARAMETRICALLY LESS ENTROPY FOR GIVEN MASS!

3)

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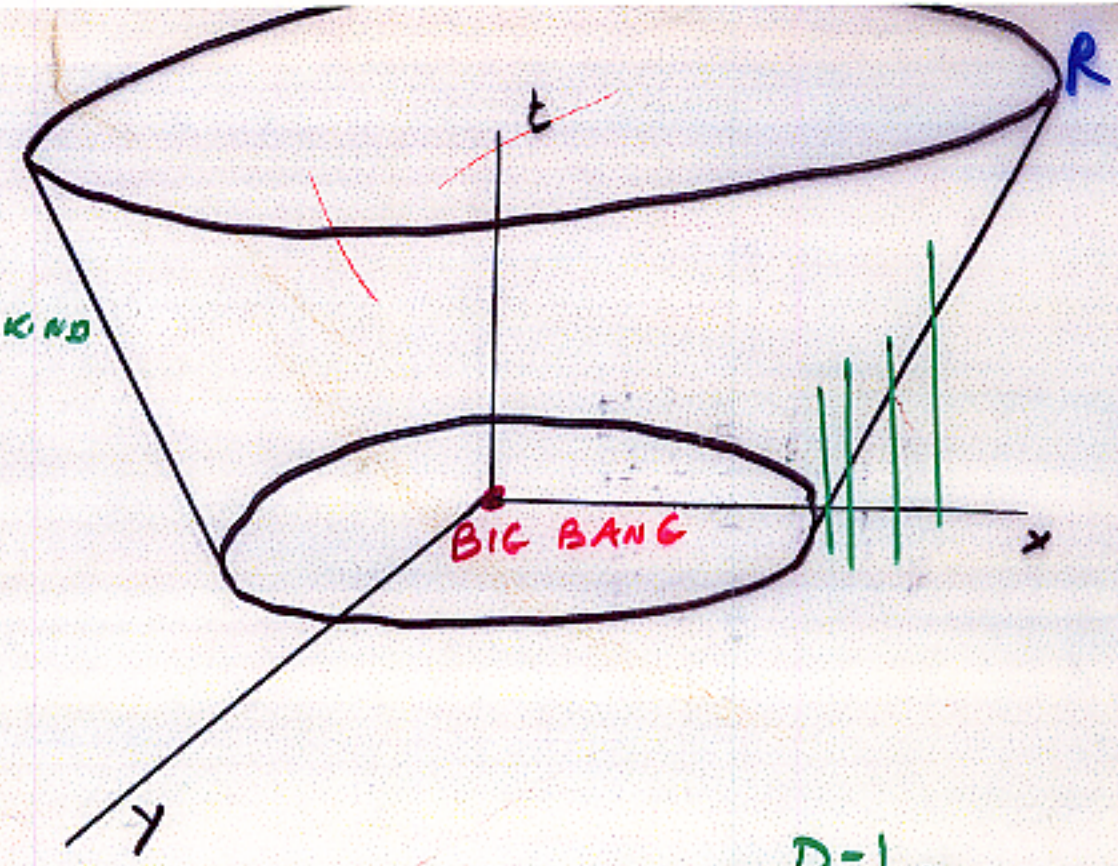


NAIVE

$$S \sim \sigma R^{D-1}$$

σ ↑ CONSTANT COMOVING ENTROPY DENSITY

$$\text{AREA} \sim R^{D-2}$$

INSTEAD USE COMOVING ENTROPY ENCOUNTERED IN BACK LIGHT CONE

$$\sigma \left(\int \frac{dt}{a(t)} \right)^{D-1} \leq a^{D-2} \left(\int \frac{dt}{a(t)} \right)^{D-1}$$

$$\left(\frac{\dot{a}}{a}\right)^2 \sim \rho \quad \text{AND} \quad \rho \sim V^{-(1+K)}$$

$$\downarrow$$

$$a(t) \sim t^{\frac{2}{(D-1)(1+K)}}$$

$$\sigma \left(\int \frac{dt}{a(t)}\right)^{D-1} \leq a^{D-2}(t) \left(\int \frac{dt}{a(t)}\right)^{D-2}$$

$$\downarrow$$

$$\boxed{K \leq 1}$$

$K = 1 \longrightarrow \dfrac{\text{ENTROPY}}{\text{AREA}}$ INDEPENDENT OF t !

