Ruth Gregory, Durham CPT

Braneworlds

COSMO-02, Chicago, September 2002

Outline:

① BASIC BRANEWORLDS, LOCALIZATION & GRAVITY.

What is a braneworld?
Why all the interest?
How does gravity work? (Randall-Sundrum)

② COSMOLOGICAL BRANEWORLDS

Cosmological solns ⇔ moving walls

Higher codimension soles.

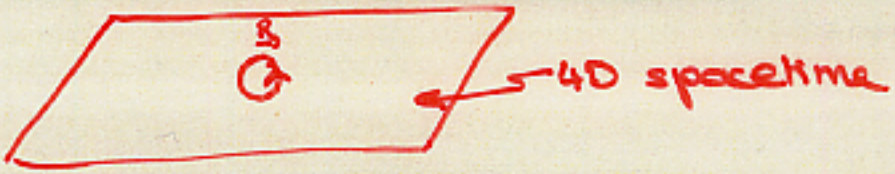
③ BRANEWORLDS WITHIN STRING/M-THEORY

Heterotic M-cosmology

Ekpyrosis

① Extra Dimensions and G_N.

Standard KK theory gives a direct geometric representation of U(1)em:



Gravity on $\mathbb{R}^4 \times S^1$ splits into

graviton	$(\mathbb{R}^4, \mathbb{R}^4)$
photon	$(\mathbb{R}^4, S^1)$
dilaton	(S^1, S^1)

Taking the 5D action and integrating out:

$$M_5^3 \int d^5x \sqrt{g_5} R_5 \rightarrow \underbrace{2\pi R M_5^3}_{M_{pl}^2} \int d^4x \sqrt{g_4} R_4 \ldots\ldots$$

* 4D Planck mass depends on size/volume of internal dimensions as well as Planck

Arkani-Hamed
Dimopoulos
Dvali
Antoniadis

$$M_{pl}^2 \sim M_{4+n}^{2+n} R^n$$

or $$R \sim 10^{\frac{32}{n} - 17} M_{(TeV)}^{-\frac{2+n}{n}} \text{ cm.}$$

- Two extra dimensions compactified at the mm scale can generate a hierarchy of ~16 orders of magnitude between M_{pl} & M_6.

How do we see the extra dimensions?

- Corrections to Newton's law ($R \lesssim 0.2$ mm)
- Missing energy in reactions, e.g.

$e^+e^- \to \gamma G$ or ZG

$$\sigma \sim \frac{\alpha}{M_{pl}^2} N(E) \quad \text{(ER)}$$

$$\sim \frac{\alpha}{E^2} \left(\frac{E}{M_{4+n}}\right)^{n+2}$$

$M_6 >$ TeV (LEP)

30 TeV (SN1987a + cosmo bounds)

② LIVING ON A BRANEWORLD

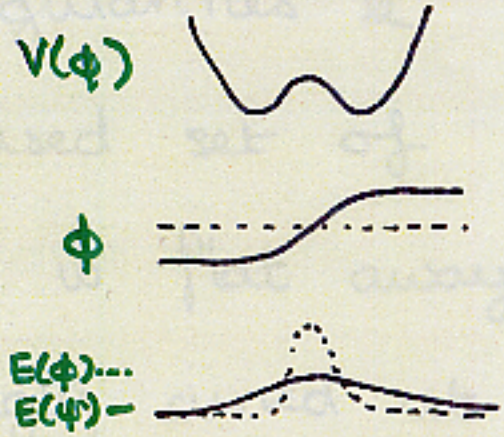
Q: How can we not be aware of 'macroscopic' extra dimensions?

A: We need to be localized on a much smaller scale.

e.g. DOMAIN WALL (Rubakov & Shaposhnikov)

$\lambda\phi^4$ kink coupled to scalars / fermions

$$\mathcal{L}_\Psi = i\bar{\Psi}\Gamma^a\partial_a\Psi - g\phi\bar{\Psi}\Psi$$



gives 4D massless chiral fermion zero modes

$$\Psi = \psi(x^\mu)\left[\operatorname{sech}\frac{z}{\sqrt{2}\eta}\right]^{\sqrt{2}\eta^2 g} \qquad (\psi = -\gamma^5\psi)$$

* Localization comes from condensates in a solitonic background.

