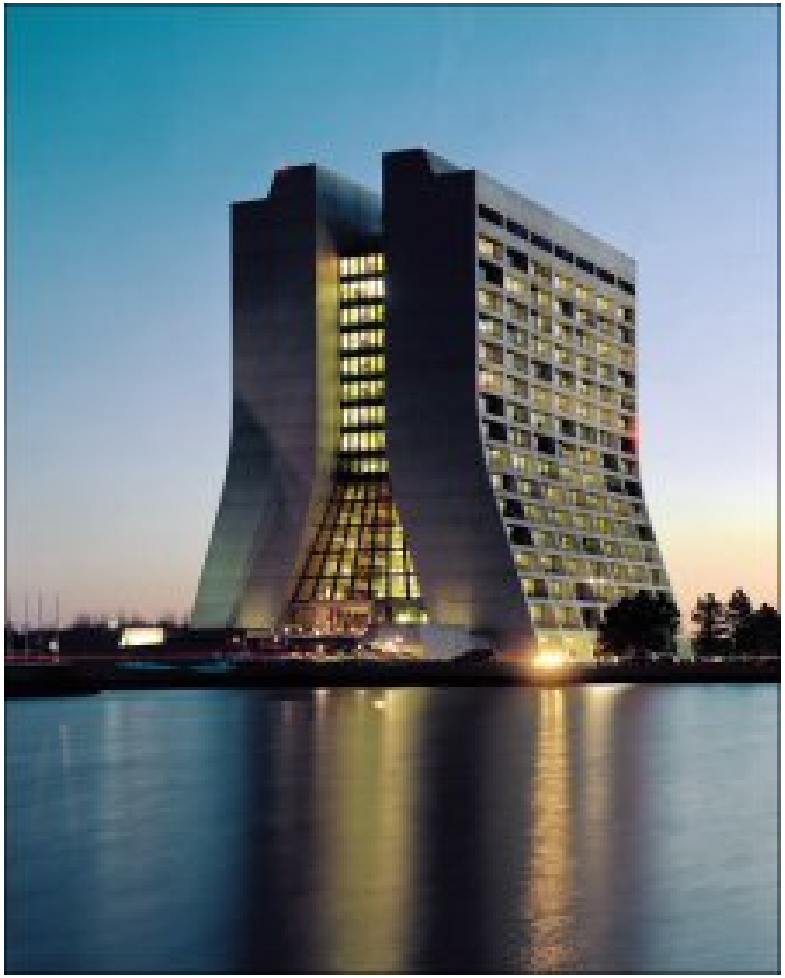
Tests of the Electroweak Theory

- History/introduction
- Weak charged current
- QED
- Weak neutral current
- Precision tests
- Rare processes
- CP violation and B decays
- Neutrino mass



Announcements

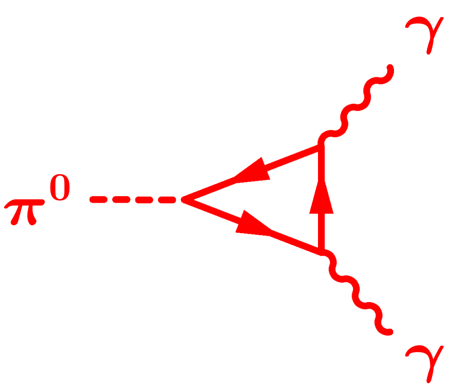
- **e-mail questions to** `carena@fnal.gov` **or** `pgl@fnal.gov` **(between lectures)**
- **Slides and video at** `http://theory.fnal.gov/AcademicLectures.html`
- **Today and 12/6 in Curia II; others in One West**

π and K Decays, and the Strong Interactions

- Charged pion decay

$\pi^+ \rightarrow \mu^+ \nu_\mu$	$(\pi_{\mu 2})$	99.99%	
$\rightarrow e^+ \nu_e$	(π_{e2})	1.23×10^{-4}	
$\rightarrow e^+ \nu_e \pi^0$	(π_{e3})	1.025×10^{-8}	π beta decay

- $\pi^0 \rightarrow 2\gamma$: 98.8%
  - Electromagnetic
  - Color counting via global anomaly



- Space reflection (parity): violated maximally by WCC, conserved by strong interactions, since $J_\mu = V_\mu - A_\mu$ (also C; $CP \sim$ conserved)

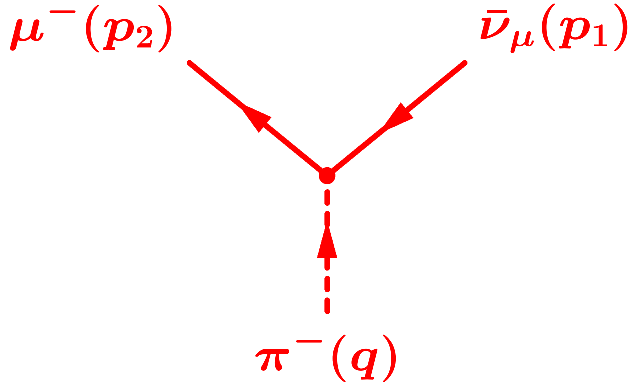
$$
\begin{aligned}
P|\pi(\vec{q})\rangle &= -|\pi(-\vec{q})\rangle \\
PV_\mu P^{-1} &= V^\mu \qquad (P\vec{V}P^{-1} = -\vec{V}, \quad PV_0P^{-1} = V_0) \\
PA_\mu P^{-1} &= -A^\mu \qquad (P\vec{A}P^{-1} = \vec{A}, \quad PA_0P^{-1} = -A_0)
\end{aligned}
$$

