Extra Dimensions from the Bottom Up

Christopher T. Hill

Theoretical Physics Dept.

Fermilab

HUTP-01/A015
BUHEP-01-05
LBNL-47676
UCB-PTH-01/11

(De)Constructing Dimensions

Nima Arkani-Hamed[*], Andrew G. Cohen[†] and Howard Georgi[‡]

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

Abstract

We construct renormalizable, asymptotically free, four dimensional gauge theories that dynamically generate a fifth dimension.

[*] arkani@bose.harvard.edu
Permanent address: *Department of Physics, UC Berkeley, Berkeley, CA 94720*

[†] cohen@andy.bu.edu
Permanent address: *Physics Department, Boston University, Boston, MA 02215*

[‡] georgi@physics.harvard.edu

FERMILAB-Pub-01/043-T
October, 2000

Gauge Invariant Effective Lagrangian for Kaluza-Klein Modes

Christopher T. Hill[1]
Stefan Pokorski[1,2]
Jing Wang[1]

[1] *Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, Illinois 60510, USA *
[2] *Institute for Theoretical Physics*
Hoza 69, 00-681, Warsaw, Poland

September 25, 2001

Abstract

We construct a manifestly gauge invariant Lagrangian in $3+1$ dimensions for N Kaluza-Klein modes of an $SU(m)$ gauge theory in the bulk. For example, if the bulk is $4+1$, the effective theory is $\Pi_{i=1}^{N+1} SU(m)_i$ with N chiral $(\overline{m}, m)$ fields connecting the groups sequentially. This can be viewed as a Wilson action for a transverse lattice in x^5, and is shown explicitly to match the continuum 4+1 compactified Lagrangian truncated in momentum space. Scale dependence of the gauge couplings is described by the standard renormalization group technique with threshold matching, leading to effective power law running. We also discuss the unitarity constraints, and chiral fermions.

*e-mail: hill@fnal.gov, jingw@fnal.gov, Stefan.Pokorski@fuw.edu.pl

Back to Basics: How Did Space Get Its Dimensions?

By GEORGE JOHNSON

In the beginning, there was little eed for physicists. The universe, oised at the moment of creation, as ruled not by the four different orces observed in later times, but by ust a single superforce. Instead of a mattering of different particles, here was only the tiny primordial nass — the grandmother of all partiles — ready to explode in the Big lang.

Then came the detonation, giving cientists some messiness to study. As the newborn universe cooled, the ingle force splintered into four orces, radiating through the four limensions of space and time, and he various classes of particles prang forth one by one.

This scientific creation story, emraced by physicists and cosmologists alike, seems to account for just bout everything — with one glaring exception: Where did the dimensions ome from?

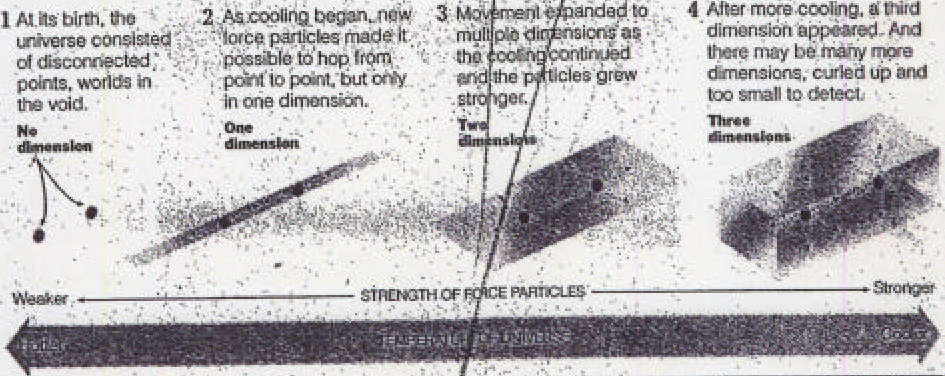
Now some physicists are adding a ew touch to the Edenic tale. A paper ublished last month in Physical Review Letters suggests that the dimensions, like the forces and particles, may have popped into being as he universe cooled. Though the idea s still in a very preliminary form, physicists are intrigued by the implication that reality may have started with just a single dimension: time. As temperatures dropped in the moments after the primordial explosion, the spatial dimensions — height, length and breadth — crystallized into existence. Creation was not ust a matter of "let there be light," out "let there be space" as well.

"For the first time, this gives us an opportunity to investigate what we mean by space," said Dr. Andrew G. Cohen, a physicist at Boston University who wrote the paper with Dr. Nima Arkani-Hamed of the University of California at Berkeley and Dr. Howard Georgi of Harvard. "We all got very excited when we realized that this was a way to get a handle on that idea."

For decades physicists have been inclined to take space for granted as an eternal backdrop, the stage on which particles and forces commune and reality unfolds. The new study is part of a growing attempt to reverse figure and ground. In this emerging view, it is space that is secondary, arising from the interaction of new kinds of particles — particles that came into existence only as the universe cooled. Throughout the field, theorists are re-examining what they had taken for granted: the nature of space, and even time. A team based at the Fermi National Accelerator Center in Batavia, Ill., has come up with its own version of dimension-generating particles.

Beginnings of Space

A new theory suggests that space, as well as matter and energy, can be described in terms of particles.



The New York Times

"Everybody thinks space-time should be an output rather than an input of a fundamental theory," said Dr. Nathan Seiberg, a theorist at the Institute for Advanced Study in Princeton, N.J.

The title of the new work, "(De)-Constructing Dimensions," has a postmodernist ring that one might expect in the table of contents of a journal of literary criticism. But the densely mathematical paper, with its talk of "Weyl fermions," "cyclic symmetries," "Kaluza-Klein excitations" and other arcana, is a precise attempt to get to the heart of what is meant by a dimension. If the theory is right, Dr. Arkani-Hamed said, "Space can appear and disappear, depending on the energy."

The research was motivated by an attempt to explain the origin of the weird extra dimensions required by string theory, an attempt to unite the laws of physics into a single framework. According to this view, everything is generated from the interactions of tiny stringlike objects, but the theory works only if they are allowed 10 dimensions to play around in. Dr. Arkani-Hamed and his colleagues set out to shed some light on where the extra ones might have come from. But, he said, the ultimate aim is to take the idea even further: "It is tempting to think that our own familiar dimensions are also made this way."

Dr. Steve Giddings of the University of California at Santa Barbara suggested that even more interesting possibilities could exist. "Perhaps one can get even more bizarre models with different dimension in different places," he said.

Explaining how fundamental ideas, like space, emerge from particles is a tradition in physics. Atoms are made of protons, neutrons and electrons. Light is carried by photons, gravity by gravitons, the strong nuclear force by gluons. At the extreme energies and temperatures of the Big Bang, the matter-making particles and the force-carrying particles were fused together into a single primal substance, separating into individual entities only later on.

As physicists reach higher and higher energies with their particle accelerators, they feel that they are approaching temperatures and energies that existed earlier in the universe, edging closer to the origins of the particles and forces.

> Physicists are no longer taking the idea of space for granted.

So far, space has been resistant to this approach. It remains the ineffable something that is just there.

To picture how space might have been created on the fly by particles, imagine an early world with no dimensions, consisting of a single point. Motion would not exist in this claustrophobic realm. There would be no directions in which to move. There might be other point worlds as well, each isolated in the void.

Suppose that as the cooling began, it led to the appearance of a set of particles that carried a force allowing an inhabitant imprisoned in a dot world to move from its own point to the next and the next, following the path of steppingstones. The result would be a single-dimensional world shaped like a wavy line.

The inhabitants could go forward and backward but nowhere else. Other line worlds might also have formed at this stage, but all would be mutually inaccessible — until the universe cooled further and the dimension-creating particles grew strong enough to allow motion in a new direction.

Now the one-dimensional-line worlds could link up to form two-dimensional worlds shaped like undulating planes. The inhabitants of each realm would be able to move left and right and back and forth, but not up and down.

Let the universe cool another notch, and particles would grow stronger, opening up motion in the third dimension. The disconnected planar worlds would be linked to form familiar three-dimensional space, where one can strike out in three different directions.

But why stop there? Before the universe reached its present state, perhaps other dimensions emerged as well — maybe even the extra ones required by string theory.

As physicists usually explain it, these additional dimensions aren't evident because they are curled up too small to detect. Imagine again a single dimension as a line, but join the ends to form a loop. A being living on one of these tiny circles could hop from point to point around the circumference. But when it returned to the starting point, it would have covered a distance only about one-thousandth the size of a proton.

From a unifier's point of view, one of the most interesting things about the new theory is the way it conflates the seemingly very different notions of dimensions and forces. A dimension arises when a force is gener that allows movement into a domain.

Dr. Arkani-Hamed said the h thetical particles behind these fo would be similar to the gluons carry the strong nuclear force. gluons, the dimension-creating p cles might be very weak at energies and stronger at lower e gies. That would explain why t were fewer dimensions in the he the Big Bang.

Of course it is possible that theory explains nothing at all — its array of hypothetical part and forces is just a mesmeri mathematical plaything. "T ideas are extremely speculative, in a generic way they should be able," said Dr. Joseph Lykke theorist at Fermilab. "High-en colliders might turn up evidence quarks or gluons or photons are ing in fewer dimensions, at the h est energies that we are now ducing. That would be relati straightforward to see."

