Cosmo – 02

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Models of inflation and primordial perturbations

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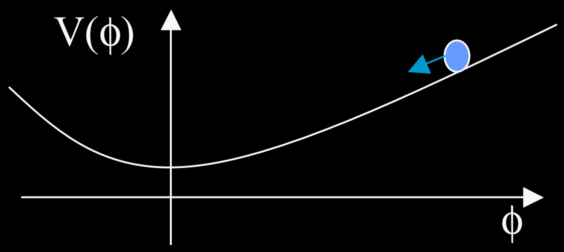
Cosmological inflation:

Starobinsky (1980)

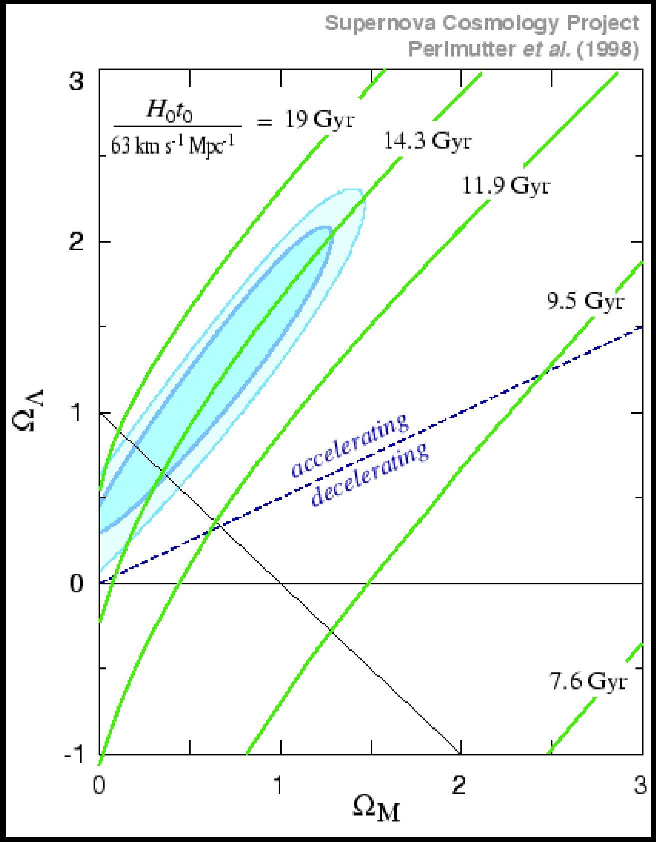
Guth (1981)

- period of accelerated expansion in the very early universe
- requires negative pressure

  e.g. self-interacting scalar field

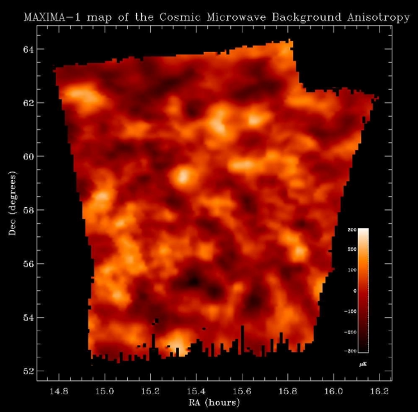


- speculative and uncertain physics
- just the kind of peculiar cosmological behaviour we observe today



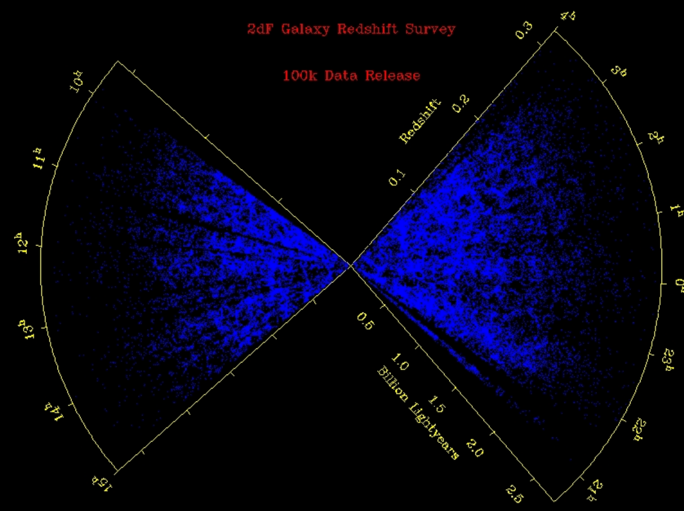
Models of inflation:

- inflation in the very early universe
- observed through primordial perturbation spectra
    - gravitational waves *(we hope)*
    - matter/radiation perturbations



gravitational

instability



new observational data offers precision tests of cosmological parameters and the nature of the primordial perturbations

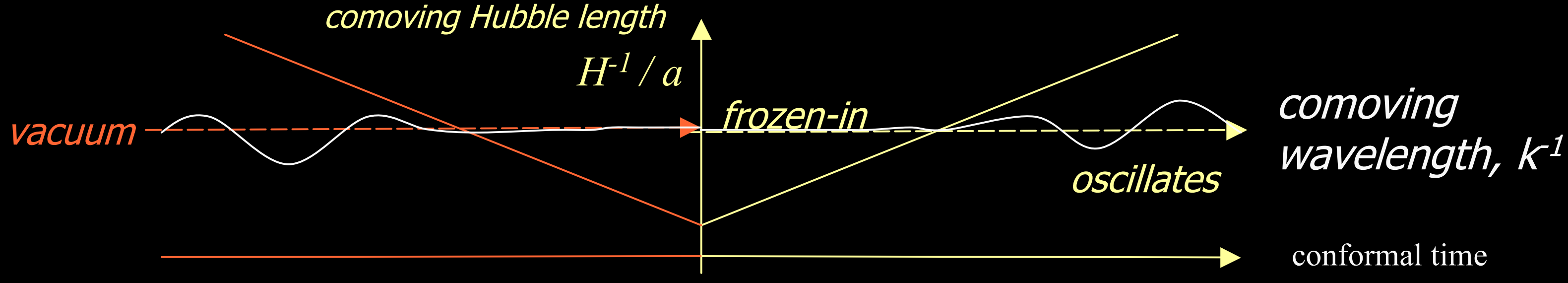
Perturbations in FRW universe:

wave equation $$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2 \delta\phi = 0$$

Characteristic timescales for comoving wavemode k

- *oscillation period/wavelength* a / k
- *Hubble damping time-scale* H^{-1}

- *small-scales* $k > aH$ *under-damped oscillator*
- *large-scales* $k < aH$ *over-damped oscillator*



inflation

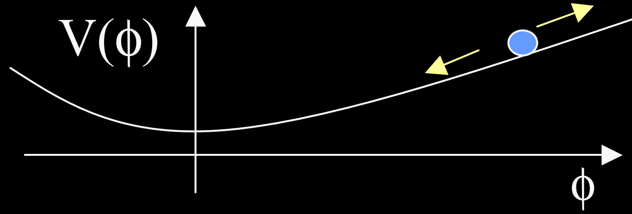
accelerated expansion
(or contraction)

radiation or matter era

decelerated expansion

Vacuum fluctuations

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations* $\delta\phi_k \approx \frac{e^{-ik\eta}}{\sqrt{2k}}$
- *large-scale/overdamped perturbations in growing mode*

  linear evolution $\Rightarrow$ *Gaussian random field*

$$\left\langle \delta\phi^2 \right\rangle_{k=aH} \approx \frac{4\pi k^3 \left|\delta\phi_k{}^2\right|}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any light fields (m<3H/2) `frozen-in' on large scales

*** assumes Bunch-Davies vacuum on small scales ***

probe of trans-Planckian effects for $k/a > M_{Pl}$ Brandenberger & Martin (2000)

effect likely to be small for $H << M_{Pl}$ Starobinsky; Niemeyer; Easther et al (2002)