- Semi-leptonic processes for light hadrons/leptons

$$
\begin{aligned}
H &= \frac{G_F}{\sqrt{2}} \left[J^{\ell\mu} J^{h\dagger}_\mu + J^{\ell\mu\dagger} J^h_\mu \right] \\
&= \frac{G_F}{\sqrt{2}} \left[\bar{\mu}\gamma^\mu(1-\gamma^5)\nu_\mu + \bar{e}\gamma^\mu(1-\gamma^5)\nu_e \right] \; \bar{u}\gamma_\mu(1-\gamma^5)d' + HC \\
d' &\equiv \sum_{i=d,s,b} V_{ui} d_i \simeq d \; \cos\theta_c + s \; \sin\theta_c
\end{aligned}
$$

($\theta_c =$ Cabibbo angle; $\sin\theta_c \simeq 0.22$)

- For $\pi^- \to \mu^- \bar{\nu}_\mu$



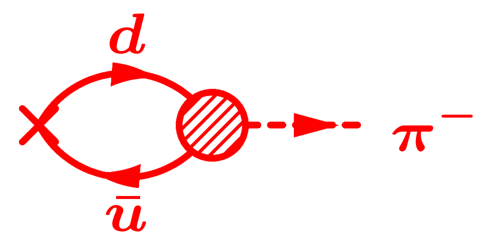
$$
\begin{aligned}
J_\mu^{h\dagger} &= \bar{u}\gamma_\mu(1-\gamma^5)\; d\;\; \cos\theta_c \\
&\equiv V_\mu^\dagger - A_\mu^\dagger
\end{aligned}
$$

$$
\begin{aligned}
M &\equiv -i\langle \mu^-(p_2)\bar{\nu}_\mu(p_1)|H|\pi^-(q)\rangle \\
&= -i\frac{G_F}{\sqrt{2}}\; \bar{u}_\mu\gamma^\mu(1-\gamma^5)v_{\bar{\nu}_\mu}\; \langle 0|J_\mu^{h\dagger}|\pi^-(q)\rangle
\end{aligned}
$$

- $\langle 0|J_\mu^{h\dagger}|\pi^-(q)\rangle$ involves strong interaction bound state $\longrightarrow$ hard to calculate (recent: lattice QCD calculation)

- However, $J_\mu^{h\dagger}$ is Lorentz vector $\longrightarrow$

$$\langle 0|J_\mu^{h\dagger}|\pi^-(q)\rangle = \underbrace{-i\cos\theta_c}_{\text{convention}} f_\pi \; q_\mu$$



  - $q = p_1 + p_2$ is only 4-vector available
  - $f_\pi \equiv$ "pion decay constant"
  - Related to pion $\bar{u}d$ "wave function" (expect $f_\pi = O(\Lambda_{QCD}) = O(100$ MeV$))$
  - f_π could depend on q^2, but $q^2 = m_\pi^2 =$ fixed

- Can show (exercize) that $\langle 0|V_\mu^\dagger|\pi^-(q)\rangle = 0$ using parity invariance (weak P violation already included explicitly to 1^{st} order$\longrightarrow$ strong interaction calculation)

- **Rate (exercize)**

$$\Gamma(\pi\rightarrow\mu\nu) = \underbrace{\frac{G_F^2 \cos^2\theta_c}{8\pi}\ f_\pi^2\ m_\mu^2}_{\text{matrix element}} \underbrace{m_\pi\ \left(1-\frac{m_\mu^2}{m_\pi^2}\right)}_{\text{phase space}}$$

  - **Experiment:** $\tau_{\pi^\pm} \sim 2.60\times10^{-8}$ **s**
  - $G_F \sim 1.17\times10^{-5}$ **GeV**$^{-2}$ **from** μ **lifetime**
  - $\cos\theta_c \sim 0.975$ **from superallowed** β **decay**

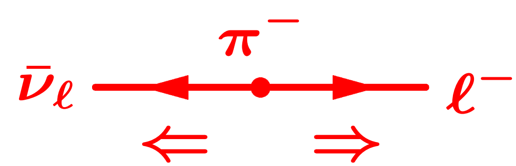
$\Rightarrow\ \ f_\pi \sim 132$ **MeV** $\sim 0.95\ m_\pi$

- e/μ ratio

$$\frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{m_e^2}{m_\mu^2} \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right) (1 + \underbrace{O(\alpha)}_{\text{rad. corr.}})$$

$$= 1.28 \times 10^{-4} \, (1 + O(\alpha)) = 1.23 \times 10^{-4}$$

  - Experiment: $(1.23 \pm 0.02) \times 10^{-4}$
  - $V - A$ favors $h_{\ell^-} = -1$
  - Angular momentum forces "wrong helicity"



  - Amplitude suppressed by m_ℓ / E_ℓ

- **Kaon decays**

$K^+ \to \mu^+ \nu_\mu$	$(K_{\mu 2})$	63.5%	f_K/f_π
$\to e^+ \nu_e$	(K_{e2})	1.6×10^{-5}	
$\to \mu^+ \nu_\mu \pi^0$	$(K_{\mu 3})$	3.2%	universality test
$\to e^+ \nu_e \pi^0$	(K_{e3})	4.8%	universality test
$\to \pi^+ \pi^0$	$(K_{2\pi})$	21.2%	nonleptonic
$\to \pi^+ \pi^+ \pi^-$	$(K_{3\pi})$	5.6%	
$\to \pi^+ \pi^0 \pi^0$	$(K_{3\pi})$	1.9%	

- K is also pseudoscalar, related to π by flavor $SU(3)$

$$\langle 0|J_\mu^{h\dagger}|K^-(q)\rangle = \underbrace{-i\sin\theta_c}_{\text{convention}} f_K\ q_\mu$$