Gravity on a single brane is 4D

Einstein gravity (+KK):

$$h_{\mu\nu} = \int \kappa D_0(x,x') \left[T_{\mu\nu} - \tfrac{1}{3}T\eta_{\mu\nu}\right] \quad \text{tensor zero mode}$$

$$+ \int dz' \int dm\, N_m D_m(x,x') [\ \text{"}\ ] \quad \text{massive KK ctm}$$

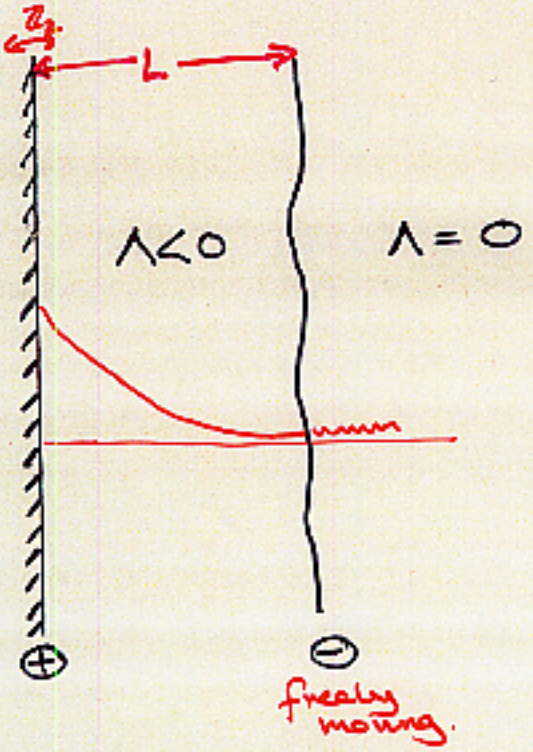
$$+ \ 2\kappa a^2 \eta_{\mu\nu} \int D_0(x,x') \frac{T}{6} \quad \text{(gauge) position of brane}$$

Garriga + Tanaka

Although at first sight the tensor structure of the propagator looks 5D, the position of the brane is not a physical observable and gives a compensating term cancelling off the 5D part.

* Adding a second brane adds a scalar THE RADION, representing brane seperation.

Variations on a theme - modified gravity.



RG, Rubakov+Sibiryakov

Gravity is quasi-localised - 4D on intermediate scales but 5D on large scales. The 'radion' or brane bending term gives a ghost scalar at large scales. Taking $L \to 0$ removes this problem giving the DGP braneworlds in Minkowski space.

BRANE WORLD COSMOLOGY:

Recall that in 4D, ISOTROPY & HOMOGENEITY ⇒ a very simple form for the metric of our U.

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega_{II}^2\right]$$

- dt^2: cosmic time
- $a^2(t)$: scale factor
- $\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega_{II}^2\right]$: const. curv. 3-space
  - $\kappa = 0$ FLAT
  - $\kappa = \pm 1$ CLOSED (S^3) OPEN (H^3)

With a simple form for the energy-momentum tensor

$$T^a{}_b = \text{diag}(\rho, -p, -p, -p)$$

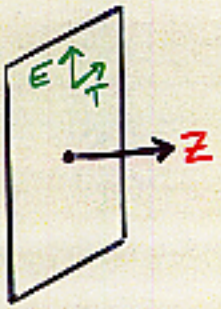
The U obeys the Friedmann eqn:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2}$$

and an energy conservation eqn:

A brane cosmology soln will also have these symmetries parallel to the brane with a similar energy-momentum

$$T^a{}_b \sim \delta(z)\,(\underbrace{\lambda+\rho}_{E},\ \underbrace{\lambda-\rho}_{T},\ \lambda-\rho,\ \lambda-\rho,\ 0)$$



Cosmological soln:
Wall at $z=0$, energy E & tension T. Constant curv spatial sections.

Expect t & z dependence in metric

$$ds^2 = e^{2\nu(t,z)} B^{-2/3}(t,z)\,[dt^2 - dz^2] - B^{2/3}(t,z)\,dx_k^2$$

Easiest to use lightcone gauge for Einstein eqns

$u = (t-z)/2$
$v = (t+z)/2$
$\Lambda = -6k^2$

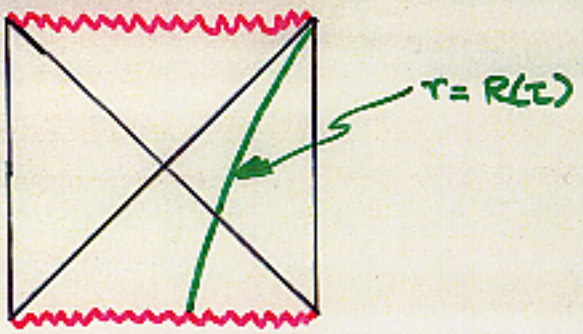
$$B_{,uv} = 6e^{2\nu}\,[2k^2 B^{1/3} - \kappa B^{-1/3}]$$

$$\nu_{,uv} = e^{2\nu}\,[2k^2 B^{-2/3} + \kappa B^{-4/3}]$$

$$B_{,u|v}\left\{\mp \ln B_{,u|v}\right]_{,u|v} = 2\nu_{,u|v}\right\}$$

The most general cosmology consists of patches of Sch-ads glued across the braneworld boundaries.