$K < 1 \longrightarrow \dfrac{\text{ENTROPY}}{\text{AREA}} \searrow \quad t \nearrow$

$\dfrac{\text{ENTROPY}}{\text{AREA}} \nearrow \quad t \searrow$

HEURISTIC PICTURE OF

$p = \rho$ DOMINATED UNIVERSE.

IMAGINE ONE BLACK HOLE/HORIZON

WHOSE SIZE SCALES WITH

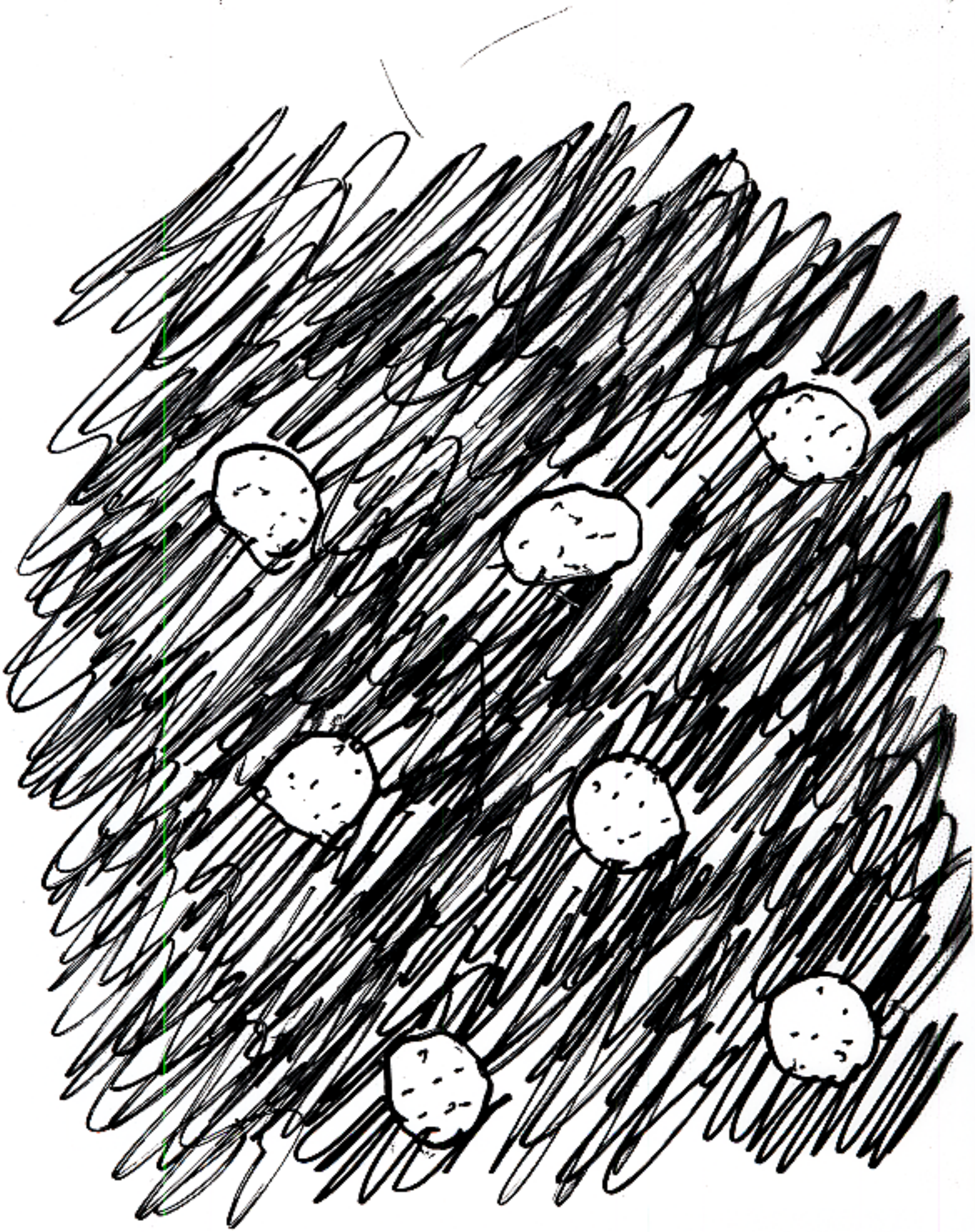
HORIZON

$$\rho = n_{BH} M_{BH} \sim n_{BH} R^{D-3}$$

$$\rho \sim \frac{R^{D-3}}{R^{D-1}} \sim \frac{1}{R^2}$$

$$s = n_{BH} R^{D-2} \sim \frac{R^{D-2}}{R^{D-1}} \sim \frac{1}{R}$$

$$\longrightarrow \quad s = \sqrt{\rho}$$



$$\rho_{1/3} \sim \frac{1}{V^{4/3}} \qquad a \sim t^{1/2}$$

$$\rho_1 \sim \frac{1}{V^2} \qquad a \sim t^{1/3}$$

EVENTUALLY UNIVERSE DOMINATED BY "NORMAL" REGION (RADIATION

HERE AND THERE LARGE BLACK HOLE WHICH EVENTUALLY EVAPORATE REHEATING UNIVERSE

4) FLUCTUATION SPECTRUM

$$ds^2 = dt^2 - t^{2/3} d\bar{x}^2$$

CONFORMAL TIME: $\tau \sim t^{2/3}$

