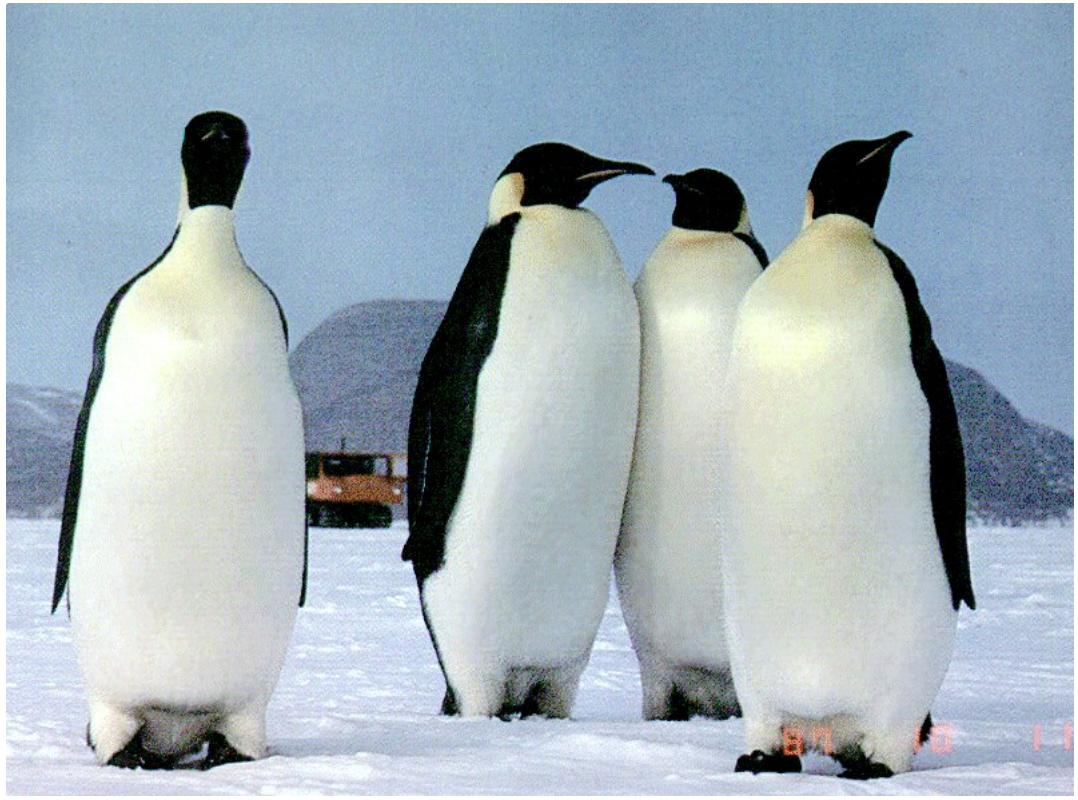
Tests of the Electroweak Theory

- History/introduction
- Weak charged current
- QED
- Weak neutral current
- Precision tests
- Rare processes
- CP violation and B decays
- Neutrino mass



References (mainly CP, mixing)

- **E.D. Commins and P.H. Bucksbaum,** *Weak Interactions of Leptons and Quarks*, **(Cambridge Univ. Press, Cambridge, 1983)**
- **P. Renton,** *Electroweak Interactions*, **(Cambridge Univ. Press, Cambridge, 1990)**
- **S. Eidelman** *et al.* **[Particle Data Group], Phys. Lett. B 592, 1 (2004)** **(CKM, CP, and $B^0 - \bar{B}^0$ reviews)**
- **J. Charles** *et al.* **[CKMfitter Group], Eur. Phys. J. C 41, 1 (2005); CKMFITTER page,** `http://ckmfitter.in2p3.fr/`
- **J. L. Rosner, hep-ph/0410281**
- **A. Ali and D. London, Eur. Phys. J. C 9, 687 (1999)**

The Cabibbo-Kobayashi-Maskawa (CKM) matrix

- **Charge-raising current**

$$J_W^{\mu\dagger} = (\bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_\tau)\gamma^\mu(1-\gamma^5)\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t})\gamma^\mu(1-\gamma^5)V\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V = A_L^{u\dagger} A_L^d$$

- **Mismatch between weak and quark masses, and between $A_L^{u,d}$, which relate weak and mass eigenstates**

