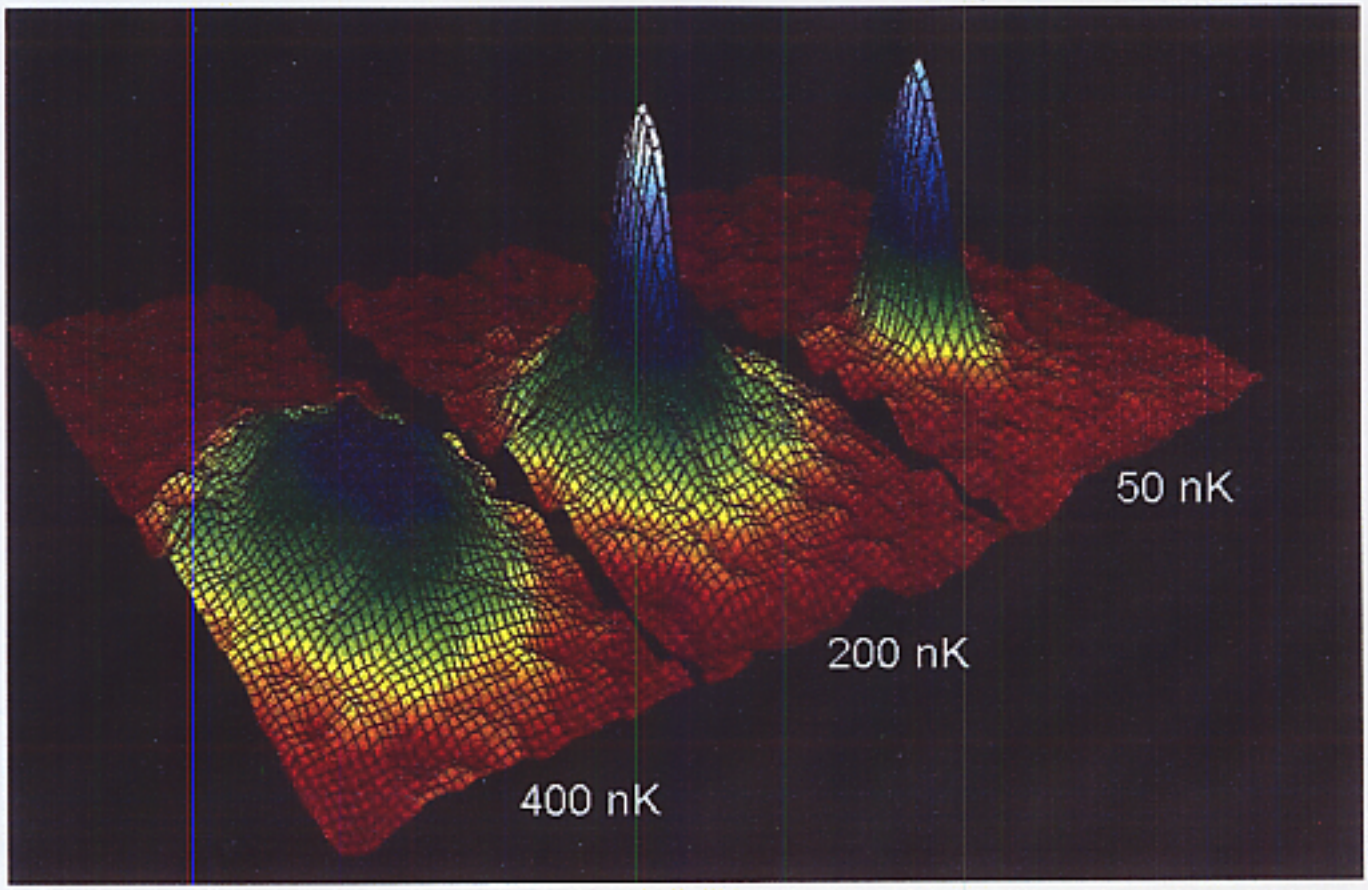
Cornell-Weiman group (1995)
http://jilawww.colorado.edu/BEC



COLD ATOMS

A New Playground for Field Theorists

Eric Braaten
Ohio State University
& Fermilab

Bosonic Atoms that have been Bose-condensed

1995	^{87}Rb
	^{7}Li
	^{23}Na
1998	H
2000	^{85}Rb
2001	$He^{*}(2^{3}S_{1})$
	^{41}K
2002	^{133}Cs

~ 40 laboratories

Fermionic Atoms that have been cooled well below Fermi temperature

1999	^{40}K
2001	^{6}Li

~ 6 laboratories

COLD ATOMS

A New Playground for

Field Theorists

1. Cold Atom Physics is Particle Physics

$\Longrightarrow$ Quantum Field Theory

2. Classical Field Theory

applied to Bose-Einstein Condensates

3. Quantum Field Theory

a. A Tale of T_c (for BEC)

b. Strongly-interacting atoms

c. Another Tale of T_c (for fermions)

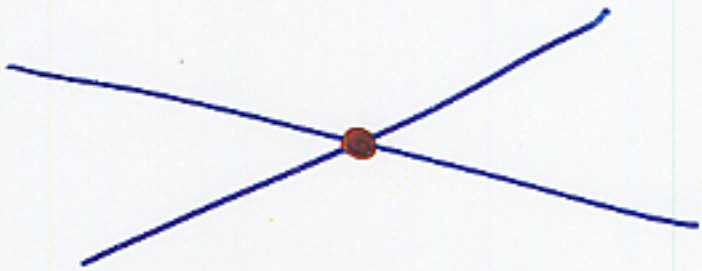
My Definition of <u>Cold Atoms</u>:

atoms with <u>low</u> <u>enough</u> <u>energy</u>

that they behave

like <u>point</u> <u>particles</u>

with <u>point</u> <u>interactions</u>



$$\Longrightarrow \quad E \lesssim 10^{-3}\,K \approx 10^{-7}\,eV$$

High Energy Physics:

particles with Tera-eV energies

and pico-barn cross sections

$(10^{-20}\,m)^2$

Cold Atom Physics:

particles with pico-eV energies

and Tera-barn cross sections

$(10^{-8}\,m)^2$

pico $= 10^{-12}$ Tera $= 10^{+12}$

Energy scales

Cold atoms are as far removed

from experimental High Energy Physic

as HEP is from Grand Unified Theorie

BEC in cold atoms is as for removed

from experimental HEP

as HEP is from the Planck scale

"Don't tell Keith Ellis,

but Braaten is working on low energy physics again."

FERMI NATIONAL ACCELERATORY LABORATORY

EMPLOYEE HANDBOOK

IMMEDIATE DISMISSALS — MISCONDUCT

Any Employee whose conduct, actions or performance violates or conflicts with Fermilab policies may be terminated immediately and without warning.

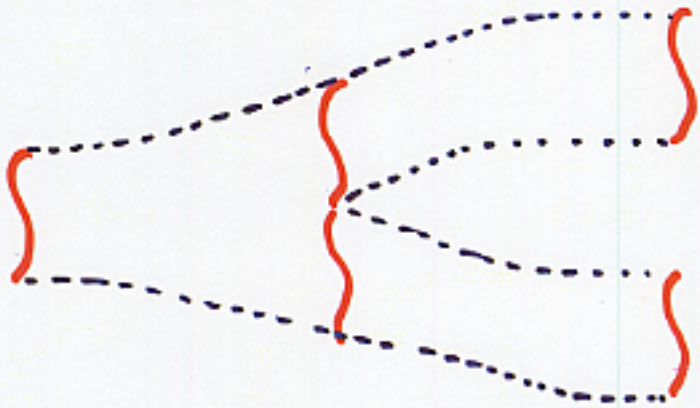
The following are some examples of grounds for immediate dismissal of an employee:

- Unauthorized posting or removal of notices from bulletin boards
- Gambling, conducting games of chance
- Sleeping on duty

Elementary particles may have <u>structure</u>

experiment: $\ell \ll 10^{-19}\,m$

string theory: $\ell \sim 10^{-35}\,m$??



de Broglie wavelength: $\hbar/p$

Structure cannot be resolved if $\ell \ll \hbar/p$

At energies $E \ll \hbar c/\ell$

elementary particles behave like point particle
with point interactions

Atoms do have <u>structure</u>

electron clouds extend to $\ell \sim 5$ Å

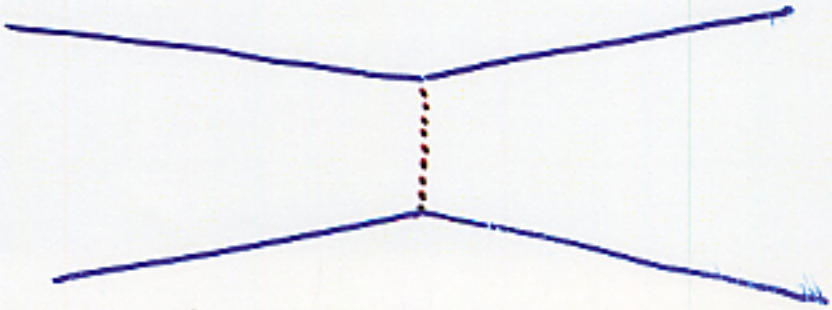
de Broglie wavelength: $\frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mE}}$

Structure cannot be resolved if $\ell \ll \frac{\hbar}{p}$

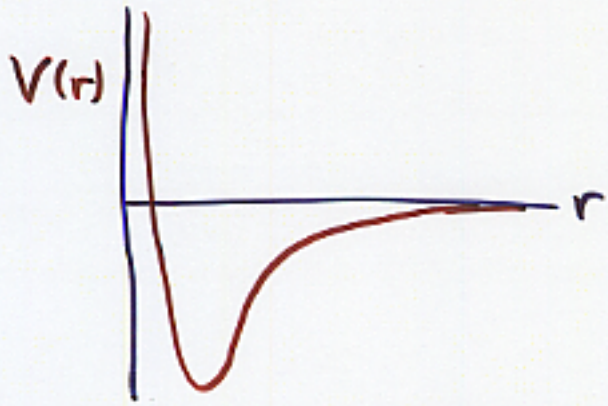
At energies $E \ll \hbar^2/m\ell^2$ $(\sim 10^{-1}K \approx 10^{-5}eV)$

atoms behave like point particles

interacting through a potential $V(\vec{r}_1-\vec{r}_2)$



Interatomic potential has long-range tail



$$V(r) \longrightarrow -\frac{C_6}{r^6}$$

van der Waals length scale:

$$\ell_{vdW} = (mC_6/\hbar^2)^{1/4} \sim 50\,\text{Å}$$

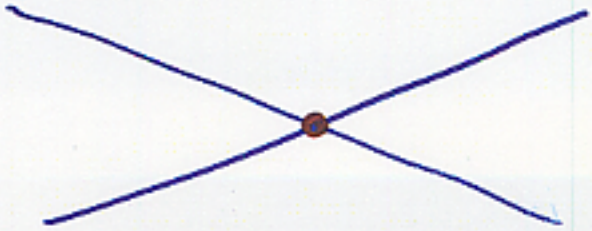
Shape of potential

cannot be resolved if $\ell_{vdW} \ll \frac{\hbar}{p}$

At energies $E \ll \hbar^2/m\ell_{vdW}^2$ ($\sim 10^{-3}\,K \approx 10^{-7}\,eV$

atoms behave like point particles

with point interactions



High Energy Physics

Elementary particles behave like point particles
with point interactions

$\Longrightarrow$ can be described by
Relativistic Local Quantum Field Theory

Cold Atom Physics

Cold atoms behave like point particles
with point interactions

$\longrightarrow$ can be described by
Nonrelativistic Local Quantum Field Theory

quantum field operator $\psi(\vec{r},t)$
annihilates atom at point $\vec{r}$

interaction: $\mathcal{L}_{int} = -g\,(\psi^{\dagger}\psi)^2$

QED

describes low-energy electrons, positrons, photons

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\not{D} - m_e)\psi$$

$$= -ie\gamma^\mu$$

2 parameters: m_e = electron mass

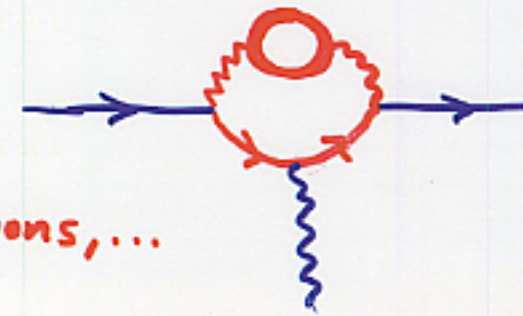
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137.036}$$

<u>limited domain of validity</u>

does not describe muons, pions, ...