Even if the theory doesn't mately survive as a descriptio the way space really came abo might serve as a tool for stud what particles do at extremely energies. Generally, the higher energy, the harder a problem solve. With the new technique, cr ing up the energy makes some o dimensions melt away, simplif the calculations.

"To coin a bad metaphor, it is like television, in that you can the world of extra dimensions out leaving the comfort of your world," said Dr. Christopher T. a theorist at Fermilab. He a colleague, Dr. Jing Wang, along Dr. Stefan Pokorski of Warsaw versity and Dr. Hsin-Chia Che the University of Chicago, inde ently came up with a similar sc for generating dimensions from ticles. But Dr. Hill has a more matic perspective.

"We are very excited about t he said. "However, I differ s what philosophically with my petitors. I see this as a very u tool, but it is not yet clear wheth is of fundamental importance."

Dr. Arkani-Hamed said what ticularly attracted him was th tion that space might have eme from a universe that started just time. He hopes it may be ble someday to explain how arose as well. "It sure looks space is not going to survive in ultimate theory," he said, "and makes it likely that time wil survive either."

Discovery of Higgs Boson will lead to new organizing principles for short-distance physics.

Candidate Operating Principles come from theories with extra dimensions.

Supersymmetry $\leftrightarrow$ Extra Fermionic Dimensions:

- (Graded) extension of Lorentz Group
- New "rotations" and "translations:" fermion $\leftrightarrow$ boson.
- Broken SUSY lifts partners; (*SUSY partners are analogues of Kaluzsa-Klein modes*).
- High energy Theory: Superstring Theory.
- Low energy Effective Theory: Perturbative Minimal Supersymmetric Standard Model.

Extra c-number Space-Time Dimensions?

- Conventional extension of Lorentz Group.
- New "rotations" and "translations."
- Broken Lorentz Invariance by compactification: *Kaluzsa-Klein modes.*
- High energy Theory: Superstring Theory?
- Extra dimensions can be associated with the electroweak scale, e.g., ~ 1 TeV, rather than M_{Planck} ?
  (D. Jackson, ca 1972; J. Lykken, 1996)

Extra x-number Space-Time Dimensions?

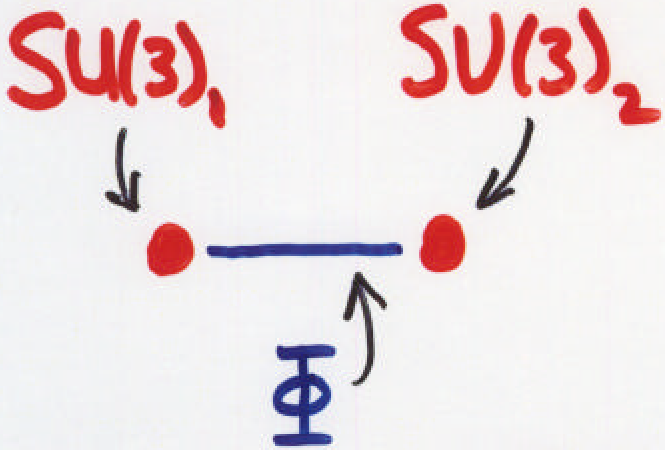
A Gedanken Experiment:

An experimentalist discovers a 0.5 TeV, $J = 1$, $I = 0$, Degenerate Color= **8** particle ("coloron"), produced in perturbative QCD, $G \to VV$ at the Tevatron. How do we describe this with a Lagrangian?

A Sufficient Solution, i.e., "Hidden Local Symmetry of Vector Mesons." (Bando, Kugo, Yamawaki, et.al.)

- QCD Kinetic terms: $SU(3)_1$, g_1:
  $-(1/2)Tr(G_{\mu\nu}G^{\mu\nu})$
- Coloron Kinetic terms: $SU(3)_2$, g_2:
  $-(1/2)Tr(F_{\mu\nu}F^{\mu\nu})$;

  spin-1 + Lorentz Invariance + Quantum Mechanics $\to A_0$ cannot propagate $\to$ Gauge invariance.
- Must give mass to the coloron $\to$ Need a Higgs: $D_\mu \Phi^\dagger D^\mu \Phi$, require $\langle \Phi \rangle \neq 0$.
- Coloron must have QCD color; must be degenerate $\longrightarrow$. Φ: $(\bar{3}_1, 3_2)$ under $SU(3)_1 \times SU(3)_2$. $\langle \Phi \rangle = vI_3$
- Extra Higgs Degrees of Freedom?



"Ethane"

$$\langle 0|\Phi|0\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$D_\mu\Phi \to \frac{v}{\sqrt{2}}\left[g_1 A^{a1}_\mu \frac{\lambda^a}{2} I_3 - g_2 A^{a2}_\mu I_3 \frac{\lambda^a}{2}\right]$$

$$D_\mu\Phi^\dagger D^\mu\Phi \to \frac{v^2}{2}\left[g_1 A^{1a}_\mu - g_2 A^{2a}_\mu\right]^2$$

Coloron: $H^a_\mu = \dfrac{g_1 A^{1a}_\mu - g_2 A^{2}_\mu}{\sqrt{g_1^2+g_2^2}}$; $M_H = \sqrt{g_1^2+g_2^2}\, v^2$

Gluon: $\mathcal{G}^a_\mu = \dfrac{g_2 A^{1a}_\mu + g_1 A^{2a}_\mu}{\sqrt{g_1^2+g_2^2}}$; $M_{\mathcal{G}} = 0$

QCD Coupling: $\dfrac{1}{g^2_{QCD}} = \dfrac{1}{g_1^2} + \dfrac{1}{g_2^2}$

Result of $SU(3) \times SU(3)$, Φ (Topcolor):

- QCD gluon (zero-mode):

$$G = \frac{1}{\sqrt{g_1^2+g_2^2}}[g_2 A_1 + g_1 A_2]$$

- Coloron:

$$H = \frac{1}{\sqrt{g_1^2+g_2^2}}[g_2 A_1 - g_1 A_2]$$

- $M_H = v\sqrt{g_1^2 + g_2^2}$; $\quad M_G = 0$
- QCD coupling constant:

$$\frac{1}{g_{QCD}^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}$$

Note: if $g_1 = g_2$ then $g_1^2 = 2g_{QCD}^2$! ("Ethane")

- Some Phenomenology: $G \to VV$, $GG \to VV$, but no $G \to GV$, etc.

Experimentalists discover a second 1.0 TeV, $J = 1$, $I = 0$, coloron:

$\longrightarrow$ Expanded effective Lagrangian:

- $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ (assume $g_1 = g_2 = g_3 \equiv g_L$);
- 2 Higgs Fields: $D_\mu \Phi_1^\dagger D^\mu \Phi_1 + D_\mu \Phi_2^\dagger D^\mu \Phi_2$;
- Φ_1: $(\bar{3}_1, 3_2)$ $\langle \Phi \rangle = vI_3$; Φ_2: $(\bar{3}_2, 3_3)$ $\langle \Phi \rangle = vI_3$;
- Unambiguous Structure: "Propane"

Result:

- QCD coupling $g_L^2 = 3g_{QCD}^2$; gluon (zero-mode)

$G = \frac{1}{\sqrt{3}}[A_1 + A_2 + A_3]$

- Two Colorons, orthogonal normal modes:

$H_1 = \frac{1}{\sqrt{2}}[A_1 - A_3]$:

$H_2 = \frac{1}{\sqrt{6}}[A_1 - 2A_2 + A_3]$:

- $M_G = 0$; $M_{H1} = g_L v/\sqrt{2} = 0.5$ TeV; predict: $M_{H2} = g_L v \sqrt{3/2} = 0.86$ TeV!?.

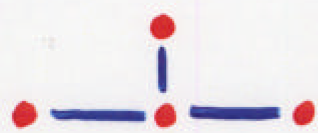
Experimentalists discover third 1.5 TeV, $J = 1$, $I = 0$, coloron:

Again, expand Effective Lagrangian:

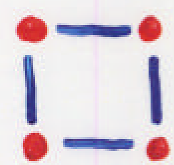
- QCD: $SU(3)_1 \times SU(3)_2 \times SU(3)_3 \times SU(3)_4$; g_L
- 3 Higgs: Φ_1, Φ_2, Φ_3; v.
- $g_L^2 = 4g_{QCD}^2$; gluon + 3 heavy H's.
- Equivalent to Geometrical Configurations and Normal Modes of Butane Isomers:

Aliphatic Butane:

[redacted]Methylpropane



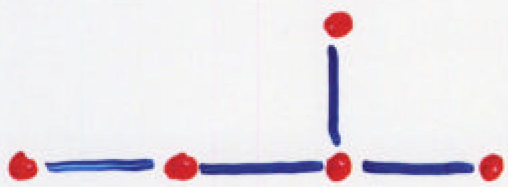
Cyclobutane (requires 4 Higgs):
(also methyl-cyclopropane !!!)



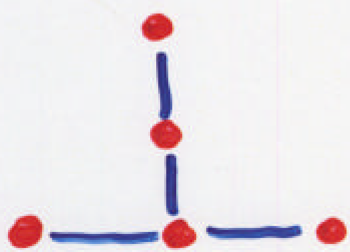
What is the organizing principle here?