- **$V =$ unitary $\rightarrow$ 3 angles and 1 phase after removing unobservable overall phases of fields**

$$VV^\dagger = V^\dagger V = I$$

- **Magnitudes of elements (PDG)**

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$|V_{ij}| = \begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix}$$

  - $|V_{ud}|$ from superallowed β decay; $|V_{us}|$ from $K_{\ell 3}$
  - $|V_{cd,cs}|$ from c decay and from c production (dimuons) in νN
  - $|V_{cb,ub}|$ from $b \to c$ and $b \to u$ decay rates (theory uncertainties)
  - $|V_{td,ts,tb}|$ from $B^0 - \bar{B}^0$ mixing, $t \to b$, $b \to s\gamma$, unitarity

- Various exact parametrizations consistent with unitarity

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} \equiv \cos\theta_{ij}, \;\; s_{ij} \equiv \sin\theta_{ij}$

- $s_{12} \sim \sin\theta_C \sim 0.23$, $s_{13} \ll s_{23} \ll s_{12}$; $\delta = CP$ phase

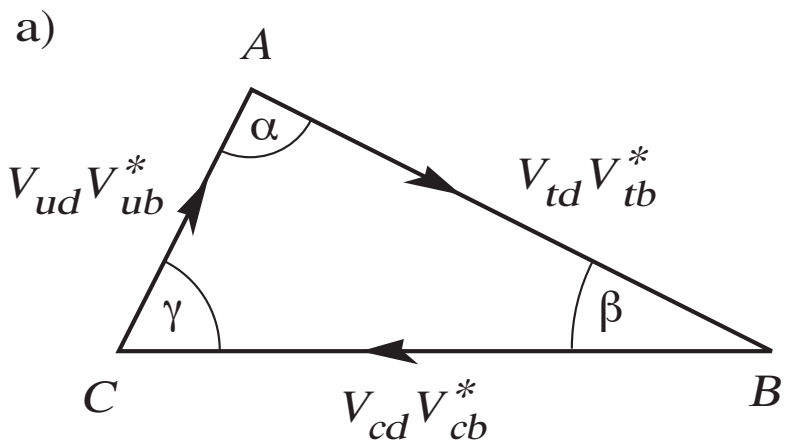
- Want to test unitarity $VV^\dagger = V^\dagger V = I$. Apparent violation would signal new physics.

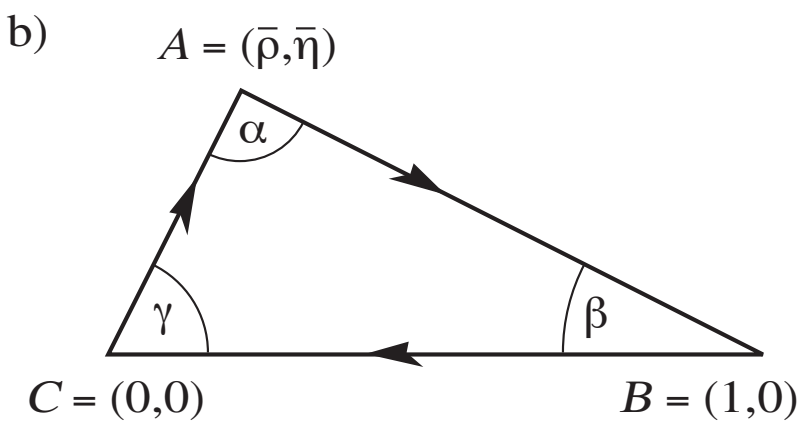
$$\left(VV^\dagger\right)_{11} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1? \text{ (Universality)}$$

$$\left(V^\dagger V\right)_{31} = V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0?$$

- Unitarity triangle: each complex number in $\left(V^\dagger V\right)_{31}$ is a vector in complex plane. Should sum to zero, i.e., form triangle.