$\Longrightarrow$ good approximation only for $E < m_\mu = 108$ M

<u>limited accuracy</u>



omits effects of virtual muons, ...

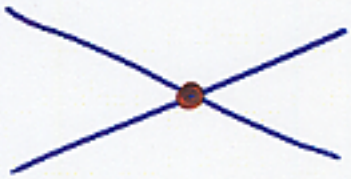
$$\Longrightarrow \text{errors} \sim \alpha^2 \frac{m_e E}{m_\mu^2}$$

The "QED" for Cold Bosonic Atoms

describes single hyperfine spin state
for atoms with sufficiently low energy

$$\mathcal{L} = \psi^{+}\left(i\hbar\frac{\partial}{\partial t}\right)\psi - \mathcal{H}$$

$$\mathcal{H} = \psi^{+}\left(-\frac{\hbar^2}{2m}\nabla^2\right)\psi + \frac{2\pi\hbar^2 a}{m}\left(\psi^{+}\psi\right)^2$$



$$= -\frac{8\pi\hbar^2 a}{m}$$

2 parameters: $m =$ mass of atom

$a =$ S-wave scattering length

(typically, $|a| \sim \ell_{vdW} \sim 50\,\text{Å}$)

<u>limited range of validity</u>

$$E < E_{vdW} = \frac{\hbar^2}{m\,\ell_{vdW}^2}$$

<u>limited accuracy</u>

$$\text{errors} \sim \frac{E}{E_{vdW}}$$

Classical Field Theory

approximate quantum field operator $\psi(\vec{r},t)$

by its expectation value: $\phi(\vec{r},t) = \langle \psi(\vec{r},t) \rangle$

(Mean-Field Approximation)

Gross-Pitaevski Equation

$$i\frac{\partial}{\partial t}\phi = \left(-\frac{\hbar^2}{2m} - \mu + U(\vec{r})\right)\phi + \frac{4\pi\hbar^2 a}{m}|\phi|^2\phi$$

μ = chemical potential
(adjust to get correct number N of atoms)

$U(\vec{r})$ = trapping potential

describes Bose-Einstein condensate

almost all atoms in same quantum state
with wavefunction $\frac{1}{\sqrt{N}}\phi(\vec{r},t)$

Gross-Pitaevski equation

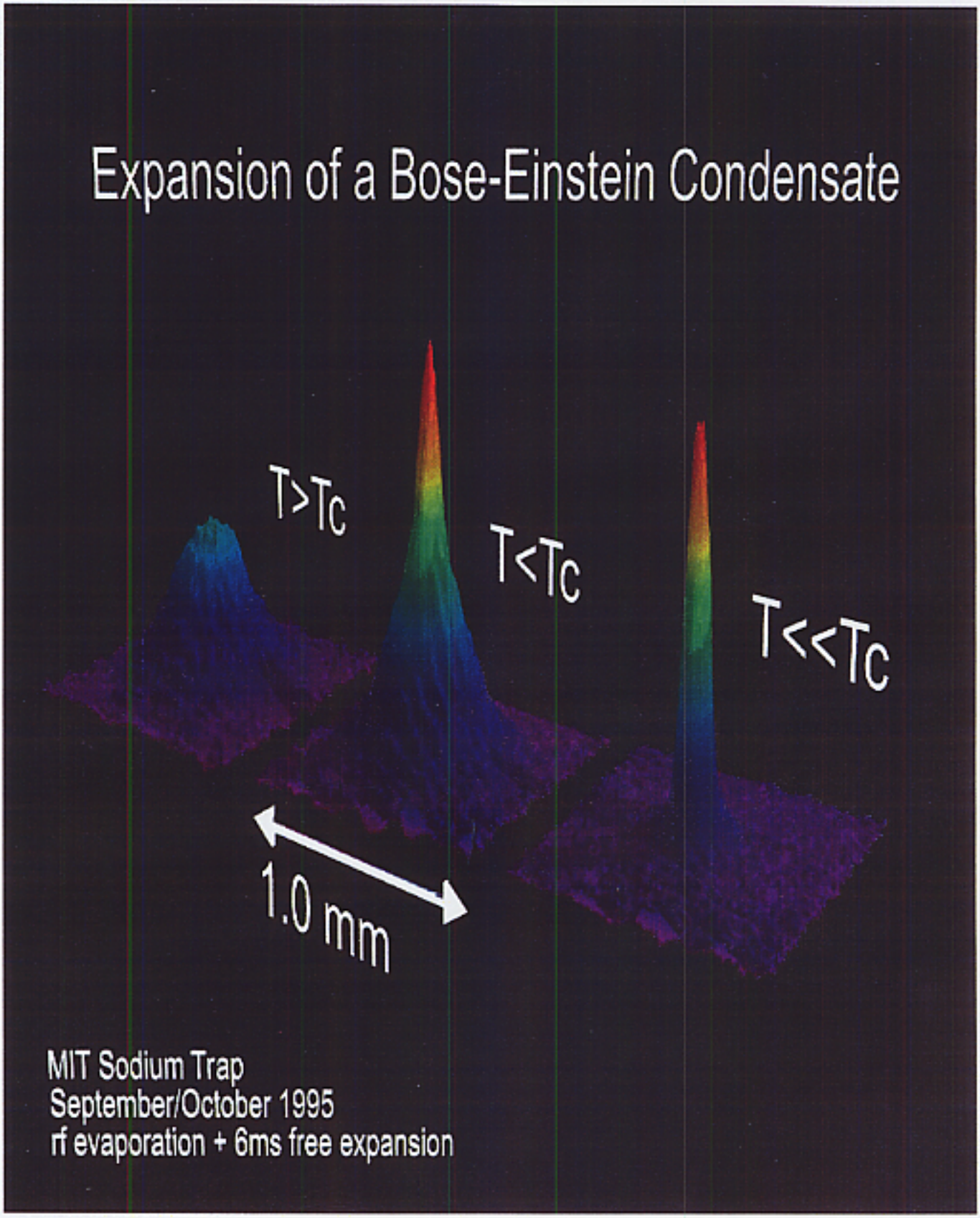
$$i\frac{\partial}{\partial t}\phi = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + U(\vec{r})\right)\phi + \frac{4\pi\hbar^2 a}{m}|\phi|^2\phi$$

accurate description of

Bose-Einstein condensate

- ground state
- collective excitations
- extended objects – solitons
  – vortices

interaction term is important!



Ketterle group – MIT

http://cua.mit.edu/ketterle_group

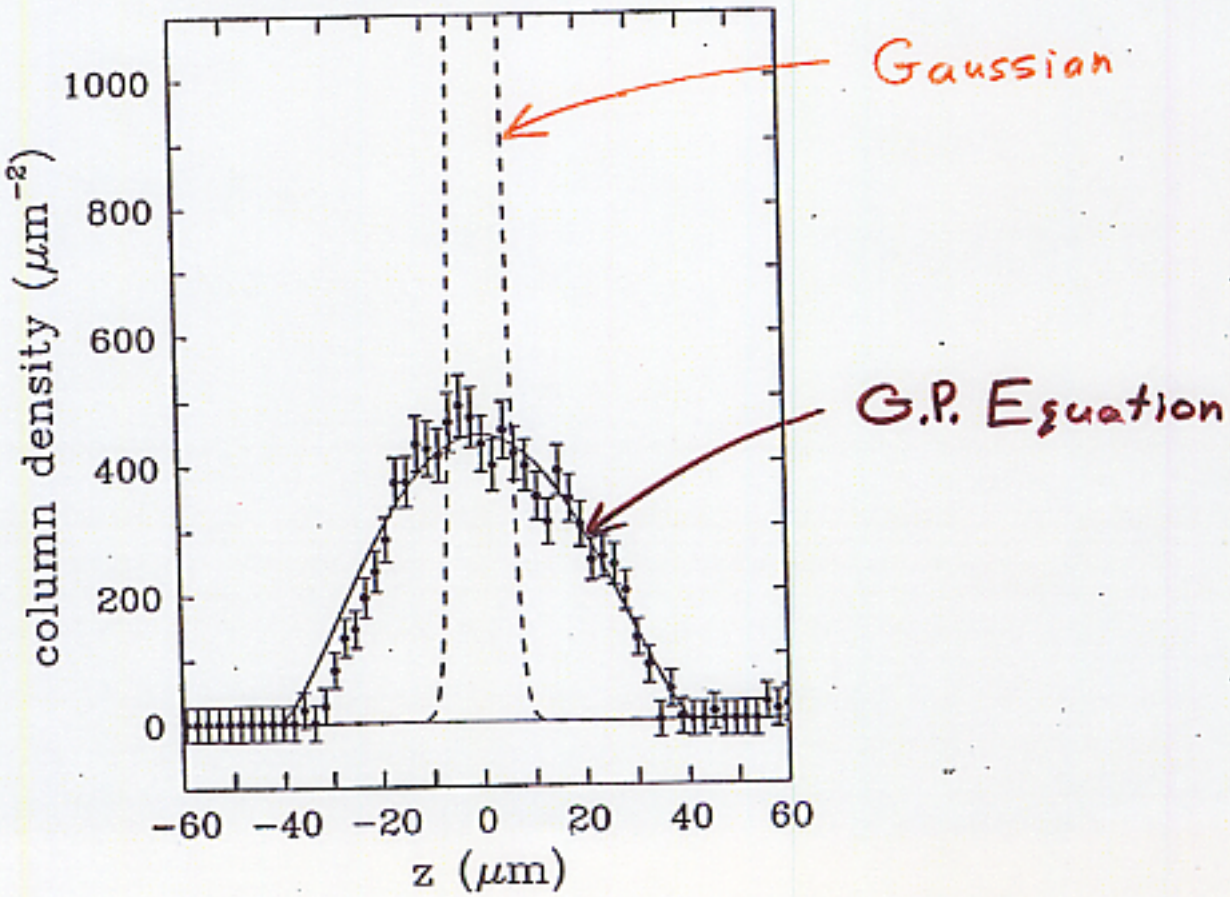
Ground state of a Bose-Einstein condensate

Gross-Pitaevski equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + U(\vec{r})\right]\phi + \frac{4\pi\hbar^2 a}{m}|\phi|^2\phi = 0$$