A fourth coloron:
$(SU(3)^5, \Phi^4)$

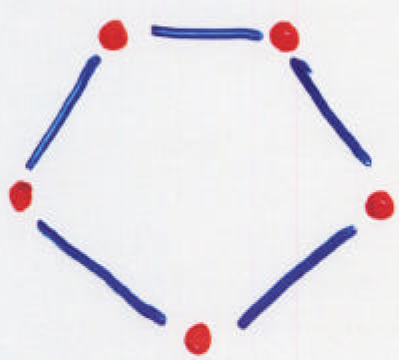
Aliphatic pentane



methyl-butane



ethyl-propane



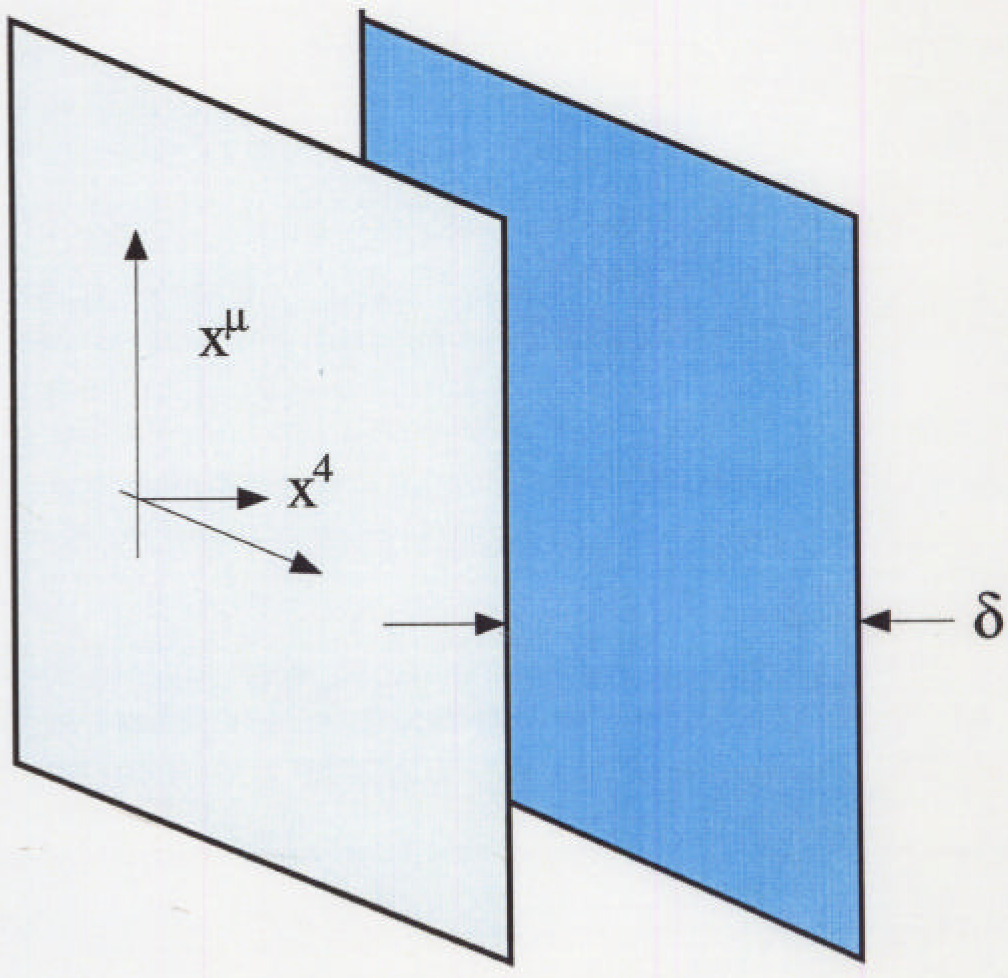
(extra Φ:)

cyclo-pentane

methyl-cyclobutane

(dimethyl or ethyl) cyclopropane

A Geometrical Extra Dimension



Fields Propagate in the Extra Dimensions

Compactification Boundary Conditions:

- Periodic:

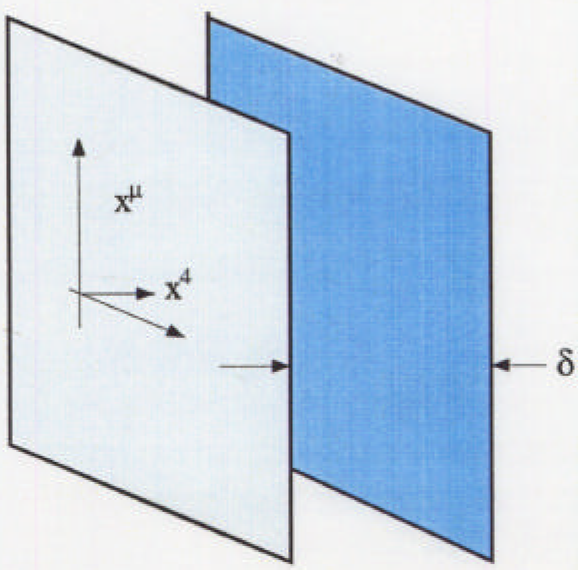
All basis functions periodic under $x^4 \to x^4 + \delta$

- Orbifold:

Divide basis functions into even F_+ and odd F_- under $x^4 \to 2\delta - x^4$

E.g., vector field A_μ: A^μ even; A^4 odd.

Assign $A^\mu \sim F_+$; $A^4 \sim F_-$.



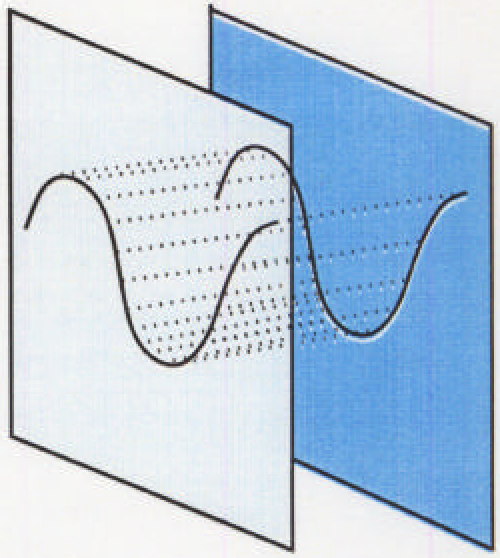
Kaluzsa-Klein Modes

Zero-mode (Gluon):

Vanishing $p^4 \rightarrow M_g = 0$

Orbifold: No A^4 zero mode, only A^μ.

Periodic: Both A_μ and A^4 zero-modes.



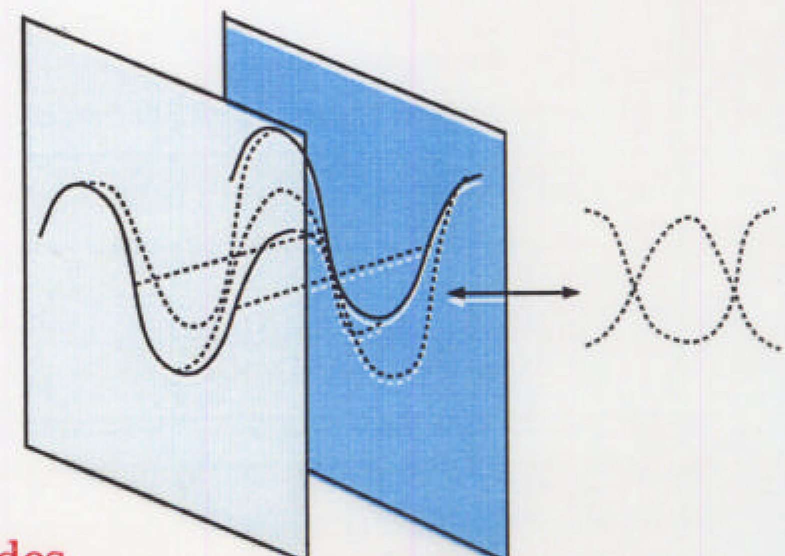
First KK Mode (Gluon):

Has mass:

$\partial_B \partial^B A = (\partial_\mu \partial^\mu + M^2) A$

$M = p^4 = 2\pi/\delta$:

A^4 is "eaten"
becomes longitudinal mode.



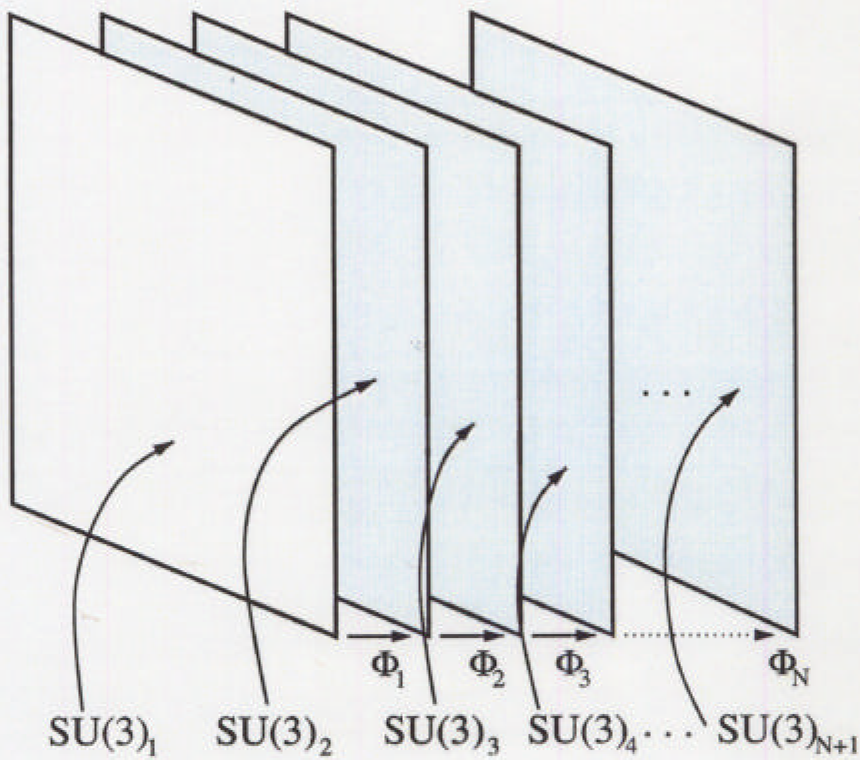
Sequential Higher KK Modes...