  - $f_K =$ kaon decay constant
  - In $SU(3)$ limit $f_K = f_\pi$
  - However, $SU(3)$ typically broken by 20-30%

$$\Gamma(K\to\mu\nu) = \frac{G_F^2 \sin^2\theta_c}{8\pi}\ f_K^2\ m_\mu^2\ m_K\ \left(1 - \frac{m_\mu^2}{m_K^2}\right)$$

- Observed rate $+\sin\theta_c \simeq 0.221$ from $K_{\ell 3}$ or hyperon decay:

$$f_K \sim 160 \text{ MeV} \ \sim (1.22 \pm 0.01) f_\pi$$

(20% $SU(3)$ breaking)

- **Pion beta decay (π_{e3}), $\pi^{\pm}\to\pi^0 e^{\pm} \overset{(-)}{\nu}_e$**

$$M = -i\frac{G_F}{\sqrt{2}}\, \bar{u}_e\gamma^{\mu}(1-\gamma^5)v_{\bar{\nu}_e}\ \langle\pi^0(p_2)|J_{\mu}^{h\dagger}|\pi^-(p_1)\rangle$$

  - Parity: only vector current $V_{\mu}^{\dagger} \equiv \cos\theta_c \bar{u}\gamma_{\mu}d$ contributes
  - Lorentz invariance: only two momenta, $p_{1,2}$ in hadronic matrix element $\longrightarrow$

$$\langle\pi^0(p_2)|V_{\mu}^{\dagger}|\pi^-(p_1)\rangle = \cos\theta_c[f_+(q^2)(p_{1\mu}+p_{2\mu})+f_-(q^2)(p_{1\mu}-p_{2\mu})]$$

  - $f_{\pm}(q^2)$ are *form factors*, which can depend on $q^2 \equiv (p_2-p_1)^2$

- **Are $f_{\pm}(q^2)$ totally unknown because of strong interactions? No!**

- **Symmetry rescues us. Strong interactions almost invariant under global $SU(2)$ isospin symmetry** (broken at 1% level by electromagnetism *and* by $m_d - m_u \sim$ few MeV $\neq 0$)

- **$V_\mu^\dagger / \cos\theta_c \equiv \bar{u}\gamma_\mu d$ is generator of isospin**

$$\langle \pi^0(p_2)|V_\mu^\dagger|\pi^-(p_1)\rangle = \cos\theta_c[f_+(q^2)(p_{1\mu}+p_{2\mu})+f_-(q^2)(p_{1\mu}-p_{2\mu})]$$

- **If isospin were exact, $|f_+(0)| = \sqrt{2}$. Also, $\partial^\mu V_\mu^\dagger = 0 \Rightarrow f_-(q^2) = 0$** (cf non-renormalization of electric charge)

- **Conserved vector current (CVC):** $\bar{u}\gamma_\mu d$, $\bar{d}\gamma_\mu u$ **and** $\frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$ (isovector part of $J_\mu^{elm} = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)$) **are related by isospin and have same form factors** (Was hypothesis. Natural in quark model.)

- **Ademollo-Gatto theorem: corrections to** $f_+(0)$ **from isospin breaking are** *second order*, $\sim 10^{-3}$ **(negligible)**

- **For** π_{e3}, $\Delta = m_{\pi^+} - m_{\pi^0} \sim 4.6$ **MeV**$\rightarrow$ **can neglect** $p_2 - p_1$ **term and take** $f_+(q^2) \sim f_+(0)$. **Neglecting** m_e,

$$\Gamma(\pi^\pm \rightarrow \pi^0 e^\pm \overset{(-)}{\nu}_e) = \frac{G_F^2 \cos^2\theta_c |f_+(0)|^2 \Delta^5}{60\pi^3} \Rightarrow |f_+(0)| = 1.37 \pm 0.02$$

Isospin: $|f_+(0)| = \sqrt{2}\ +$ second order

- $K_{\ell 3}$ **decays:** $K^+ \rightarrow \ell^+ \nu_\ell \pi^0$, $K^0 \rightarrow \ell^+ \nu_\ell \pi^-$, **etc**

$$\langle \pi^0(p_2)|V_\mu|K^+(p_1)\rangle = |V_{us}|[f_+^{K^+}(q^2)(p_{1\mu}+p_{2\mu})+f_-^{K^+}(q^2)(p_{1\mu}-p_{2\mu})]$$

