Parton Distribution Functions

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The proton is described by QCD – the theory of the strong interactions. This makes an understanding of its structure a difficult problem.

However, it is also a very important problem – not only as a question in itself, but also in order to search for and understand new physics.

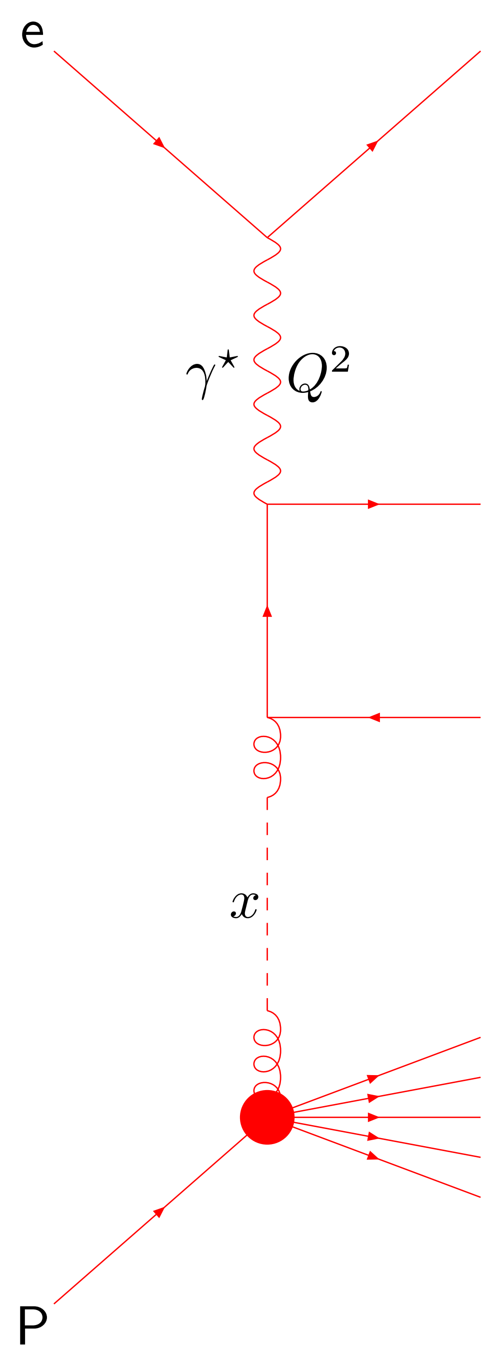
Many important particle colliders use hadrons – HERA is an ep collider, the Tevatron is a $p\bar{p}$ collider, the LHC (large hadron collider) at CERN will be a pp collider. An understanding of proton structure is essential in order to interpret the results.

Fortunately, when one has a relatively large scale in the process, in practice only $> 1\mathrm{GeV}^2$, the proton is essentially made up of the more fundamental constituents – quarks and gluons (partons), which are relatively independent.

Hence, the fundamental quantities one requires in the calculation of scattering processes involving hadronic particles are the **parton distributions**.

These can be derived from, and then used within, the **Factorization Theorem** – separates processes into nonperturbative parts which can be determined from experiment, and perturbative parts which can be calculated as a power-series in the strong coupling constant α_S.

Hadron scattering with an electron factorizes.



$\alpha_s(Q^2)$ – Strong Coupling

$x = \frac{Q^2}{2m\nu}$ – Momentum fraction of Parton (ν=energy transfer)

The cross-section for this process can be written in the factorized form

$$\sigma(ep \to eX) = \sum_i C_i^P(x, \alpha_s(Q^2)) \otimes f_i(x, Q^2, \alpha_s(Q^2))$$

where $f_i(x, Q^2, \alpha_s(Q^2))$ are the parton distributions, i.e the probability of finding a parton of type i carrying a fraction x of the momentum of the hadron.

Corrections to above formula of size $\Lambda^2_{\mathrm{QCD}}/Q^2$ - Higher Twist.

The parton distributions are not easily calculable from first principles. However, they do evolve with Q^2 in a perturbative manner

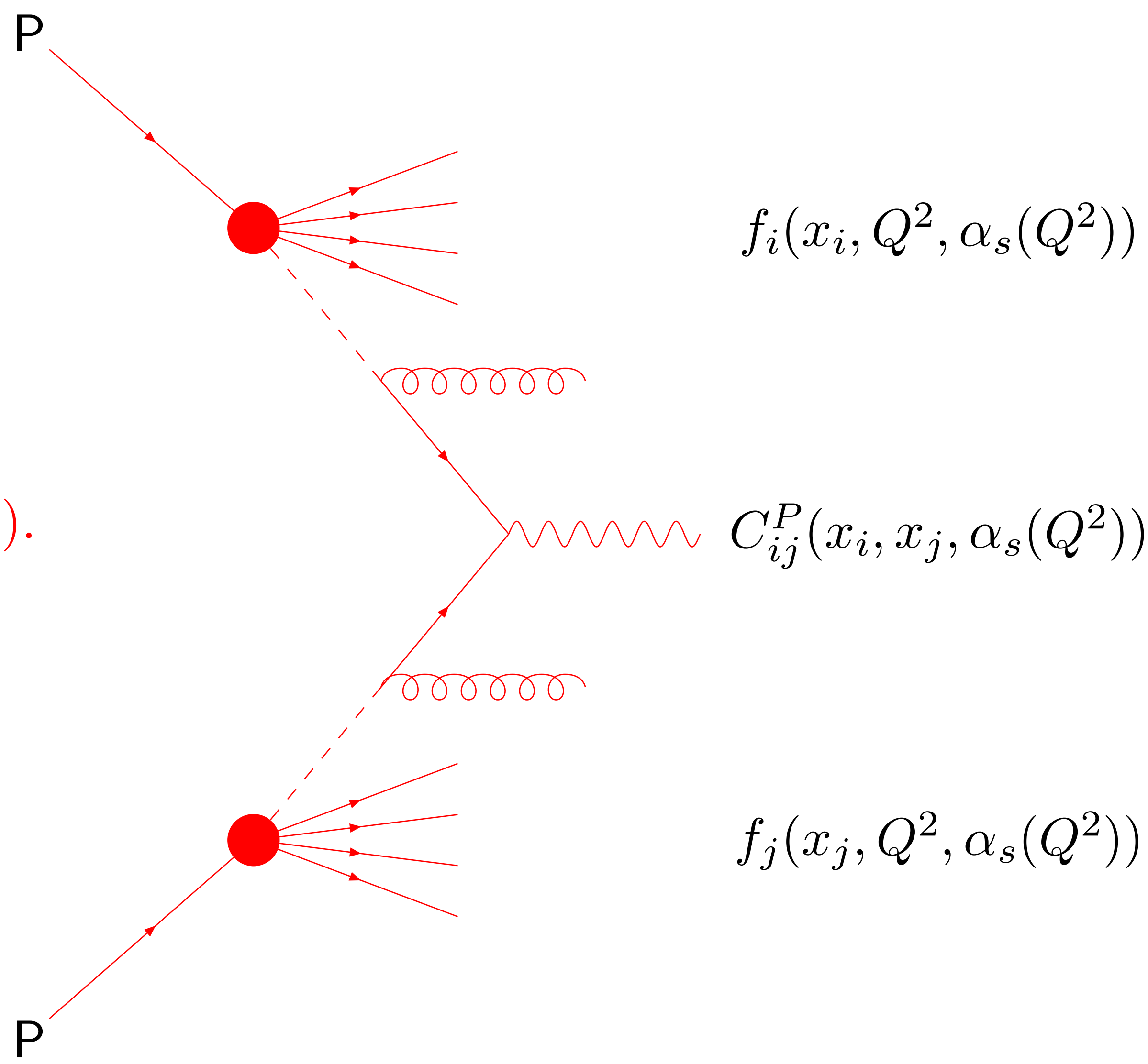
$$\frac{df_i(x, Q^2, \alpha_s(Q^2))}{d\ln Q^2} = \sum_i P_{ij}(x, \alpha_s(Q^2)) \otimes f_j(x, Q^2, \alpha_s(Q^2))$$

where the splitting functions $P_{ij}(x, Q^2, \alpha_s(Q^2))$ are calculable order by order in perturbation theory.

The coefficient functions $C_i^P(x, \alpha_s(Q^2))$ describing the hard scattering process are process dependent but are calculable as a power-series.

$$C_i^P(x, \alpha_s(Q^2)) = \sum_k C_i^{P,k}(x)\alpha_s^k(Q^2).$$

Since the $f_i(x, Q^2, \alpha_s(Q^2))$ are process-independent, i.e. **universal**, once they have been measured at one experiment, one can predict many other scattering processes.



Global fits to determine parton distributions use all available data - largely $ep \rightarrow eX$ (Structure Functions), and the most up-to-date QCD calculations to best determine **parton distributions** and their consequences. (Also $\rightarrow$ good determination of strong coupling constant.)

Currently use NLO–in–$\alpha_s(Q^2)$, i.e.

$$\begin{aligned} C_i^P(x,\alpha_s(Q^2)) &= \alpha_S^P(Q^2)(C_i^{P,0}(x)+\alpha_S(Q^2)C_i^{P,1}(x)). \\ P_{ij}(x,\alpha_s(Q^2)) &= \alpha_S(Q^2)P_{ij}^0(x)+\alpha_S^2(Q^2)P_{ij}^1(x). \end{aligned}$$

NNLO coefficient functions are known for some processes, e.g. structure functions, and NNLO splitting functions have considerable information (see later).

General procedure. Start parton evolution at low scale $Q_0^2 \sim 1\mathrm{GeV}^2$. Input partons parameterized as, e.g.

$$xf(x,Q_0^2) = a_1(1-x)^{a_2}(1+a_3x^{0.5}+a_4x)x^{a_5}.$$

Evolve partons upwards using NLO DGLAP equations. Fit data for scales above $2-5\mathrm{GeV}^2$.

In principle 11 different parton distributions to consider

$$u, \bar{u}, \quad d, \bar{d}, \quad s, \bar{s}, \quad c, \bar{c}, \quad b, \bar{b}, \quad g$$

$m_c, m_b \gg \Lambda_{\rm QCD}$ so heavy parton distributions determined perturbatively. Assume $s = \bar{s}$. Leaves 6 independent combinations. Relate s to $1/2(\bar{u} + \bar{d})$ and use

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad {\rm sea} = 2 * (\bar{u} + \bar{d} + \bar{s}), \quad \bar{d} - \bar{u}, \quad g.$$

Assuming isospin symmetry $p \rightarrow n$ leads to

$$d^p(x) \rightarrow u^n(x) \qquad u^p(x) \rightarrow d^n(x).$$

Various sum rules constraining parton inputs and conserved order by order in α_S for evolution, i.e. conservation of number of valence quarks.

$$\int_0^1 u_V(x)\, dx = 2 \qquad \int_0^1 d_V(x)\, dx = 1$$

Also conservation of momentum carried by partons – important constraint on the gluon, which is only probed indirectly.

$$\int_0^1 x\Big(\sum_i (q_i(x) + \bar{q}_i(x)) + g(x)\Big)\, dx = 1.$$

In determining partons need to consider that not only are there 6 different combinations of partons, but also wide distribution of x from 0.75 to 0.00003. Need many different types of experiment for full determination.

H1 $F_2^{e^+p}(x,Q^2)$ 1996-97 moderate Q^2 and 1996-97 high Q^2, and $F_2^{e^-p}(x,Q^2)$ 1998-99 high Q^2 small x. ZEUS $F_2^{e^+p}(x,Q^2)$ 1996-97 small x wide range of Q^2. (1999-2000)

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$, E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$ medium x.

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$, SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$ large x.

CCFR (NuTeV) $F_2^{\nu(\bar{\nu})p}(x,Q^2), F_3^{\nu(\bar{\nu})p}(x,Q^2)$ large x , singlet, valence.

E605 (E866) $pN \to \mu\bar{\mu} + X$ large x sea.

E866 Drell-Yan asymmetry $\bar{u}, \bar{d}$ $\bar{d} - \bar{u}$.

CDF W-asymmetry u/d ratio at high x.

CDF D0 Inclusive jet data high x gluon.

CCFR (NuTev) Dimuon data constrains strange sea.

Large x.

Quark distributions are determined mainly by structure functions. Dominated by non-singlet valence distributions.

Simple evolution of non-singlet distributions and conversion to structure function

$$\begin{aligned}\frac{df^{NS}(x,Q^2)}{d\ln Q^2} &= P^{NS}(x,\alpha_s(Q^2))\otimes f^{NS}(x,Q^2)\\ F_2^{NS}(x,Q^2) &= C^{NS}(x,\alpha_s(Q^2))\otimes f^{NS}(x,Q^2,\alpha_s(Q^2)).\end{aligned}$$

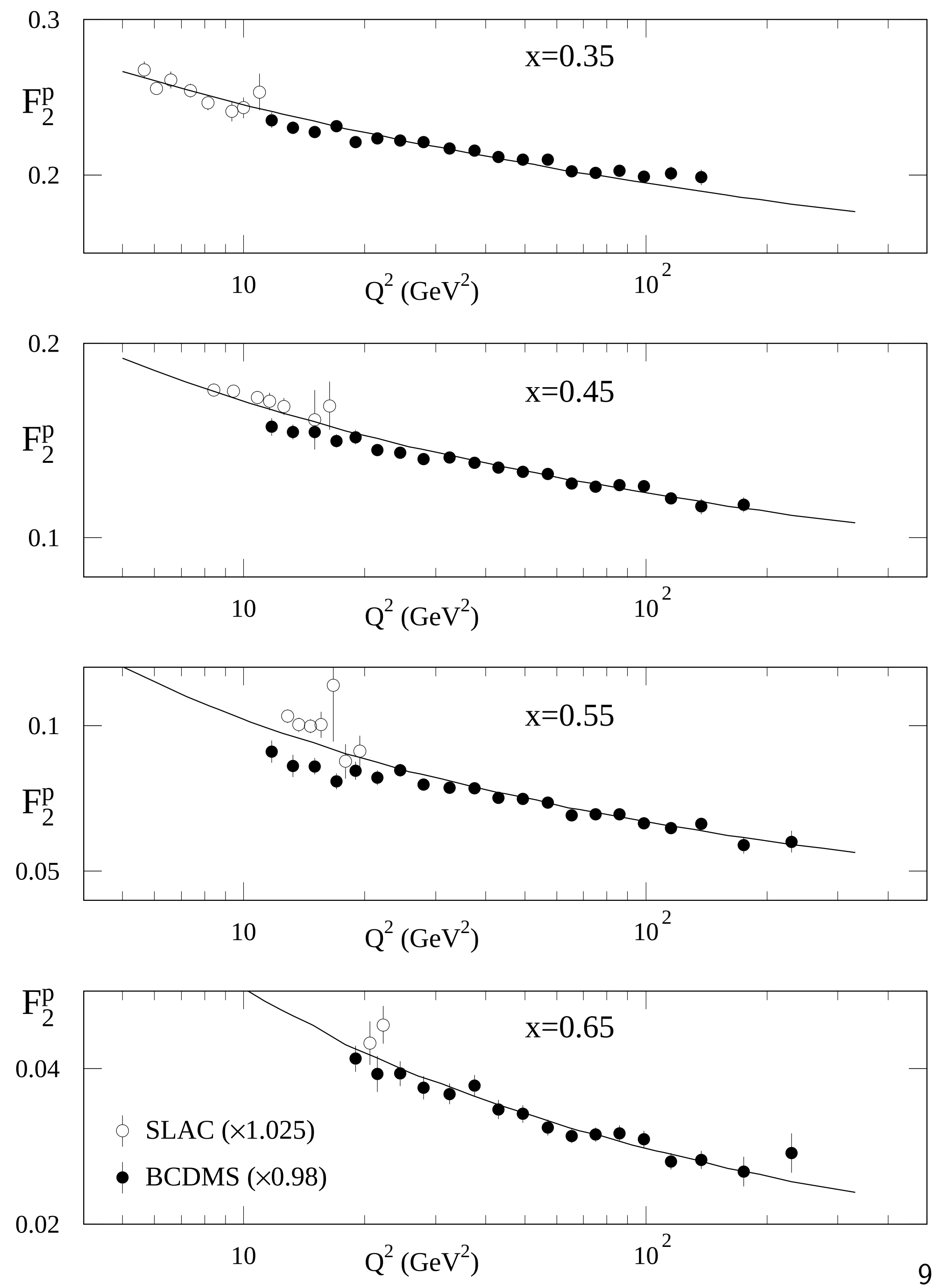
So evolution of high x structure functions good test of theory and of $\alpha_S(Q^2)$.

However - perturbation theory involves contributions to coefficient function $\sim \alpha_S^n(Q^2)\ln^{2n-1}(1-x)$ and higher twist known to be enhanced as $x \to 1$. Hence to avoid contamination of NLO theory make cut

$$W^2 = Q^2(1/x-1) + m_p^2 \leq 10-15\mathrm{GeV}^2.$$

Description of large x BCDMS and SLAC measurements of F_2^p.

Determines $\alpha_S(M_Z^2)$.



Small x.

The extension to very low x has been made in the past decade by HERA. In this region there is very great scaling violation of the partons from the evolution equations and also interplay between the quarks and gluons.

At each subsequent order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e.

$$P_{ij}(x, \alpha_s(Q^2)), \quad C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x),$$

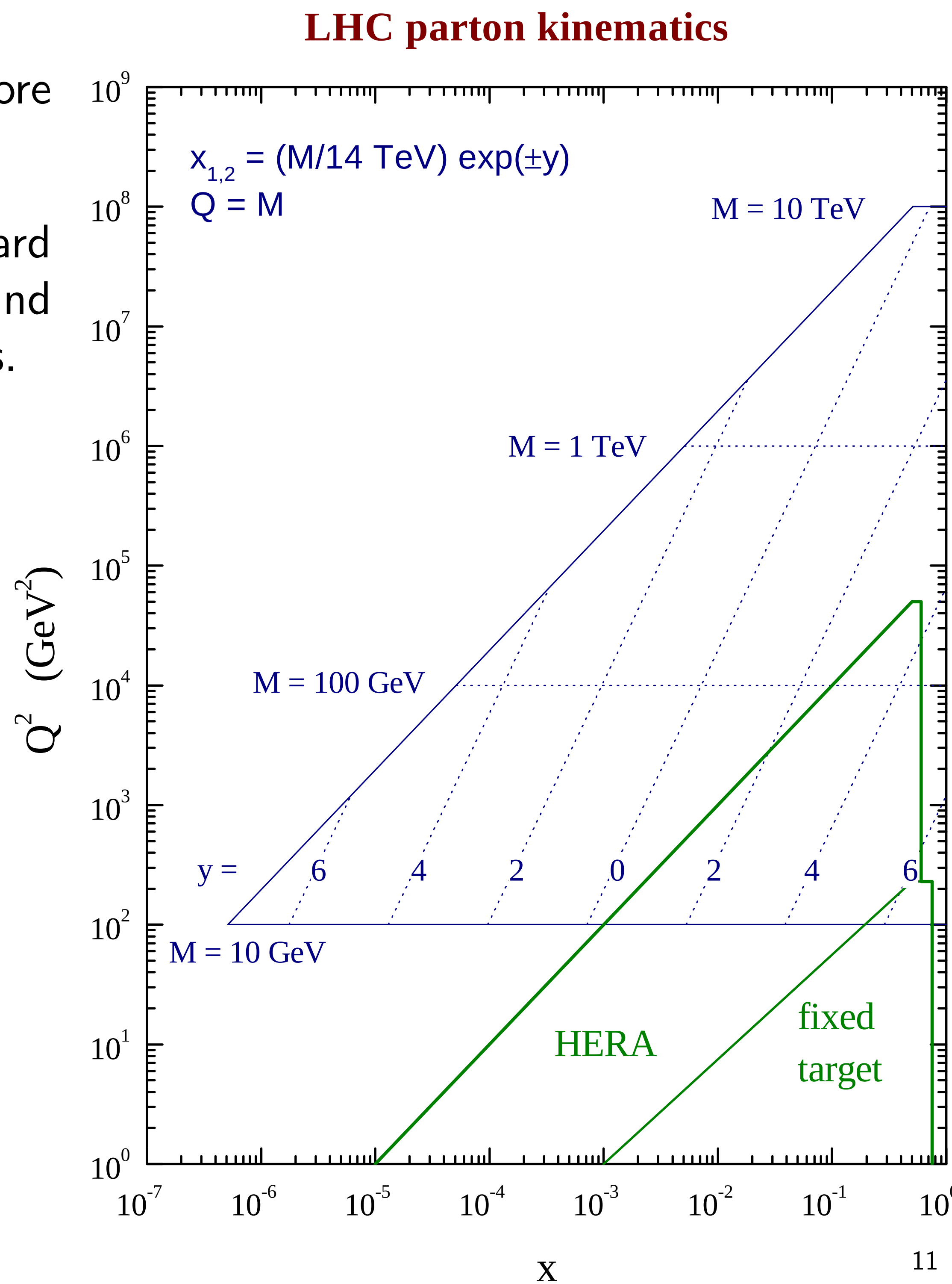
and hence the convergence at small x is questionable.