An Organizing Principle for $3 + 1$

Kaluzsa-Klein spectrum of an extra dimension of unknown geometry and topology.

How do we describe it in $3 + 1$ Dimensions?

Aliphatic case → Effective Lagrangian of a Transverse Wilsonian Lattice of an Extra Dimension.



Continuum $SU(3)_n$ on nth "brane."
(Transverse lattice: Bardeen, Pearson, Rabinovici)

Wilson Links are equivalent to Higgs fields in spontaneously broken phase:
$\Phi_n = v \exp(ig \int_n^{n+1} A_4 dx^4)$

"Aliphatic" Theory has gauge group $\Pi_{n=1}^{N+1} SU(3)_n$ with N Φ_n sequential links.
Analogue of Aliphatic Hydrocarbon

(CTH, Pokorski, Wang)

Manifestly Gauge Invariant Low Energy Effective Theory Constructed Without Knowing Complete High Energy Theory.
$\longrightarrow$ *Universality.*

Theory with compact extra dimensions is indistinguishable from a particular $3+1$ theoretical structure!

QCD $\rightarrow SU(3) \times SU(3) \times SU(3)$...

Standard Model $\rightarrow [SU(3) \times SU(2) \times U(1)]^N$

Generic "Aliphatic" $(SU(3)^{N+1}, \Phi^N)$ model $\longrightarrow$

Yang-Mills Lagrangian in $1+3$ dimensions for $N+1$ copies of QCD:

$$\mathcal{L}_{QCD} = -\sum_{j=1}^{N+1} \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu j} F_j^{\mu\nu} + \sum_{j=1}^{N} D_\mu \Phi_j^\dagger D^\mu \Phi_j$$

$N+1$ gauge groups $SU(3)_j$ with common g_L

N link-Higgs fields, Φ_j forming $(3_j, \bar{3}_{j-1})$ reps.

Note: Ordinary $3+1$ coordinates x^μ, μ runs from 0 (time) through (1, 2, 3) space;
$4+1$ coordinates, x^A, A runs from $0, 1, .., 4$; x^4 is the 5th dimension.

The Kaluzsa-Klein Spectrum

Renormalizable potential for the Higgs fields:

$$V(\Phi_j) = \sum_{j=1}^{N} \left[-M^2 \mathrm{Tr}(\Phi_j^2) + \lambda_1 \mathrm{Tr}(\Phi_j^4) + \lambda_2 \mathrm{Tr}(\Phi_j^2)^2 + M' \, det(\Phi_j) \right] ,$$

$\rightarrow \Phi_j$ develops diagonal VEV $= vI_3$;

Higgs and $U(1)$ PNGB must be heavy.

Thus:

$$\Phi_j \rightarrow vI_3 \exp(i\phi_i^a \lambda^a / 2v)$$

Φ_j have become Wilson links.

ϕ_i^a are $\sim A_n^4$ modes and are eaten.

Φ_j kinetic terms $\rightarrow$ gauge field mass matrix:

$$\sum_{j=1}^{N} \frac{1}{2} g_L^2 v^2 (A^a_{(j-1)\mu} - A^a_{j\mu})^2$$

Mass term has the structure of a nearest neighbor coupled oscillators (e.g. phonons in crystal).

Diagonalize Mass Matrix:

$$M_n = \sqrt{2} g_L v \sin\left[\frac{n\pi}{2(N+1)}\right] \qquad n = 0, 1, 2, \ldots N$$

For small n this system fakes a "KK tower:"

$$M_n \approx \frac{g_L v \pi n}{\sqrt{2}(N+1)} \qquad n << N$$

$n = 0$ corresponds to $M_0 = 0$, the gluon "zero-mode."

To match onto geometrical KK modes:

$$\frac{g_L v}{\sqrt{2}(N+1)} = \frac{2\pi\hbar}{\delta} = \frac{1}{R}.$$

R is "radius" of extra dimension, $\delta = 2\pi R$.

Aliphatic system $SU(3)^{N+1}$ and N Φ_i provides gauge invariant description of gluon + N KK modes.

Spectrum of KK modes reveals the geometry/topology of the "extra dimensions;"

Extra Geometrical Dimensions lose absolute meaning!

"DeConstruction"

"DeConstruction"

(Arkani-Hamed, Cohen, Georgi)

The link-Higgs-fields can be generated dynamically, e.g. fermion bilinear condensates (Technicolor). Asymptotically Free Theory in $3+1$.

Periodic Compactification:

Can use the extra A^4 zero mode (pseudo-Nambu-Goldstone Boson) to engineer a low mass Higgs Boson for Standard Model Electroweak Symmetry Breaking.

Running of the Coupling Constants

Above first coloron threshold:

$$\frac{1}{g_{QCD}^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}$$

If $g_1 = g_2 = \bar{g}$ we have: $\bar{g}^2 = \sqrt{2} g_{QCD}$ $= 2 g_{QCD}^2$

Above N coloron thresholds:

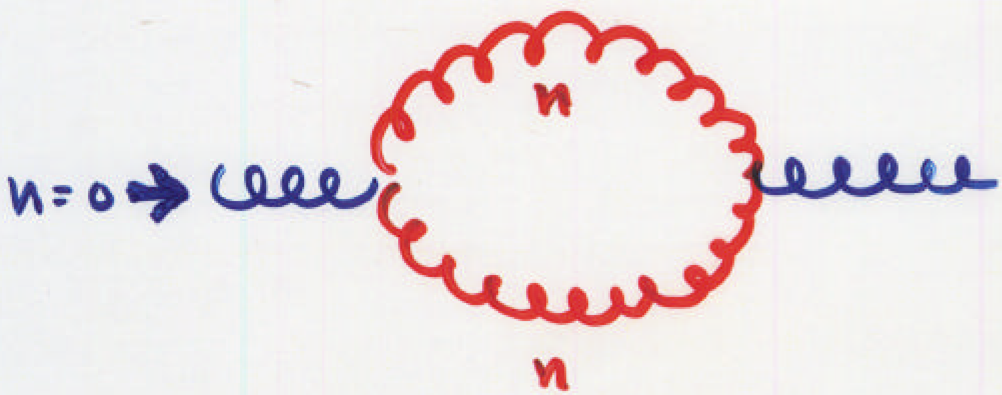
$$g_{eff} = \sqrt{N} g_{QCD}.$$

Thresholds spaced linearly → "power law running."

Renormalization Group simple at 1-loop level:

Running of $\bar{g}$ between (M_n, M_{n-1}) involves n modes lighter than M_n:

$$\frac{d\bar{g}}{d\log\mu} = -[n\frac{b_0}{16\pi^2}]\,\bar{g}^3, \qquad M_{n-1} \le \mu \le M_n;$$



Linear spectrum: $n \approx \left(\frac{\mu}{M_K}\right)$ (or $n \approx \left(\frac{\mu}{M_K}\right)^p$ for p extra dimensions):

$$\frac{d\bar{g}}{dt} \approx -\frac{b_0 e^{pt}}{16\pi^2}\,\bar{g}^3, \qquad t = \ln(\mu/M_K)$$

Exhibits power law running of coupling constant characteristic of p extra dimensions:

$$\frac{1}{\bar{g}^2(M_{\tilde{N}})} - \frac{1}{\bar{g}^2(M_K)} = \frac{b_0}{8\pi^2 p}\left[\left(\frac{M_{\tilde{N}}}{M_K}\right)^p - 1\right]$$

Sum series for generic spectrum:

$$\alpha^{-1}(\mu) = \alpha^{-1}(M_Z) - \frac{b_0}{4\pi}\ln\left(\frac{\mu}{M_Z}\right) - \frac{b_0}{4\pi}\tilde{N}\ln\left(\frac{\mu}{M_1}\right) + \frac{b_0}{4\pi}F(M_{\tilde{N}}).$$

Linear KK spectrum (from 4 + 1 theory) gives:

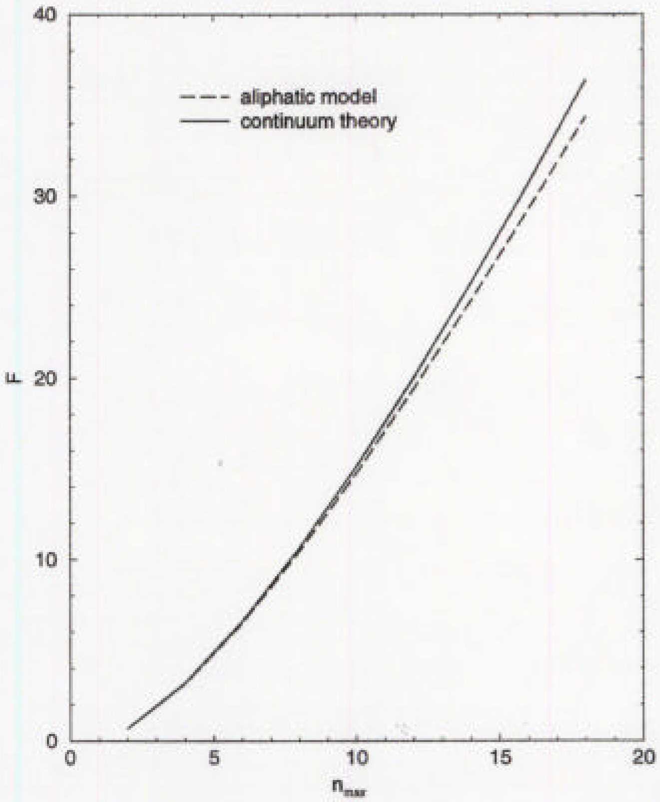
$$F_{lin} = \ln(\tilde{N}!) \sim \tilde{N}\ln(\tilde{N});$$

(Dienes, Dudas, Gerghetta; H.C. Cheng, Dobrescu, CTH)

Aliphatic theory gives:

$$F_{ali} = \ln\left(\frac{\Pi_{n=1}^{\tilde{N}}\sin\left(\frac{n\pi}{2(\tilde{N}+1)}\right)}{\sin\left(\frac{\pi}{2(\tilde{N}+1)}\right)^{\tilde{N}}}\right).$$

Difference between F_{lin} and F_{ali} provides measure on how much aliphatic theory deviates from continuum theory:



F_{lin} (solid line) and F_{ali} (dashed line) as functions of $\tilde{N} \equiv n_{max}$. $N = 20$ is chosen for the plot. Deviation of F_{ali} from F_{lin} is less than 10% at the high end.

Fermions

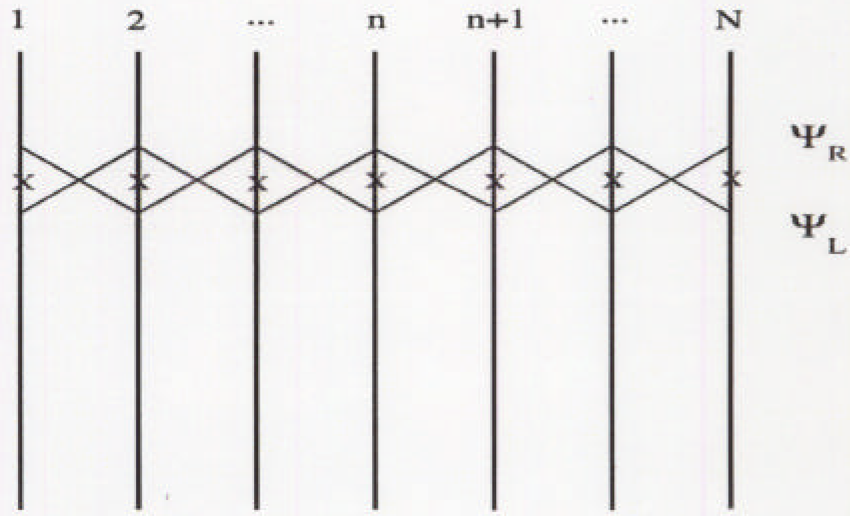
Free fermion action:

$$\int d^5x \; \bar{\Psi}(i\gamma_\mu \partial^\mu - \gamma^5 \partial_4 - \phi(x^4))\Psi$$

ϕ is a background field giving mass.

Decompose ∂_4 into "fast" components (high momentum) and "slow" components (low momentum).
Latticize the "slow" components.

If background field is $\sim$constant then impose (fast)$\partial_4 \Psi = 0$, i.e., "integrate out" high momentum components.



A general Fermionic Lattice Action with gauge fields and link-Higgs-fields:

$$-\frac{1}{2a}\sum_{n=1}^{N}\Big[\Big(\bar{\psi}(n)_L i\gamma_\mu D^\mu \psi(n)_L - \phi_n \bar{\psi}(n)_L \psi(n)_R + (L \leftrightarrow R)\Big)$$
$$+\Big(\bar{\psi}(n+1)_L \Phi_n \psi(n)_R + \bar{\psi}(n)_L \Phi_n^\dagger \psi(n+1)_R - (L \leftrightarrow R)\Big)\Big]$$

Describes a Mass Matrix for a $3+1$ Theory of N Dirac Fermions coupled to Aliphatic Gauge Fields!

Domain Wall Fermions (Jackiw + Rebbi)

$$(\gamma^5 \partial_4 + \phi(x^4))\psi = 0$$

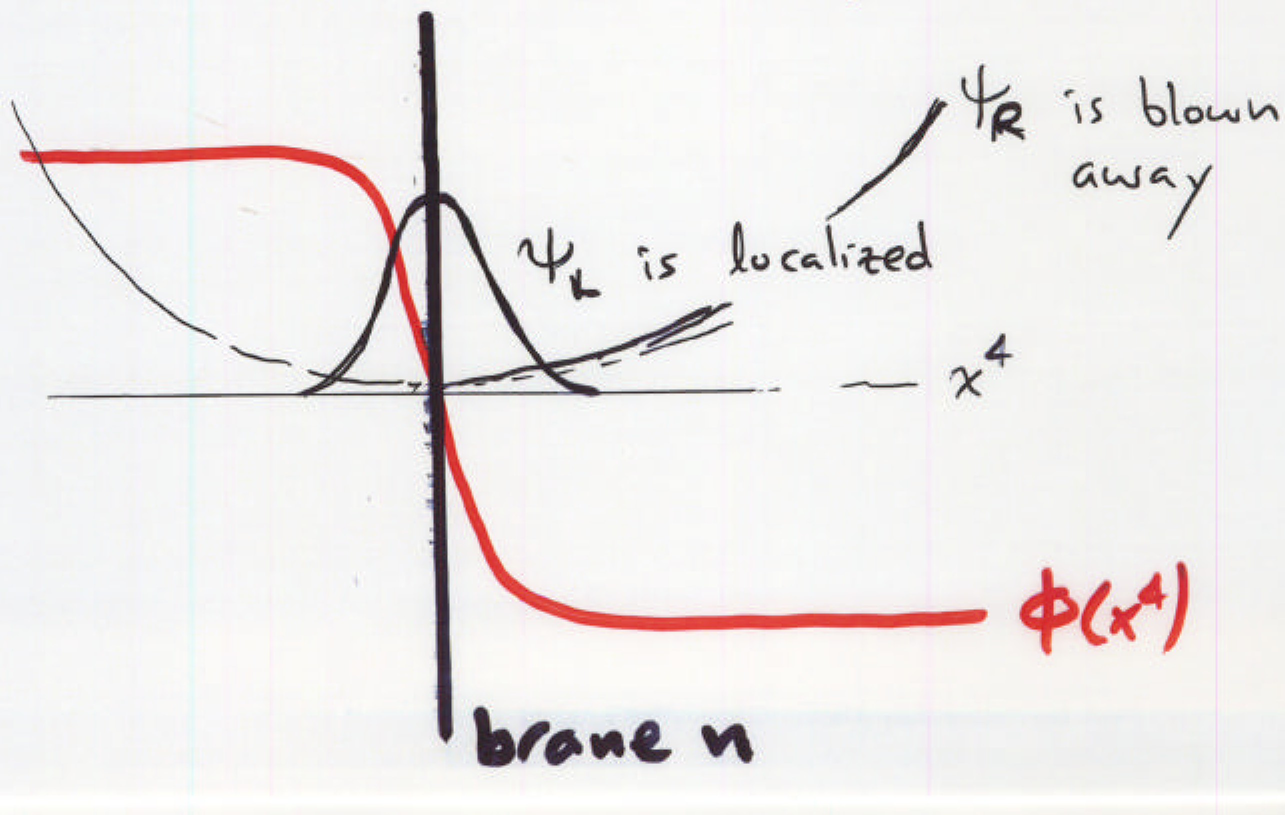
$$\Rightarrow \quad \partial_4 \psi_R + \phi(x^4)\psi_R = 0$$

$$-\partial_4 \psi_L + \phi(x^4)\psi_L = 0$$

$$\Rightarrow \quad \psi_L \propto \exp\left(\int_0^{x^4} \phi(z)\,dz\right)$$

$$\psi_R \propto \exp\left(-\int_0^{x^4} \phi(z)\,dz\right)$$

$\Rightarrow$



Chiral Fermions

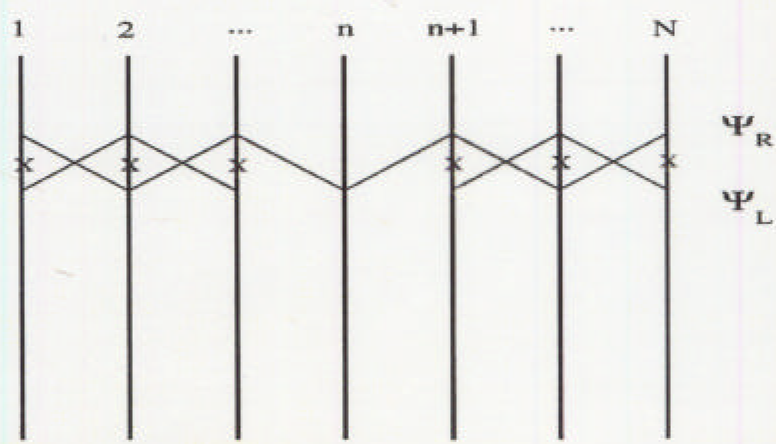
Chiral fermions are localizable in fifth dimension by background fields.