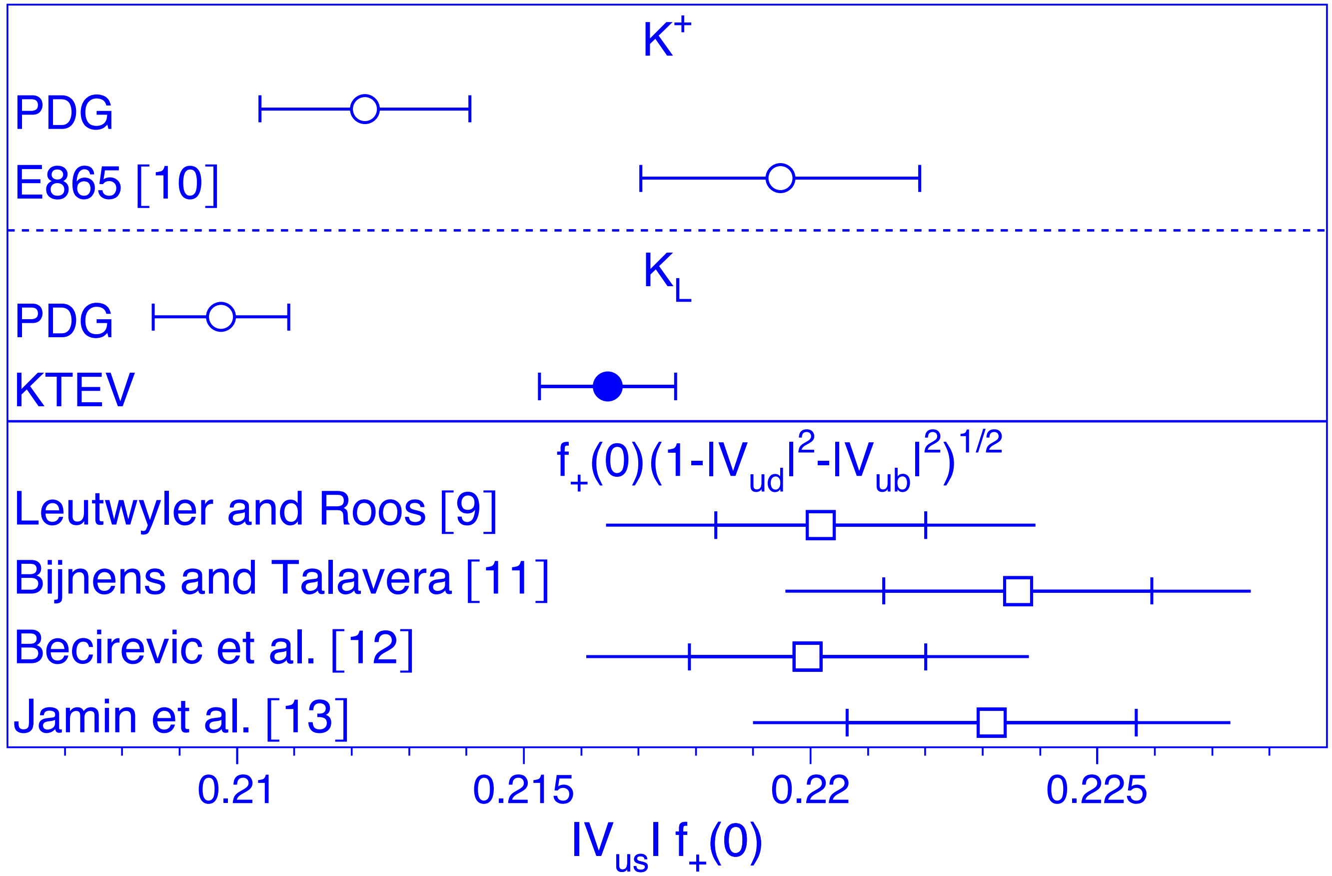
  - Write $|V_{us}|$ rather than $\sin\theta_c$ (most precise determination)
  - Large energy release. Cannot neglect q^2 dependence of form factors or f_- term
  - Can measure them (linear or quadratic approximation) from decay distributions, but need $|f_+^{K^+}(0)|$ and $|f_+^{K^0}(0)|$
  - V_μ are $SU(3)$ generators $\longrightarrow$

$$\sqrt{2}|f_+^{K^+}(0)| \sim |f_+^{K^0}(0)| \sim 1 + O(\epsilon^2)$$

    where $\epsilon \sim 20\%$ is typical $SU(3)$ breaking (Ademollo-Gatto)
  - Detailed estimates (chiral perturbation theory): $|f_+^{K^0}(0)| \sim 0.961(8)$, $\sqrt{2}|f_+^{K^+}(0)| \sim 1.022|f_+^{K^0}(0)|$

- V_{us} **from** K_{l3} **extremely important for weak universality test**

- **Recent significant shift, KTEV (FNAL) and E865 (BNL)**



(from KTEV, PRL 93, 181802)

β Decay and Related Processes

$n \rightarrow pe^- \bar{\nu}_e$	neutron
$(N, Z) \rightarrow (N-1, Z+1) e^- \bar{\nu}_e$	nuclear (heavy)
$(N, Z) \rightarrow (N+1, Z-1) e^+ \nu_e$	nuclear (light, e.g., Sun)
$\ell^- p \rightarrow n \nu_\ell$	atomic e^- or μ^- capture
$\nu_e n \leftrightarrow e^- p, \quad \bar{\nu}_e p \leftrightarrow e^+ n$	inverse β decay

$$H = \frac{G_F}{\sqrt{2}} V_{ud} \, \bar{e} \gamma^\mu (1 - \gamma^5) \nu_e \, \bar{u} \gamma_\mu (1 - \gamma^5) d + HC$$

- Radiative corrections divergent in Fermi theory, finite in $SU(2) \times U(1)$

- **Nuclear filter:** V_μ (Fermi transition), A_μ (Gamow-Teller transition), or both relevant

- **Superallowed:** $0_i^+ \to 0_f^+$ in same isomultiplet
  - Pure Fermi transition
  - $\langle 0_f^+ | \bar{u}\gamma_0 d | 0_i^+ \rangle = 1 + O(\delta^2)$, where δ = isospin breaking (Ademollo-Gatto). Corrections tiny but critical.
  - Best determination of $|V_{ud}| = 0.97377(11)(15)(19)$ (various theoretical uncertainties) (Marciano, Sirlin, hep-ph/0510099)