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless. The fit is good, but could be improved.

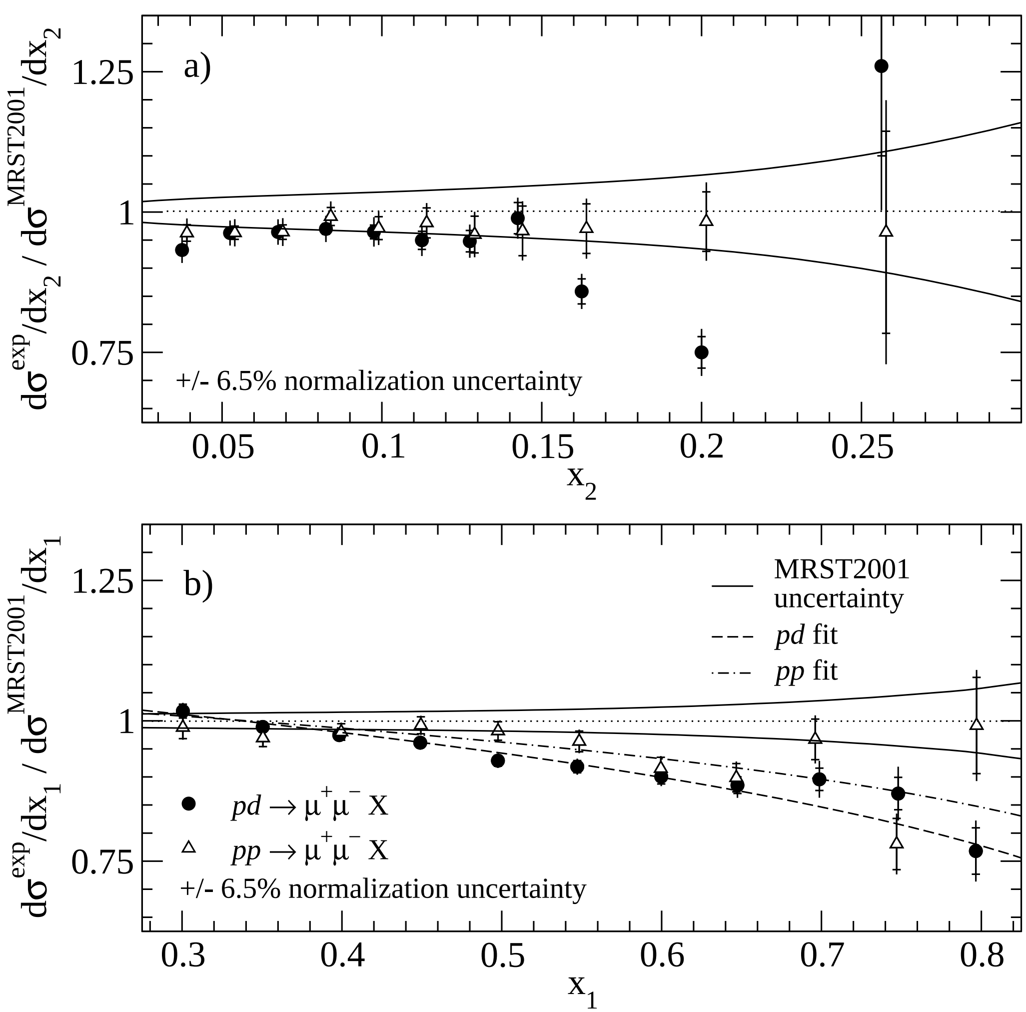
Small x predictions somewhat uncertain. Very active area of research (later).

Small x parton distributions therefore interesting within QCD.

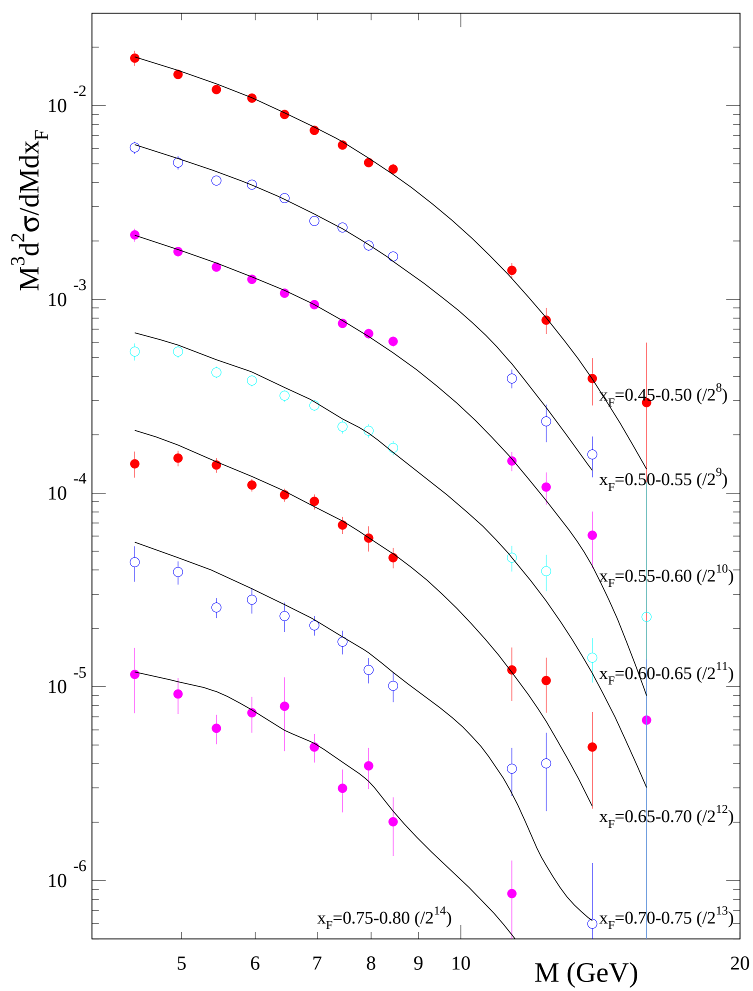
Also vital for understanding the standard production processes at the LHC, and perhaps some of the more exotic ones.



High-x Sea quarks determined by Drell-Yan data (assuming knowledge of valence quarks). Recent suggested discrepancy by fit to E866/NuSea collaboration. Implies larger high-x valence quarks.



E866 pd data and MRST2001 ($x_F > 0.45$)



Not observed by MRST or by CTEQ.

$s(x)$ **and** $\bar{s}(x)$ **distributions** can now be probed separately using NuTeV dimuon data

$\nu + s \to \mu^- + c(\mu^+)$, $\bar{\nu} + \bar{s} \to \mu^+ + \bar{c}(\mu^-)$.

Examinied in detail by CTEQ.

$\to s(x) < \bar{s}(x)$ at quite small x.

$\int (s(x) - \bar{s}(x))\, dx = 0$, (zero strangeness number).

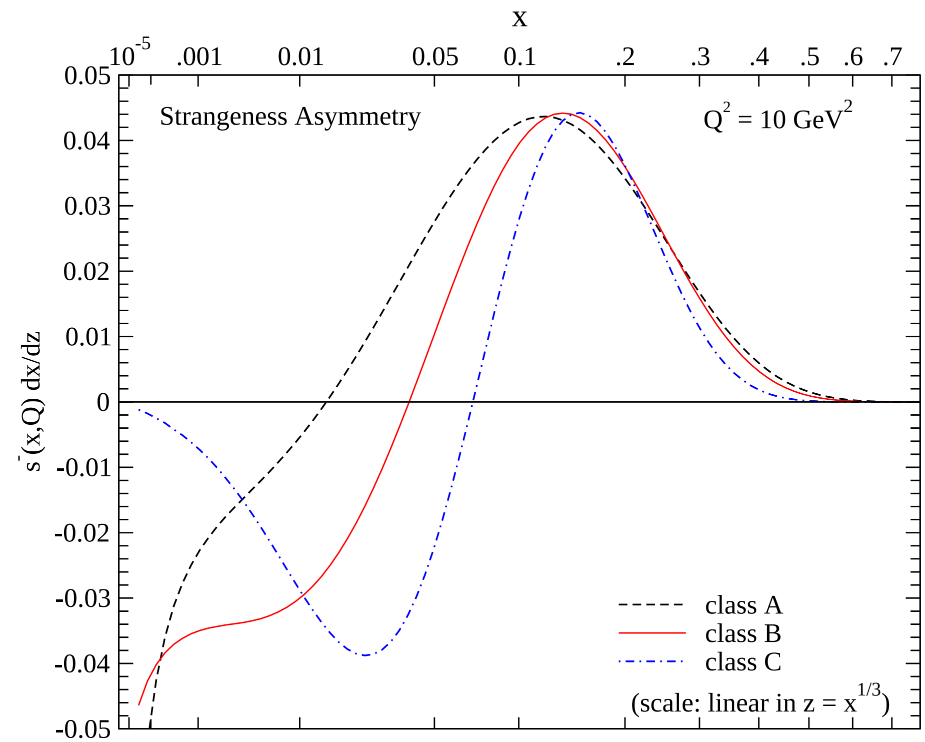
$\to \int x(s(x) - \bar{s}(x))\, dx = [S^-] > 0$.

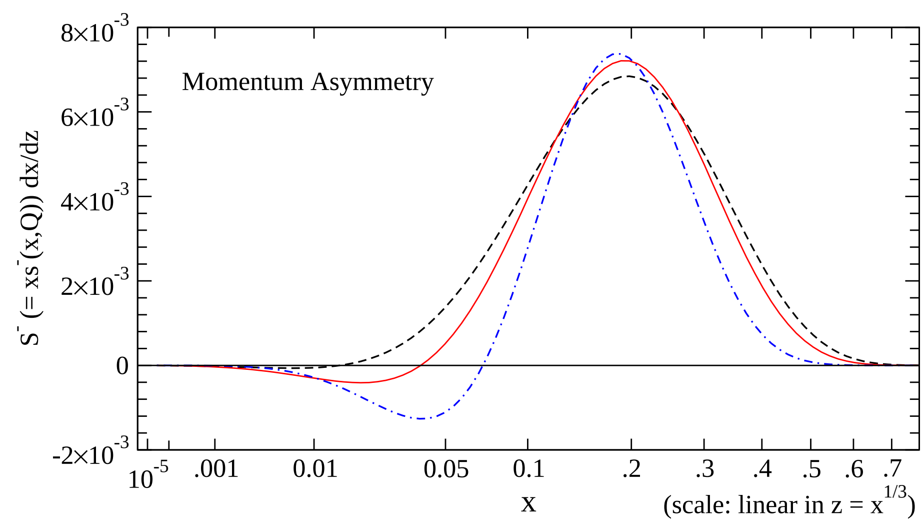
$0 < [S^-] < 0.004$.

$NuTeV$ measure $R^- = \frac{\sigma^{\nu}_{\rm NC} - \sigma^{\bar{\nu}}_{\rm NC}}{\sigma^{\nu}_{\rm CC} - \sigma^{\bar{\nu}}_{\rm CC}}$.

$R^- = \frac{1}{2} - \sin^2\theta_W - (1 - \frac{7}{3}\sin^2\theta_W)\frac{[S^-]}{[V^-]}$.

$[S^-] = 0.002$ reduces NuTeV anomaly from 3σ to 1.5σ.





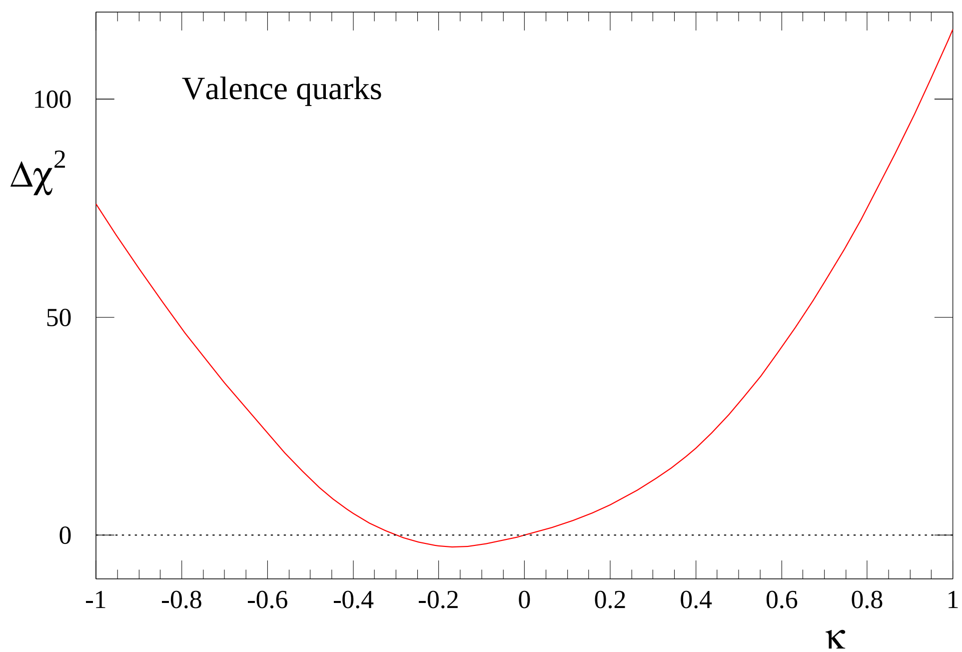
MRST also look at effect of **isospin violation**.

$$R^- = \frac{1}{2} - \sin^2\theta_W + (1 - \frac{7}{3}\sin^2\theta_W)\frac{[\delta U_v] - [\delta D_v]}{2[V^-]}.$$

$$[\delta U_v] = [U_v^p] - [D_v^n], \qquad [\delta D_v] = [D_v^p] - [U_v^n].$$

$$u_v^p(x) = d_v^n(x) + \kappa f(x), \qquad d_v^p(x) = u_v^n(x) - \kappa f(x).$$

$\kappa = -0.2 \rightarrow$ a similar reduction of the NuTeV anomaly, i.e. $\Delta sin^2\theta_W \sim -0.002$. Larger (more negative) κ allowed.



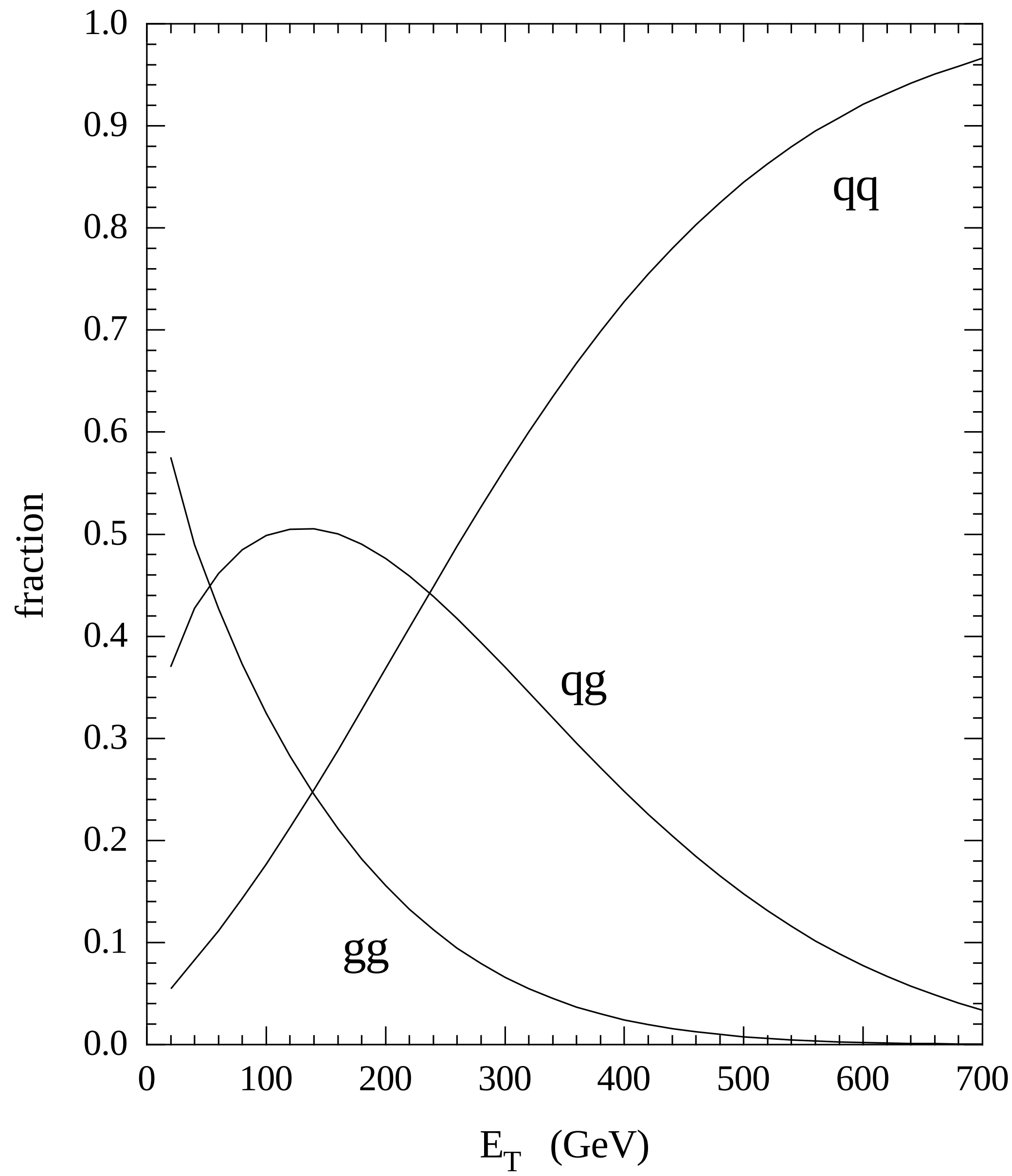
High-x Gluon distribution.

Best determination from inclusive jet measurements by D0 and CDF at Tevatron. Measure $d\sigma/dE_T d\eta$ for central rapidity CDF or in bins of rapidity D0.

At central rapidity $x_T = 2E_T/\sqrt{s}$, and measurements extend up to $E_T \sim 400\text{GeV}$, i.e. $x_T \sim 0.45$, and down to $E_T \sim 60\text{GeV}$, i.e. $x_T \sim 0.06$.

Gluon-gluon fusion dominates, but $g(x, \mu^2)$ falls off more quickly as $x \rightarrow 1$ than $q(x, \mu^2)$ so there is a transition from gluon-gluon fusion at small x_T, to gluon-quark to quark-quark at high x_T. However, even at the highest x_T gluon-quark contributions are significant.

Jet photoproduction at HERA will be another constraint in the future.



Results.

Above procedure completely determines parton distributions at present. Total fit reasonably good, e.g CTEQ6. $\alpha_S(M_Z^2)$ fixed at 0.118. Total $\chi^2 = 1954/1811$.