The unitarity triangle





- Divide each side by (well-measured) $|V^*_{cb}V_{cd}|$
- Overconstrain by redundant tests
- Sides by (CP-conserving) rates, e.g., $b \to u$ decay rate, $B^0 - \bar{B}^0$ mixing
- Angles by CP violating effects
- Deviation: e.g., extra family, fermion mixing, new sources of CP-violation (e.g., SUSY loops), new FCNC interactions (extended Higgs, Z')

- The Wolfenstein parametrization. Empirical expansion in $\lambda \sim \sin\theta_C \sim 0.225$

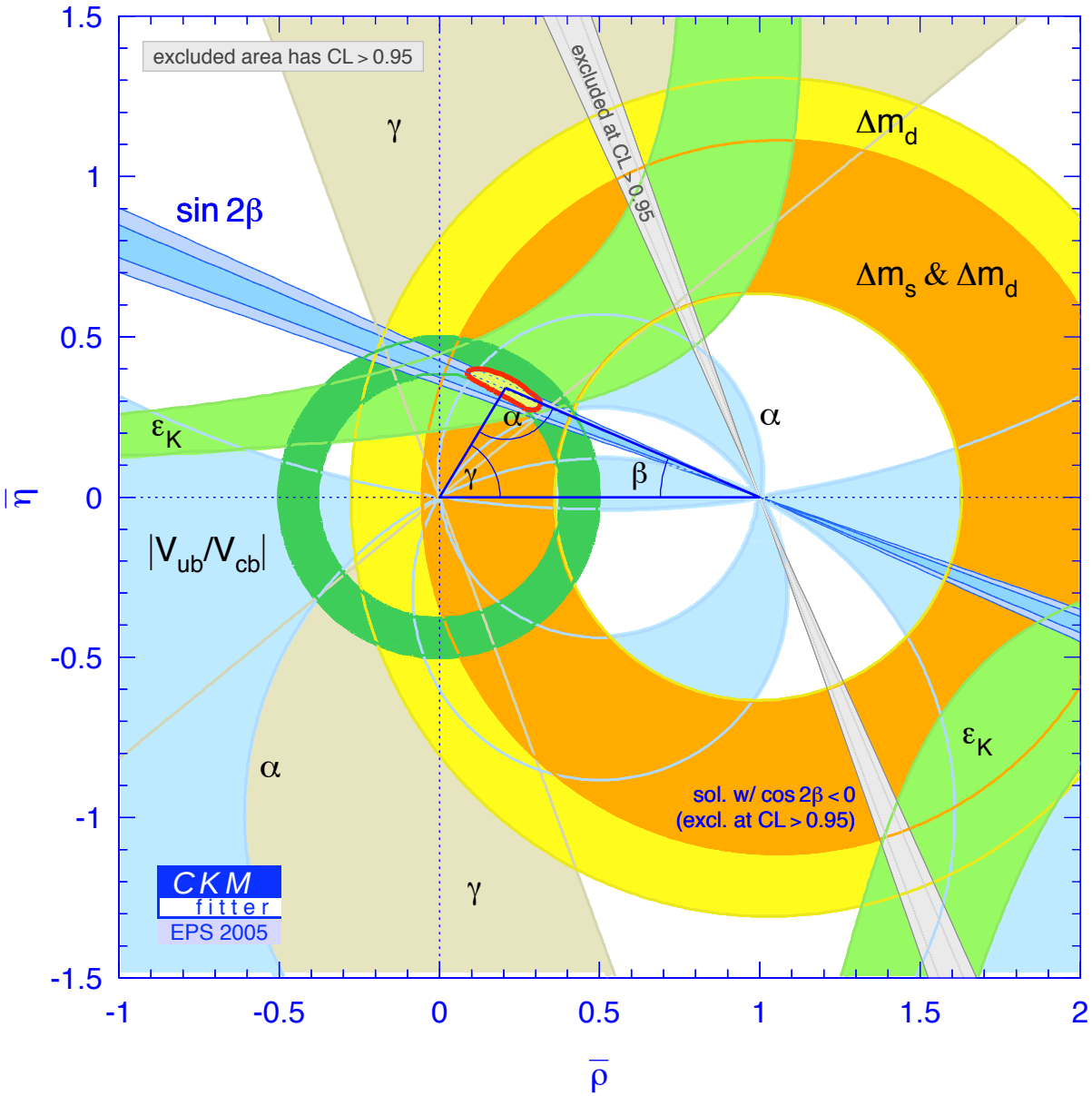
- Simple to remember and work with

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- $A \sim 0.8$ from $b \to c$ decay rate; (ρ, η) are real, expected to be $O(1)$

- Vertices of triangle at $(0,0)$, $(1,0)$, and $(\bar\rho, \bar\eta)$, where $\bar\rho = \rho(1-\lambda^2/2)$, $\bar\eta = \eta(1-\lambda^2/2)$