- **Neutron β decay**

$$\langle p|\bar{u}\gamma_\mu(1-\gamma^5)d|n\rangle$$

$$= \bar{u}_p \left[\gamma_\mu f_1(q^2) + \underbrace{\frac{i\sigma_{\mu\nu}}{2m} q^\nu f_2(q^2)}_{\text{weak magnetism}} + \underbrace{q_\mu f_3(q^2)}_{\text{vanishes by CVC}} \right.$$

$$\left. - \gamma_\mu\gamma^5 g_1(q^2) - \underbrace{\frac{\sigma_{\mu\nu}\gamma^5}{2m} q^\nu g_2(q^2)}_{\text{T}-\text{odd}} - \underbrace{q_\mu\gamma^5 g_3(q^2)}_{\text{induced pseudoscalar}} \right] u_n$$

$$\sim \bar{u}_p\gamma_\mu(g_V - g_A\gamma^5)u_n$$

- $q = p_p - p_n$, $g_V \sim 1 + O(\delta^2)$
- $g_A \neq 1$, since no symmetry prevents large strong interaction effect (Adler-Weisberger estimate: $g_A = O(1.25)$)

- Lifetime $\propto G_F^2 |V_{ud}^2| m_e^5 (g_V^2 + 3g_A^2)$

- Asymmetries w.r.t. n polarization, $e - \bar{\nu}$ correlation
  (e.g., ILL (Grenoble)) $\rightarrow g_A/g_V$

$$|V_{ud}| = 0.9725(13) \qquad\qquad g_A/g_V = 1.2695(29)$$

CKM Universality

- **Universality of quark and lepton couplings** (required by gauge theory) **and unitarity of CKM relate μ, β, $K_{\ell 3}$, and $b \rightarrow u \ell^- \bar{\nu}_\ell$**

- **Measure $G_F |V_{ui}|$. Divide by G_F from μ decay$\rightarrow$ $|V_{ui}|$**

- **Expect**

$$\underbrace{|V_{ud}|^2}_{\beta} + \underbrace{|V_{us}|^2}_{K_{\ell 3}} + \underbrace{|V_{ub}|^2}_{\text{negligible}} = 1$$

- Severe constraint on new interactions, W_R, $W_L - W_R$ mixing, fermion-exotic mixing, neutrino-exotic mixing (excludes some explanations of NuTeV)
  - For many years, appeared to be 2σ discrepancy, $\sum |V_{ui}|^2 \sim 0.9969(15)$
  - Particle physicists doubted nuclear physics ($|V_{ud}|$)
  - But problem was really in V_{us} from $K_{\ell 3}$!
- Current: $\sum |V_{ui}|^2 \sim 0.9992(5)_{ud}(4)_{us}(8)_{f_+(0)}$ (M.S., hep-ph/0510099)
- Without radiative corrections, would find ~ 1.04! These are only finite and meaningful in full $SU(2)\times U(1)$ gauge theory

Quantum Electrodynamics (QED)

Incorporated into standard model

Lagrangian:

$$\mathcal{L} = -\frac{gg'}{\sqrt{g^2 + g'^2}} J_Q^\mu (\cos\theta_W B_\mu + \sin\theta_W W_\mu^3)$$

Photon field ($A = \gamma$):

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

Positron electric charge: $e = g \sin\theta_W$, **where** $\tan\theta_W \equiv g'/g$

Electromagnetic current:

$$
\begin{aligned}
J_Q^\mu &= \sum_{m=1}^{F}\left[\frac{2}{3}\bar{u}_m^0\gamma^\mu u_m^0 - \frac{1}{3}\bar{d}_m^0\gamma^\mu d_m^0 - \bar{e}_m^0\gamma^\mu e_m^0\right] \\
&= \sum_{m=1}^{F}\left[\frac{2}{3}\bar{u}_m\gamma^\mu u_m - \frac{1}{3}\bar{d}_m\gamma^\mu d_m - \bar{e}_m\gamma^\mu e_m\right]
\end{aligned}
$$

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

Purely vector (parity conserving): L and R fields have same charge

Quantum Electrodynamics

Experiment	Value of α^{-1}		Difference from $\alpha^{-1}(a_e)$
Deviation from gyromagnetic ratio, $a_e = (g-2)/2$ for e^-	137.035 999 58 (52)	$[3.8 \times 10^{-9}]$	–
ac Josephson effect	137.035 988 0 (51)	$[3.7 \times 10^{-8}]$	$(0.116 \pm 0.051) \times 10^{-4}$
h/m_n (m_n is the neutron mass) from n beam	137.036 011 9 (51)	$[3.7 \times 10^{-8}]$	$(-0.123 \pm 0.051) \times 10^{-4}$
Hyperfine structure in muonium, $\mu^+ e^-$	137.035 993 2 (83)	$[6.0 \times 10^{-8}]$	$(0.064 \pm 0.083) \times 10^{-4}$
Cesium D_1 line	137.035 992 4 (41)	$[3.0 \times 10^{-8}]$	$(0.072 \pm 0.041) \times 10^{-4}$

Spectacularly successful:

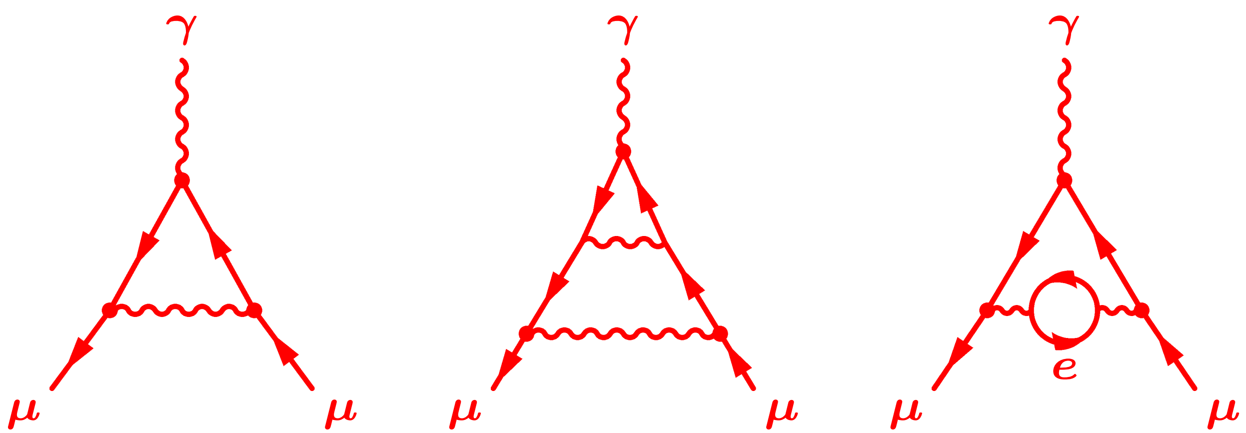
- Most precise: e anomalous magnetic moment $\rightarrow \alpha$
- Many low energy tests to few $\times 10^{-8}$
- $m_\gamma < 6 \times 10^{-17}$ eV
- $q_\gamma < 5 \times 10^{-30} |e|$
- High energy well-measured (PEP, PETRA, TRISTAN, LEP)