$$ds^2 = \left(\kappa + k^2 r^2 - \frac{\mu}{r^2}\right)dt^2 - \frac{dr^2}{(\;)} - r^2 d\Sigma_k^2$$



The Israel matching eqns give the analogue of a Friedmann eqn

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{E^2}{36} + \frac{9(\Delta k^2)^2}{4E^2} - \overline{k^2} - \frac{\kappa}{R^2} + \frac{\overline{\mu}}{R^4} - \frac{\Delta k^2 \Delta\mu}{2E^2R^4} + \frac{9(\Delta\mu)^2}{4E^2R^8}$$

for the induced metric

$$ds^2 = d\tau^2 - R^2(\tau)\, d\Sigma_k^2$$

COSMOLOGY ↔ MOVING BRANES.

Taking $\mathbb{Z}_2$ symmetry and writing

$E = 6R + \rho$ gives the nonconventional cosmological eqns of Binetruy et al:

$$\left(\frac{\dot{R}}{R}\right)^2 = \underbrace{\frac{k\rho}{3} - \frac{\kappa}{R^2}}_{\text{standard FRW}} + \underbrace{\frac{\rho^2}{36}}_{\text{NON-CONV}^{\text{L}}} + \underbrace{\frac{M}{R^4}}_{\substack{\text{Holographic/} \\ \text{Weyl term}}}$$

- example of braneworlds and holography.

Pure domain wall in Schwarzschild AdS looks like a radiation cosmology. Gubser $H^2 \propto 1/R^4$ Verlinde

A careful calculation of ρ_{CFT} & P_{CFT} however gives a nonstandard eqn of state and a nonconventional cosmology!

J. Gregory + Padilla

HIGHER CODIMENSION BRANEWORLDS

($n \geq 3$) Main difference is that gravity can propagate in more than 2 spatial dimensions, or **BRANEWORLDS CURVE SPACE.**

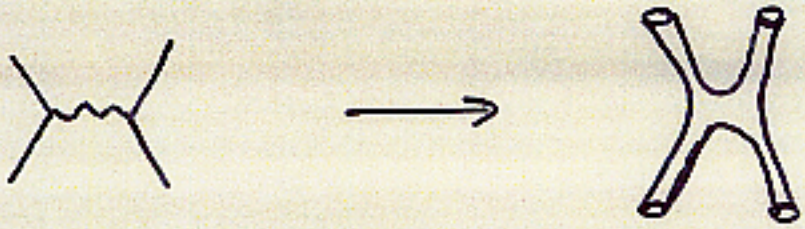
- Can find exact solns which have singular horizons, yet particle wavefns in the bulk are well-defined.
- Explicit core models seem desirable, yet messy!
- Cosmological braneworlds problematic

$n=2$: Can use the global vortex to construct a braneworld with 'diffusely' localized gravity

- Fine tuning not necessary – can have cosmological braneworlds.

④ STRINGS, BRANES & M-THEORY.

Branes occur v. naturally in String Theory



Gravity now corresponds to a closed string excitation

$g_{\mu\nu}$, $B_{\mu\nu}$, ϕ NSNS

TypeIIA

A_μ $C_{\mu\nu\lambda}$

RR

TypeIIB

ϕ' $B'_{\mu\nu}$ $F^{(4D)}_{\mu\nu\lambda\rho\sigma}$

Low energy string theory ↔ SUGRA

?∃ BRANE-LIKE SUGRA SOLNS?

There are many solutions of SUGRA which are charged (electrically or magnetically) under the various form fields.

Gibbons, Maeda
Horowitz, Strominger

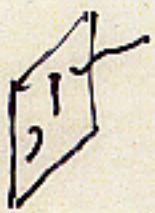
e.g. $B_{\mu\nu}$: E H_{ozr} — FUNDAMENTAL STRING

M $H_{o\varphi\psi}$ — NS 5-BRANE

A_μ : E F_{or} — 0-BRANE / 'BLACK HOLE'

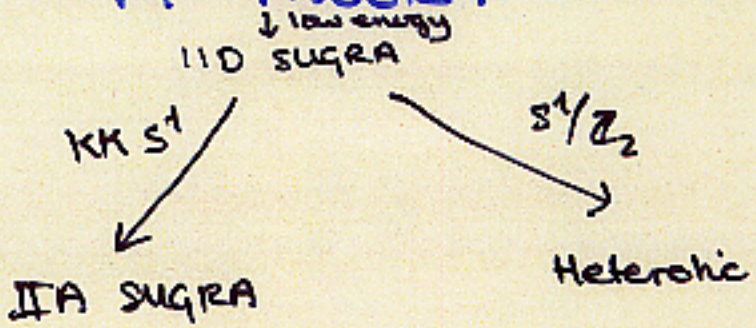
M $F_{o\varphi}$ — 6-BRANE

Those charged under RR fields are 'classical' SUGRA D-branes. A D-brane is an ending plane for open strings.