Inflation -> matter perturbations

for adiabatic perturbations on super-horizon scales $\dot{R} = 0$

during inflation

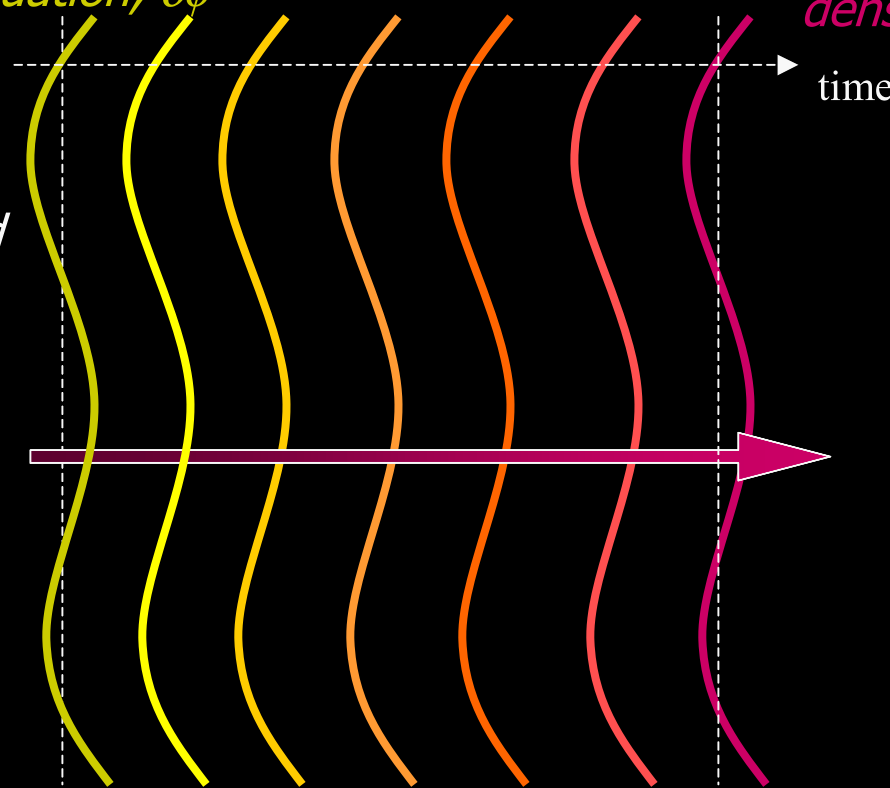
scalar field fluctuation, $\delta\phi$

during matter+radiation era

density perturbation, $\delta\rho$

scalar curvature on uniform-field hypersurfaces

$$R = \frac{H\delta\sigma}{\dot{\sigma}}$$



scalar curvature on uniform-density hypersurfaces

$$R = \frac{H\delta\rho}{\dot{\rho}}$$

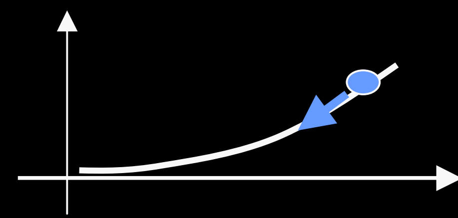
Single-field models:

see, e.g., Lyth & Riotto
Kinney, Melchiorri & Riotto (2001)

high energy / not-so-slow roll

1. **large field** ($\Delta\varphi < M_{Pl}$)

   e.g. chaotic inflation

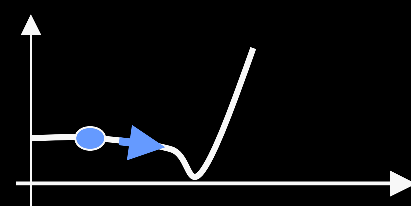


$$0 < \eta < \varepsilon$$

not-so-high energy / very slow roll

2. **small field**

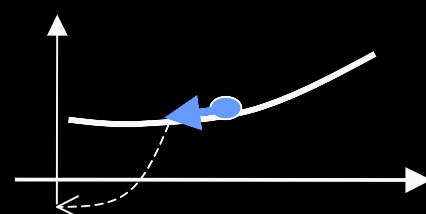
   e.g. new or natural inflation



$$\eta < 0$$

3. **hybrid inflation**

   e.g., susy or sugra models



$$0 < \varepsilon < \eta$$

slow-roll solution for potential-dominated, over-damped evolution gives useful approximation to growing mode for *{ ε , $|\eta|$ } << 1*

$$\varepsilon \equiv \frac{M_P^{\ 2}}{16\pi}\left(\frac{V_\phi}{V}\right)^2 \approx -\frac{\dot{H}}{H^2} \qquad \eta \equiv \frac{M_P^{\ 2}}{8\pi}\left(\frac{V_{\phi\phi}}{V}\right) = \frac{m^2}{H^2}$$