The Anomalous Magnetic Moment of the Muon

- Muon $a_\mu \equiv \frac{g_\mu - 2}{2}$ sensitive to new physics (usually $\sim (m_\mu/M_X)^2$)

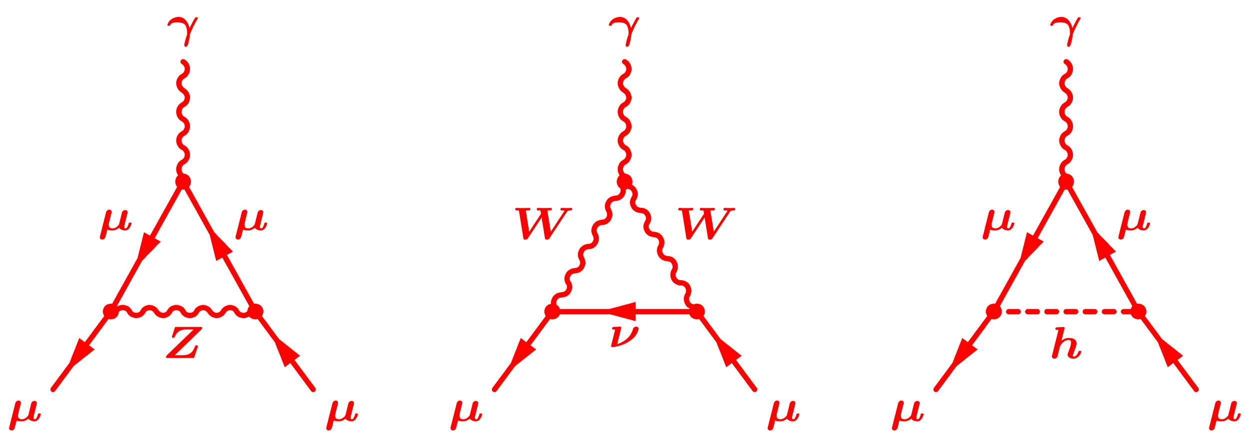
$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm Had} + a_\mu^{\rm EW}$$

- $a_\mu^{\rm QED}$ known to four loops (3 analytic); leading logs to five



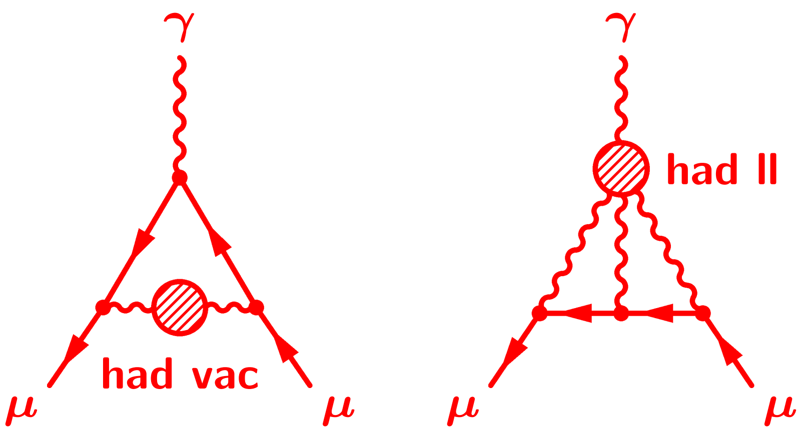
$$
\begin{aligned}
a_\mu^{\text{QED}} \;&=\; \frac{\alpha}{2\pi} + 0.765857376(27)\left(\frac{\alpha}{\pi}\right)^2 \\
&+24.05050898(44)\left(\frac{\alpha}{\pi}\right)^3 + 126.07(41)\left(\frac{\alpha}{\pi}\right)^4 \\
&+930(170)\left(\frac{\alpha}{\pi}\right)^5 = 1165847.06(3)\times 10^{-9}
\end{aligned}
$$

- $a_\mu^{\text{EW}} = 1.52(3)\times 10^{-9}$ (goal of experiments) includes leading 2 and 3 loops (cancellation)