- Each constraint determines allowed region in $\bar\rho, \bar\eta$ plane



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- $|V_{ub}|$ determined by rate for exclusive or inclusive $b \to u l \nu$ (theory uncertainties)
- $(|V_{ub}^* V_{ud}|)^2/(|V_{cb}^* V_{cd}|)^2 = \bar{\rho}^2 + \bar{\eta}^2 \to$ radius^2 of circle centered at $(0,0)$

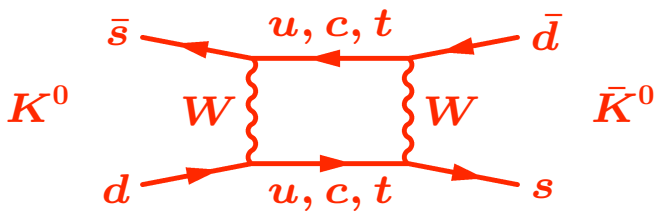
$K^0 - \bar{K}^0$ mixing

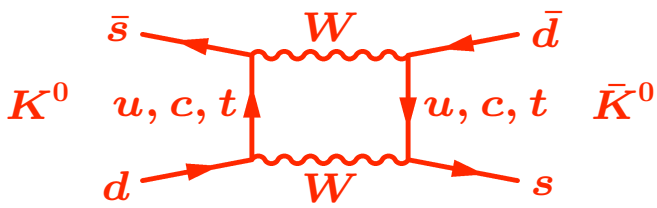
- **Strong interactions conserve strangeness**

$$|K^0\rangle = |d\bar{s}\rangle \qquad\qquad |\bar{K}^0\rangle = |\bar{d}s\rangle$$

$$CP\,|K^0\rangle = |\bar{K}^0\rangle \qquad\qquad CP\,|\bar{K}^0\rangle = |K^0\rangle$$

- **K^0 and $\bar{K}^0$ are states relevant to strong interaction processes, e.g., $\pi^- p \to K^0 \Lambda$** $\quad (|\Lambda\rangle = |sdu\rangle)$





- Need GIM
- $m_c \sim 1.5$ GeV predicted
- Severe constraint on tree-level Z', FCNC Higgs

- However, WCC can violate strangeness and lead to $K^0 - \bar{K}^0$ mixing at second order

$$M \sim \begin{pmatrix} M_{K^0} & M_{12} \\ M_{12}^* & M_{\bar{K}^0} \end{pmatrix}$$

- $M_{K^0} = M_{\bar{K}^0}$ by $CPT \rightarrow$ tiny M_{12} important
- 2 families $\rightarrow$ real $M_{12} \rightarrow$ eigenstates $K^0_{1,2} = \frac{K^0 \pm \bar{K}^0}{\sqrt{2}} \simeq K_{S,L}$

$$\Delta m = m_{K_2} - m_{K_1} = -2M_{12}$$
$$\sim \frac{G_F^2}{4\pi^2} m_c^2 |V_{cd}|^2 |V_{cs}|^2 f_K^2 m_K$$
$$\sim 3.5 \times 10^{-6} \text{ eV}$$

CP violation

- Assume CP is conserved

$$CP\ |K^0\rangle = |\bar{K}^0\rangle \qquad CP\ |\bar{K}^0\rangle = |K^0\rangle$$

$$CP\ |K^0_{1,2}\rangle = CP\ \frac{|K^0 \pm \bar{K}^0\rangle}{\sqrt{2}} = \pm K^0_{1,2}$$

$$CP|\pi^+\pi^-\rangle = +|\pi^+\pi^-\rangle \qquad CP|\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle \text{ for } J = 0$$

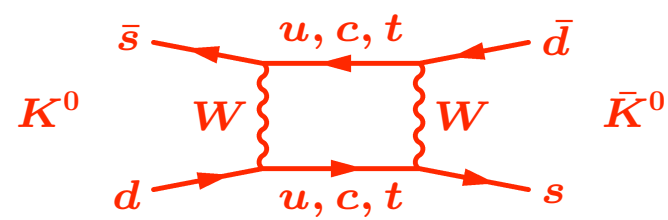
$$CP|3\pi\rangle = -|3\pi\rangle \text{ for } J = Q = 0$$

- Thus, $K_S \equiv K^0_1 \to 2\pi$ ($\tau_S = 8.9\times10^{-11}$ s); $K_L \equiv K^0_2 \to 3\pi$ ($\tau_S = 5.2\times10^{-8}$ s)