Data set	No. of data pts	χ^2
H1 ep	230	228
ZEUS ep	229	263
BCDMS μp	339	378
BCDMS μd	251	280
NMC μp	201	305
E605 (Drell-Yan)	119	95
D0 Jets	90	65
CDF Jets	33	49

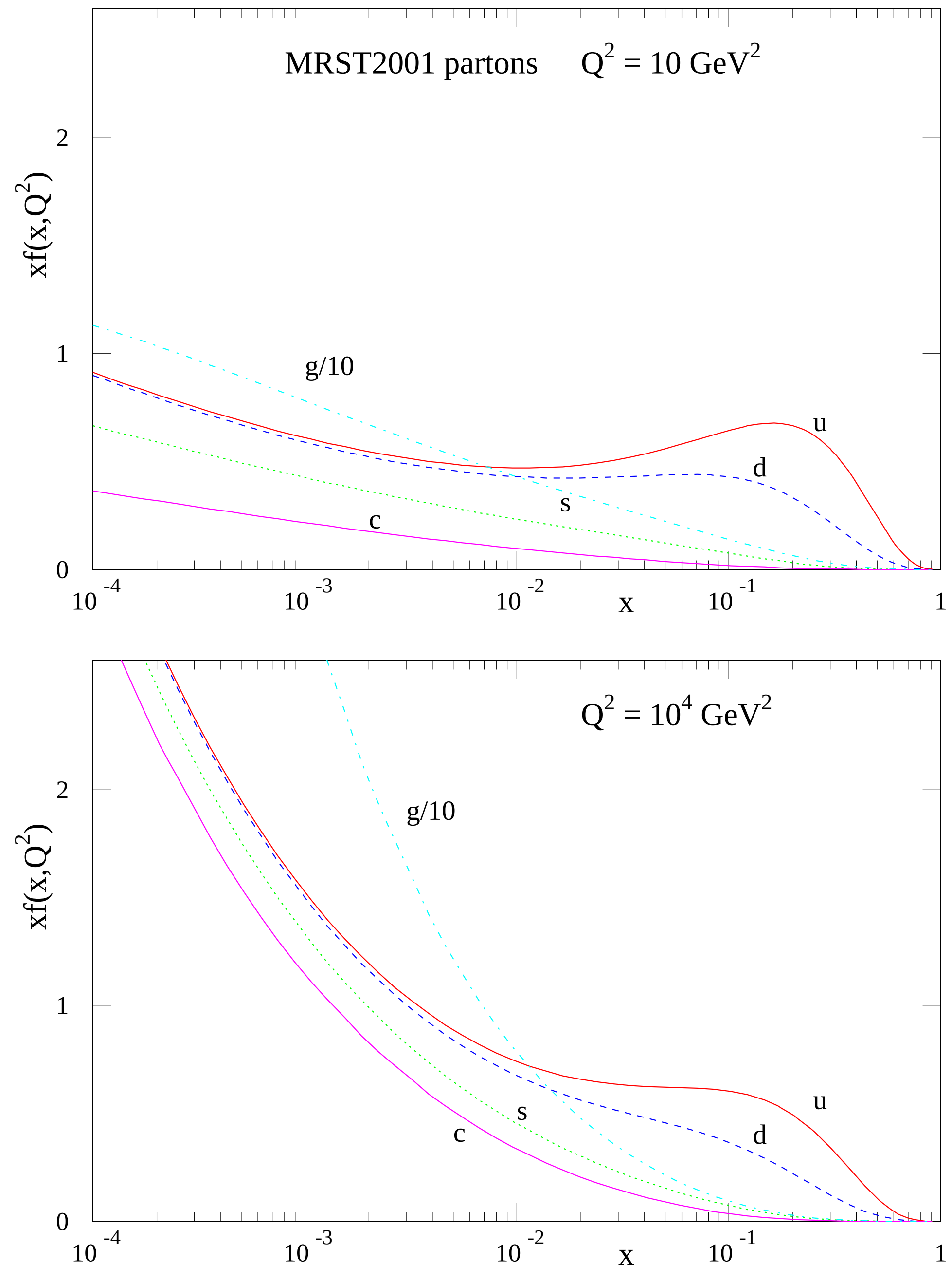
For MRST $\alpha_S(M_Z^2) = 0.119$. Compromise between different data sets. Total $\chi^2 = 2328/2097$ – but errors treated differently, and different data sets and cuts.

Same sort of conclusion for other *global fits* (H1, ZEUS, Alekhin, GKK) (with rather fewer data).

Some areas where theory perhaps needs to be improved. (See later.)

MRST2001 partons.

CTEQ *etc.* (generally) very similar.



Parton Uncertainties – currently an issue attracting a lot of work. Number of approaches.

Hessian (Error Matrix) approach first used by H1 and ZEUS, recently extended by CTEQ.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

We can then use the standard formula for linear error propagation.

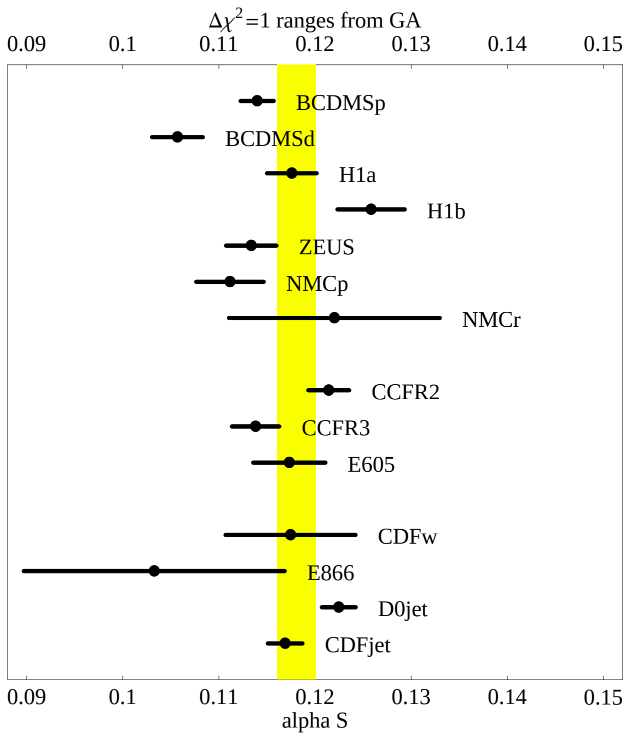
$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j},$$

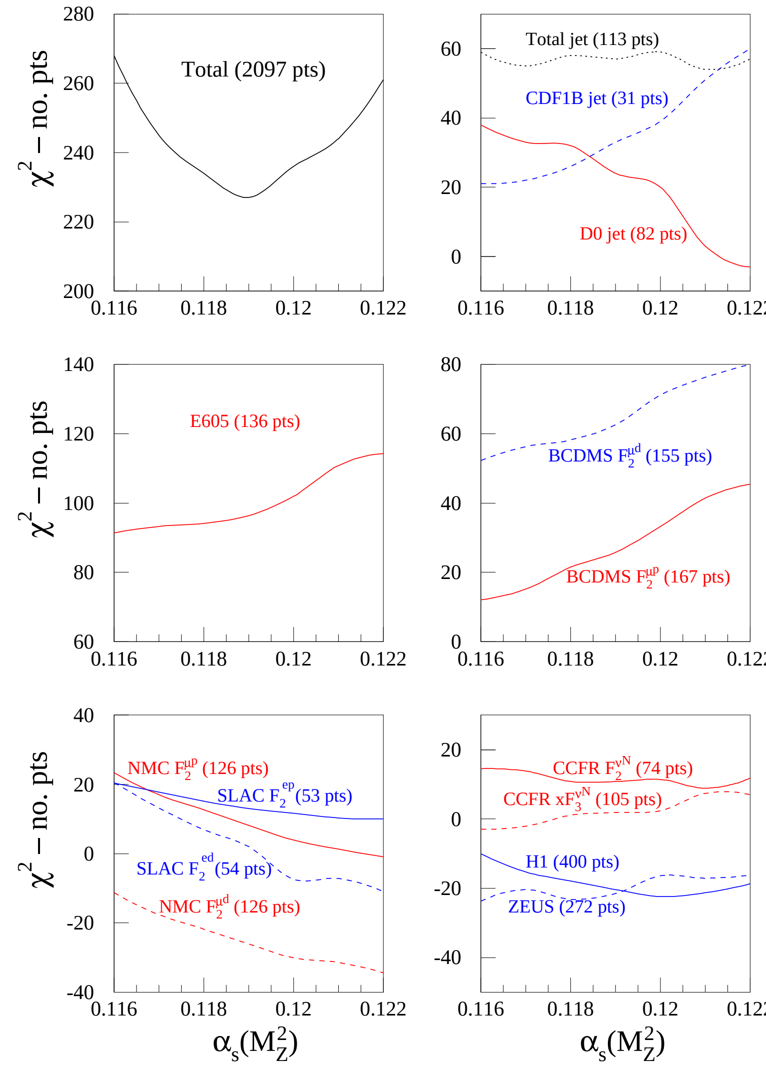
This has been used to find partons with errors by Alekhin and H1, each with restricted data sets.

Simple method problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space - particularly with more parameters (more data). $\rightarrow$ numerical instability.

Solved (helped) by finding and rescaling eigenvectors of H leading to diagonal form $\Delta\chi^2 = \sum_i z_i^2$. First used by CTEQ. Now used in slightly weaker form by MRST and ZEUS.

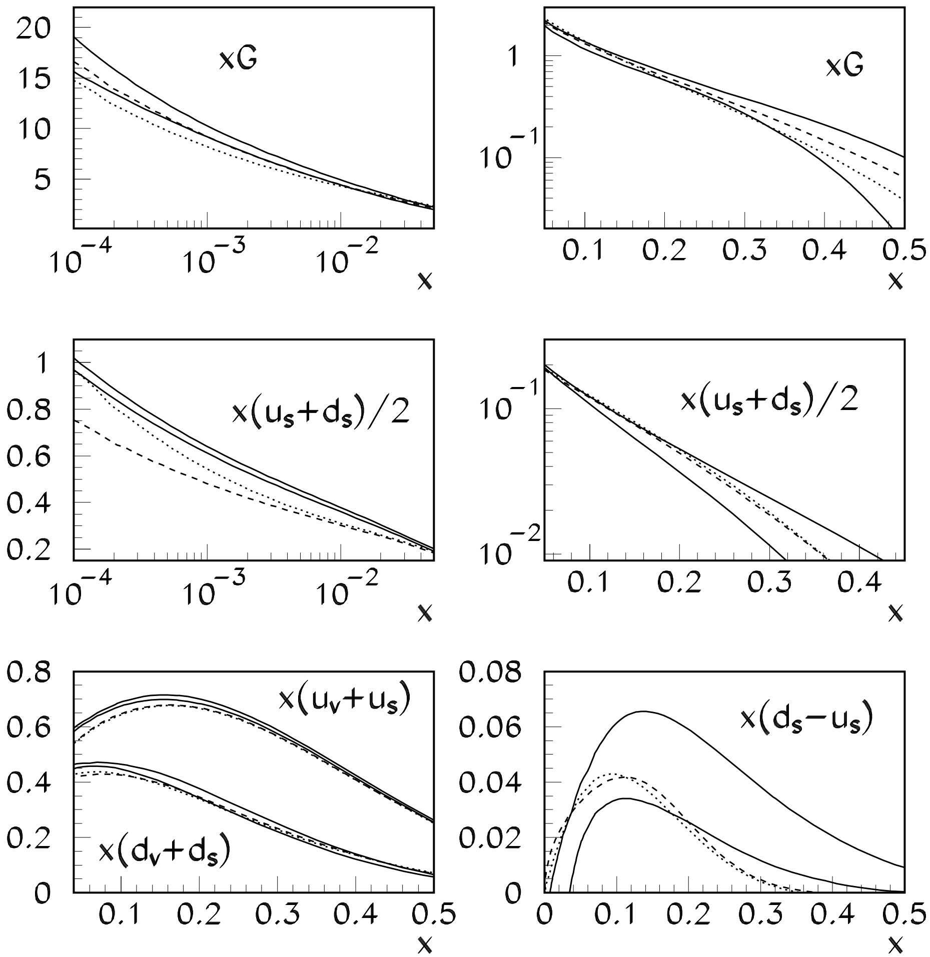
In full **global** fit art in choosing "correct" $\Delta\chi^2$ given complication of errors. Ideally $\Delta\chi^2 = 1$, but unrealistic.

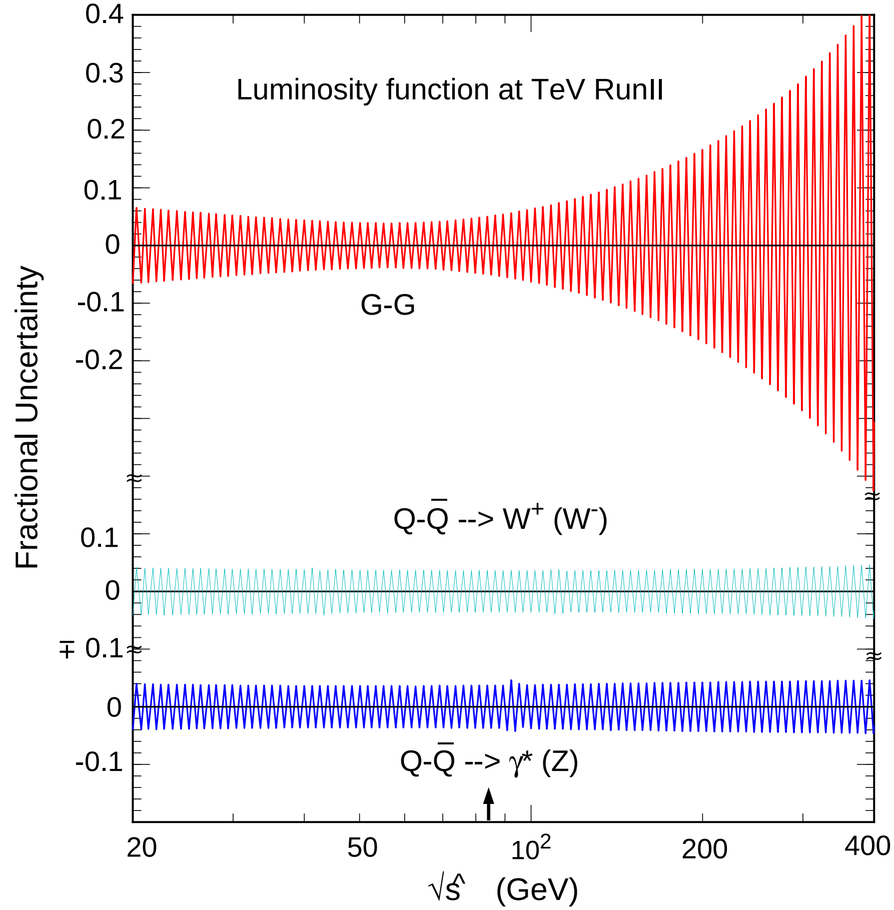




Many approaches use $\Delta\chi^2 \sim 1$. CTEQ choose $\Delta\chi^2 \sim 100$ (conservative?). MRST choose $\Delta\chi^2 \sim 20$ for $1-\sigma$ error.

Results for Alekhin partons (left) at $Q^2 = 9\,\mathrm{GeV}^2$ with uncertainties (solid lines), (dashed lines – CTEQ5M, dotted lines – MRST01), and CTEQ Hessian approach for luminosity uncertainty (right).





Other Approaches.

Statistical Approach (Giele, Keller and Kosower) constructs an ensemble of distributions labelled by $\mathcal{F}$ each with probability $P(\{\mathcal{F}\})$. Can incorporate full information about measurements and their error correlations in the calculation of $P(\{\mathcal{F}\})$. Calculate by summing over N_{pdf} different distributions with unit weight but distributed according to $P(\{\mathcal{F}\})$. (N_{pdf} can be made as small as 100). Mean μ_O and deviation σ_O of observable O then given by

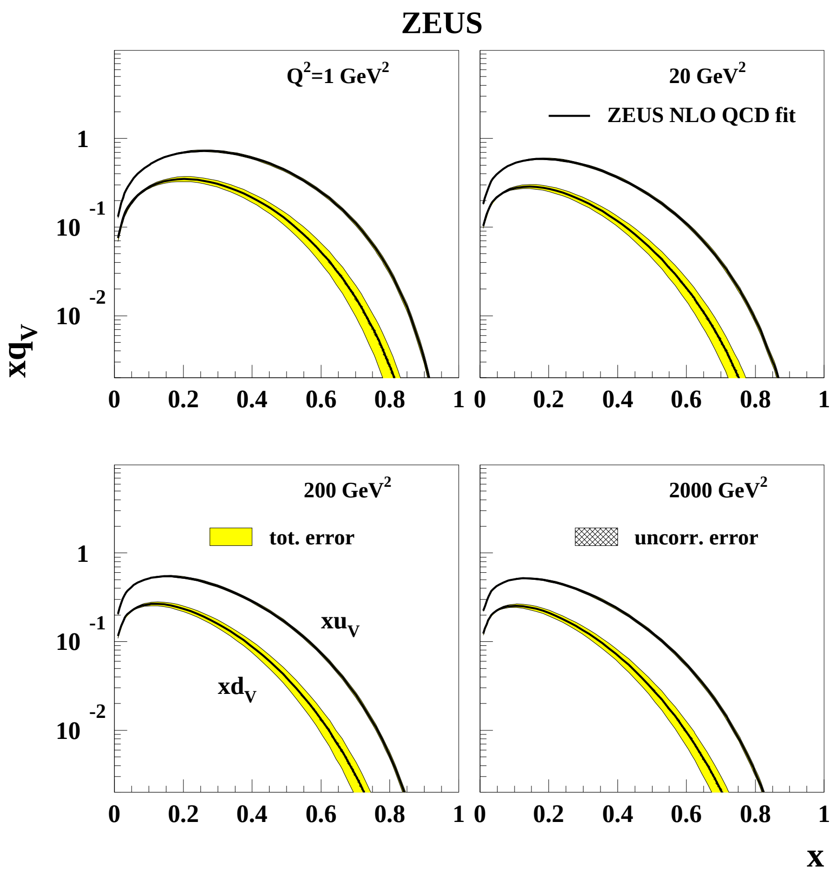
$$\mu_O = \sum_{\{\mathcal{F}\}} O(\{\mathcal{F}\}) P(\{\mathcal{F}\}), \quad \sigma_O^2 = \sum_{\{\mathcal{F}\}} (O(\{\mathcal{F}\}) - \mu_O)^2 P(\{\mathcal{F}\}).$$

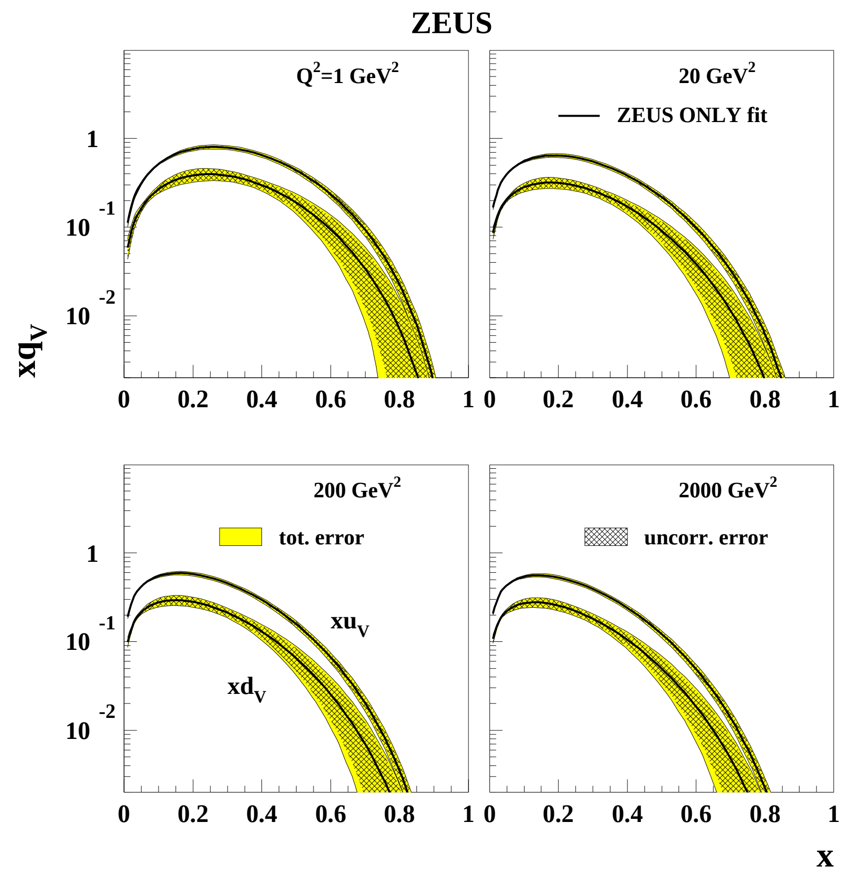
Currently uses only proton DIS data sets in order to avoid complicated uncertainty issues such as shadowing effects for nuclear targets. Demands strict consistency between data sets. It is difficult to find many compatible DIS experiments. Fermi2001 partons determined by only H1(94), BCDMS, E665 data sets.