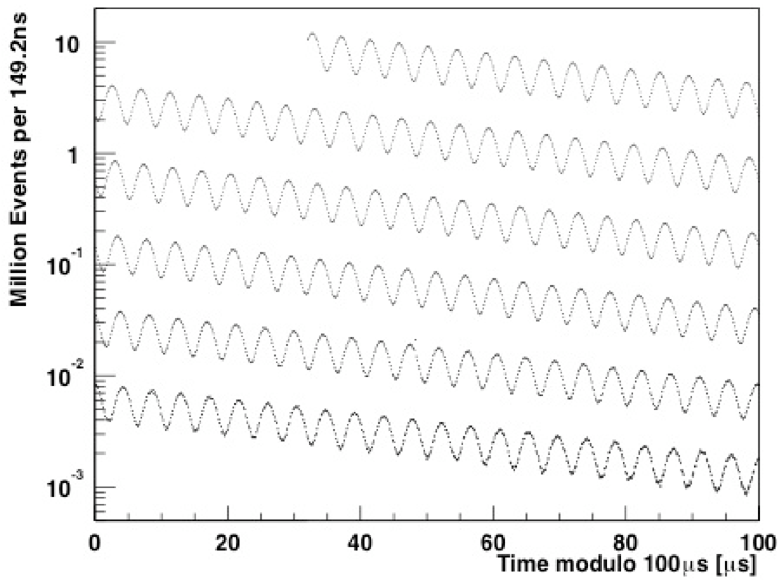
- **Biggest uncertainty: $a_\mu^{\rm Had}$ = hadronic vacuum polarization (2 loop) and hadronic light by light (3 loop)**

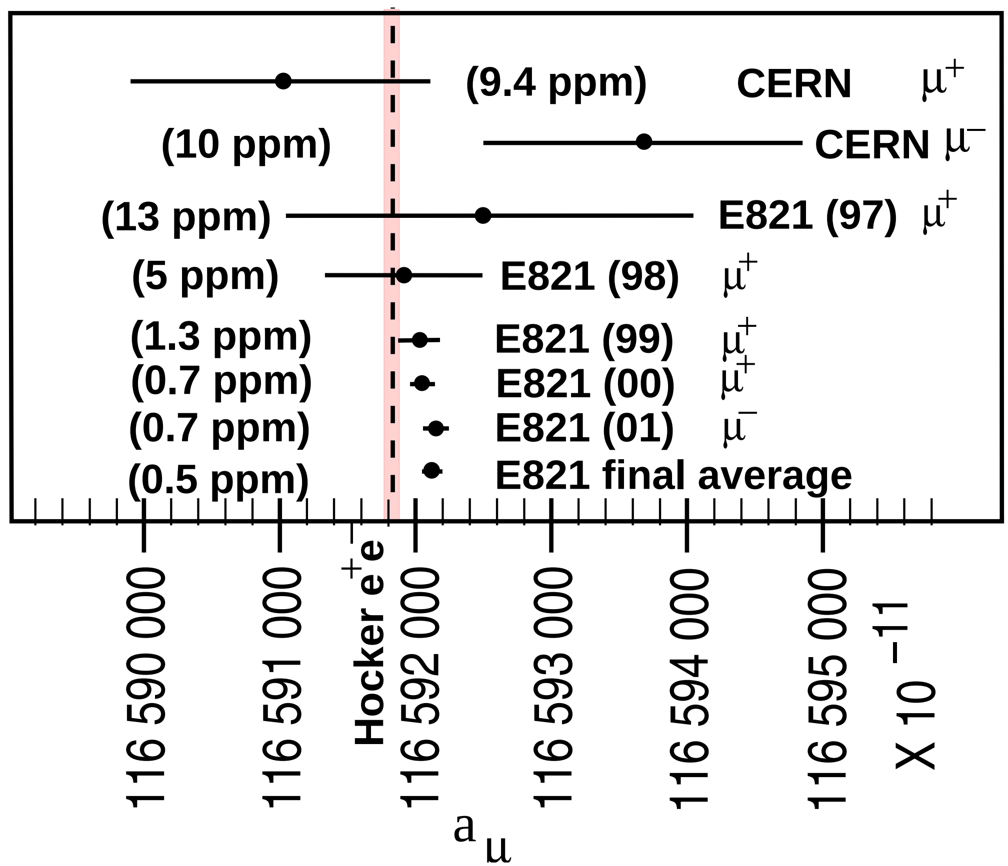
$$a_\mu^{\rm Had\ \ vac} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \underbrace{K(s)}_{\text{fnc of } m_\mu^2/s} \frac{\sigma(e^+e^-\to \text{had})}{\sigma(e^+e^-\to \mu^+\mu^-)}$$

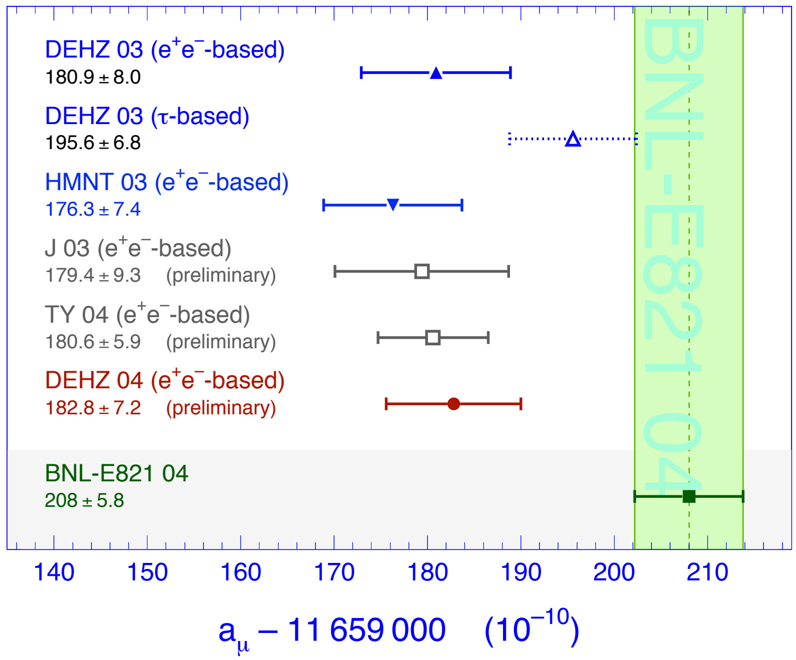


  - $a_\mu^{\rm Had\ \ vac}$: discrepancy between e^+e^- (new KLOE, SND) and τ decay (isospin violation?)
  - $a_\mu^{\rm Had\ \ l.l.}$ sign now settled down. Small but non-negligible