(Jackiw, Rebbi; Kaplan; Arkani-Hamed, Schmaltz; et.al.)

If $\phi(x^4)$ swings through zero rapidly in the vicinity of brane n, then impose $(\gamma^5 \partial_{4\ fast} + \phi(x^4))\Psi = 0$

One chiral component of Ψ (corresponding to the non-normalizable solution) projected to zero on the brane. The zero-mode is kept.

The chiral zero mode is a dislocation in the lattice:



Finite renormalization effects due to the ϕ field:

$$\left[-\frac{1}{2g^2} + \sum_p c_p \left(\frac{\phi(x^5)}{M}\right)^p\right] \mathrm{Tr}(G_{\mu\nu})^2$$

These renormalize g_n

$$\frac{1}{2g^2_{renorm}} = \left[\frac{1}{2g^2} - \sum_p c_p \left(\frac{\phi(x^5)}{M}\right)^p\right]$$

$\longrightarrow$ Not a coincidental effect.

(e.g., chiral fermion feed-back.)

Gauge coupling strength g_n of $SU(3)_n$ on the n-th brane can be supercritical!

Dynamical Chiral condensates

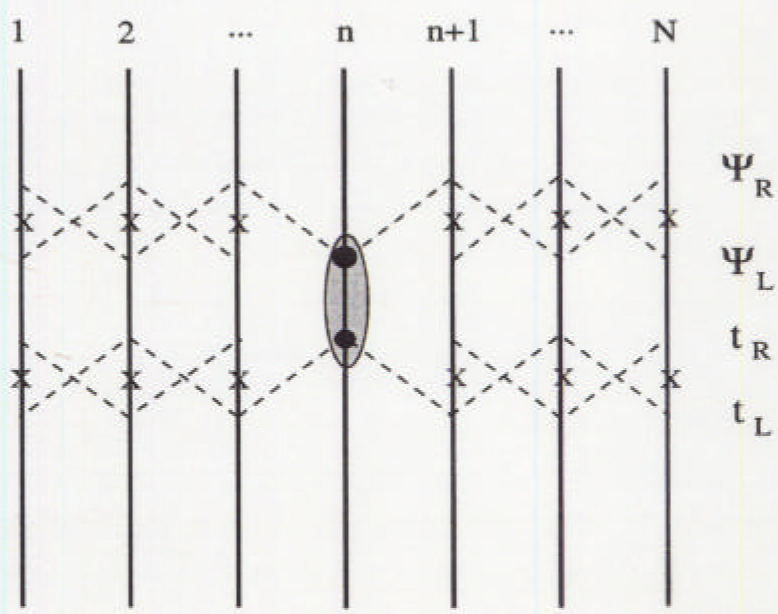
Introduce two flavors of fermions.

(Anomaly free if also include b_R.)

Identify $\Psi = (t, b)_L$, t_R as the chiral zero-modes on brane n; neighbor links large and decouple at low energies. Recover a Topcolor model with pure top quark condensation

(Nambu; Miransky, Tanabashi, Yamawaki; Bardeen, CTH, Lindner; Dobrescu, CTH).

Top Quark Condensation via Topcolor:

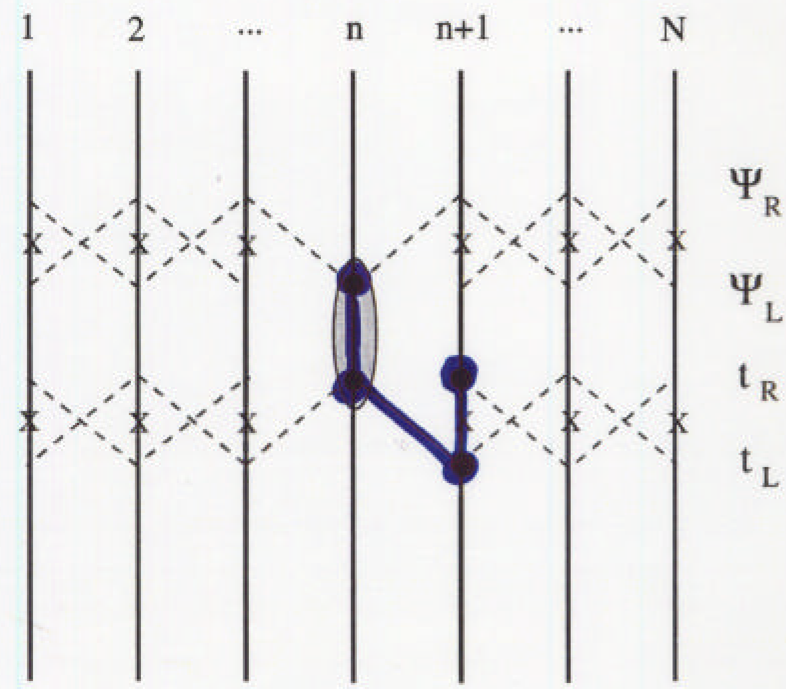


Topcolor Top Seesaw

Some links to nearest neighbors may not be completely decoupled.
Background field renormalization; to warping.

Thus the mixing with heavy vectorlike fermions occurs in addition to the chiral dynamics on brane n.

Top Quark Seesaw Model:



Top Seesaw model is prime example of new organizing principle leading to completely different paradigm for the Higgs mechansm!

Phenomenology of Top Seesaw Model

Identify: $\Psi_n \sim \Psi_L$, $t_{Rn} \sim \chi_R$, $t_{L,n+1} \sim \chi_L$, $t_{R,n+1} \sim t_R$; χ is usually introduced as vectorlike copy of t_R.

Mass matrix for $t - \chi$ system is,

$$-\left(\overline{t_L}\ \ \overline{\chi_L}\right)\begin{pmatrix} 0 & m_{t\chi} \\ \mu_{\chi t} & \mu_{\chi\chi} \end{pmatrix}\begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{h.c.}$$

700 GeV ~ 4 TeV

Diagonalized:

$$m_t^2 = \frac{1}{2}\left[\mu_{\chi\chi}^2 + \mu_{\chi t}^2 + m_{t\chi}^2 - \sqrt{\left(\mu_{\chi\chi}^2 + \mu_{\chi t}^2 + m_{t\chi}^2\right)^2 - 4\,\mu_{\chi t}^2\, m_{t\chi}^2}\right]$$

$$\longrightarrow \left.\frac{m_{t\chi}^2 \mu_{\chi t}^2}{\mu_{\chi t}^2 + \mu_{\chi\chi}^2}\right|_{(\mu_{\chi\chi} \gg \mu_{\chi t},\, m_{t\chi})}$$

$m_t \sim \frac{m_{t\chi}\mu_{\chi t}}{\mu_{\chi\chi}} = 175$ GeV

$$m_\chi^2 = \frac{1}{2}\left[\mu_{\chi\chi}^2 + \mu_{\chi t}^2 + m_{t\chi}^2 + \sqrt{\left(\mu_{\chi\chi}^2 + \mu_{\chi t}^2 + m_{t\chi}^2\right)^2 - 4\,\mu_{\chi t}^2\, m_{t\chi}^2}\right]$$

$$\longrightarrow \left.\mu_{\chi\chi}^2 + \mu_{\chi t}^2\right|_{(\mu_{\chi\chi} \gg \mu_{\chi t},\, m_{t\chi})},$$

$\sim \mu_{\chi\chi}^2 \sim 4$ TeV

Top Seesaw proposed in 1998. It was DOA (dead on arrival) since outside of the S-T ellipse by several σ. (Chivukula, Dobrescu, Georgi, CTH)

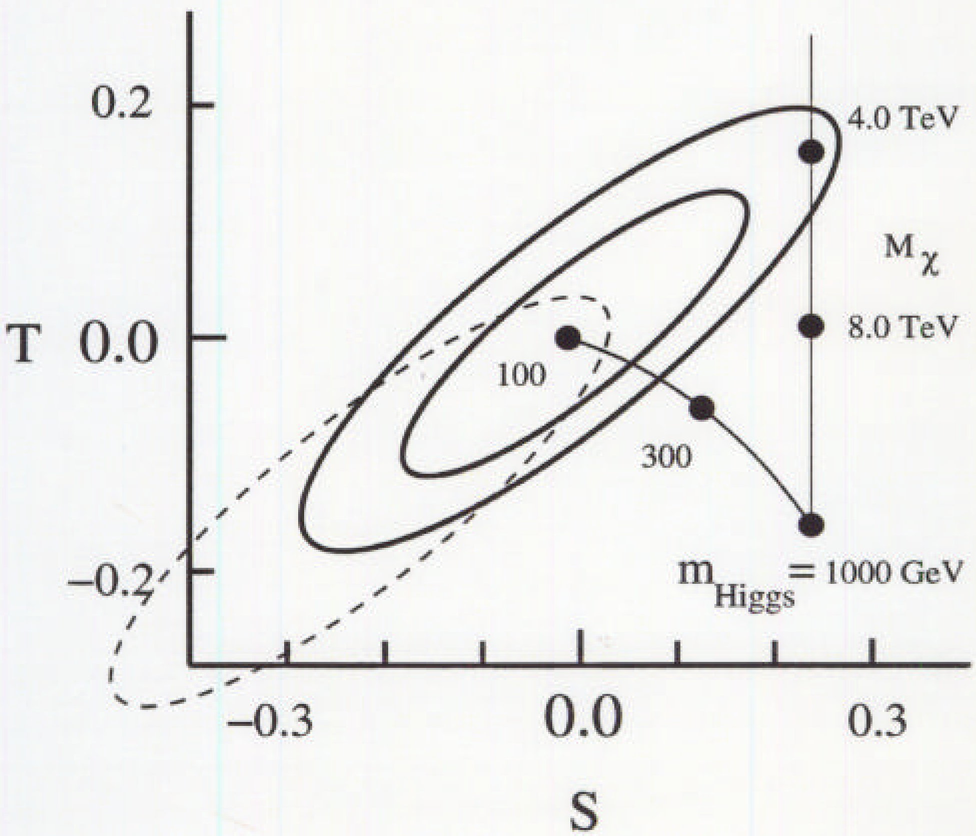
In 1999, S-T error ellipse shifted along major axis toward upper right.