D branes have condensates & zero-modes (open strings carry gauge fields),

A web of dualities relate the 10D string theories, and it is believed that they are all derivable from an underlying (as yet unknown) 11D theory -



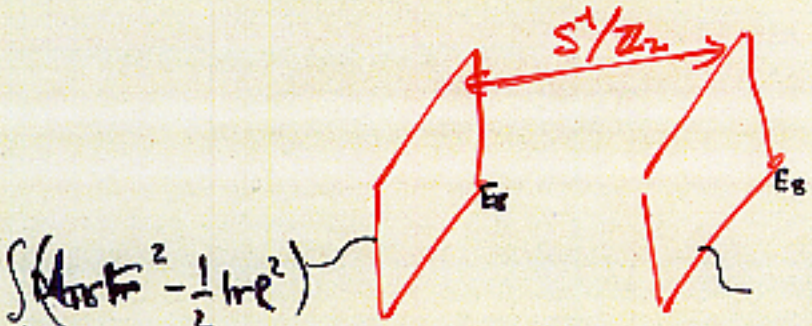
HETEROTIC M-COSMOLOGY

Lukas
Ovrut
Waldram
Stelle

Uses 11D SUGRA

$$S \sim \int \sqrt{-g}\left[-R - F_{IJKL}^2\right] - C \wedge F \wedge F$$

compactified on an interval



which modifies the Bianchi identity for F_{IJKL}

$$dF \sim J^{(1)}\delta(x^{11} - x^{(1)}) + J^{(2)}\delta(x^{11} - x^{(2)})$$

$$\downarrow$$

$$\mathrm{tr} F^{(1)} \wedge F^{(1)} - \frac{1}{2}\mathrm{tr} R \wedge R$$

Nonzero source terms correspond to M5 branes wrapped on the CY.

Having these 5-branes modifies the bulk soln. See this by looking at effective 5D action:

$$ds_{11}^2 = \underbrace{V^{-2/3} g_{\alpha\beta} dx^\alpha dx^\beta}_{\text{5D "RS" UNIVERSE}} + \underbrace{V^{1/3} \Omega_{AB} dx^A dx^B}_{CY}$$

$$F_{ABCD} \sim \frac{\alpha}{6} \epsilon_{ABCD}{}^{EF} \omega_{EF}(x^{11})$$ harmonic (2,2) form

zero modes:

$g_{\alpha\beta}$ graviton

V dilaton

$C_{\alpha\beta\gamma}$ $G_{\alpha\beta\gamma 8}$

$C_{\alpha AB} = \frac{1}{6} A_\alpha \omega_{AB}$ → $\mathcal{F}_{\alpha\beta}$

$C_{ABC} = \frac{1}{6} \xi \omega_{ABC}$ → $\partial\xi$.

SOLN: $V = \sqrt{2}\alpha z$

$a = a_0 z^{1/6}$

Randall – Sundrum

$S^1/\mathbb{Z}_2$

⊕ wall, ⊖ wall

Bulk Λ

Scalar tensor gravity
s.m. structure
Solns integrable

Lukas et al.

$S^1/\mathbb{Z}_2$

⊕ wall ⊖ wall

Bulk V

$T^0_0 = \frac{1}{8} z^{-2}$ $T^5_5 = -\frac{1}{6} z^{-2}$

Loads of z-modes
no "uniqueness"

Conclusion : RS a useful tool – but

just that – a tool.

FIRE WARS

EPISODE N: ATTACK OF THE BRANES

The ekpyrotic scenario takes the set-up of heterotic M-cosmology and adds

- Moving branes
- Nonperturbative interbrane potential
- Brane wrinkles → perturbations.

"WISH LIST"

- Modification of gravity at large scales. – Can we find compelling and consistent ways of changing gravity at v. large scales as an alternative to Λ & q-ess?
- Holography and quantum black holes.

– An exact classical b.h. / bulk solution ["C-metric"] would give us a quantum corrected black hole on the brane. Emparan, Fabbri + Kaloper

- Cosmological solns in String Theory. Do we live on a D-brane? How do we get de-Sitter space? – a real challenge for string theorists.