can be distinguished by observations

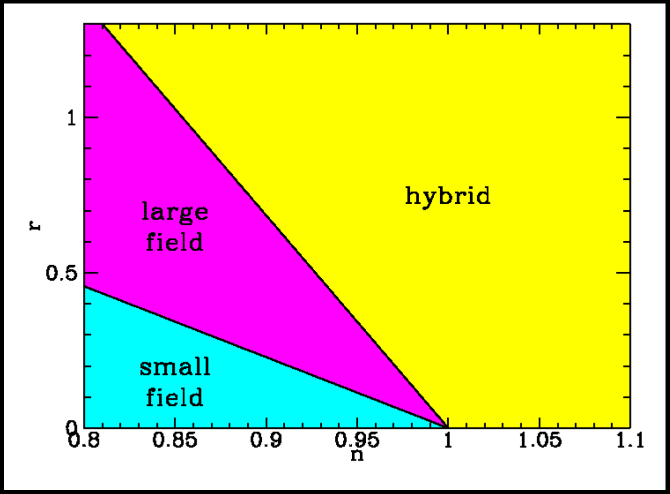
- slow time-dependence during inflation

  -> weak scale-dependence of spectra

$$n = 1 - 6\varepsilon + 2\eta$$

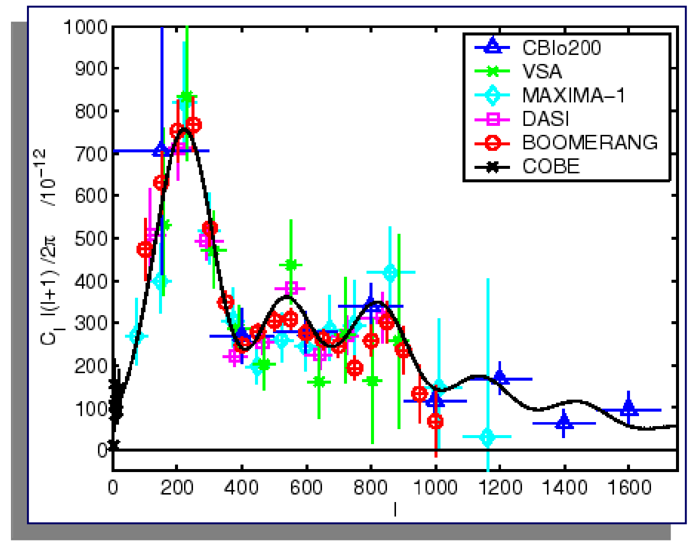
- tensor/scalar ratio suppressed at low energies/slow-roll

$$\frac{\langle T^2 \rangle}{\langle R^2 \rangle} = 16\varepsilon$$



Observational constraints

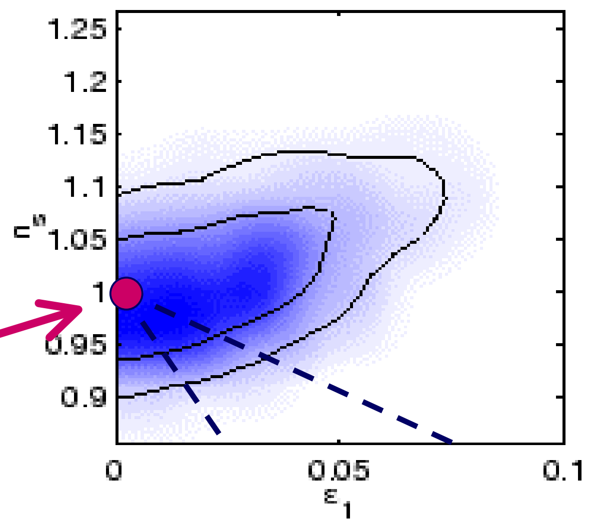
- Microwave background +2dF + BBN + Sn1A + HST



Lewis & Bridle (2002)

spectral index

$$n \approx 1 - 6\varepsilon + 2\eta$$



$$\frac{tensor}{scalar} \approx \varepsilon \quad < 0.08$$

Harrison-Zel'dovich

$$\boldsymbol{n \to 1, \quad \varepsilon \to 0}$$

digging deeper:

- additional fields may play an important role
  - initial state — new inflation
  - ending inflation — hybrid inflation
  - enhancing inflation — assisted inflation

    warm inflation

    brane-world inflation
- may yield additional information
  - **non-gaussianity**
  - **isocurvature (non-adiabatic) modes**

Inflation -> primordial perturbations (II)

scalar field fluctuations

two fields (σ,χ)

curvature of uniform-field slices

density perturbations

matter and radiation (m,γ)

curvature of uniform-density slices

$$R_* = \frac{H\delta\sigma}{\dot\sigma} \longrightarrow R = \frac{H\delta\rho}{\dot\rho}$$

$$\dot R = \alpha H S,$$

isocurvature

$$\dot S = \beta H S$$

$$S_* = \frac{H\delta\chi}{\dot\sigma} \longrightarrow S = \frac{\delta n_m}{n_m} - \frac{\delta n_\gamma}{n_\gamma}$$

$$\begin{pmatrix} R \\ S \end{pmatrix}_{\substack{\text{primordial} \\ k<<aH}} = \begin{pmatrix} 1 & T_{RS} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} R_* \\ S_* \end{pmatrix}_{\substack{\text{inflation} \\ k=aH}}$$