Good principle if theory and data good enough.

Some good predictions, e.g. σ_W and σ_Z at Tevatron. Some unusual parameters compared to other sets, e.g. low $\alpha_S(M_Z^2)$, very hard $d_V(x)$ at high x.

In the offset method the best fit and parameters a_0 are obtained using only uncorrelated errors. The quality of fit is then estimated by adding in quadrature. Systematic errors are determined (effectively) by letting each source of systematic error vary by $1-\sigma$ and adding the deviations in quadrature. Used by ZEUS. Effective $\Delta\chi^2 > 1$.



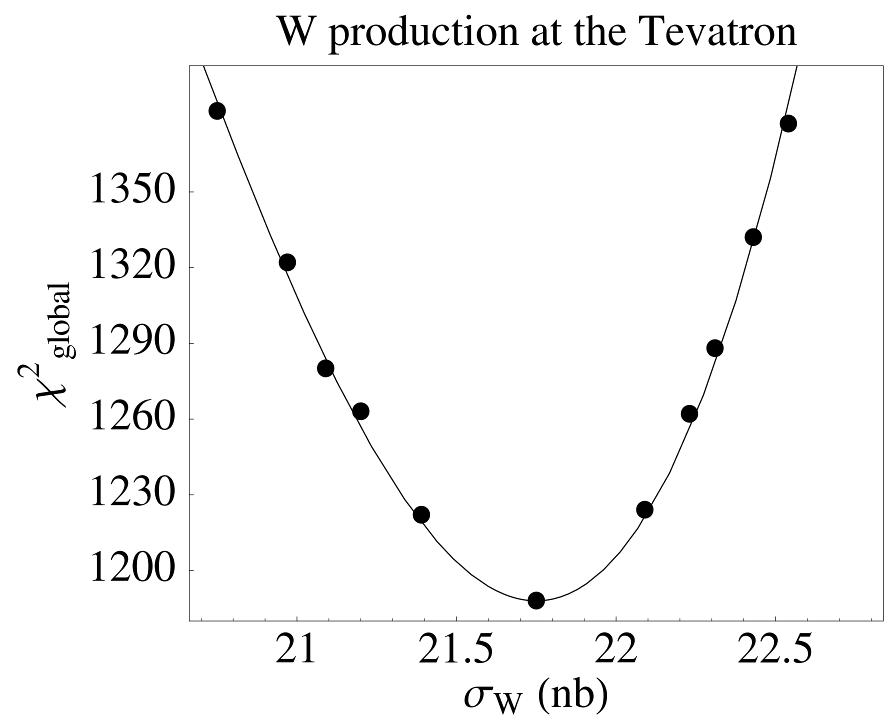


Valence partons extracted by ZEUS from *global* fit and fit to own data alone (with some input assumptions). Potential for real constraint in future.

Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ and concentrated on by MRST. Minimize

$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a).$$

Gives best fits for particular values of quantity $F(a)$ without relying on Gaussian approx for χ^2. Uncertainty then determined by deciding allowed range of $\Delta\chi^2$.



CTEQ obtain for $\alpha_S = 0.118$

$$\Delta\sigma_W(\mathrm{LHC}) \approx \pm 4\% \quad \Delta\sigma_{\rm W}(\mathrm{Tev}) \approx \pm 4$$

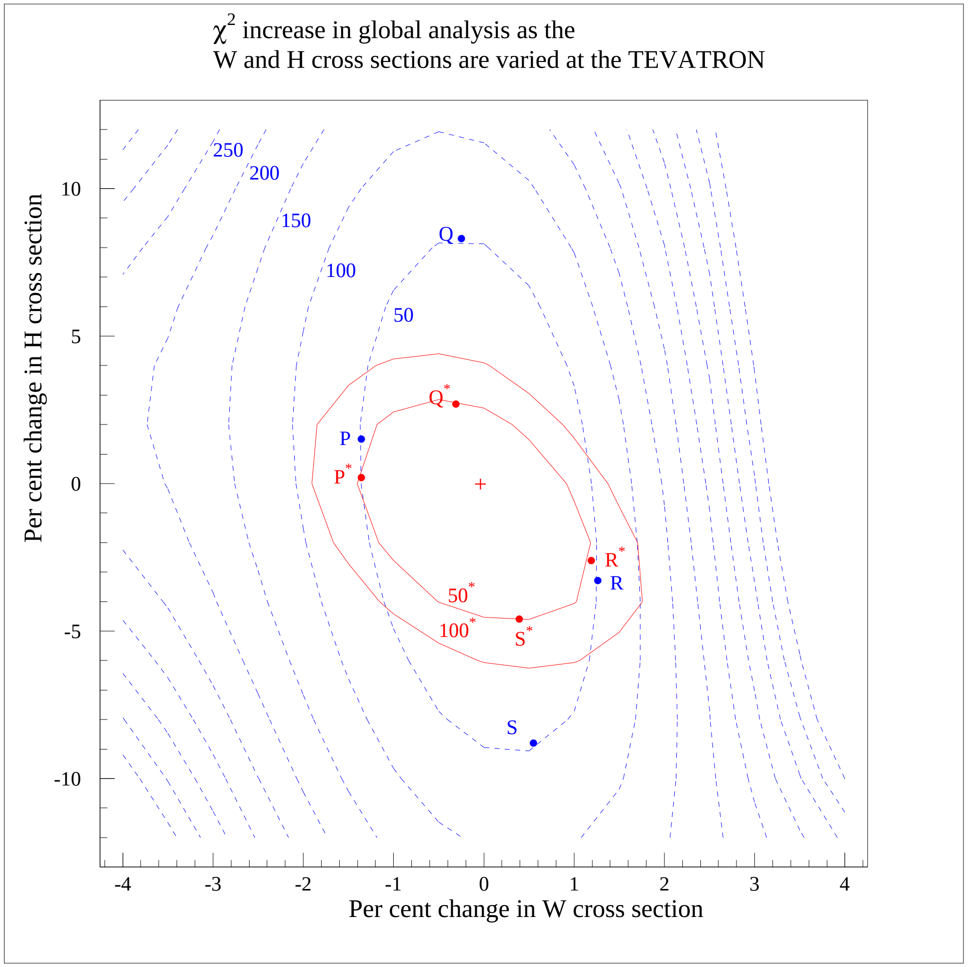
$$\Delta\sigma_H(\mathrm{LHC}) \approx \pm 5\%.$$

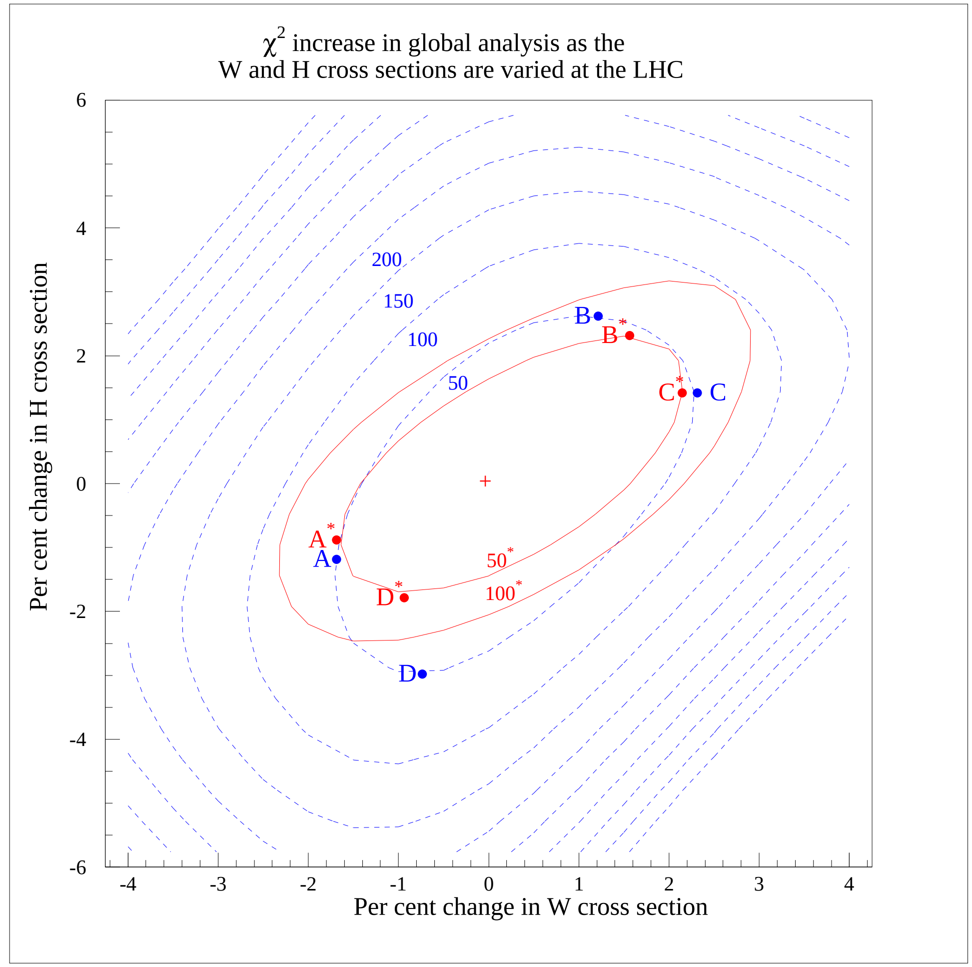
MRST use a wider range of data, and if $\Delta\chi^2 \sim 50$ find for $\alpha_S = 0.119$

$$\Delta\sigma_W(\mathrm{Tev}) \approx \pm 1.2\% \quad \Delta\sigma_{\rm W}(\mathrm{LHC}) \approx \pm 2\%$$

$$\Delta\sigma_H(\mathrm{Tev}) \approx \pm 4\% \quad \Delta\sigma_{\rm H}(\mathrm{LHC}) \approx \pm 2\%.$$

MRST also allow α_S to be free.





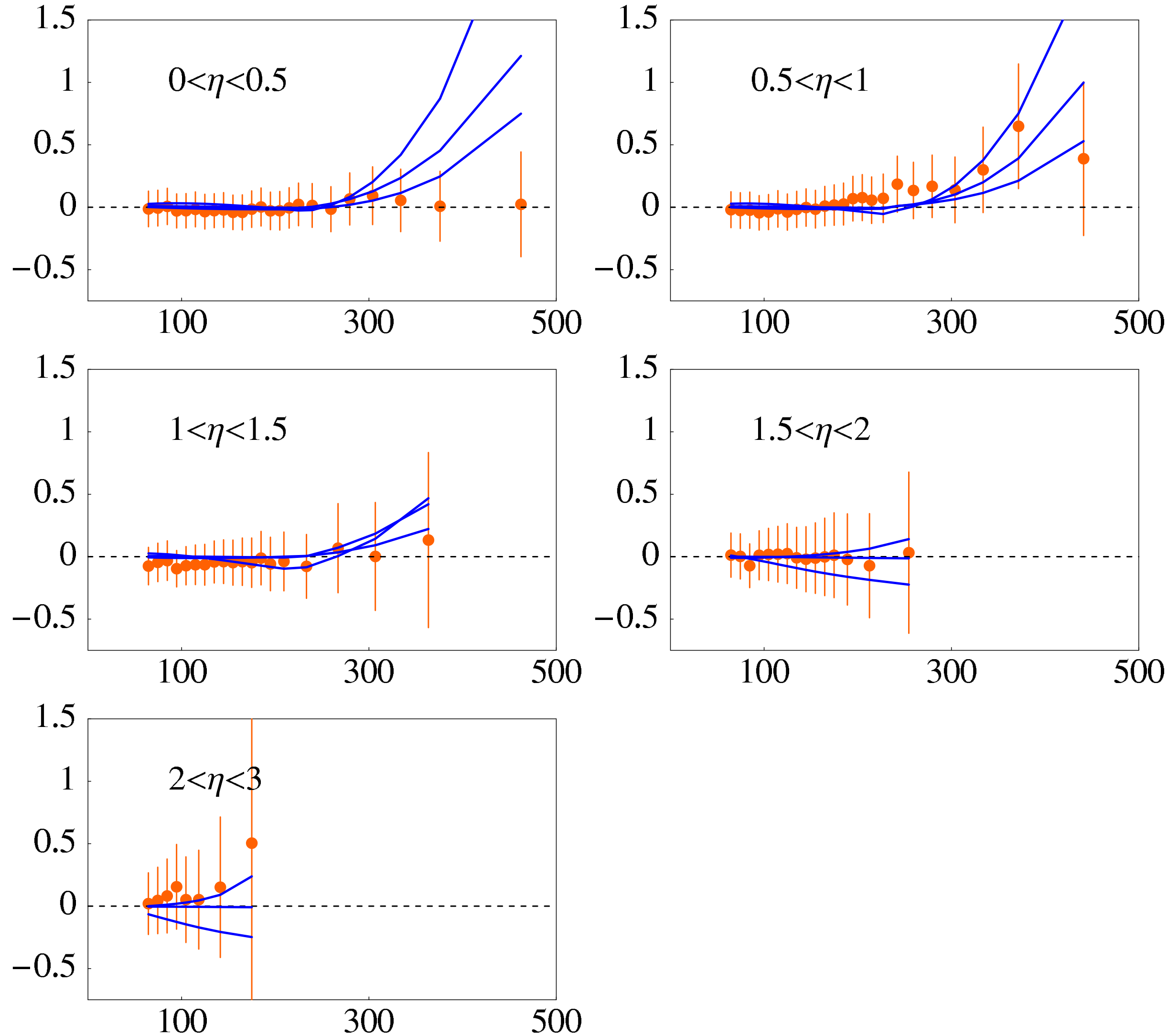
χ^2-plots for W and Higgs $(120\mathrm{GeV})$ production at the Tevatron and LHC α_S free (blue) and fixed (red) at $\alpha_S = 0.119$.

Same general procedure used by CTEQ to look at effect of new physics in contact term

$$\pm(2\pi/\Lambda^2)(\bar{q}_L\gamma^\mu q_L)(\bar{q}_L\gamma_\mu q_L).$$

Curves show fit to $D0$ jet data for $\Lambda = 1.6, 2.0, 2.4, \infty$ TeV, $A = -1$.

$\Lambda > 1.6$, TeV.



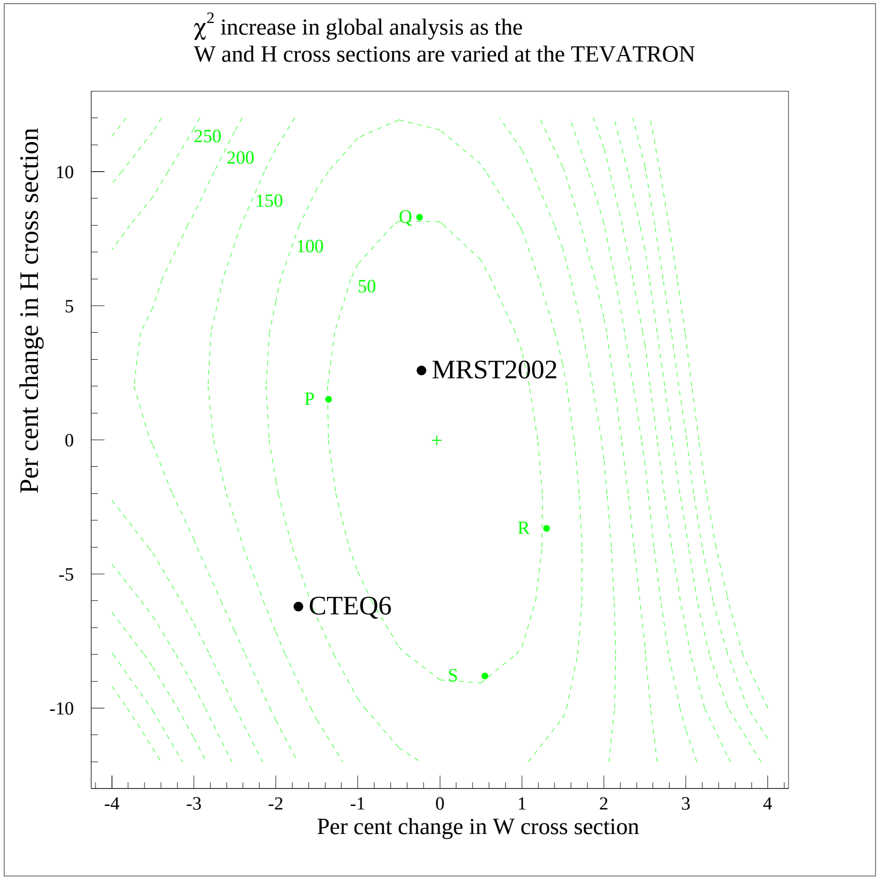
Hence, the estimation of uncertainties due to experimental errors has many different approaches and different types and amount of data actually fit. Overall conclude that uncertainty due to experimental errors only more than few $\%$ for quantities determined by high x gluon and very high x down quark.

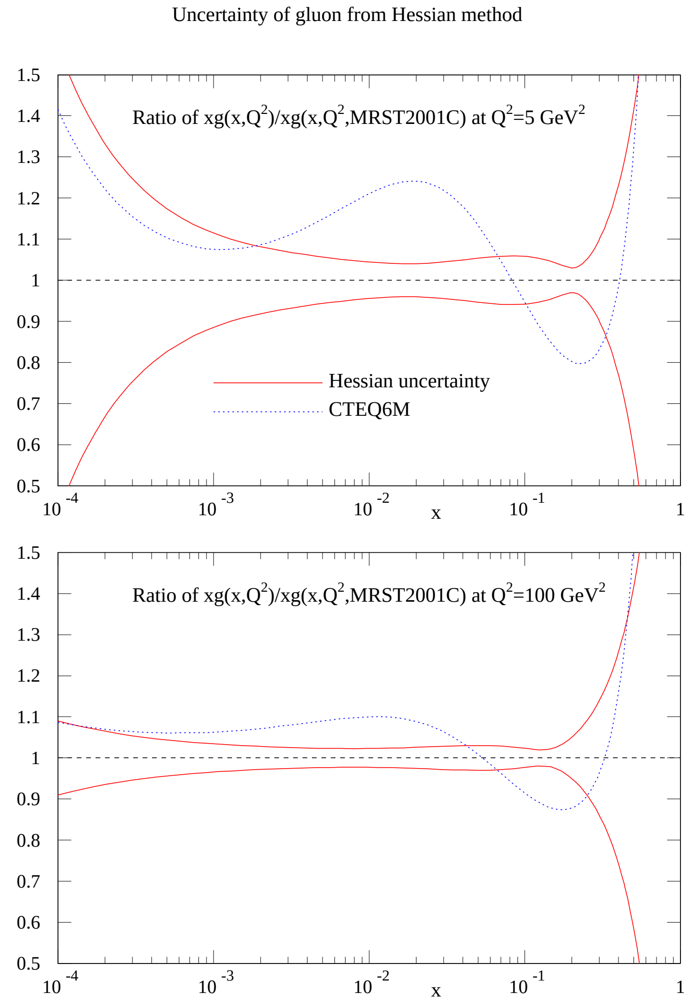
Values of $\alpha_s(M_Z^2)$ and its error from different NLO QCD fits with different error tolerances. Reasonable agreement in general – but some outliers.

CTEQ6	$\Delta\chi^2 = 100$	$\alpha_s(M_Z^2) =$	$0.1165 \pm 0.0065(exp)$
ZEUS	$\Delta\chi^2_{eff} = 50$	$\alpha_s(M_Z^2) =$	$0.1166 \pm 0.0049(exp)$
			$\pm 0.0018(model)$
			$\pm 0.004(theory)$
MRST01	$\Delta\chi^2 = 20$	$\alpha_s(M_Z^2) =$	$0.1190 \pm 0.002(exp)$
			$\pm 0.003(theory)$
H1	$\Delta\chi^2 = 1$	$\alpha_s(M_Z^2) =$	$0.115 \pm 0.0017(exp)$
			${}^{+\ 0.0009}_{-\ 0.0005}\ (model)$
			$\pm 0.005(theory)$
Alekhin	$\Delta\chi^2 = 1$	$\alpha_s(M_Z^2) =$	$0.1171 \pm 0.0015(exp)$
			$\pm 0.0033(theory)$
GKK	CL	$\alpha_s(M_Z^2) =$	$0.112 \pm 0.001(exp)$

Theory errors highly correlated.