Theory consistent for natural values of its parameters at the 2σ level (He, CTH, Tait)

With $M_\chi \sim 4$ TeV, the shift in the error ellipse was predicted by the theory — Top Seesaw has scored a predictive phenomenological success!

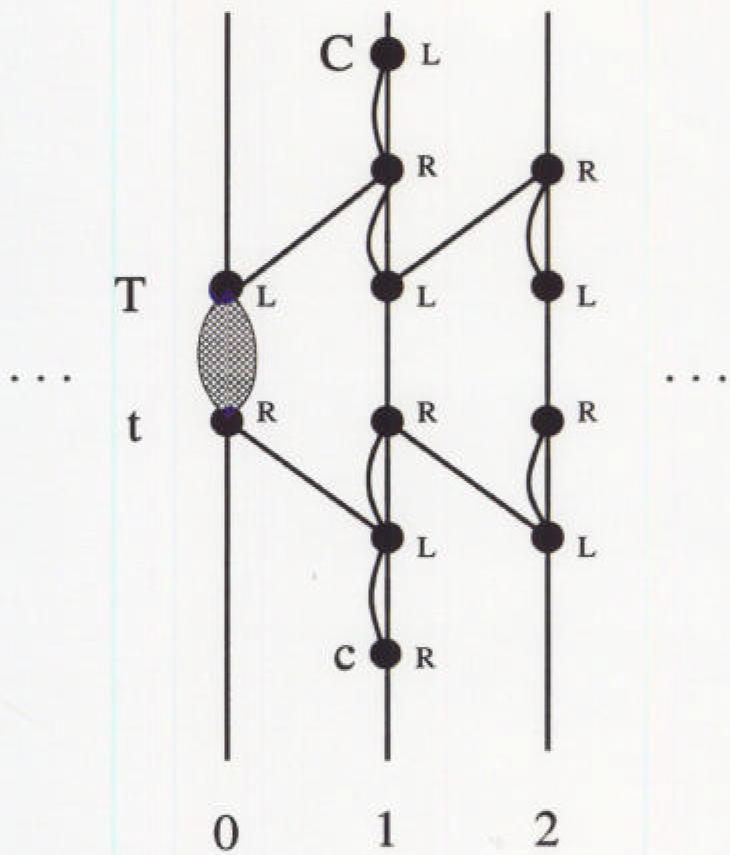
More conservatively: The measured error ellipse is a determination of heavy seesaw partner mass; obtain roughly: $M_\chi \sim 4\,\mathrm{TeV}$.

High precision electroweak measurements probing mass of heavy new particle, the χ quark, significantly above the electroweak mass scale!

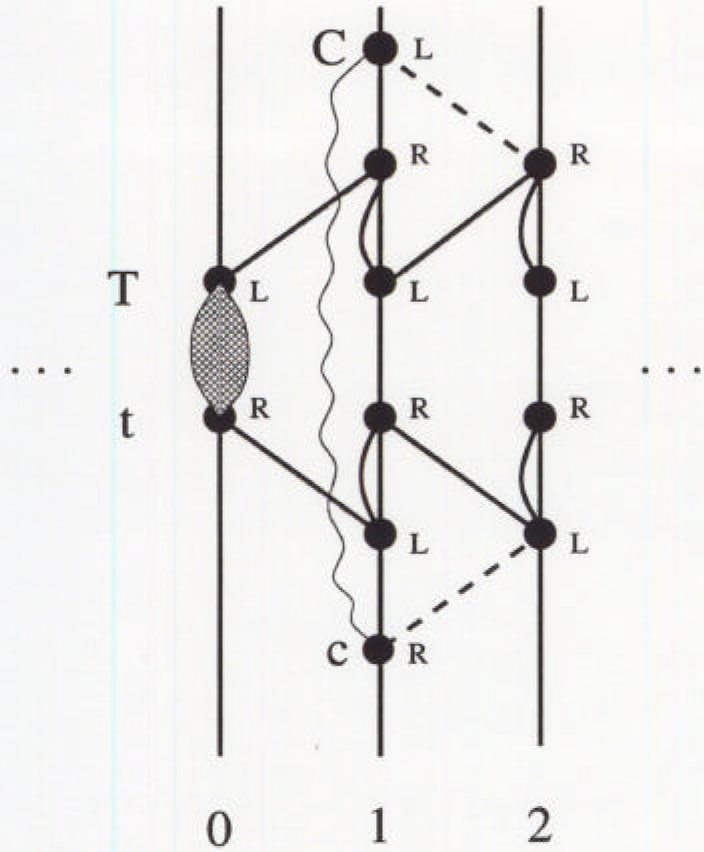


The 68% and 95% C.L. S-T contours (solid), superimposing the Standard Model curve for Higgs mass varying from 100 GeV up to 1000 GeV. The pre-1999 95% ellipse is shown with a dashed line. For the Top Seesaw model with a 1 TeV composite Higgs, we show the S-T contributions as a function of χ mass. The data is therefore consistent with a ~ 1 TeV Higgs and $M_\chi \sim 4.0$ TeV. (The S-T ellipses are taken from 1999 precision fit of M. Swartz; See (He, CTH, Tait) for expanded discussion).

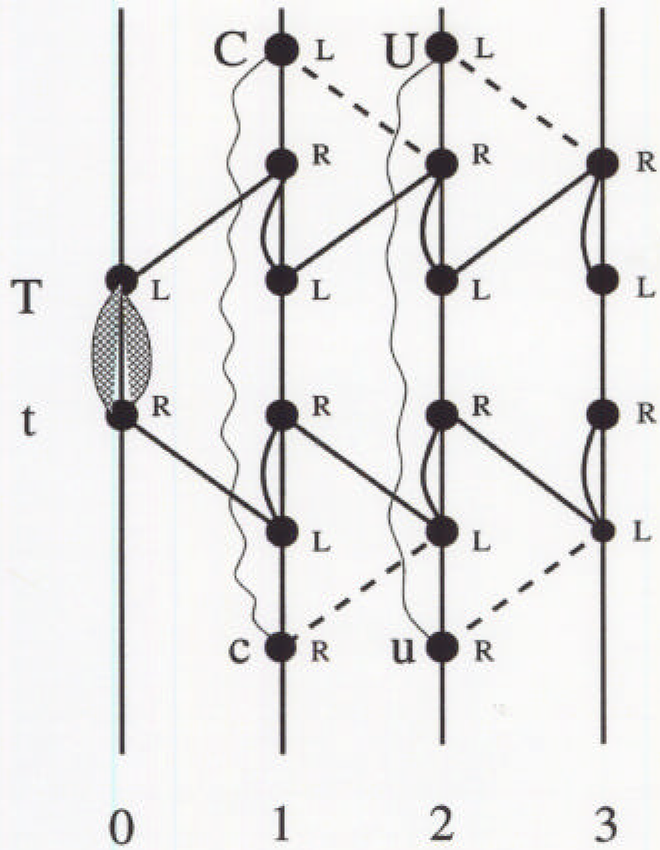
Blueprints for a Flavor Theory?