model-dependent transfer functions

initial power spectra:

$$\left\langle \delta\sigma_*^{\,2} \right\rangle = \left\langle \delta\chi_*^{\,2} \right\rangle = \left(\frac{H}{2\pi} \right)_*^2$$

$$\left\langle R_*^{\,2} \right\rangle = \left\langle S_*^{\,2} \right\rangle = \left(\frac{H^2}{2\pi\dot{\sigma}} \right)_*^2$$

uncorrelated at horizon-crossing $\left\langle \delta\sigma_* \, \delta\chi_* \right\rangle = \left\langle R_* \, S_* \right\rangle = 0$

primordial power spectra:

$$\langle R^2 \rangle = \langle R_*{}^2 \rangle + T_{RS}{}^2 \langle S_*{}^2 \rangle$$

$$\langle S^2 \rangle = T_{SS}{}^2 \langle S_*{}^2 \rangle$$

$$\langle R\,S \rangle = T_{RS} T_{SS} \langle S_*{}^2 \rangle$$

correlated! *Langlois (1999)*

correlation angle measures contribution of non-adiabatic modes to primordial curvature perturbation

$$\cos\theta = \frac{\langle RS \rangle}{\langle R^2 \rangle^{1/2} \langle S^2 \rangle^{1/2}} = \frac{T_{RS}}{\sqrt{1 + T_{RS}{}^2}}$$

can reconstruct curvature perturbation at horizon-crossing

$$\langle R_*{}^2 \rangle = \sin^2\theta \, \langle R^2 \rangle$$

consistency conditions:

gravitational waves:

$$\left\langle T^2 \right\rangle = \left\langle T_*^{\,2} \right\rangle = 32\pi \left(\frac{H}{M_{Pl}} \right)^2 = 16\,\varepsilon \left\langle R_*^{\,2} \right\rangle$$

$$n_T^{\;2} = -2\varepsilon$$

single-field inflation:

Liddle and Lyth (1992)

$$\left\langle R^2 \right\rangle = \left\langle R_*^{\,2} \right\rangle \quad \Rightarrow \quad \frac{\left\langle T^2 \right\rangle}{\left\langle R^2 \right\rangle} = \frac{\left\langle T_*^{\,2} \right\rangle}{\left\langle R_*^{\,2} \right\rangle} = -8n_T$$

multi-field inflation:

Sasaki & Stewart (1996)

$$\left\langle R^2 \right\rangle > \left\langle R_*^{\,2} \right\rangle \quad \Rightarrow \quad \frac{\left\langle T^2 \right\rangle}{\left\langle R^2 \right\rangle} < -8n_T$$

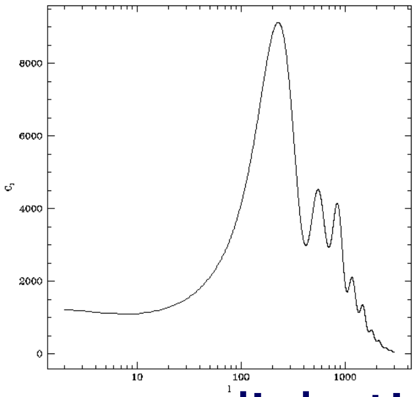
two-field inflation:

Wands, Bartolo, Matarrese & Riotto (2002)

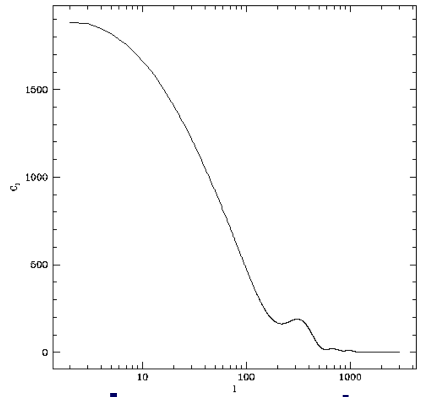
$$\frac{\left\langle T^2 \right\rangle}{\left\langle R^2 \right\rangle} = \frac{\left\langle T_*^{\,2} \right\rangle}{\left\langle R_*^{\,2} \right\rangle} \sin^2\theta = -8n_T \sin^2\theta$$