Different approaches lead to similar accuracy of measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit.





Cuts made on data, data sets fit, parameterization for input sets, form of strange sea, heavy flavour prescription, assumption of no isospin violation, strong coupling ......

Many can be as important as experimental errors on data used (or more so).

Results from LHC/LP Study Working Group (Bourilkov).

Table 1: Cross sections for Drell-Yan pairs (e^+e^-) with PYTHIA 6.206, rapidity < 2.5. The errors shown are the PDF uncertainties.

PDF set	Comment	xsec [pb]	PDF uncertainty %
$81 < M < 101$ GeV			
CTEQ6	LHAPDF	1065 ± 46	4.4
MRST2001	LHAPDF	$1091 \pm$ ...	3
Fermi2002	LHAPDF	853 ± 18	2.2

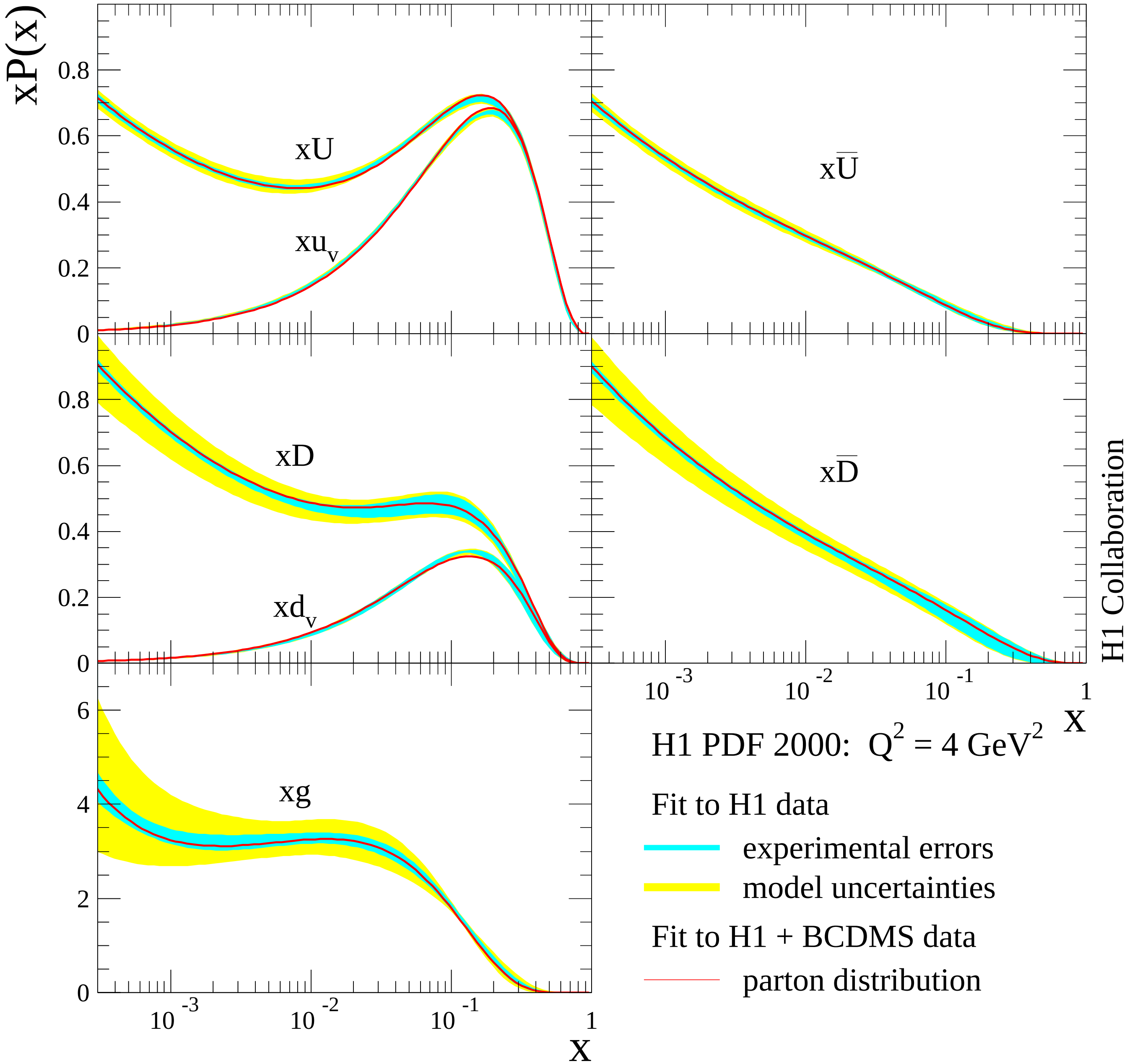
Comparison of $\sigma_W \cdot B_{l\nu}$ for MRST2002 and Alekhin partons.

PDF set	Comment	xsec [nb]	PDF uncertainty
Alekhin	Tevatron	2.73	$\pm$ 0.05 (tot)
MRST2002	Tevatron	2.59	$\pm$ 0.03 (expt)
CTEQ6	Tevatron	2.54	$\pm$ 0.10 (expt)
Alekhin	LHC	215	$\pm$ 6 (tot)
MRST2002	LHC	204	$\pm$ 4 (expt)
CTEQ6	LHC	205	$\pm$ 8 (expt)

In both cases differences (mainly) due to detailed constraint (by data) on quark decomposition.

Also demonstrated by most recent H1 fit (to own data alone) where *model error* dominates.

Again shows constraint now achieved by HERA data alone – with some assumptions.



Problems in the fit.

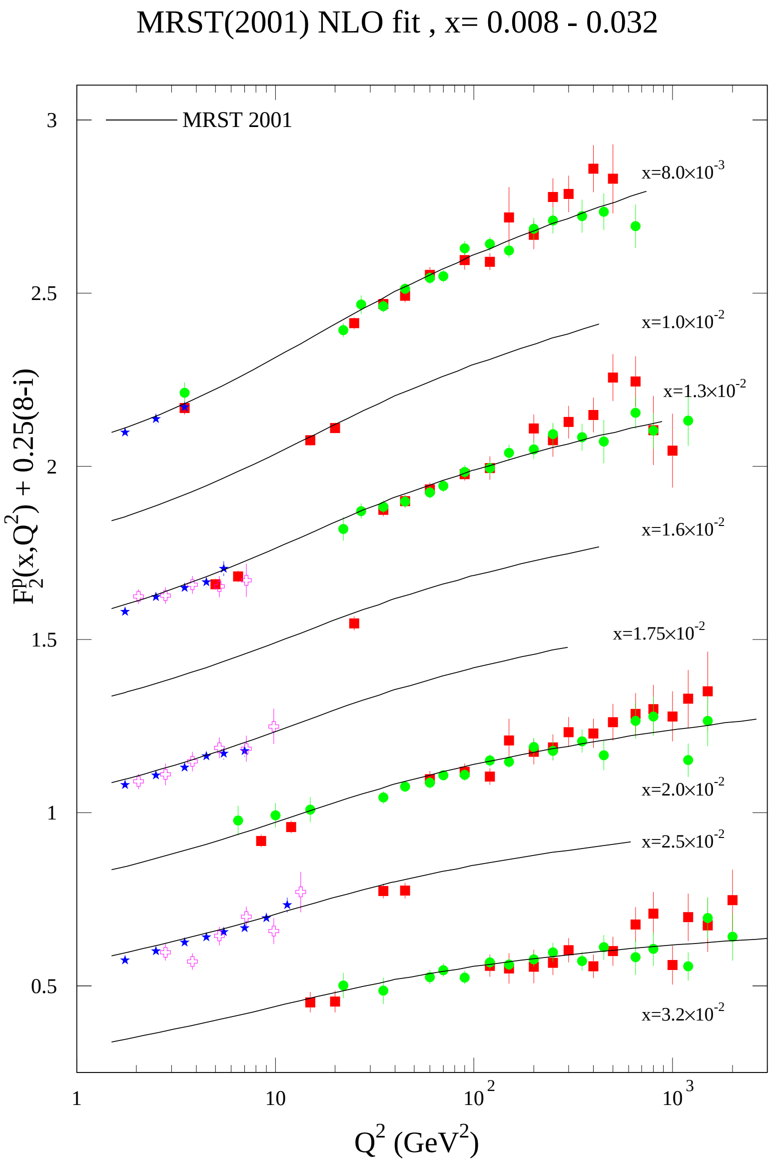
Variations from different approaches partially due to inadequacy of theory .

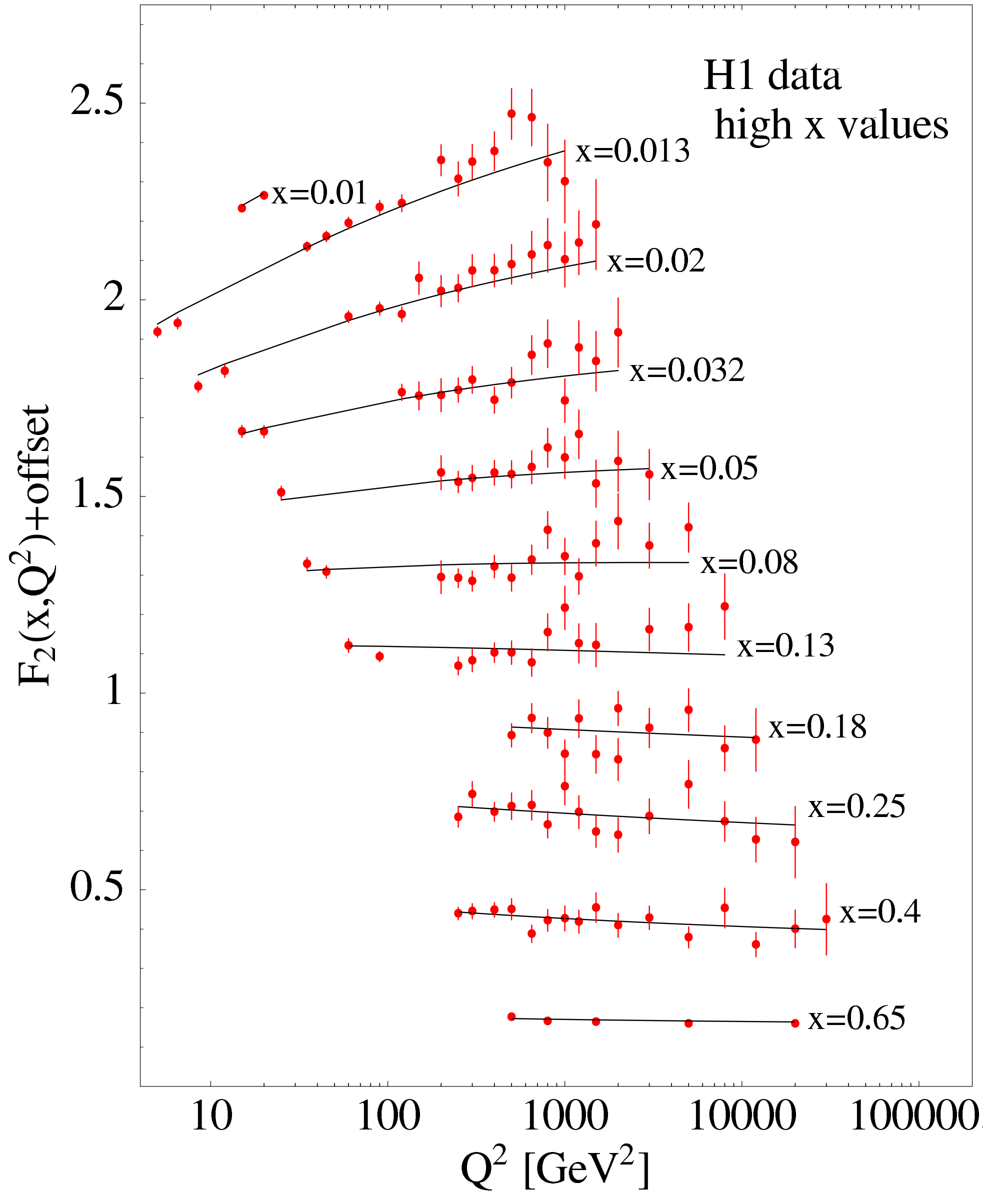
Failings of NLO QCD indicated by some areas where fit quality could be improved.

Good fit to HERA data, but some problems at highest Q^2 at moderate x, i.e. in $dF_2/d\ln Q^2$.

Want more gluon in the $x \sim 0.01$ range, and/or larger $\alpha_S(M_Z^2)$.

Possible sign of required $\ln(1/x)$ corrections.



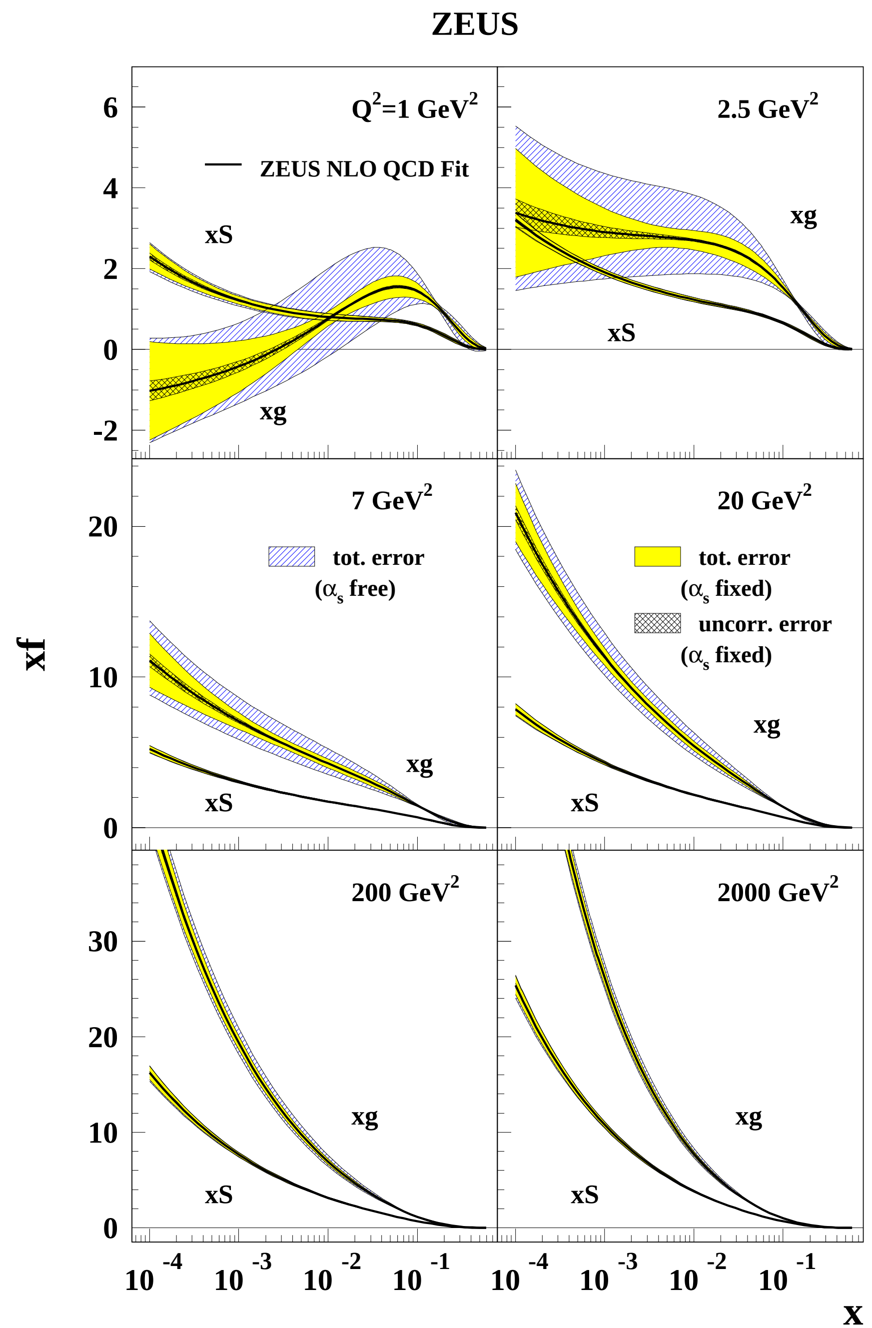


Comparison of MRST(2001) $F_2(x, Q^2)$ with HERA, NMC and E665 data (left) and of CTEQ6 $F_2(x, Q^2)$ and H1 data.

Data require gluon to be negative at low Q^2, e.g. MRST $Q_0^2 = 1\mathrm{GeV}^2$. Needed by all data (e.g Tevatron jets) not just low Q^2 low x data.

$\rightarrow F_L(x, Q^2)$ dangerously small at smallest x, Q^2.

Other groups find similar problems with gluon and/or $F_L(x, Q^2)$ at low x, e.g. ZEUS.

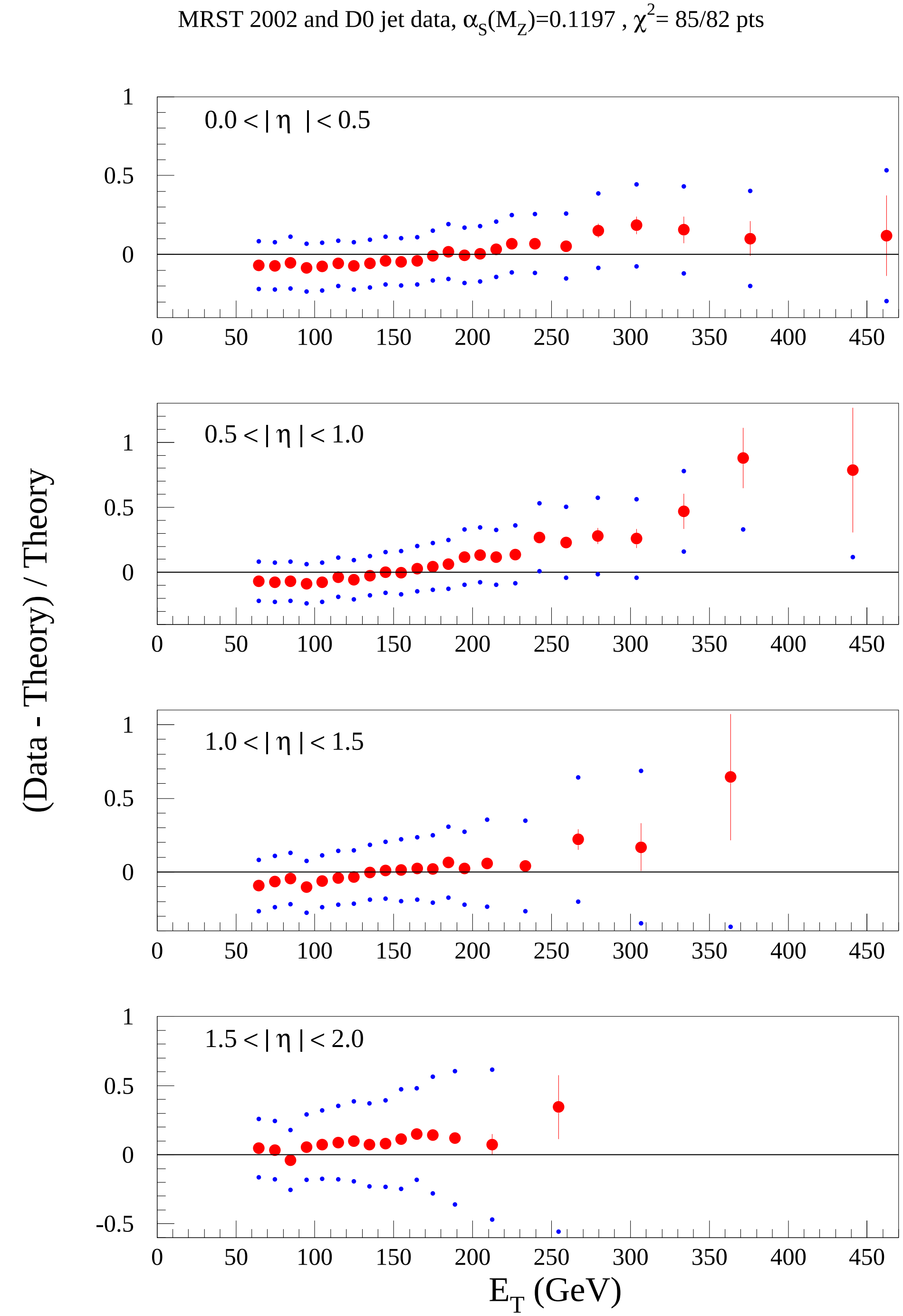


Difficult to reconcile fit to jets and rest of data.