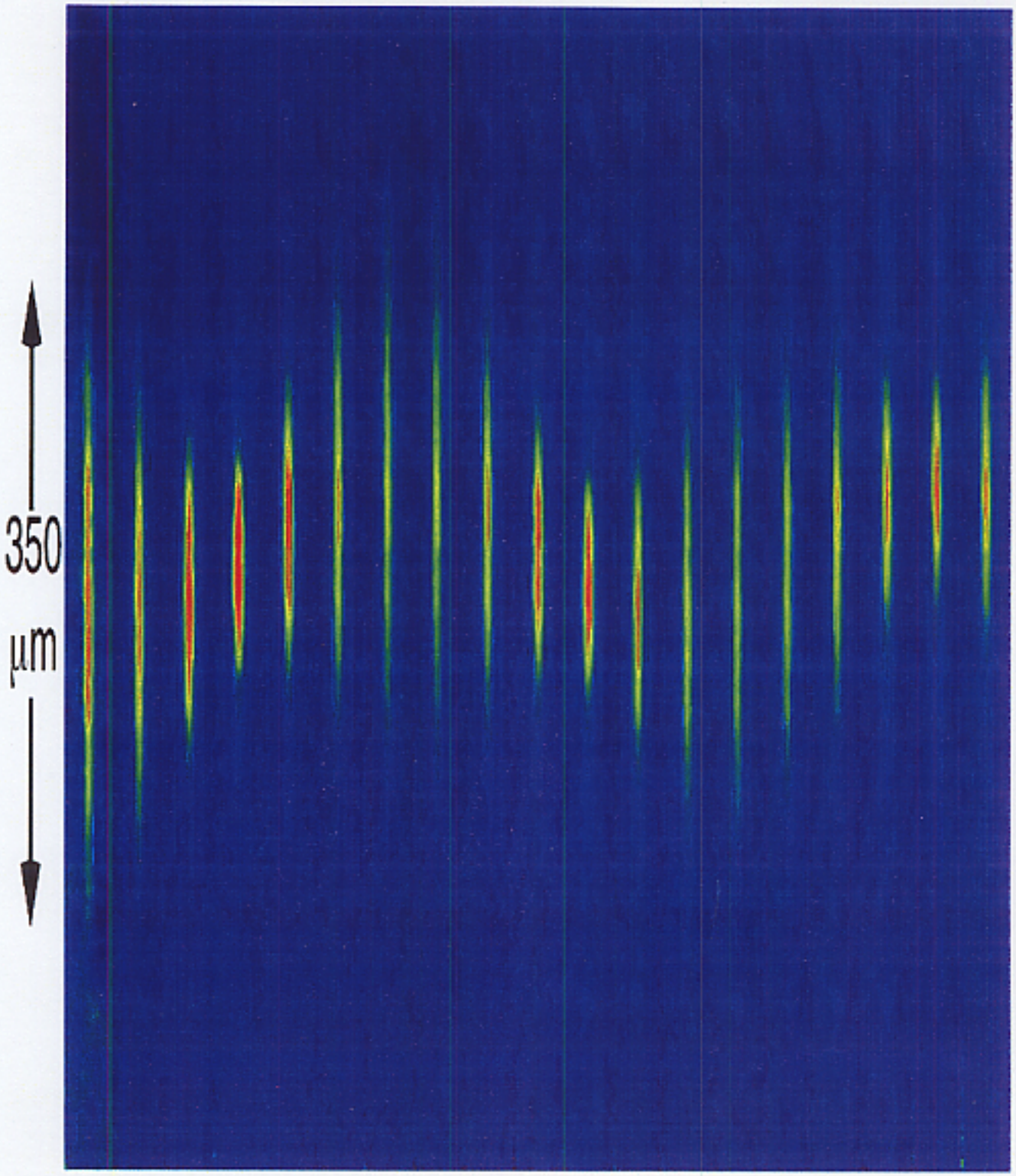
number density: $n(\vec{r}) = |\phi(\vec{r})|^2$

BEC of ^{23}Na atoms



Hau group — Harvard (1998)

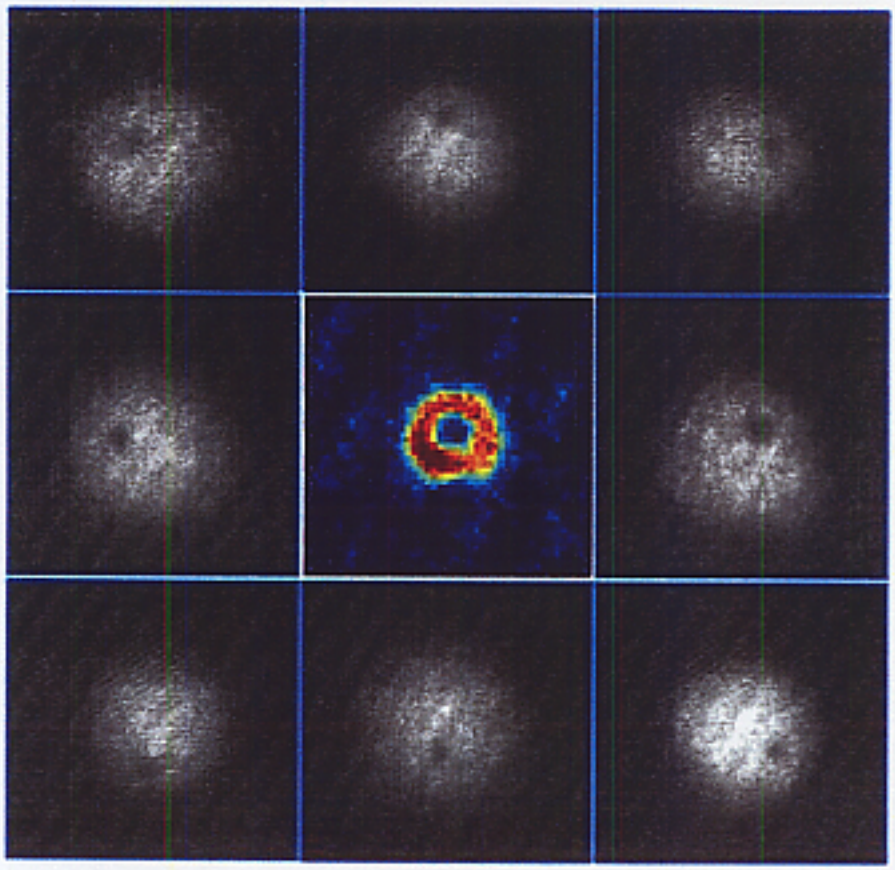
Quadrupole oscillations
in a BEC of ^{23}Na atoms



5 milliseconds per frame

Ketterle group – MIT (1998)

http://cua.mit.edu/ketterle_group

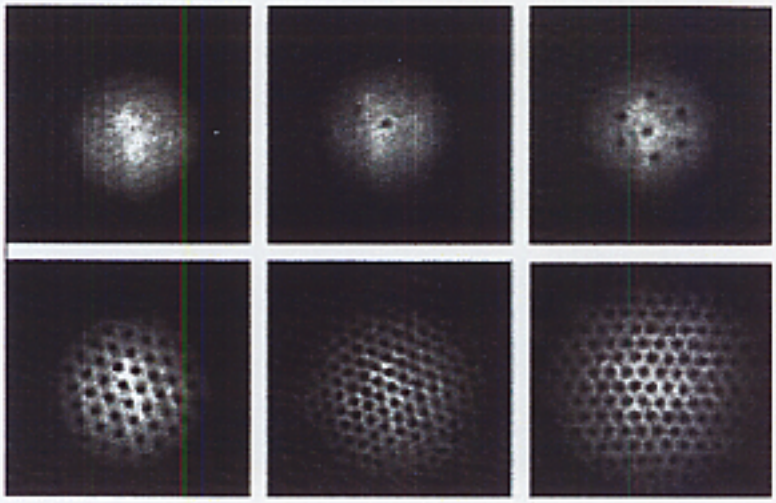


Center: Vortex in BEC of ^{87}Rb

Edges: Precession of vortex around the center of the BEC

Cornell group — JILA/Colorado (2000)

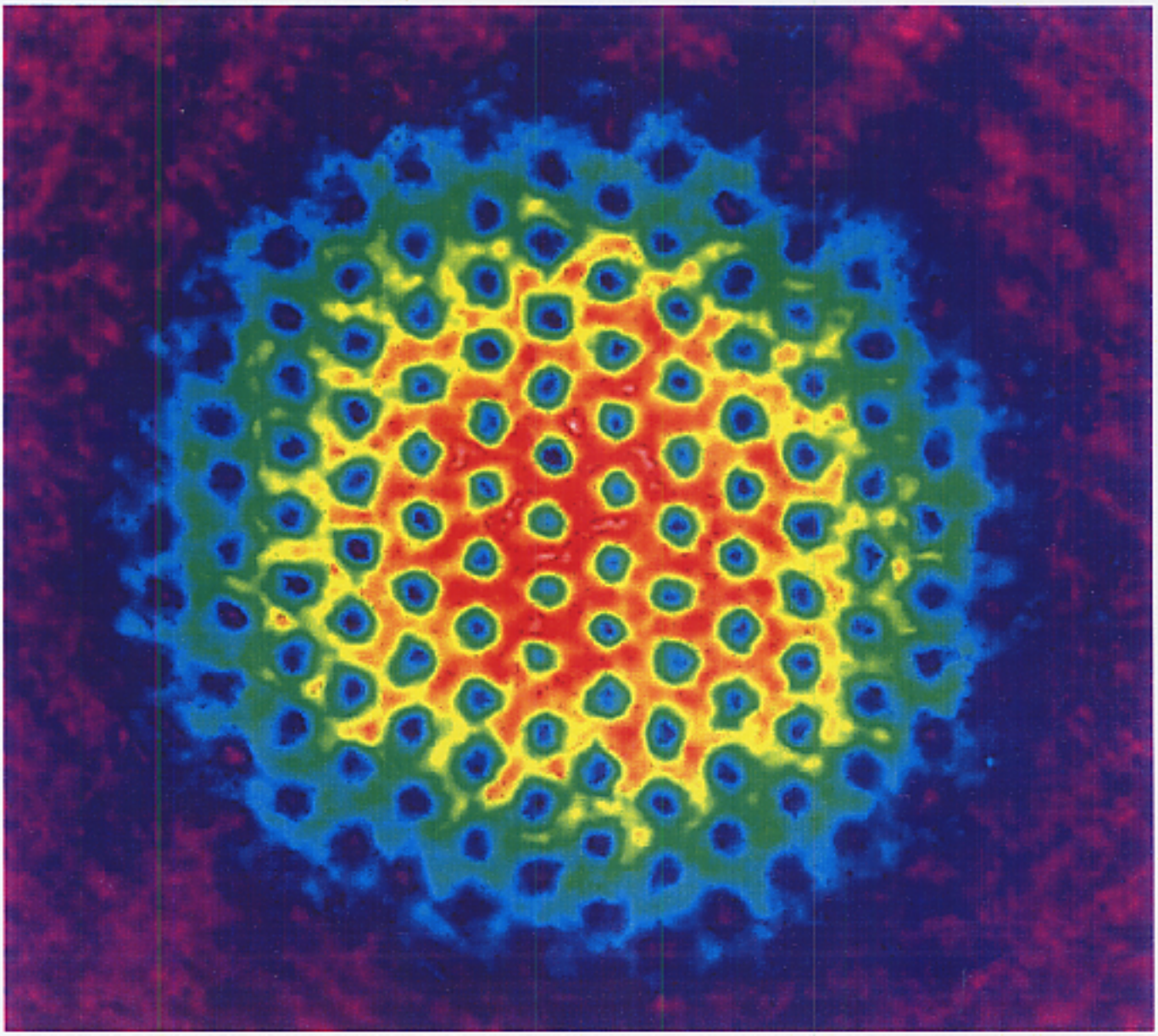
http://jilawww.colorado.edu/BEC



formation of Vortices
in BEC of ^{87}Rb

Cornell group — JILA/Colorado (2002)