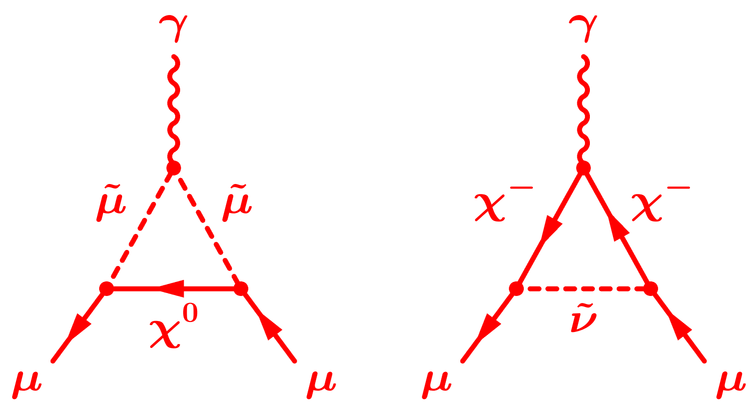
- $a_\mu^{\rm exp} = 1165920.80(63) \times 10^{-9}$ (dominated by BNL 821)







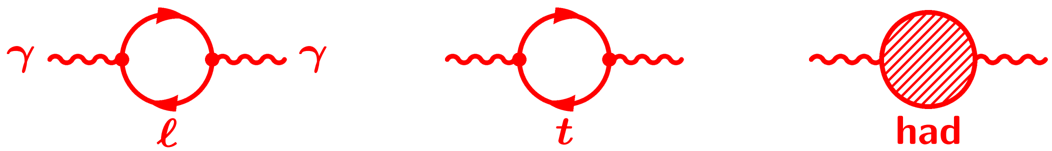
- e^+e^- **data:** $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 2.5(9) \times 10^{-9}$ **(2.7σ)**
- τ **decay data: no discrepancy (0.7σ)**
- **Supersymmetry: central value (e^+e^-) for** $m_{\rm SUSY} \sim 72\sqrt{\tan\beta}$ **GeV**



Running of α

- **Largest theory uncertainty in $M_Z - \sin^2\theta_W$ (cf. $a_\mu^{\rm had}$)**

$$\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha}$$



$$\begin{aligned}
\Delta\alpha &= \Delta\alpha_\ell + \Delta\alpha_t + \Delta\alpha^{(5)}_{\rm had} \\
&\sim 0.03149769 - 0.000070(5) + \Delta\alpha^{(5)}_{\rm had}
\end{aligned}$$

$$\alpha^{-1} \sim 137.036$$

- **Calculation of $\Delta\alpha^{(5)}_{\rm had}$**
  - **Data driven: $R_{\rm had}$ up to $\sim$ 40 GeV; PQCD above**
  - **Theory driven: PQCD + NPQCD (OPE, sum rules) above $\sim$ 2 GeV $\longrightarrow$ smaller uncertainties**
  - **e^+e^- data vs τ decays**
  - **Table for α_s= 0.120**
  - **Correlation with $g_\mu - 2$**
- $\alpha(M_Z)^{-1} = 128.954 \pm 0.031$

Reference	Result	Comment
Martin & Zeppenfeld [21]	0.02744 ± 0.00036	PQCD for $\sqrt{s} > 3$ GeV
Eidelman & Jegerlehner [22]	0.02803 ± 0.00065	PQCD for $\sqrt{s} > 40$ GeV
Geshkenbein & Morgunov [13]	0.02780 ± 0.00006	$\mathcal{O}(\alpha_s)$ resonance model
Burkhardt & Pietrzyk [24]	0.0280 ± 0.0007	PQCD for $\sqrt{s} > 40$ GeV
Swartz [25]	0.02754 ± 0.00046	use of fitting function
Alemany, Davier, Höcker [26]	0.02816 ± 0.00062	includes τ decay data
Krasnikov & Rodenberg [27]	0.02737 ± 0.00039	PQCD for $\sqrt{s} > 2.3$ GeV
Davier & Höcker [28]	0.02784 ± 0.00022	PQCD for $\sqrt{s} > 1.8$ GeV
Kühn & Steinhauser [29]	0.02778 ± 0.00016	complete $\mathcal{O}(\alpha_s^2)$
Erler [16]	0.02779 ± 0.00020	converted from $\overline{\rm MS}$ scheme
Davier & Höcker [30]	0.02770 ± 0.00015	use of QCD sum rules
Groote *et al.* [31]	0.02787 ± 0.00032	use of QCD sum rules
Martin, Outhwaite, Ryskin [32]	0.02741 ± 0.00019	includes new BES data
Burkhardt & Pietrzyk [33]	0.02763 ± 0.00036	PQCD for $\sqrt{s} > 12$ GeV
de Troconiz & Yndurain [34]	0.02754 ± 0.00010	PQCD for $s > 2$ GeV^2
Jegerlehner [35]	0.02765 ± 0.00013	converted from MOM scheme
Hagiwara *et al.* [36]	0.02757 ± 0.00023	PQCD for $\sqrt{s} > 11.09$ GeV
Burkhardt & Pietrzyk [37]	0.02760 ± 0.00035	includes KLOE data

- **Measurements: DORIS, PEP, PETRA, TRISTAN, LEP II**