MRST find a reasonable fit to jet data, but need to use the large systematic errors.

Better for CTEQ6 largely due to different cuts on other data. Usually worse for other partons (jets not in fits). General tension between HERA and NMC data and jets.

In general different data compete over the gluon and $\alpha_S(M_Z^2)$.



Theoretical Errors

Hence it is vital to consider theoretical corrections. These include ....

- higher orders (NNLO)
- small x $(\alpha_s^n \ln^{n-1}(1/x))$
- large x $(\alpha_s^n \ln^{2n-1}(1-x))$
- low Q^2 (higher twist)

In order to investigate true theoretical error must consider large and small x resummations, and/or use what we already know about NNLO.

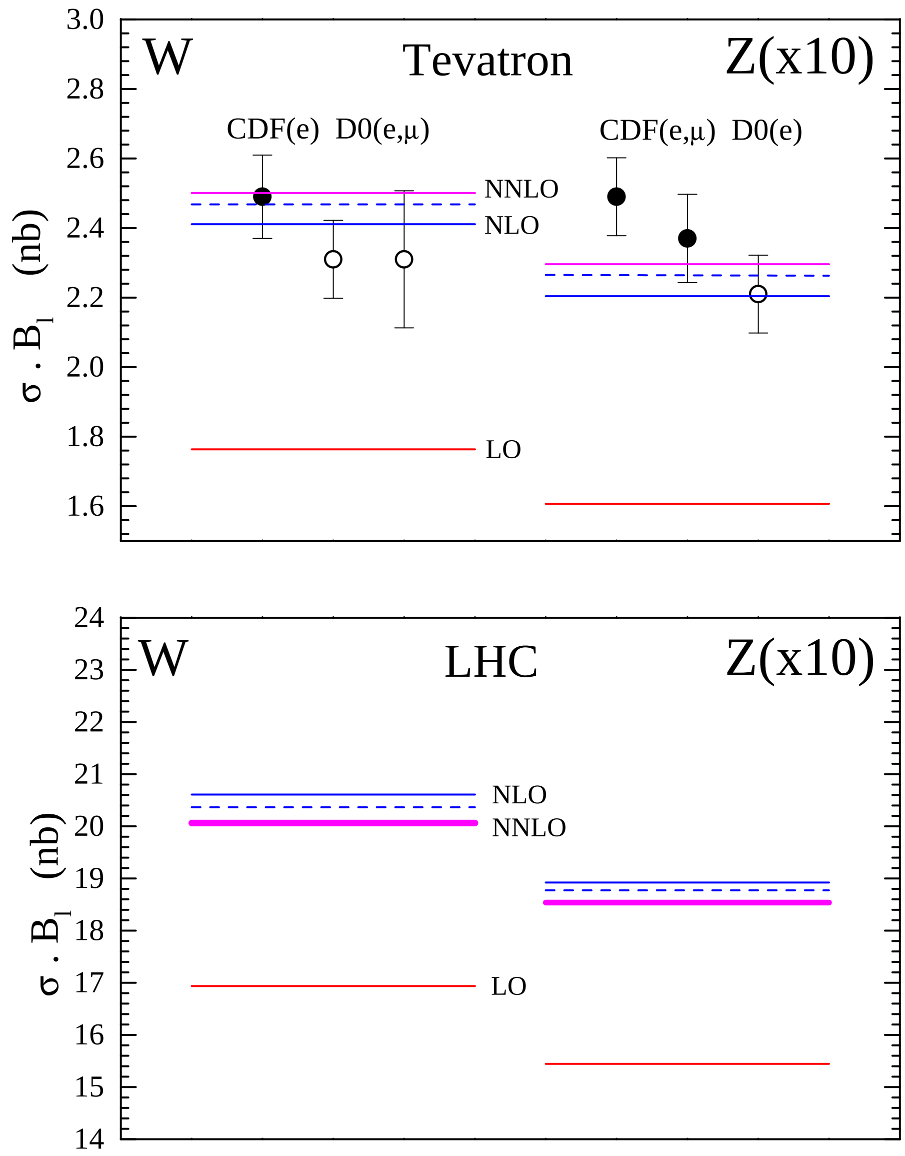
Coefficient functions known at NNLO. Singular limits $x \to 1$, $x \to 0$ known for NNLO splitting functions as well as limited moments (Larin, Nogueira, van Ritenberg Vermaseren, Retey). Complete soon. Approximate NNLO splitting functions devised by van Neerven and Vogt.

Improve quality of fit very slightly (MRST). Not much improvement at small x. Lowered value of $\alpha_S(M_Z^2) = 0.1155$ (from 0.119), determined mainly by high x data. Alekhin finds $\alpha_S(M_Z^2) = 0.1143$ at NNLO.

Reasonable stability order by order for (quark-dominated) W and Z cross-sections.

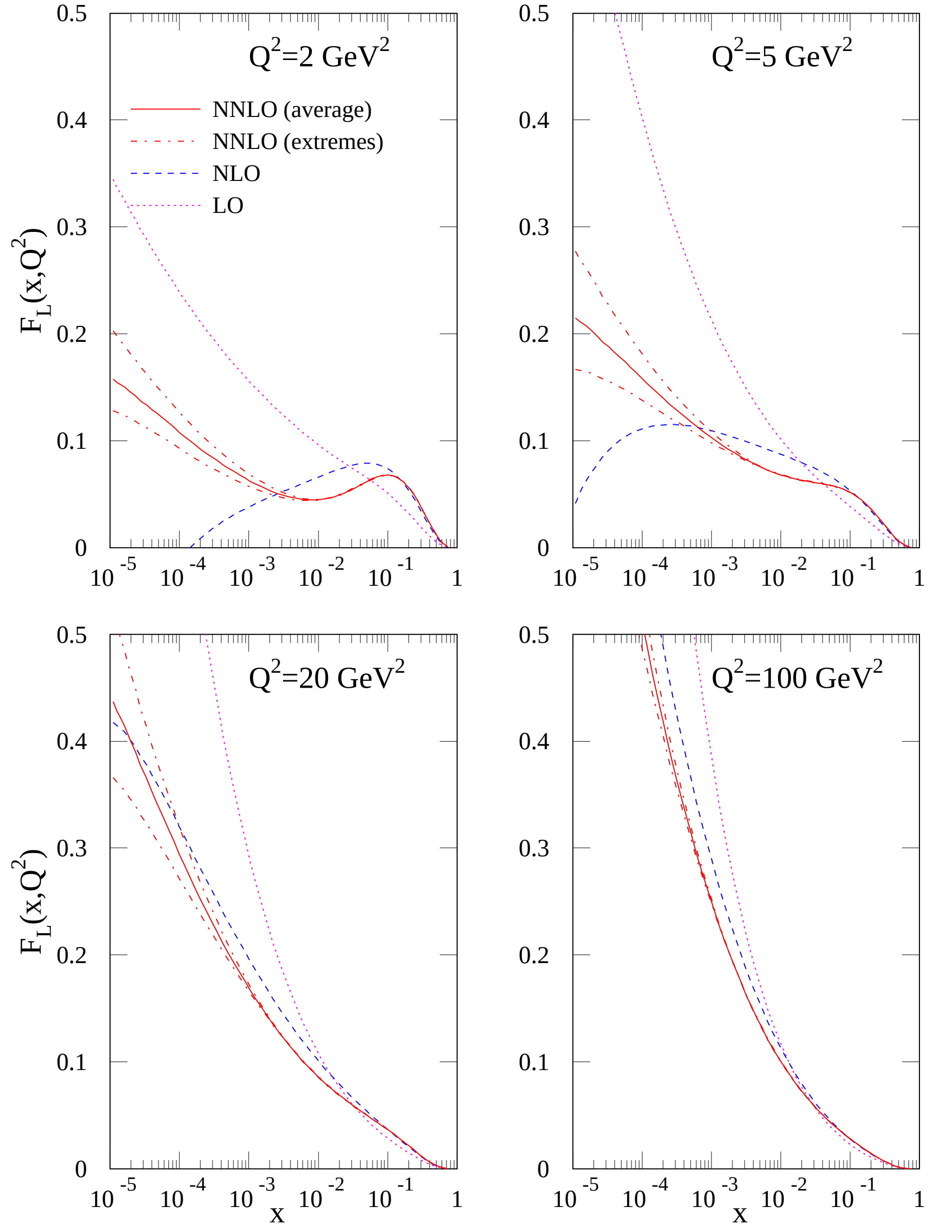
However, changes of order 4%. Much bigger than uncertainty due to experimental errors.

This fairly good convergence is largely guaranteed because the quarks are fit directly to data.



More danger in gluon dominated quantities, e.g. $F_L(x,Q^2)$.

Hence the convergence from order to order is uncertain.



Alternative approach.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons stabilize.

For W^2_{cut} raising from $12.5\mathrm{GeV}^2$ to $15\mathrm{GeV}^2$ sufficient.

Raising Q^2_{cut} from $2\mathrm{GeV}^2$ in steps there is a slow continuous and significant improvement for higher Q^2 up to $> 10\mathrm{GeV}^2$ (cut 560 data points) – suggests any corrections mainly higher orders not higher twist.

Raising x_{cut} from 0 to 0.005 (cut 271 data points) continuous improvement. At each step moderate x gluon becomes more positive.

$\rightarrow$ MRST2003 conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of x, Q^2. $\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$.

Gluon outside conservative range very negative, and $dF_2(x,Q^2)/d\ln Q^2$ incorrect, (NNLO much more stable than NLO). Theory corrections could cure this (quite plausible). Empirical resummation corrections improve global fit, e.g.

$$P_{gg} \to .... + \frac{3.86\bar{\alpha}_S^4}{x}\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right),$$

$$P_{qg} \to .... + 5.12\alpha_S \frac{N_f\bar{\alpha}_S^4}{3\pi x}\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right).$$

Saturation corrections do not help at NLO or NNLO.

Cuts suggestive of possible/probable theoretical errors for small x and/or small Q^2.

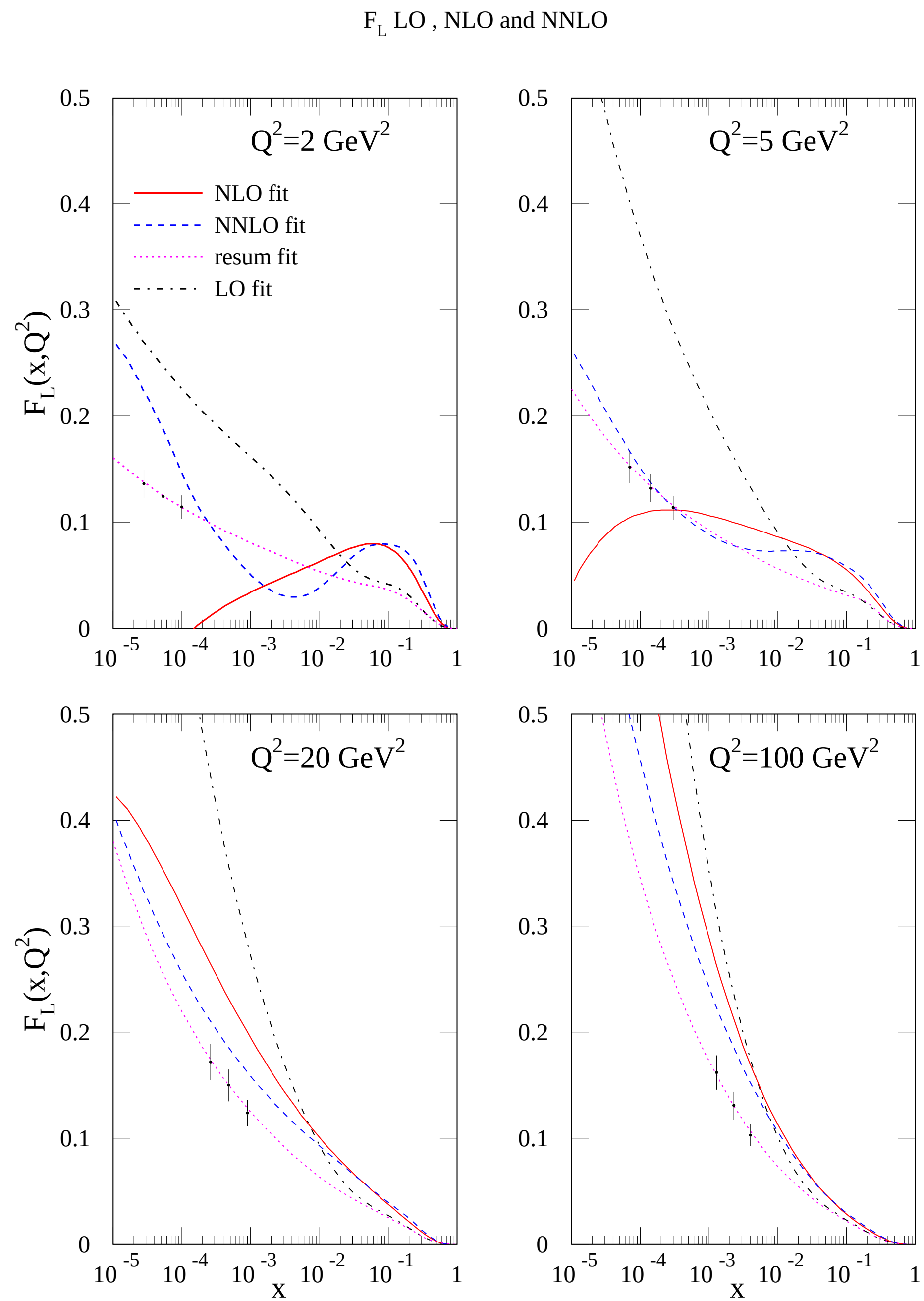
Much explicit work on $\ln(1/x)$-resummation in structure functions and parton distributions - RT, Ciafaloni, Colferai, Salam and and Stasto, Altarelli, Ball and Forte, ...

(*Also work on connecting the partons to alternative approaches at small* x, *e.g.* Golec-Biernat, Wüsthoff (dipole models), Donnachie, Landshoff (pomerons), ...)

Can suggest improvements to fit and changes in predictions.

Comparison of prediction for $F_L(x,Q^2)$ at LO, NLO and NNLO using MRST partons and also a $\ln(1/x)$-resummed prediction RT.

Accurate and direct measurements of $F_L(x,Q^2)$ and other quantities at low x and/or Q^2 (predicted range and accuracy of $F_L(x,Q^2)$ measurements at HERA III shown on the figure) would be a great help in determining whether $NNLO$ is sufficient or whether resummed (or other) corrections are necessary, or helpful for maximum precision.



Conclusions

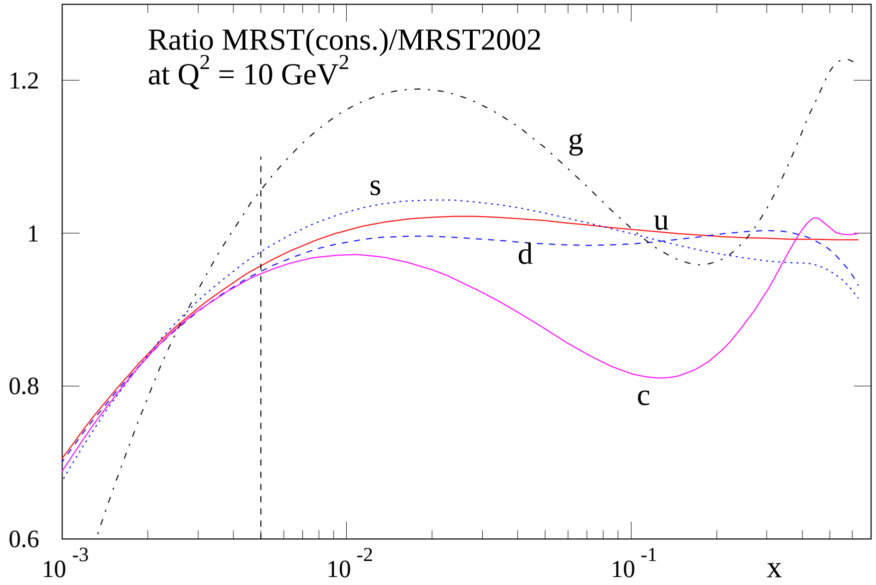
One can determine the parton distributions by performing global fits to all up-to-date data over wide range of parameter space. The fit quality using NLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data alone. No totally preferred approach – all have pros and cons. Uncertainties rather small using all approaches – $\sim 1-5\%$ except in certain regions of parameter space.

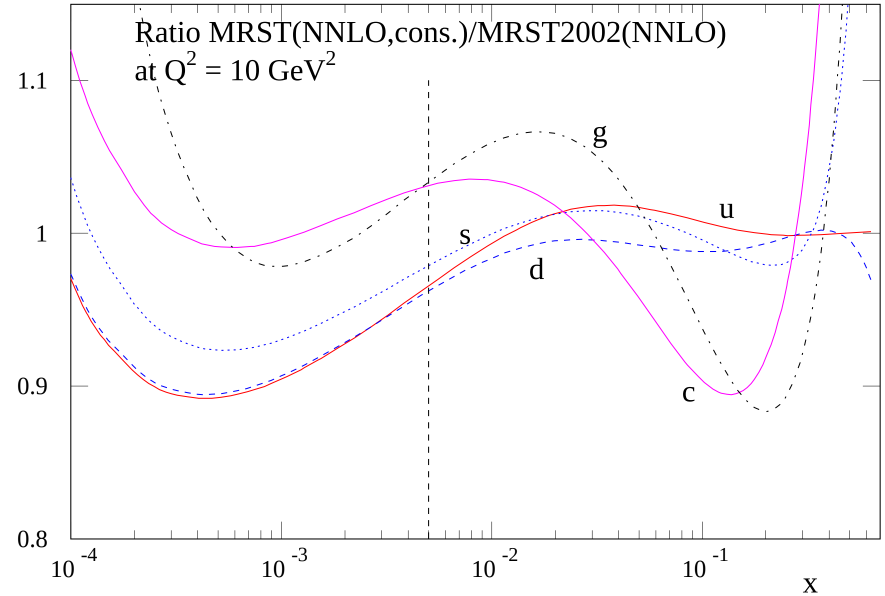
Uncertainty from input assumptions e.g. cuts on data, data used, ..., comparable and potentially larger. Can shift central values of predictions on/using partons significantly.

Errors from higher orders/resummation potentially large in some regions of parameter space, and due to correlations between partons feed into all regions. Cutting out low x and/or Q^2 allows much improved fit to remaining data, and altered partons. NNLO appears to be much more stable than NLO.