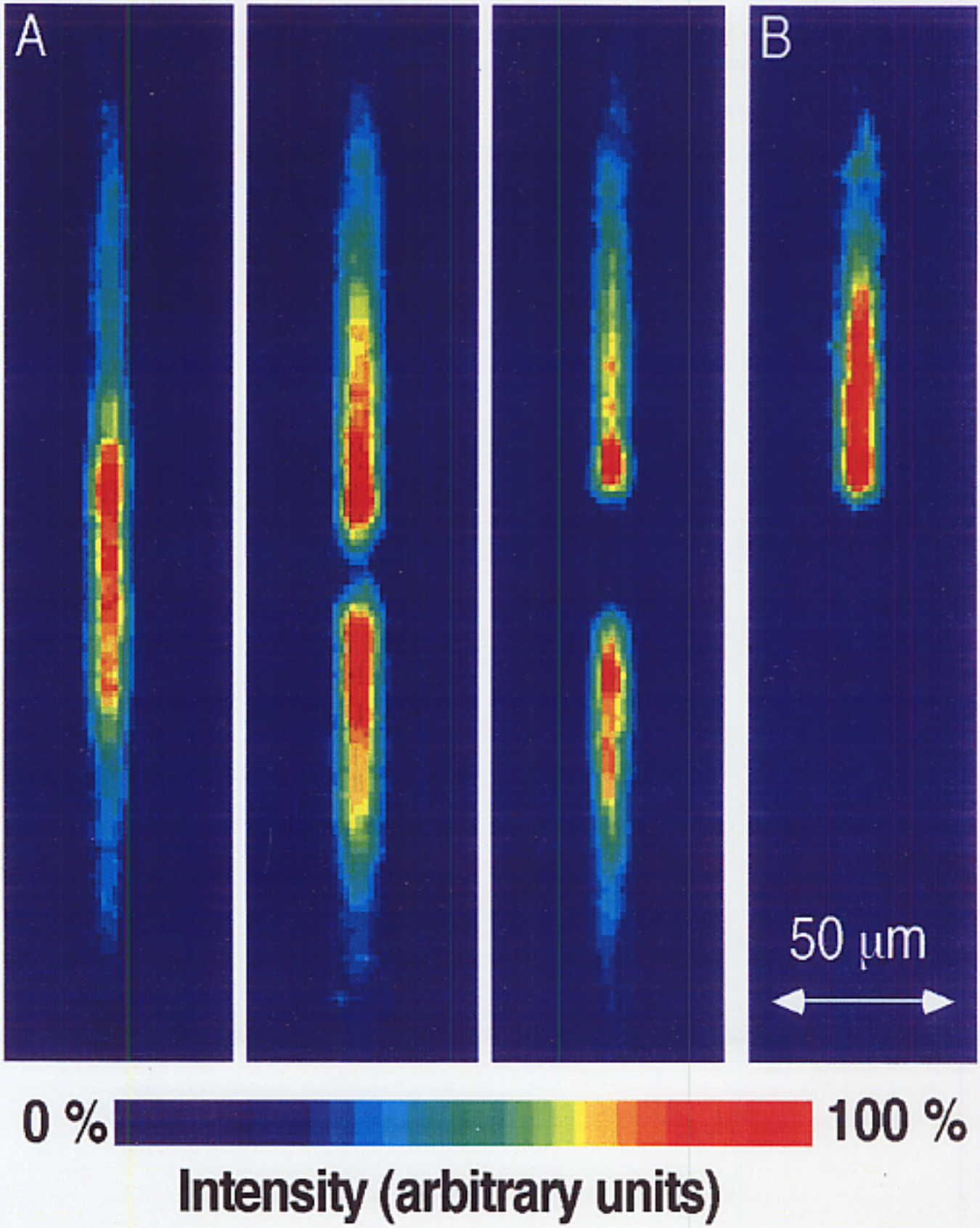
http://jilawww.colorado.edu/BEC



Abrikosov lattice of vortices
in BEC of ^{87}Rb

Cornell group – JILA/Colorado (2002)
http://jilawww.colorado.edu/BEC

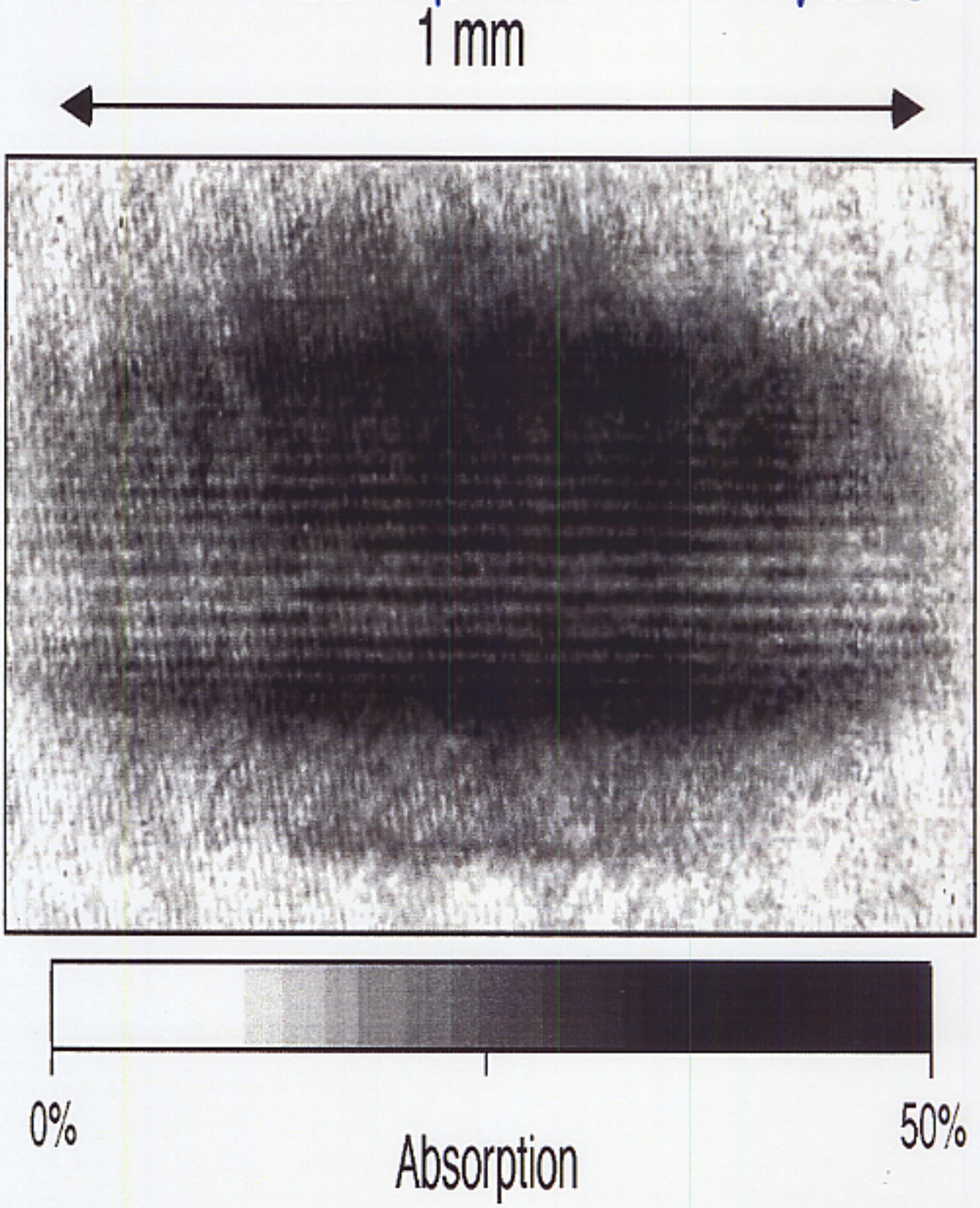
Separating a BEC of ^{23}Na atoms into two condensates



Ketterle group - MIT (1997)

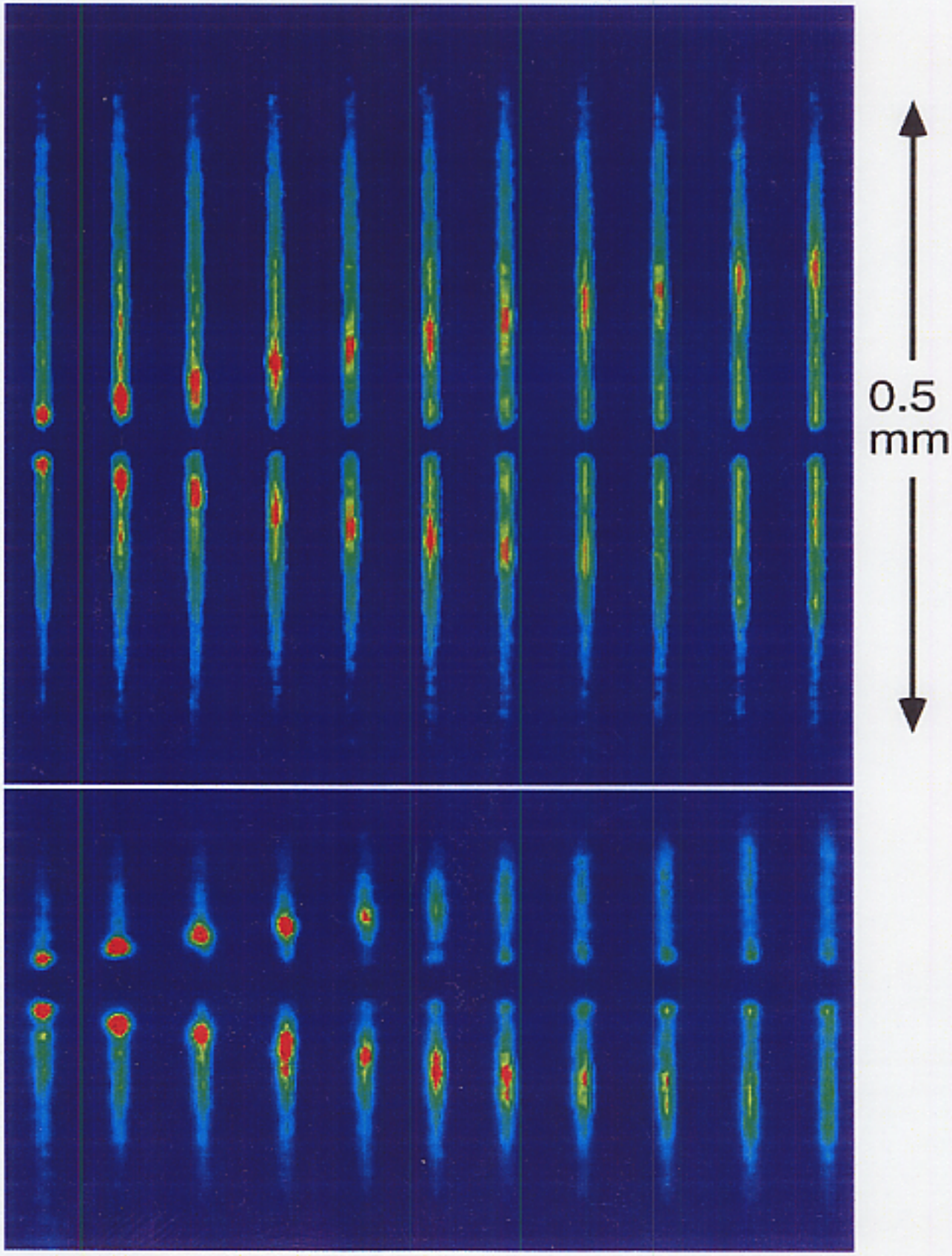
http://cua.mit.edu/ketterle_group

Interference between
two expanding clouds of ^{23}Na atoms
from BEC separated into two pieces



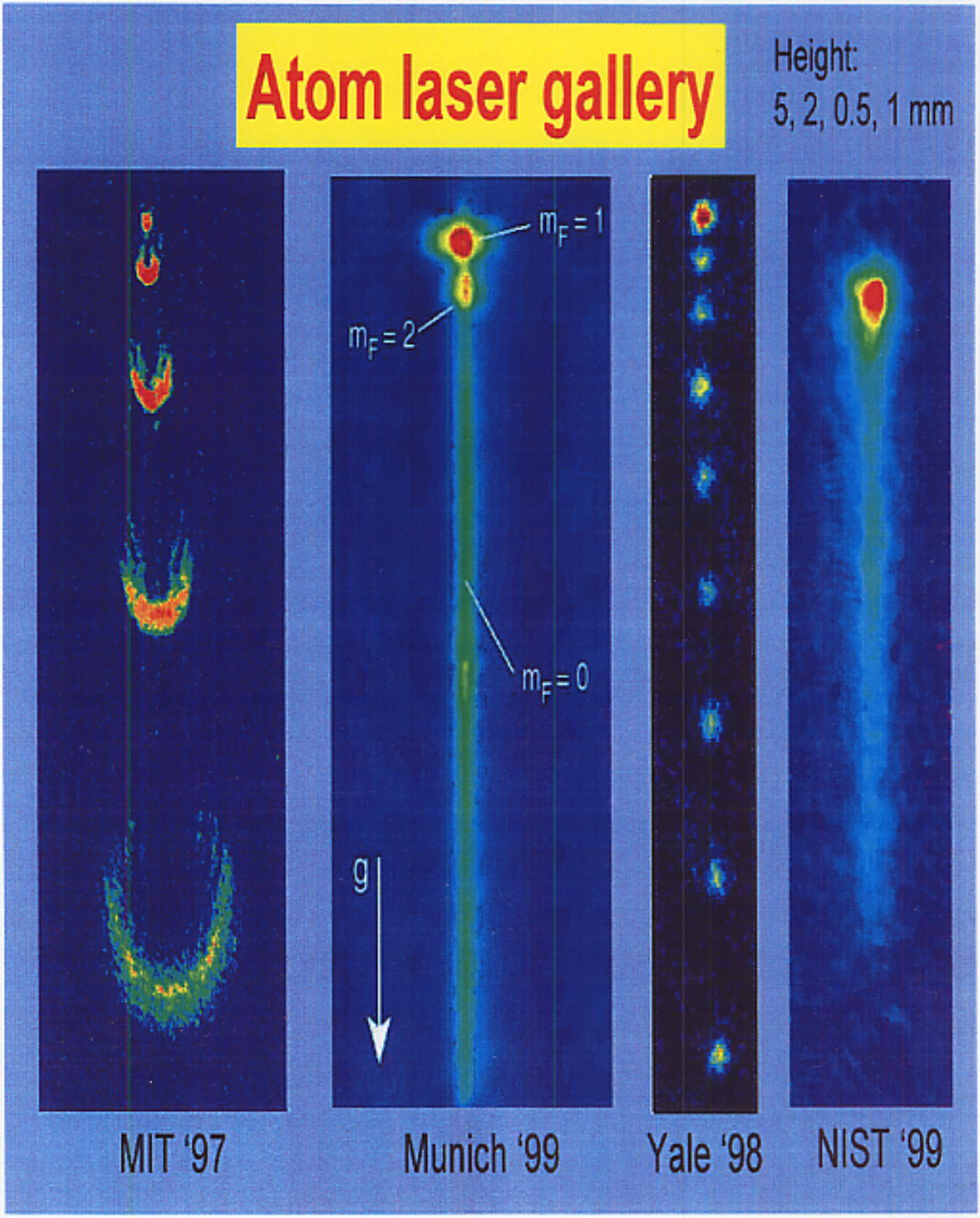
Ketterle group – MIT (1997)
http://cua.mit.edu/Ketterle_group