microwave background predictions

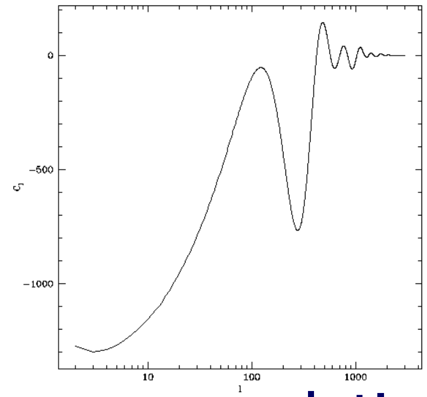
$$C_l = A^2 \times \qquad + B^2 \times \qquad + 2\,A\,B\cos\Delta \times$$



adiabatic



isocurvature



correlation

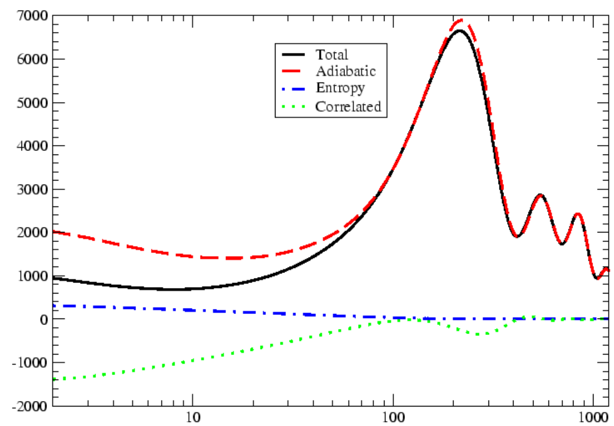
Bucher, Moodley & Turok '00
Trotta, Riazuelo & Durrer '01 $\Big\}$ $n_S = 1$

Amendola, Gordon, Wands & Sasaki '01

best-fit to Boomerang, Maxima & DASI

$B/A = 0.3$, $\cos\Delta = +1$, $n_S = 0.8$

$\omega_b = 0.02$, $\omega_{cdm} = 0.1$, $\Omega_\Lambda = 0.7$



curvaton scenario:

Lyth & Wands, Moroi & Takahashi, Enqvist & Sloth (2002)

assume negligible curvature perturbation during inflation $\left\langle R_*^{\ 2} \right\rangle = 0$

light during inflation, hence acquires isocurvature spectrum $\left\langle S_*^{\ 2} \right\rangle \approx \dfrac{\delta\rho_\chi}{\rho_\chi}$

late-decay, hence energy density non-negligible at decay $T_{RS} \approx \Omega_{\chi,decay}$

large-scale density perturbation
generated entirely by
non-adiabatic modes
after inflation

$$\left\langle R^2 \right\rangle = T_{RS}^{\ 2} \left\langle S_*^{\ 2} \right\rangle \approx \Omega^2_{\chi,decay} \left\langle (\delta\rho_\chi / \rho_\chi)^2 \right\rangle$$

- negligible gravitational waves
- 100%correlated residual isocurvature modes
- detectable non-Gaussianity if $\Omega_{\chi,decay} << 1$

primordial non-gaussianity

simple definition in terms of conformal Newtonian potential $\Phi=3R/5$

$$\Phi = \Phi_{Gauss} + f_{NL}\left(\Phi_{Gauss}{}^2 + \left\langle \Phi_{Gauss}{}^2 \right\rangle\right)$$

Komatsu & Spergel (2001)

also Wang & Kamiokowski (2000)

where $f_{NL} = 0$ for linear evolution

significant constraints on f_{NL} from all-sky microwave background surveys

- COBE $f_{NL} < 2000$
- MAP $f_{NL} < 20$
- Planck $f_{NL} < 5$

theoretical predictions

- non-linear inflaton dynamics constrained $f_{NL} < 1$ Gangui et al '94
- dominant effect from second-order metric perturbations? Acquaviva et al (2002)
- but non-linear isocurvature modes could allow $f_{NL} >> 1$ Bartolo et al (2002)
- e.g., curvaton $f_{NL} \approx 1 / \Omega_{\chi,decay}$ Lyth, Ungarelli & Wands (2002)

Conclusions:

1. ***Observations*** of ***tilt*** of density perturbations $(n \neq 1)$ and ***gravitational waves*** $(\varepsilon > 0)$ can distinguish between slow-roll models
2. ***Isocurvature*** perturbations and/or ***non-Gaussianity*** may provide valuable info
3. ***Non-adiabatic*** perturbations (off-trajectory) in multi-field models are an additional source of curvature perturbations on large scales
4. ***Consistency relations*** remain an important test in multi-field models - can falsify slow-roll inflation