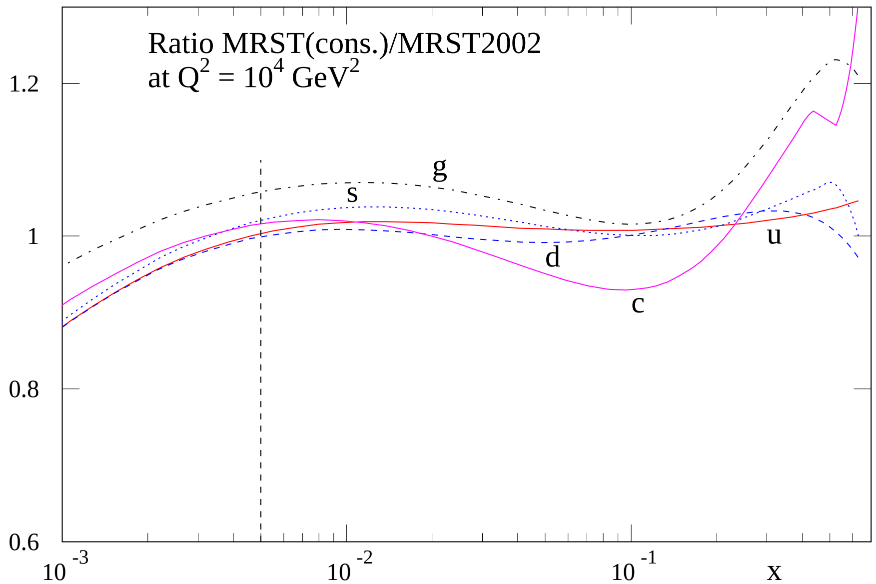
Theory often the dominant source of uncertainty at present. Systematic study needed. Much progress – NNLO, resummations ..., but much still to do. Both for theory and in obtaining useful new data (HERA III ?). Very busy and important area of research.



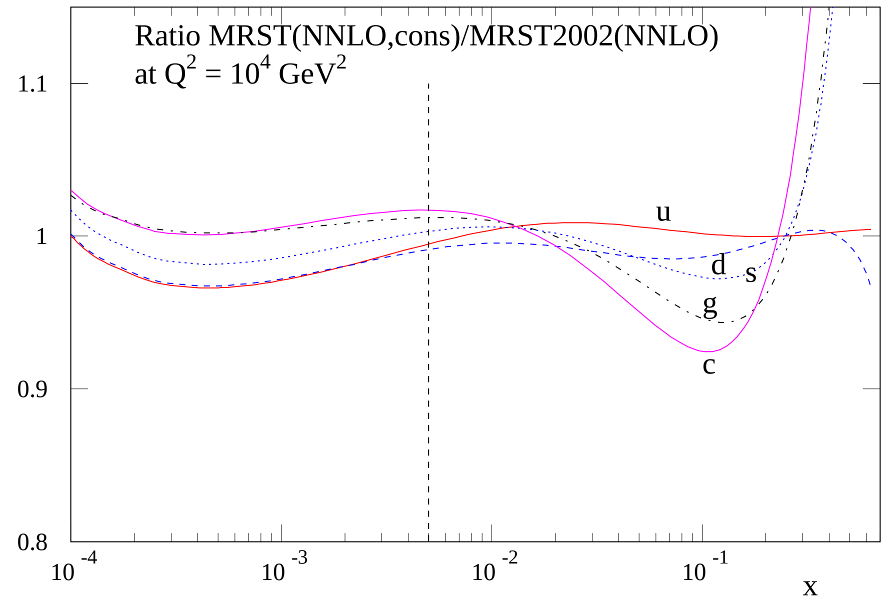
Ratio MRST(cons.)/MRST2002
at Q^2 = 10 GeV^2
g
s
u
d
c
1.2
1
0.8
0.6
10^{-3}
10^{-2}
10^{-1}
x



Ratio MRST(NNLO,cons.)/MRST2002(NNLO)
at Q^2 = 10 GeV^2
g
u
s
d
c
1.1
1
0.9
0.8
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

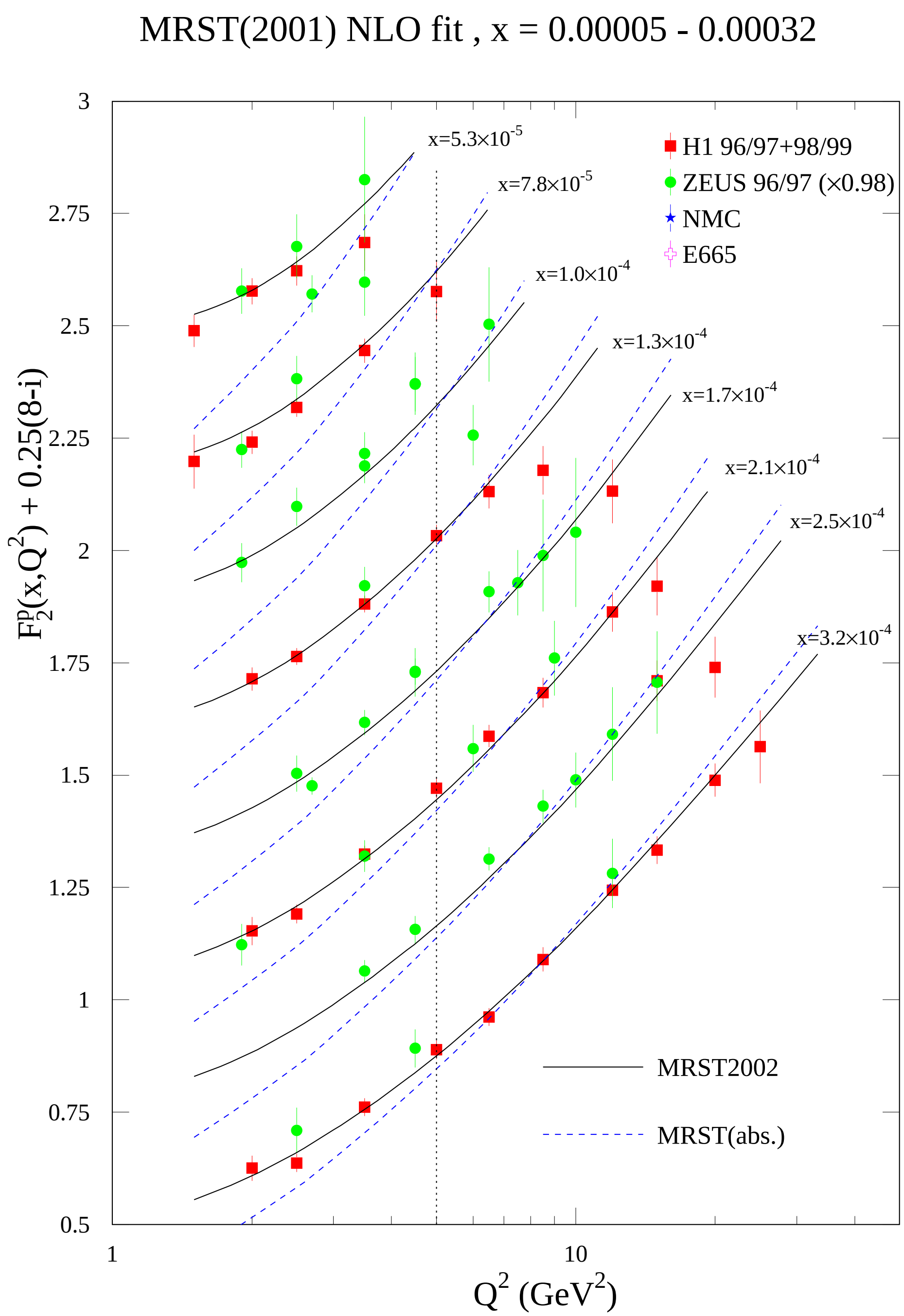


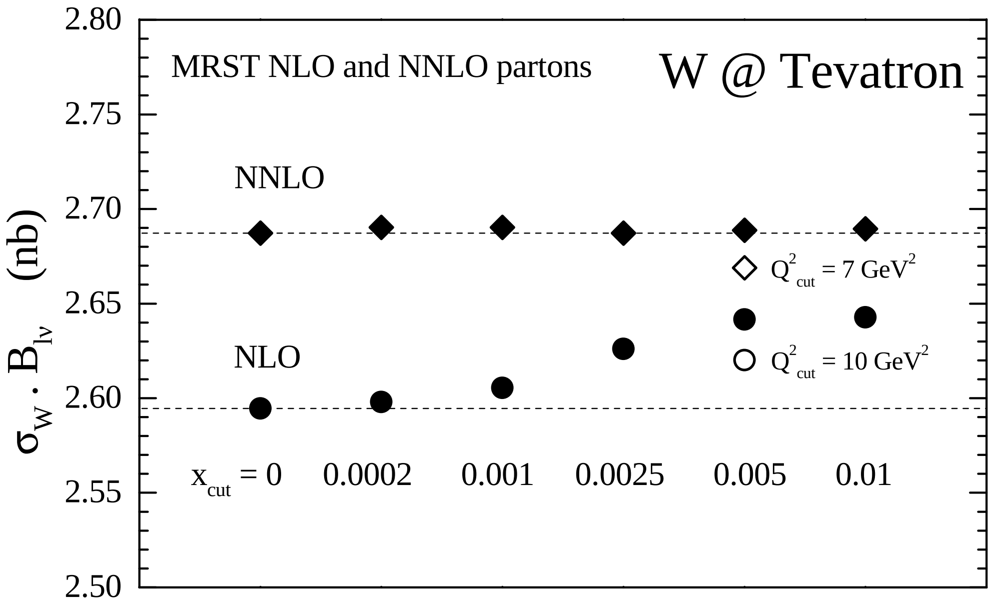
Ratio MRST(cons.)/MRST2002
at Q^2 = 10^4 GeV^2
g
s
d
u
c
1.2
1
0.8
0.6
10^{-3}
10^{-2}
10^{-1}
x

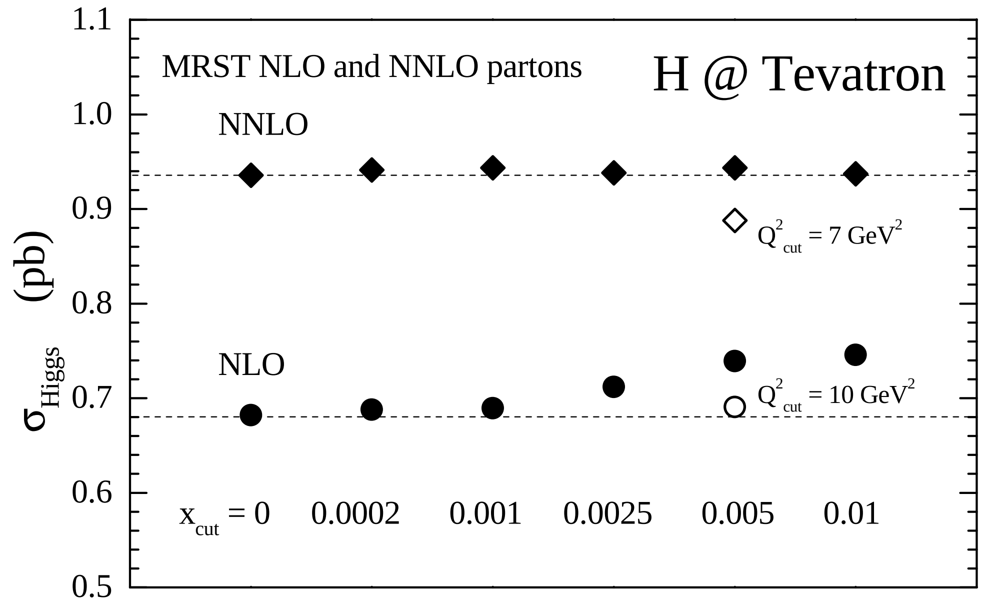


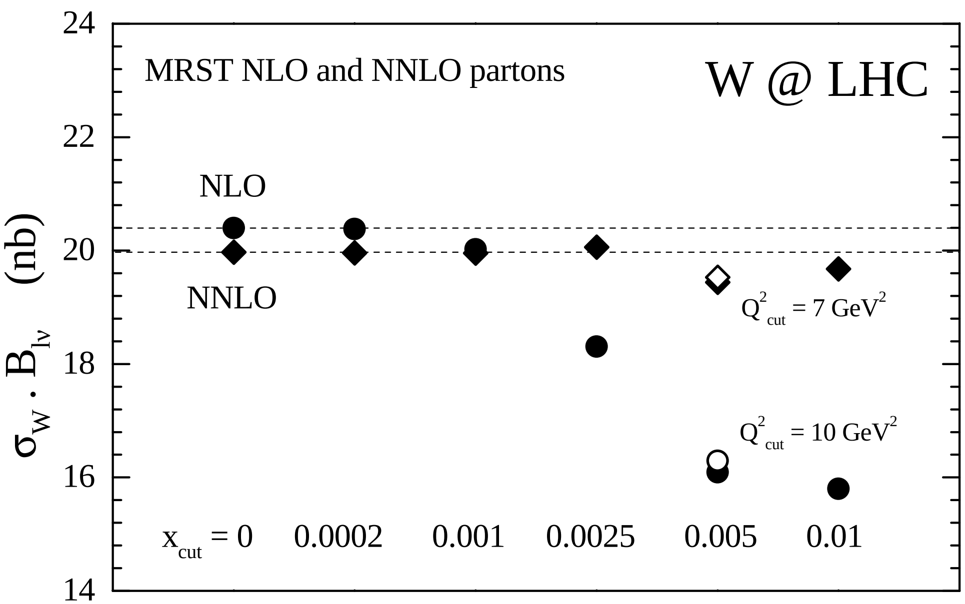
Ratio MRST(NNLO,cons)/MRST2002(NNLO)
at Q^2 = 10^4 GeV^2
u
d
s
g
c
1.1
1
0.9
0.8
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

MRST fit with shadowing corrections extraploated to $Q^2 \leq 5\mathrm{GeV}^2$




MRST NLO and NNLO partons
W @ Tevatron
NNLO
NLO
Q^2_{cut} = 7 GeV^2
Q^2_{cut} = 10 GeV^2
x_{cut} = 0
0.0002
0.001
0.0025
0.005
0.01
$\sigma_W \cdot B_{l\nu}$ (nb)
2.80
2.75
2.70
2.65
2.60
2.55
2.50


MRST NLO and NNLO partons
H @ Tevatron
NNLO
NLO
Q^2_{cut} = 7 GeV^2
Q^2_{cut} = 10 GeV^2
x_{cut} = 0
0.0002
0.001
0.0025
0.005
0.01
σ_{Higgs} (pb)
1.1
1.0
0.9
0.8
0.7
0.6
0.5


MRST NLO and NNLO partons
W @ LHC
NLO
NNLO
Q^2_{cut} = 7 GeV^2
Q^2_{cut} = 10 GeV^2
x_{cut} = 0
0.0002
0.001
0.0025
0.005
0.01
$\sigma_W \cdot B_{l\nu}$ (nb)
24
22
20
18
16
14

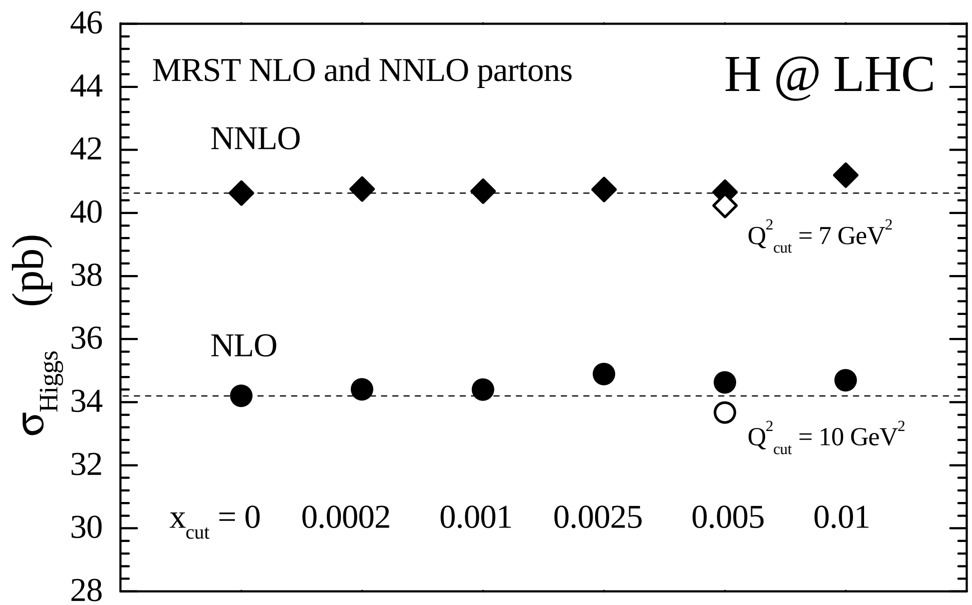
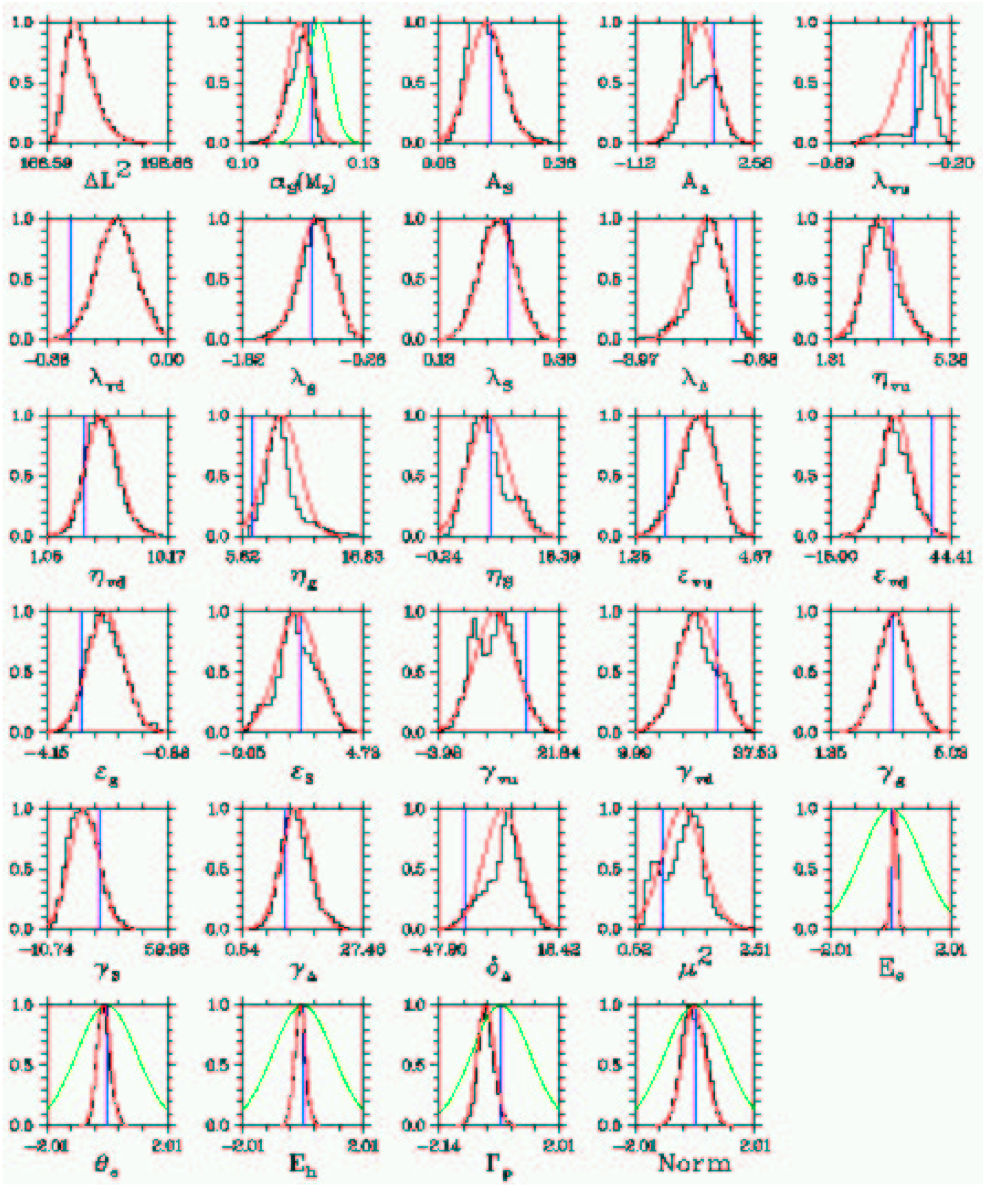

MRST NLO and NNLO partons
H @ LHC
NNLO
NLO
Q^2_{cut} = 7 GeV^2
Q^2_{cut} = 10 GeV^2
x_{cut} = 0
0.0002
0.001
0.0025
0.005
0.01
σ_{Higgs} (pb)
46
44
42
40
38
36
34
32
30
28