Sound propagation in a BEC of ^{23}Na atoms



Ketterle group – MIT (1997)

http://cua.mit.edu/ketterle_group



Ketterle group - MIT
http://cua.mit.edu/ketterle_group

A Tale of T_c

Planck (1900)

empirical black body radiation spectrum

$$S(\lambda) \sim \frac{1}{e^{\hbar c/\lambda kT} - 1}$$

Einstein (1917)

spectrum can be derived by assuming energy levels of atoms are quantized

Bose (1923)

spectrum can be derived by assuming photons are <u>identical</u> <u>massless</u> particles

Einstein (1923)

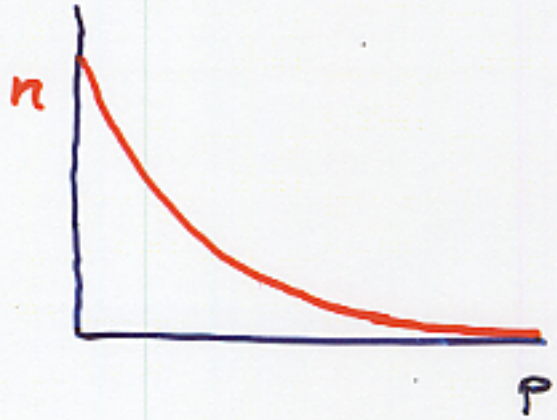
If ideal gas of <u>identical</u> <u>massive</u> particles is cooled, it will undergo a phase transition to a "Bose-Einstein Condensate"

A Tale of T_c (cont.)

N bosons of mass m
in volume V at temperature T

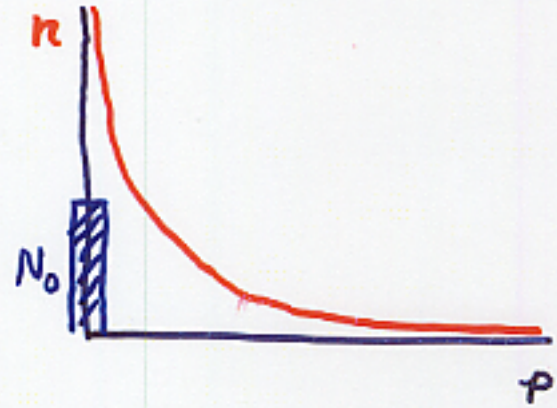
$T > T_c$

$$N = V\int_{\vec{p}} \frac{1}{e^{\beta(p^2/2m-\mu)}-1}$$



$T < T_c$

$$N = N_0 + V\int_{\vec{p}} \frac{1}{e^{\beta p^2/2m}-1}$$



critical temperature

$$N = V\int_{\vec{p}} \frac{1}{e^{\beta p^2/2m}-1}$$

$$\boxed{n = 2.612\left(\frac{mkT_c}{2\pi\hbar^2}\right)^{3/2}}$$

A Tale of T_c (cont.)

What is shift in T_c
due to interactions between atoms?

- open problem since 1923 (Einstein)
- well-posed problem since 1957 (Lee + Yang)
- solved definitively in 2001 Arnold + Moore, Prokofev + Svistunov

using $\begin{cases} \text{effective field theory} \\ \text{lattice field theory} \end{cases}$

ΔT_c depends on interatomic potential $V(r)$

but is sensitive only to scattering length a

$$\frac{\Delta T_c}{T_c} = C\,(a n^{1/3})^p + \cdots$$

at low densities $a n^{1/3} \to 0$

what is ... leading power p? $(\frac{1}{2}?\ 1?\ \frac{3}{2}?)$

... coefficient C?

Transition Temperature of a Uniform Imperfect Bose Gas

Kerson Huang
Center for Theoretical Physics and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 2 April 1999)

We calculate the transition temperature of a uniform dilute Bose gas with repulsive interactions, using a known virial expansion of the equation of state. We find that the transition temperature is higher than that of an ideal gas, with a fractional increase $K_0(na^3)^{1/6}$, where n is the density, a is the S-wave scattering length, and K_0 is a constant given in the paper. This disagrees with all existing results, analytical or numerical.

PACS numbers: 05.30.Jp, 67.20.+k, 67.40.Kh

The weakly interacting Bose gas is an old subject that has found new life since the experimental discovery of Bose-Einstein condensation in ultracold trapped atoms. Although there is a vast literature on the subject, the transition temperature of the interacting gas remains a controversial subject, even in the uniform case. In this note, we present a calculation that is simple and transparent, in the hope that it will settle the controversy.

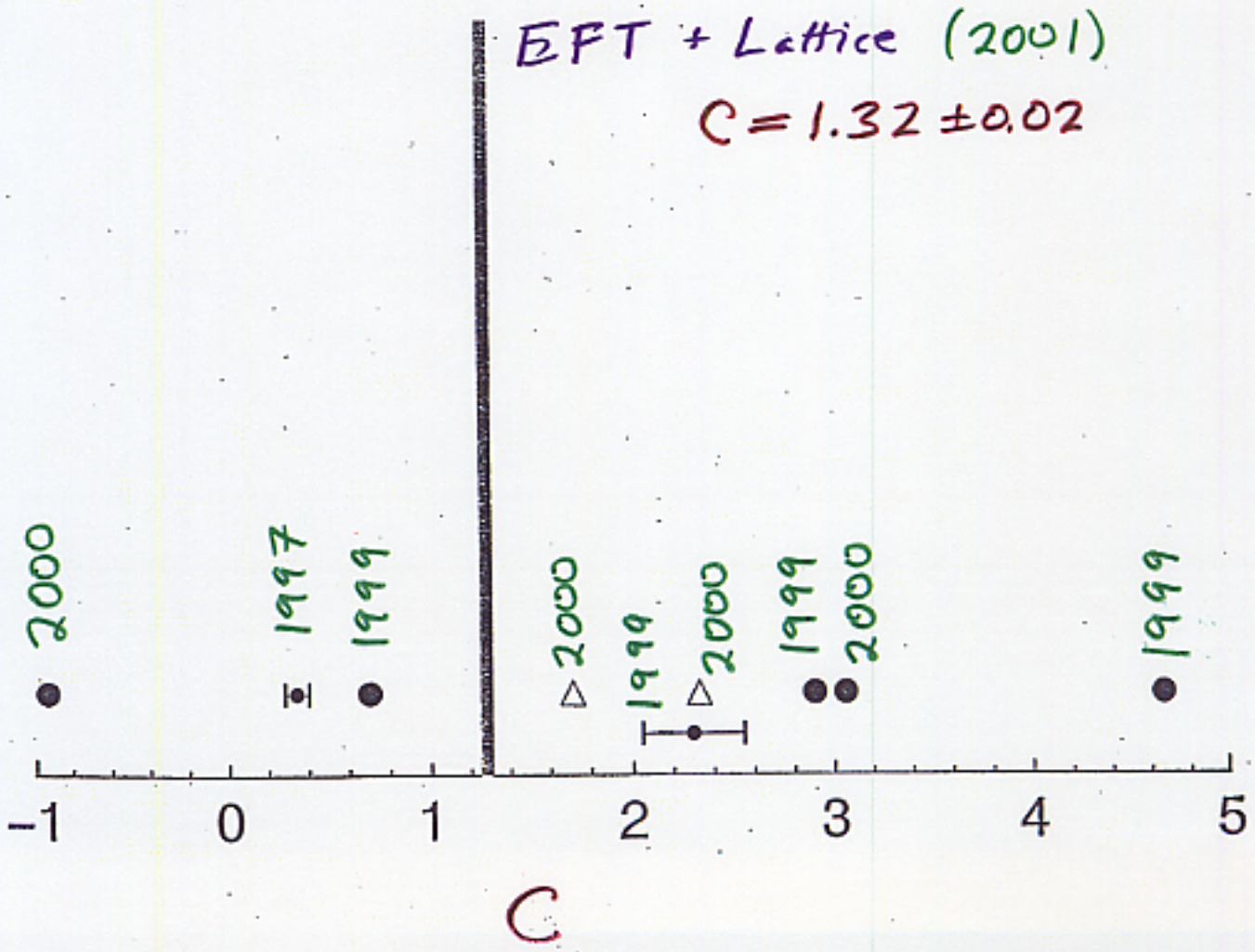
$$\frac{\Delta T_c}{T_c} = 3.527\,(a n^{1/3})^{1/2}$$

$$p = \frac{1}{2} \; ???$$

A Tale of T_c (cont.)

Correct power: $p=1$

$$\frac{\Delta T_c}{T_c} = C\, a n^{1/3}$$



A Tale of T_c (conti)

ΔT_c: nonperturbative problem in

Euclidean field theory on $\mathbb{R}^3 \times S^1$

two energy scales: $\mu \ll T$

<u>Step 1</u> <u>Effective field theory</u>

dimensional reduction to

Euclidean field theory on $\mathbb{R}^3$

one energy scale μ

Baym, Blaizot, Holzmann, Laloë, Vautherin (1999)

$$\Longrightarrow \boxed{p = 1}$$

<u>Step 2</u> <u>Lattice field theory</u>

calculate C using Monte Carlo simulations

of Euclidean field theory on $\mathbb{R}^3$

$$\Longrightarrow \boxed{C = 1.32 \pm 0.02}$$

Arnold + Moore (200
Prokofev + Svistunov

Strongly-Interacting Atoms

natural scale for scattering length a

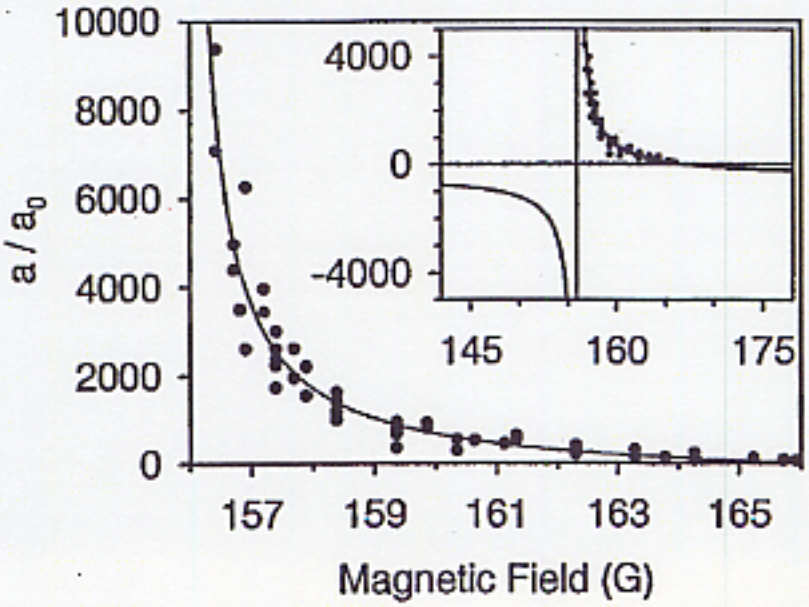
$$|a| \sim \ell_{vdw} \sim 50\,\text{Å}$$

But scattering length can be varied

by tuning magnetic field

to a "Feshbach resonance"

Feshbach resonance for ^{85}Rb

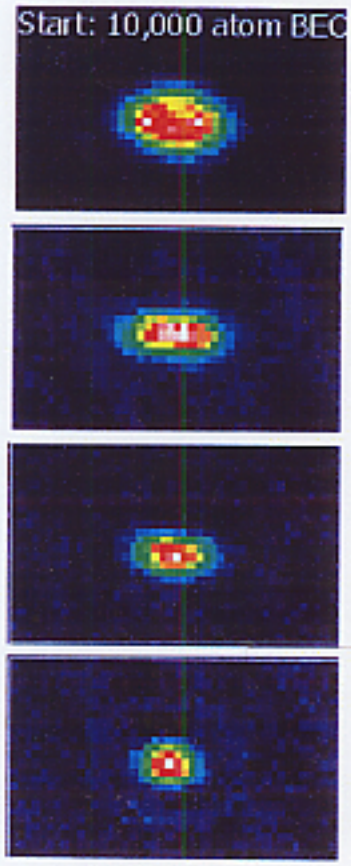


Weiman group — JILA/Colorado

Wieman group — JILA/Colora
http://jilawww.colorado.edu/BE
(2001)

The "Bosenova"

These images show the condensate collapsing, then exploding outward, leaving a small cold remnant.