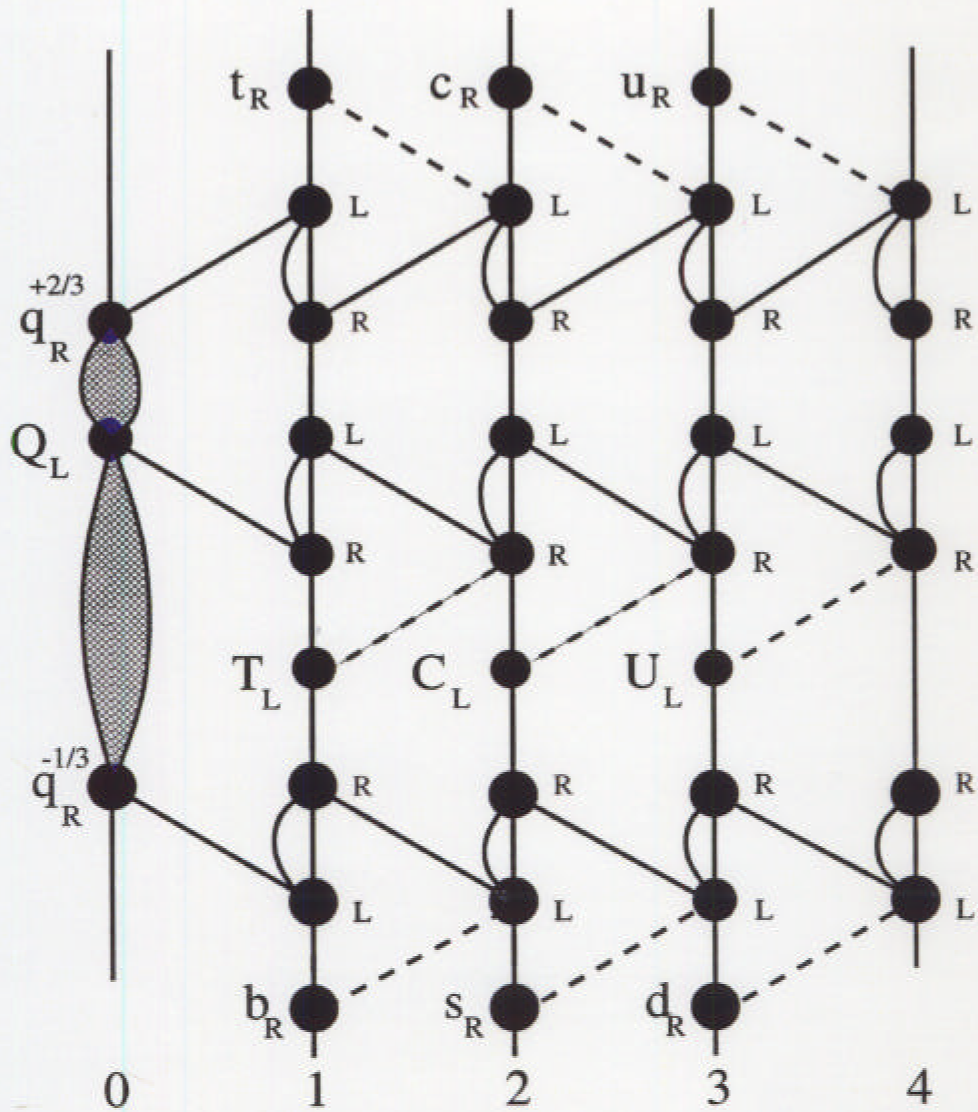
Three brane approximation incorporating charm, where $C = (c, s)_L$ is a doublet zero-mode, and $c = c_R$ is a singlet zero mode, both trapped on brane 1 (we assume the vectorlike partners of C and c are decoupled). Dirac flavor mixing between $\bar{C}_L \psi_{R1}$ and $\bar{c}_R t_{L1}$ can be rotated away by redefinition of ψ_{R1} and t_{L1}. Cheng, CTH, Wang



The flavor mixing (dashed lines) between $\bar{C}_L \prod(\Phi/v)\psi_{R12}$ and $\bar{c}_R \prod(\Phi/v)t_{L2}$ cannot be rotated away by redefinitions of ψ_{R2} and t_{L2} without generating effective kinetic term mixing which leads to non-zero-mode flavor changing gluon vertices (in the broken phase where $\Phi \to v$; this mixing is actually a higher dimension operator). The charm quark mass is thus generated when radiative corrections are included (wavy line).



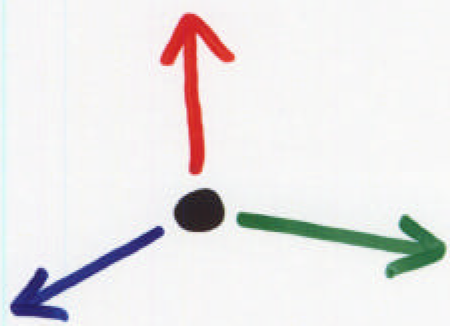
The extension to include the up quark in a 4-brane model with radiatively generated mass and mixing.



The fourth generation condensate generating the up and down type quark masses.

Topology in Extra Dimensions:

Magnetic Monopole in 3+1:



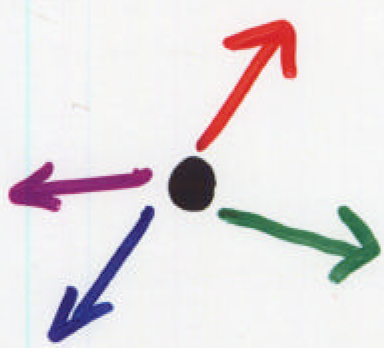
$$H^a \sim \hat{x}^a$$

$$F^a_{\mu\nu} \sim \hat{x}^a$$

$$\frac{F^a_{\mu\nu} \cdot H^a}{|H^\dagger H|^{1/2}} \sim F_{\mu\nu}$$

Instantonic Monopole in 4+1

(CTH, Ramond)



"Descent Cohomology"

∃ Matching of Currents

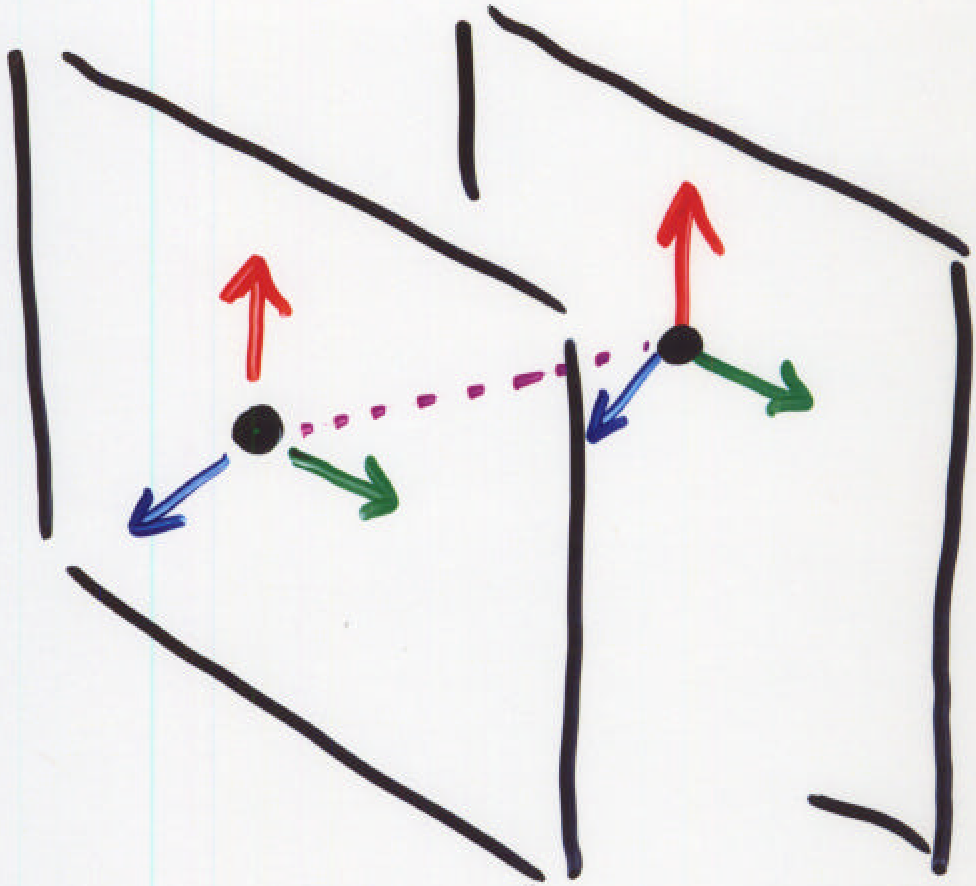
4+1: $$\varepsilon_{ABCDE} F^{BC} F^{DE} = Q_A$$

3+1: $$\mathrm{Tr}(\Phi^\dagger D_\nu \Phi\, \Phi^\dagger D_\rho \Phi\, \Phi^\dagger D_\lambda \Phi)\, \varepsilon^{\mu\nu\rho\lambda}$$

$$+ \frac{2}{3} \mathrm{Tr}(F_{\rho\nu}\, \Phi^\dagger D_\lambda \Phi)\, \varepsilon^{\mu\nu\rho\lambda} = Q^\mu$$

$$\begin{cases} \partial^A Q_A = 0 & \text{in } 4+1 \\ \partial^\mu Q_\mu = 0 & \text{in } 3+1 \end{cases}$$

Compactify the Instantonic Monopole?



Im → appears as a monopole when compactified

(1) No symmetry breaking → Gauge Bosonic Skyrmion

(2) SSB → 't Hooft Polyakov Monopole

$$M \sim \frac{m_K}{\alpha}$$

Conclusions

- Extra dimensions viewed though lattice/deconstruction generates new Lagrangian structures $\longleftrightarrow$ New Organizing Principles;
- Replicant K-K gauge groups: $G \rightarrow G \times G \times G ...$;
- Suggests New Dynamics for EWSB, e.g. dynamical condensation via QCD, Top Seesaw, etc.;
- Suggests New Directions for Flavor Physics Model Building
- Dimensional "Descent Cohomology" through Lattice, "deconstruction;"
- Rich topological structures expected in any extra dimensional theories expected at mass scale of $\sim M_{KK}/\alpha$.