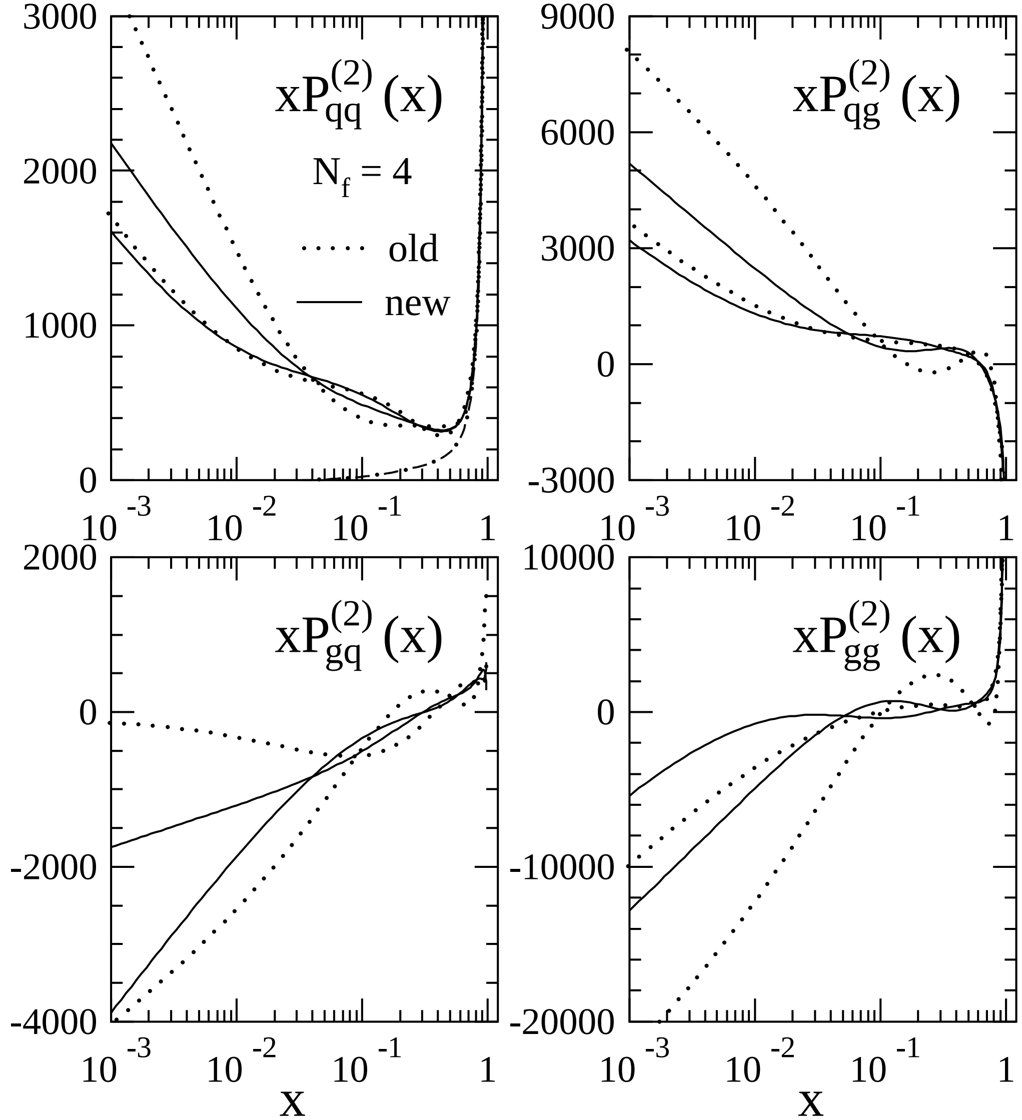
Table 2: Cross sections for Drell-Yan pairs (e^+e^-) with PYTHIA 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

PDF set	Comment	xsec
	$81 < M < 101$ GeV	
CTEQ5L	PYTHIA internal	1516 $\pm$ 5 pb
CTEQ5L	PDFLIB	1536 $\pm$ 5 pb
CTEQ6	LHAPDF	1564 $\pm$ 5 pb
MRST2001	LHAPDF	1591 $\pm$ 5 pb
Fermi2002	LHAPDF	1299 $\pm$ 4 pb
	$M > 1000$ GeV	
CTEQ5L	PYTHIA internal	6.58 $\pm$ 0.02 fb
CTEQ5L	PDFLIB	6.68 $\pm$ 0.02 fb
CTEQ6	LHAPDF	6.76 $\pm$ 0.02 fb
MRST2001	LHAPDF	7.09 $\pm$ 0.02 fb
Fermi2002	LHAPDF	7.94 $\pm$ 0.03 fb

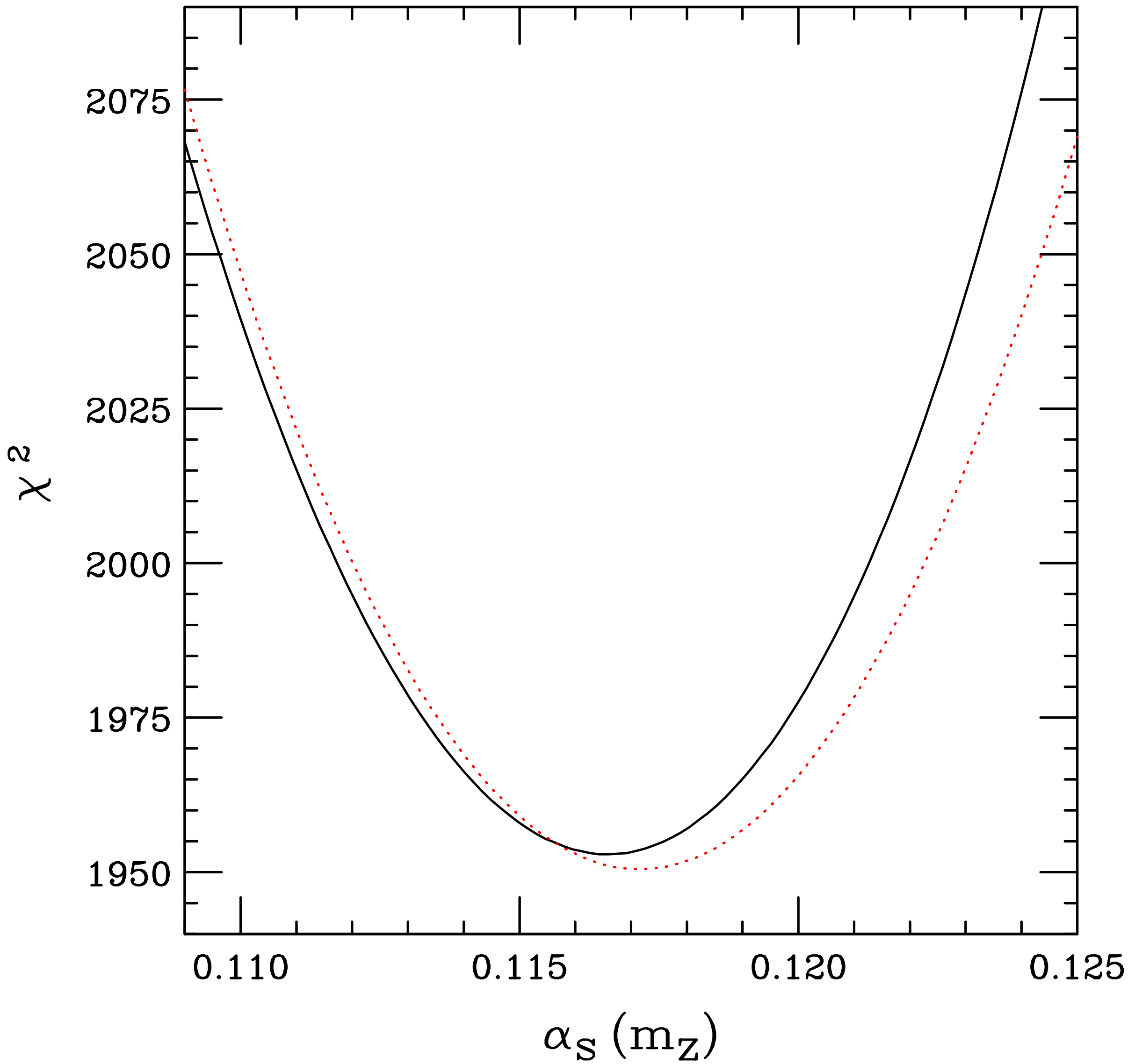
H1 set of parton parameters from GKK approach. Red curve Gaussian approx and blue line MRST value. Green curve for α_S is LEP result.

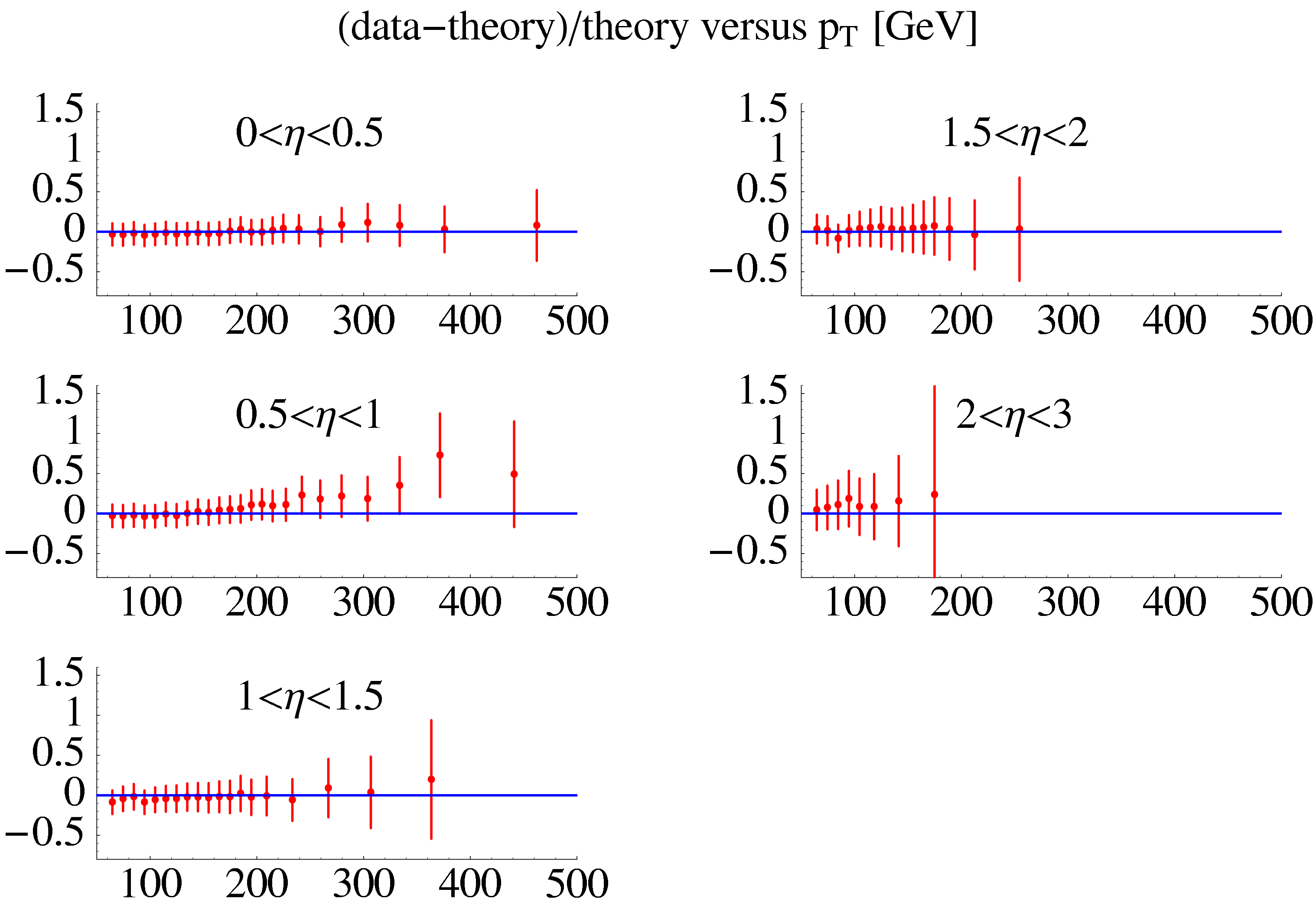


Aproximate NNLO splitting functions devised by van Neerven and Vogt.



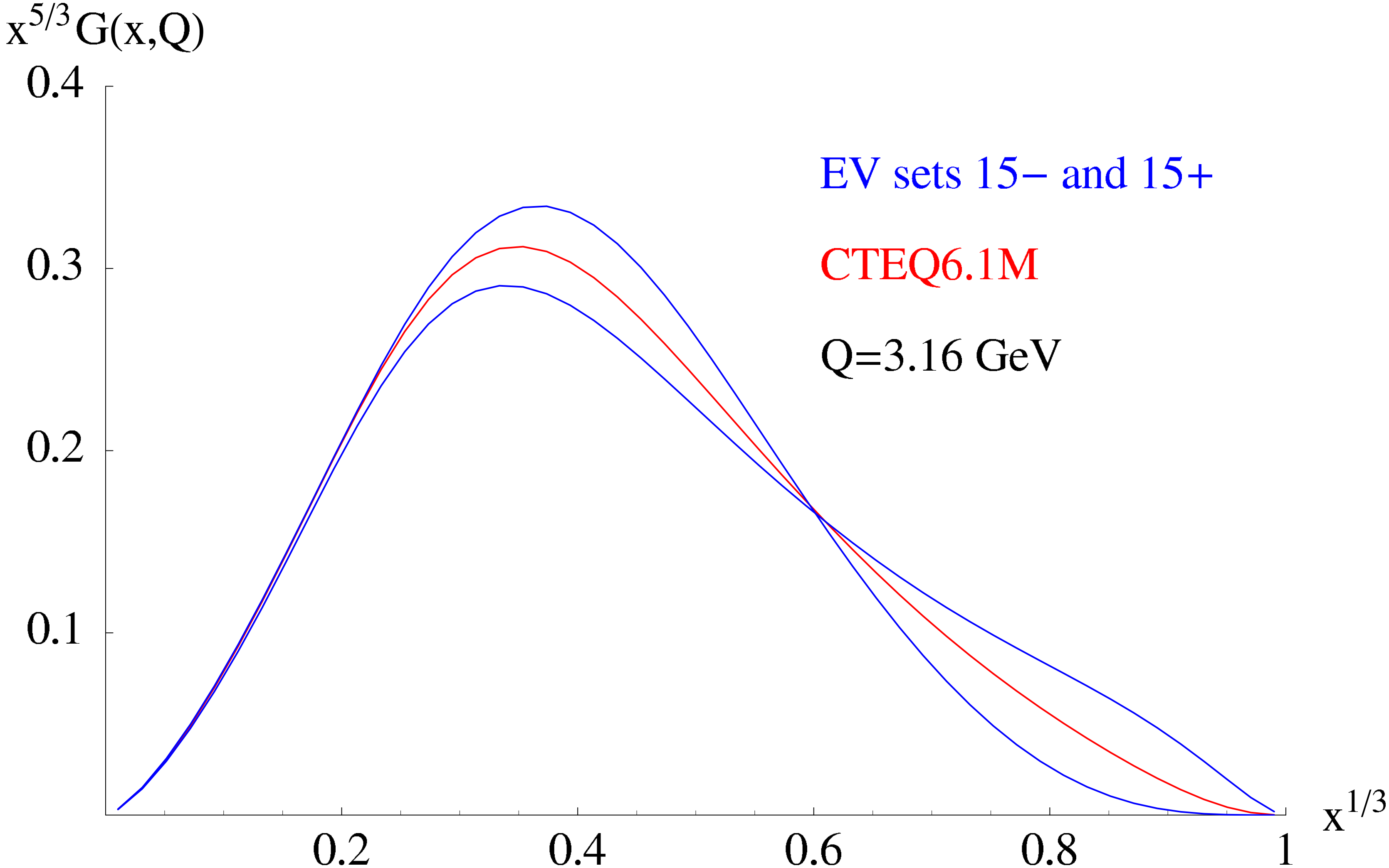
χ^2 against $\alpha_S(M_Z^2)$ for CTEQ for two choices of defintion of NLO coupling.



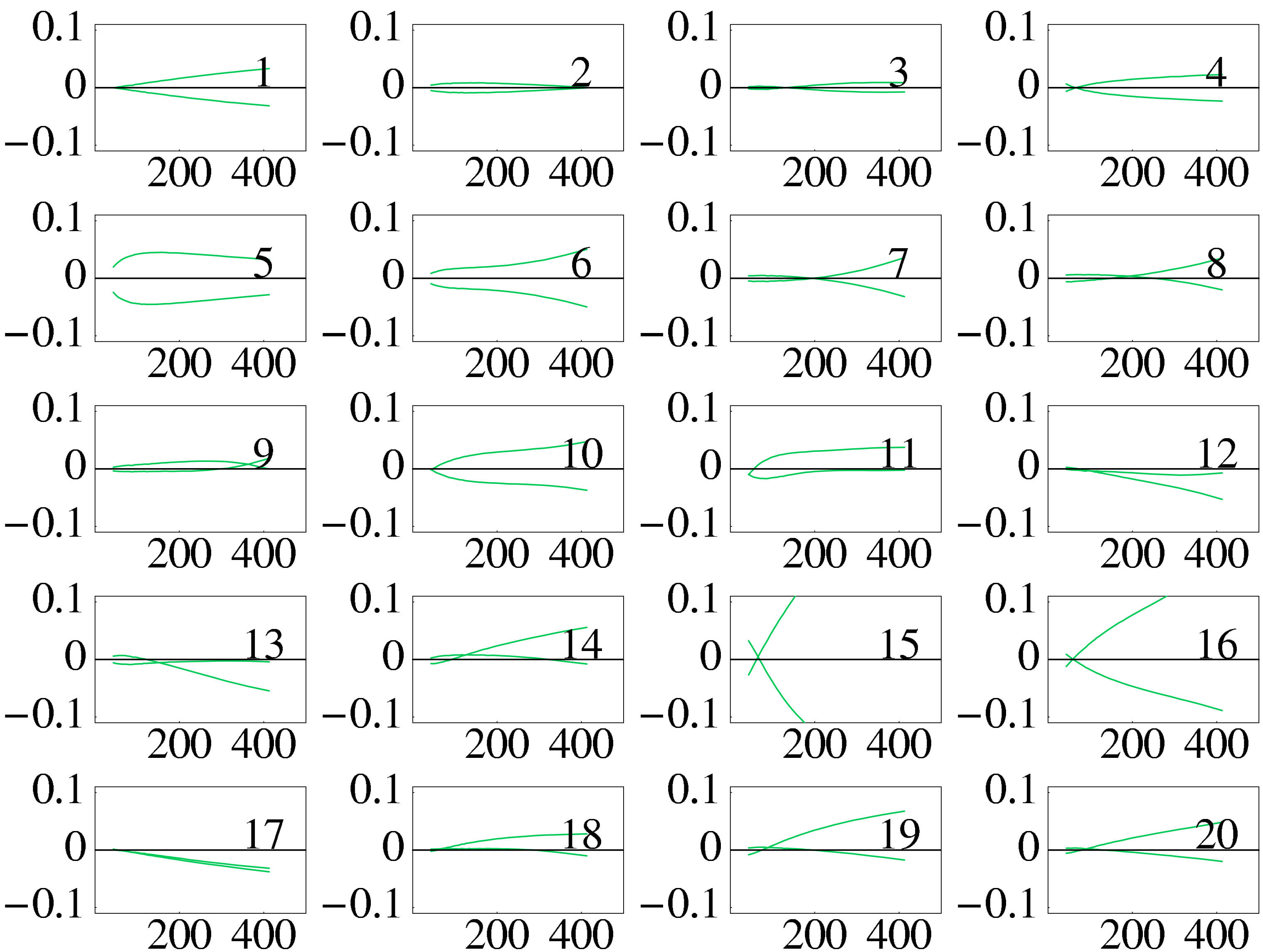


CTEQ6 fit to D0 jet data.

Variation in CTEQ6 gluon along most sensitive eigenvalue direction.



Variation in CTEQ6 jet predictions for variations in each of the 20 eigenvector directions.



Variation in χ^2 against $[S^-]$ for NuTeV dimuon data (red) and all data sensitive to strangeness asymmetry (blue).