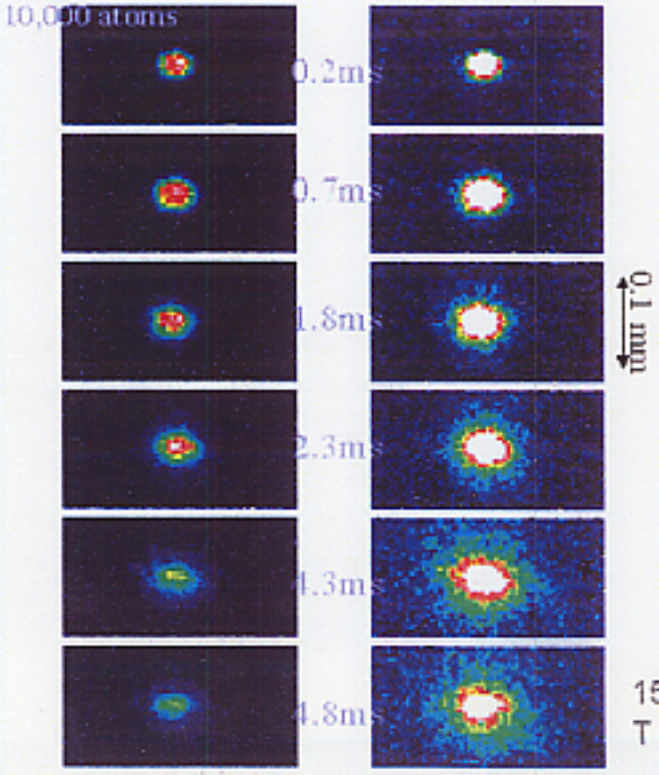


Watching the dynamics

Collapse

TIME

then…



"Bosenova"

like tiny supernova:
- collapse
- explosion
- small cold remnant

1500 atom burst
T ~ 200 nK

Strongly-interacting atoms (cont.)

Large scattering length:

$$|a| \gg \ell_{vdW}$$

$\Longrightarrow$ 2-body bound state (if $a>0$: $B_2 = \frac{\hbar^2}{ma^2}$)
or virtual state (if $a<0$)
near threshhold

$\Longrightarrow$ strong resonant interactions
between "cold atoms"

(point particles with point interactions)

contrast with liquid Helium

extended objects
interacting strongly
through nonlocal potential

Strongly-interacting atoms (cont.)

Ken Wilson (1971)

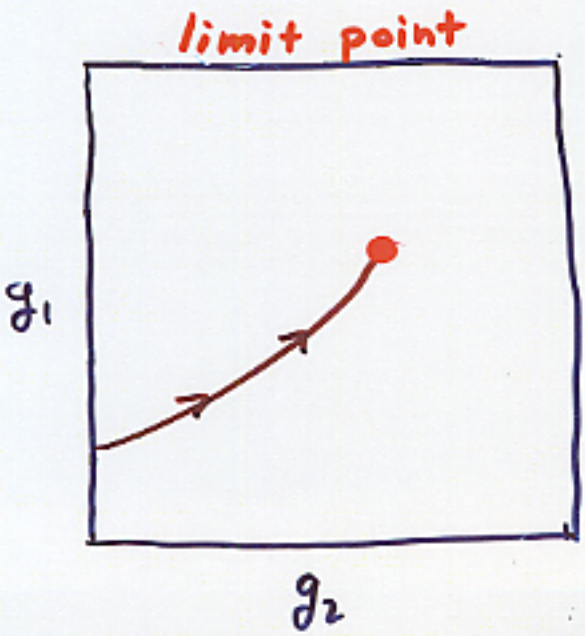
"Renormalization Group for Strong Interactions"

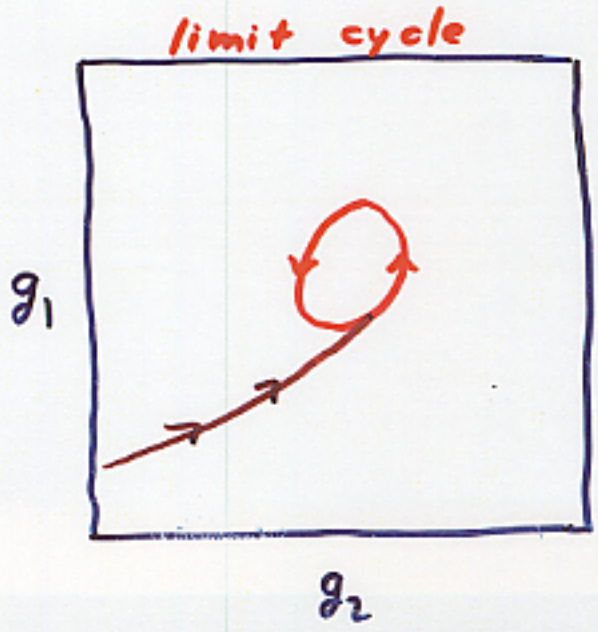
Coupling constants: $g_1, g_2, \ldots$

Renormalization group: $\Lambda \frac{d}{d\Lambda} g_i = \beta_i(g_1, g_2, \ldots)$

As $\Lambda \to \infty$,

the coupling constants could flow
to ∞
or to a limit point
or to a limit cycle
or ...



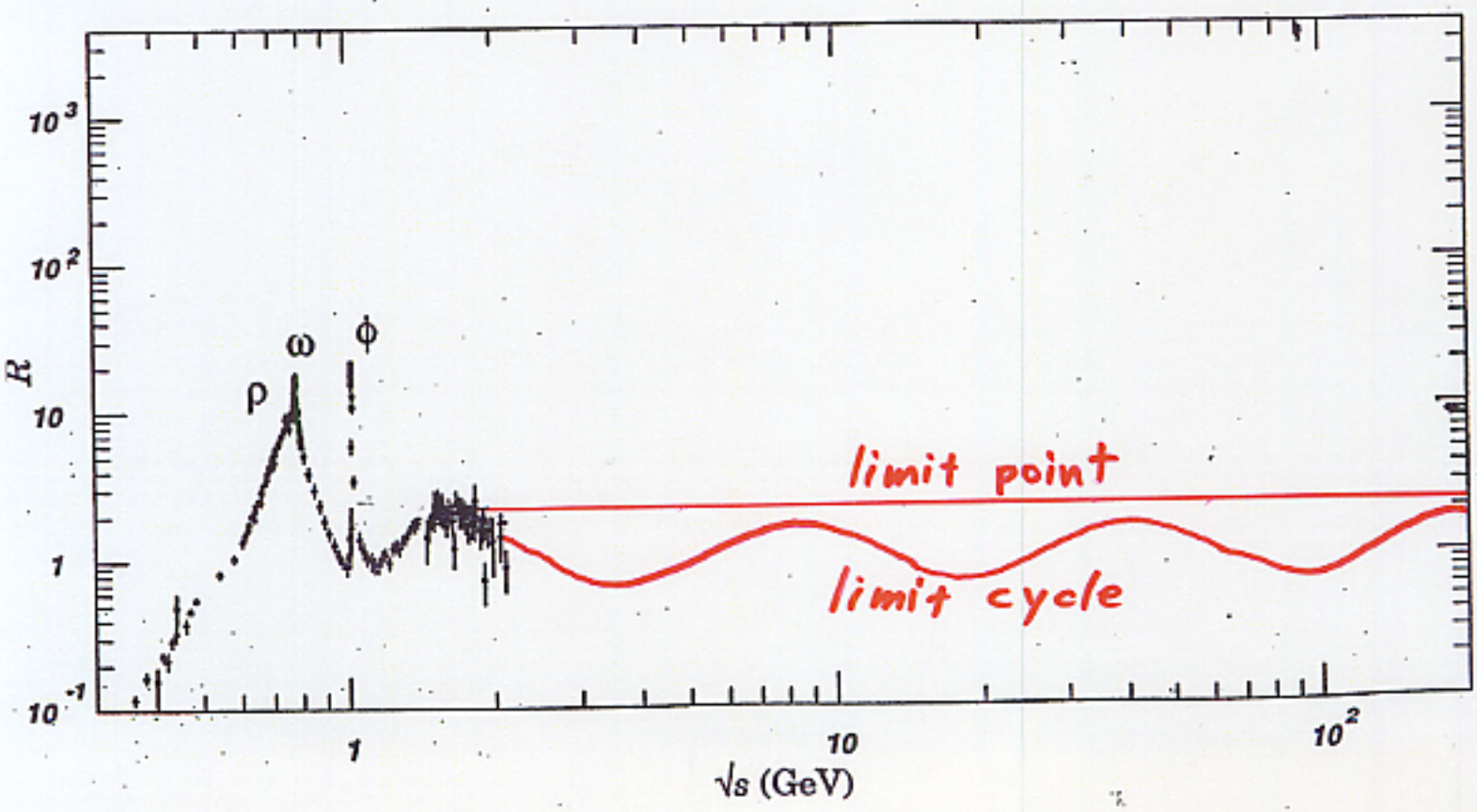


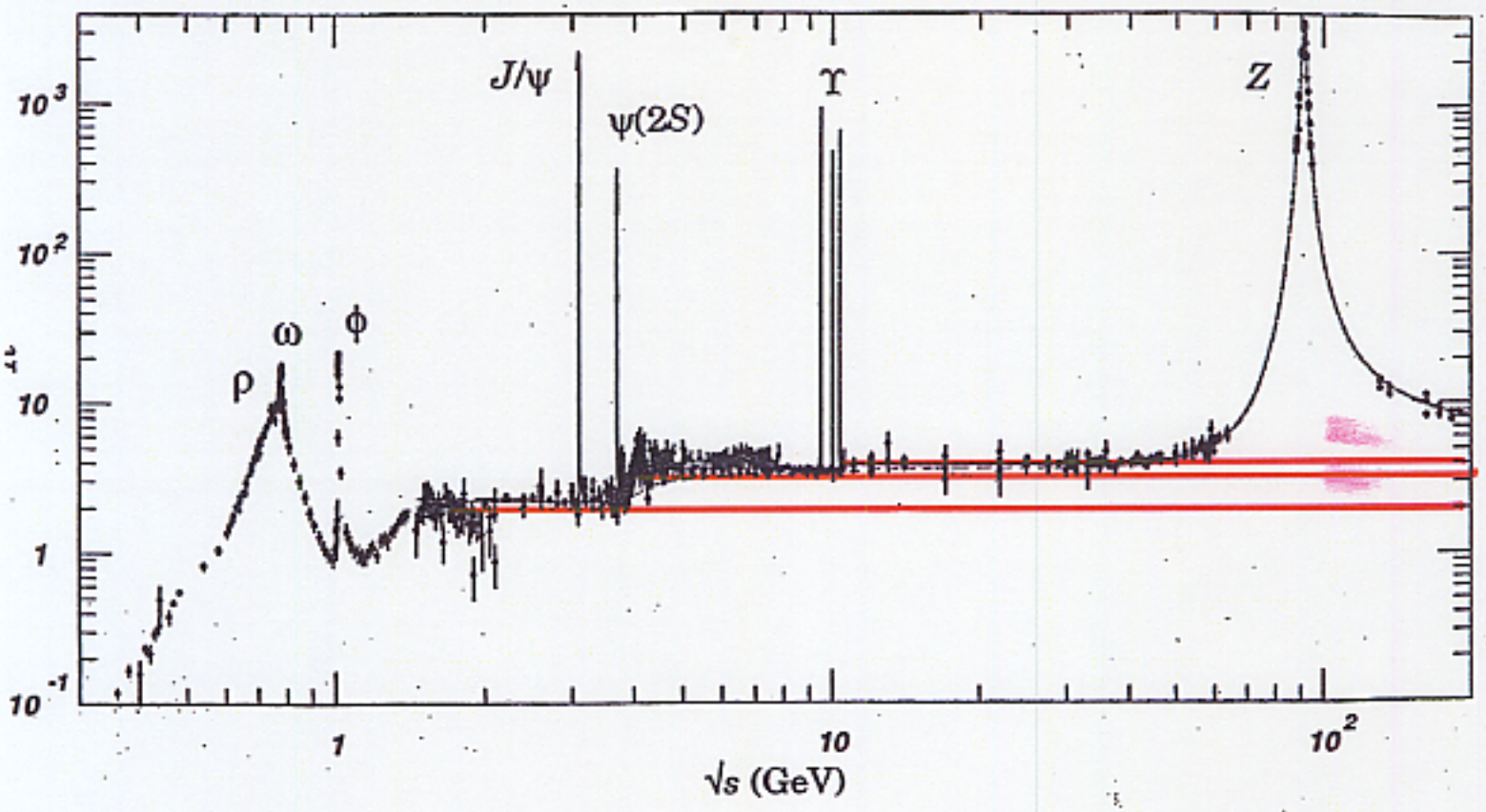
$$R(s) \equiv \frac{\sigma[e^+e^- \to \text{hadrons}]}{\sigma[e^+e^- \to \mu^+\mu^-]}$$