$$\mathcal{L}(\varphi) = \sqrt{g}\, g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$H = \sum_k \frac{p_k^2}{2} + \frac{k^2 \chi_k^2}{2} + \frac{\chi_k^2}{8\tau^2}$$

$$(\chi_k = \sqrt{\tau}\, \varphi_k)$$

GRAVITONS $\equiv$ MINIMALLY COUPLED SCALARS

$$i \frac{\partial}{\partial \tau} \psi = H \psi$$

$$\psi = \prod_k \psi_k (x_k, \tau)$$

SCHRODINGER EQUATION INVARIANT

$\tau \to \lambda \tau$ KEEPING $k\tau$ FIXED

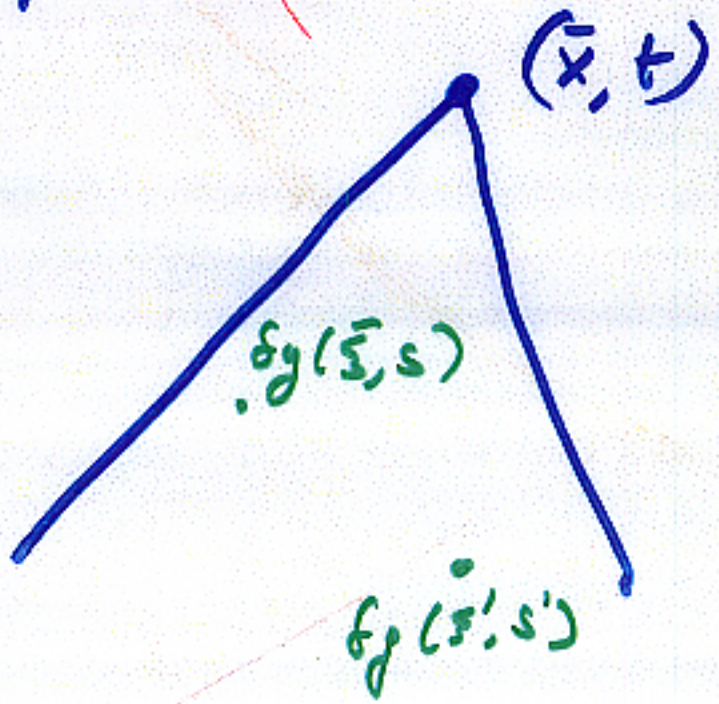
$x \to \sqrt{\lambda}\, x$ $kt^{2/3}$ FIXED

φ FIXED

$$|\psi_k(\varphi_k, t)|^2 = \rho(\varphi_k, kt^{2/3})$$

$$\sim e^{-f_k(\tau)\varphi_k^2}$$

$$\langle \delta\rho^2(\bar{x},t)\rangle$$



$$\delta\rho(\bar{x},t) = \int^{T} d\bar{\xi}\, ds\; f(\bar{x}-\bar{\xi},s)\,\delta g(\bar{\xi},\bar{s})$$

$$\langle \delta\rho_{\bar{k}}\, \delta\rho_{-\bar{k}}\rangle \sim \frac{1}{k^3}$$

⚠ PROBLEM

SCALE INVARIANCE

DOES NOT EXTEND

TO SCALE ~ SIZE OF

UNIVERSE

(CAUSALITY ?!?)

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