As $s \to \infty$,

$R(s) \longrightarrow$ constant $\Longrightarrow$ RG limit point

$R(s)$ oscillates as function of $\log s$ $\Longrightarrow$ RG limit cycle





Strong interactions is described by QCD

QCD coupling constant α_s
has asymptotically-free limit point

$$\alpha_s(\Lambda) \longrightarrow 0$$

$$R(s) \longrightarrow 3 \sum_q e_q^2$$

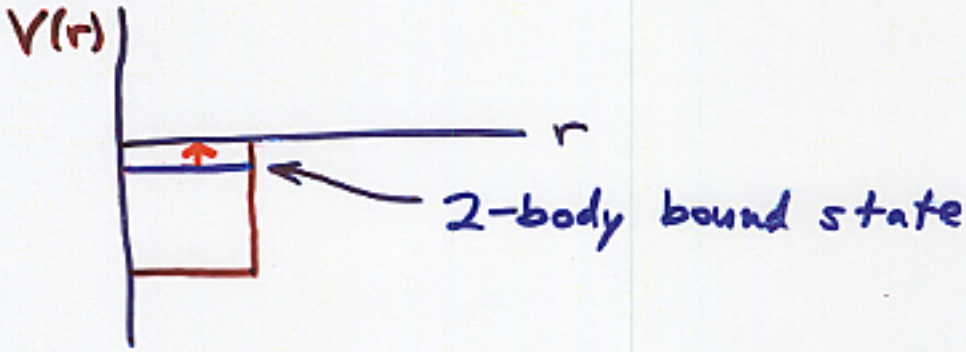
(between quark thresholds)

Strongly-interacting atoms (cont.)

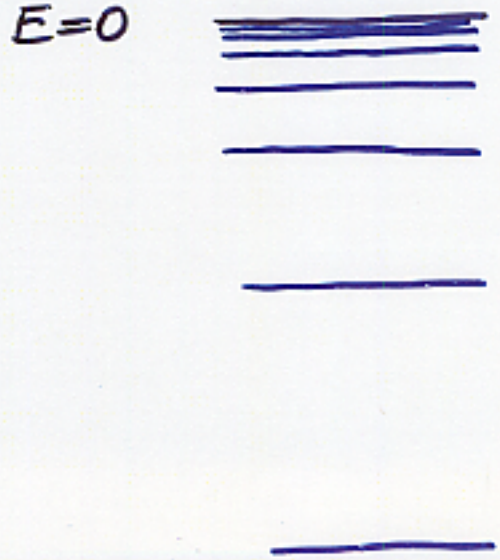
Vitaly Efimov (1970)

"Energy levels arising from Resonant 2-Body Forces in a 3-Body System

Suppose $V(r)$ is tuned so $a \longrightarrow \pm\infty$.



Then there are infinitely many 3-body bound states with an accumulation point at $E=0$ and geometric spacing: $E_{n+1} = 515\, E_n$



Strongly-interacting atoms (cont.)

Bedaque, Hammer, & van Kolck (1999)

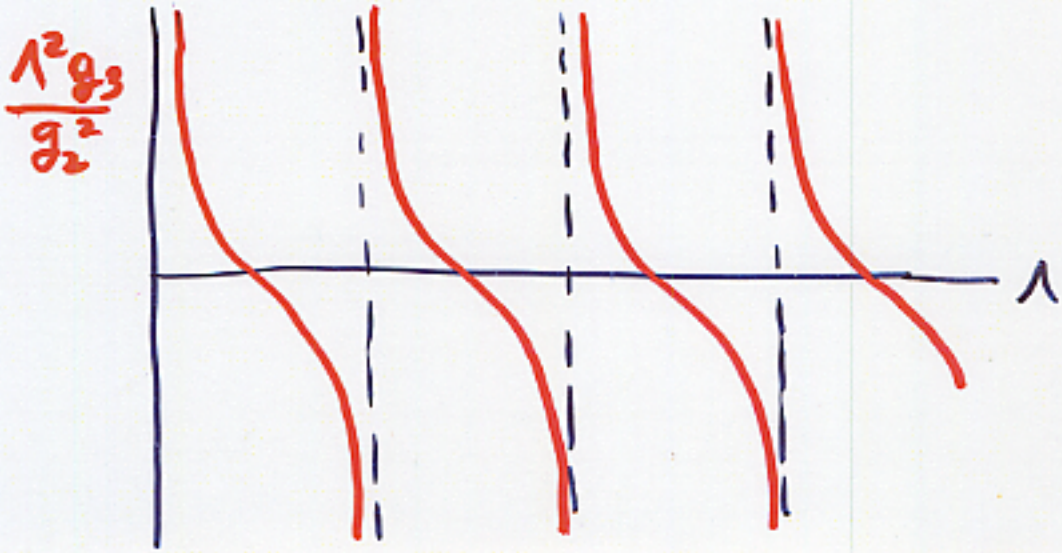
Field theory formulation
requires a 3-body interaction

Renormalization involves a RG limit cycle!

$$\mathcal{H} = \psi^\dagger\left(-\frac{\hbar^2}{2m}\nabla^2\right)\psi + \frac{g_2}{4}(\psi^\dagger\psi)^2 + \frac{g_3}{36}(\psi^\dagger\psi)^3$$

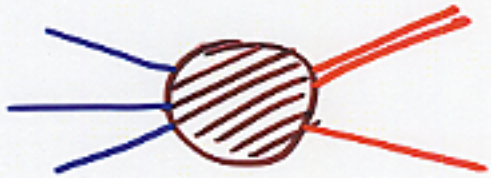
$-i\,g_2$

$-i\,g_3$



3-Body Recombination

3 atoms recombine into atom + dimer



severe loss mechanism for BEC

$$\frac{\partial}{\partial t} n = -\frac{1}{2}\alpha n^3$$

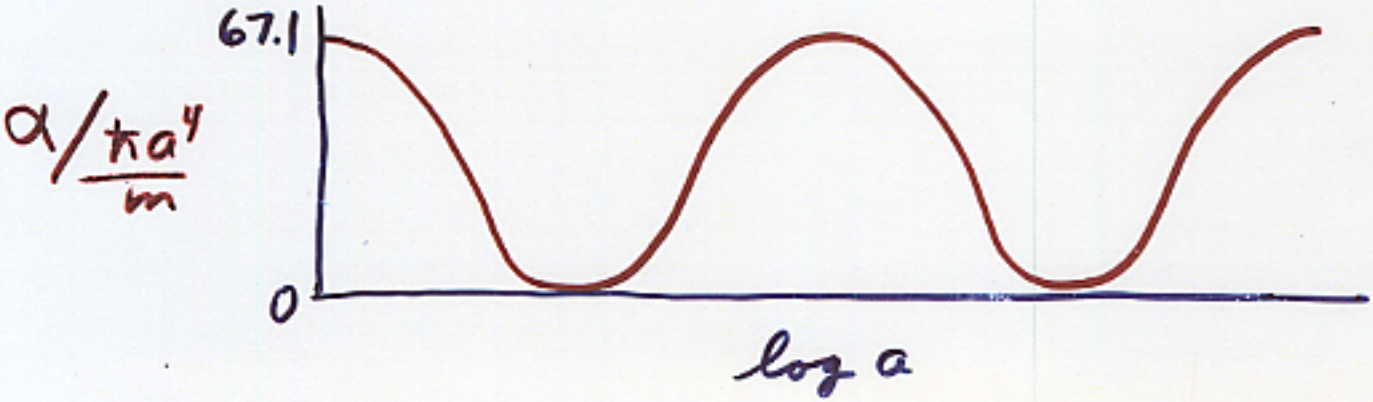
Dimensional analysis $\Longrightarrow$ $\alpha \sim \frac{\hbar a^4}{m}$

but coefficient is periodic function of $\log a$

$$\alpha = 67.1 \sin^2\left[s_0 \log(a\Lambda_*) + 0.19\right] \frac{\hbar a^4}{m}$$

$s_0 = 1.00624$

Bedaque, Braaten, Hammer (2000)



Another Tale of T_c

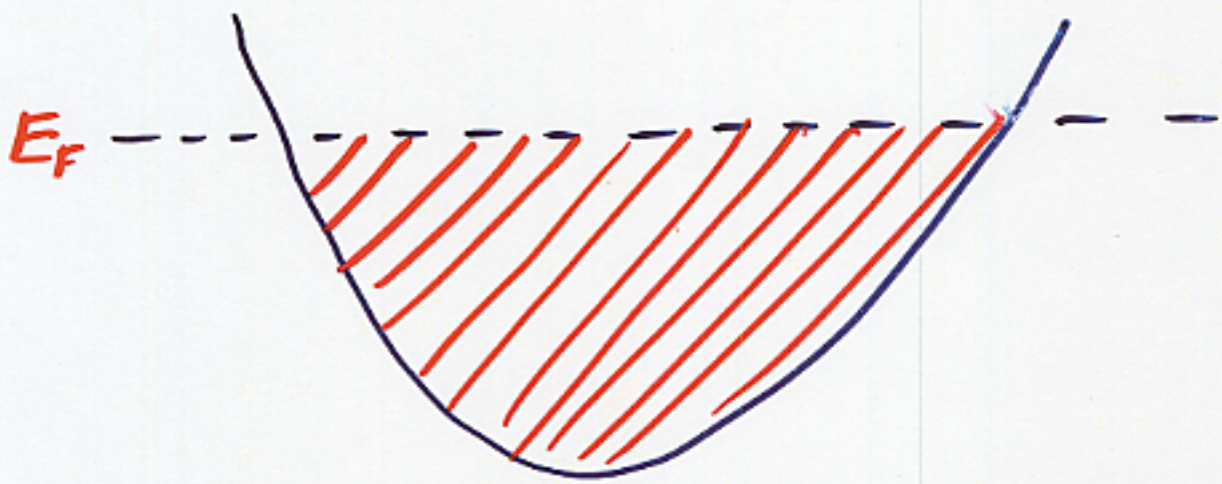
As $T \to 0$,

fermionic atoms form a "Fermi gas"

states with $E < E_F$: occupied

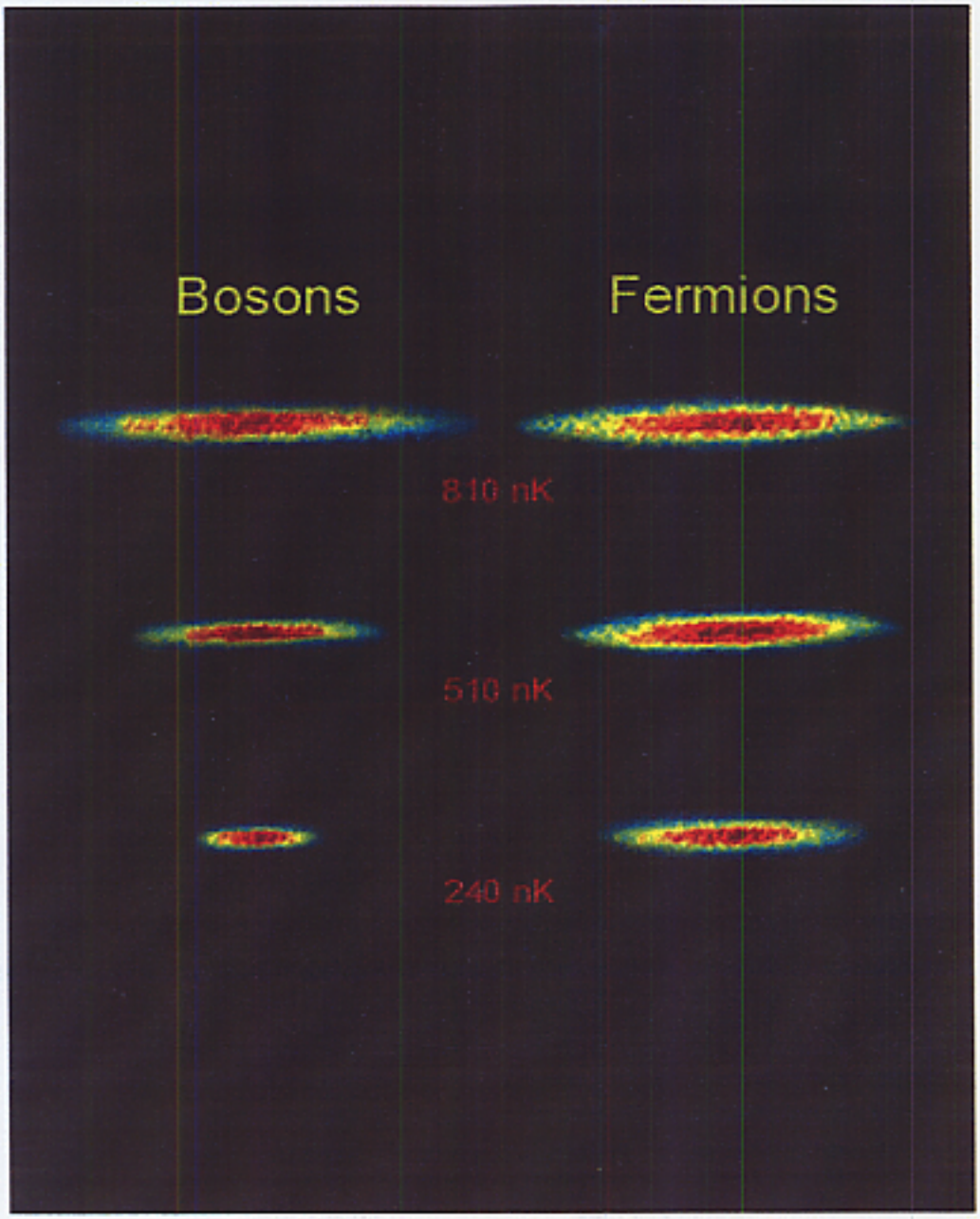
" " $E > E_F$: unoccupied

E_F = Fermi energy



Fermi temperature: $E_F = kT_F$

^7Li ^6Li



$T = T_F$

$T = 0.56\ T_F$

$T = 0.25\ T_F$

Degenerate Fermi gas of ^6Li atoms

by sympathetic cooling with ^7Li atoms

Hulett group – Rice University

http://atomcool.rice.edu/

Another Tale of k_c (cont.)

Two fermionic species of atoms
with scattering length $a < 0$

Superfluidity phase transition

Below some critical temperature T_c,
the atoms will form Cooper pairs
which condense into a superfluid.

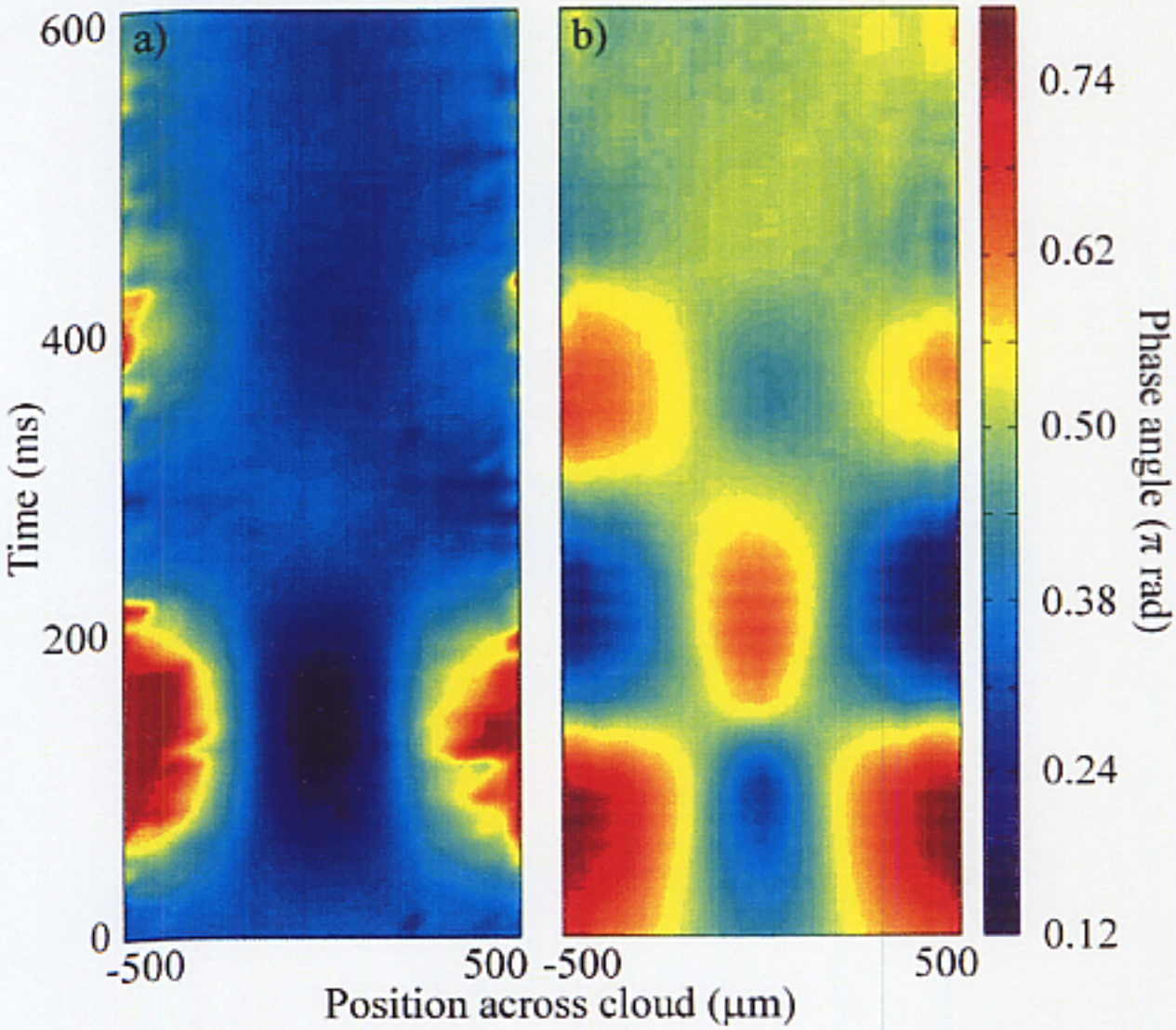
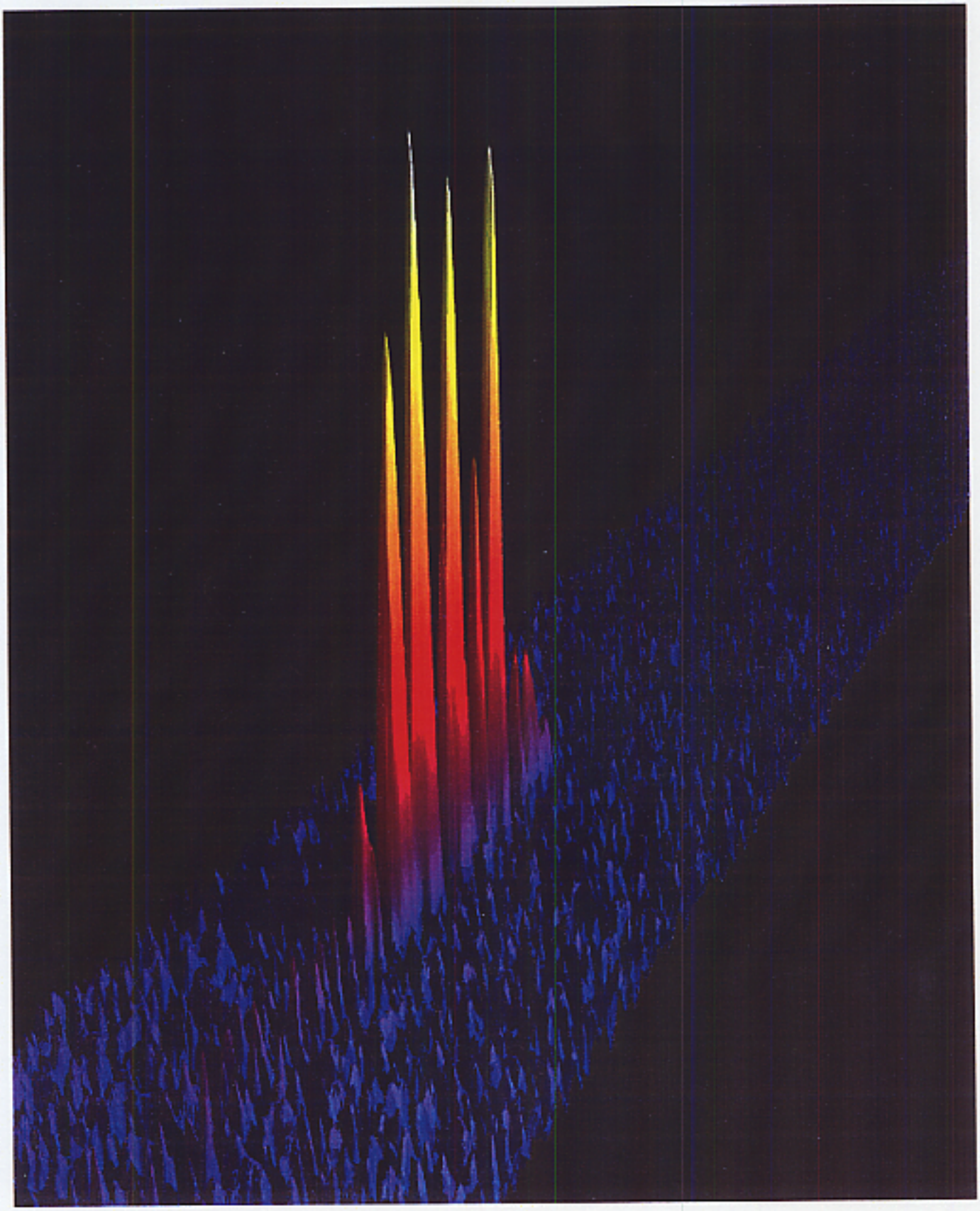
Weak coupling: $k_F|a| \ll 1$ $\qquad E_F = \frac{\hbar^2 k_F^2}{2m}$

$$T_c = 0.277\, e^{-\pi/2k_F|a|}\, T_F$$

Gorkov, Melik-Barkhudarov 1961
Heiselberg, Pethick, Smith, + Viverit 2000

Strong coupling: $k_F|a| \gtrsim 1$

What is T_c?


a)
b)
600
400
200
0
Time (ms)
-500
500
-500
500
Position across cloud (μm)
0.74
0.62
0.50
0.38
0.24
0.12
Phase angle (π rad)