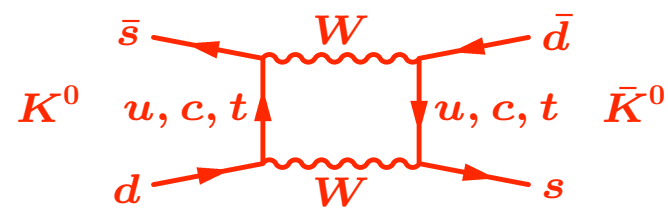
- Fitch-Cronin (1964): observed $K_L \to 2\pi$ with $B(K_L \to 2\pi) \sim 2\times10^{-3} \to \not{C}P$

CP violation and phases

- CP violation requires complex observable phases



- Some phases unobservable: only overall phase of amplitude; can absorb in phase of state



- Need interference between amplitudes, e.g., diagrams involving all three families

- Phase in CKM can generate Im $M_{12} \neq 0$. Effect small due to small *mixings* even if CKM phase large

- **Toy example**

$$H_{eff} = \underbrace{M_{K^0}\bar{K}^0 K^0}_{K^0,\bar{K}^0 \text{ mass}} + \underbrace{|M_{12}|\left(e^{2i\phi}K^0\ K^0 + e^{-2i\phi}\bar{K}^0\ \bar{K}^0\right)}_{\text{complex mixing mass (indirect)}}$$

$$+ \underbrace{|a|\left(e^{i\beta}K^0 + e^{-i\beta}\bar{K}^0\right)\pi\pi}_{\text{complex decay amplitude (direct)}}$$

- **Absorb $e^{2i\phi}$ by $K^{0\prime} \equiv e^{i\phi}K^0$ and $\bar{K}^{0\prime} \equiv e^{-i\phi}\bar{K}^0 \rightarrow$**

$$H_{eff}^{decay} = |a|\left(e^{i(\beta-\phi)}K^{0\prime} + e^{-i(\beta-\phi)}\bar{K}^{0\prime}\right)\pi\pi$$

$$= \sqrt{2}|a|\left[\cos(\beta-\phi)\underbrace{\frac{K^{0\prime}+\bar{K}^{0\prime}}{\sqrt{2}}}_{K_S} + i\sin(\beta-\phi)\underbrace{\frac{K^{0\prime}-\bar{K}^{0\prime}}{\sqrt{2}}}_{K_L}\right]\pi\pi$$

- **Observable $\not{C}P$ by phase *difference* $\beta - \phi$**

- **Actual $K_{L,S}$ system; amplitude ratios**

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \epsilon - 2\epsilon'$$

  - Difference because two isospin amplitudes for 2π, $I = 0$ (large) and $I = 2$ (small)
  - ϵ from mixing (indirect) and $\not{C}P$ in $I = 0$ amplitude (small)
  - ϵ' from difference in $I = 0$ and $I = 2$ phases (direct–suppressed)

- **Experiment:** $|\epsilon| = 2.284(14) \times 10^{-3}$

- $\epsilon'/\epsilon = 1.67(26) \times 10^{-3}$ (only recently measured, CERN and FNAL)

- **Physical states**

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle = \frac{\tilde{\epsilon}|K_1^0\rangle + |K_2^0\rangle}{\sqrt{1+|\tilde{\epsilon}|^2}}$$

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle = \frac{|K_1^0\rangle + \tilde{\epsilon}|K_2^0\rangle}{\sqrt{1+|\tilde{\epsilon}|^2}}$$

$$\frac{p}{q} \equiv \frac{1+\tilde{\epsilon}}{1-\tilde{\epsilon}}$$

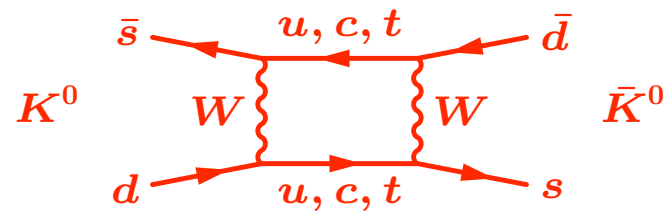
- **CP parameters**

$$\epsilon = \underbrace{\tilde{\epsilon}}_{\text{mixing}} + \underbrace{i\frac{\text{Im } A_0}{\text{Re } A_0}}_{I=0 \text{ decay}}$$

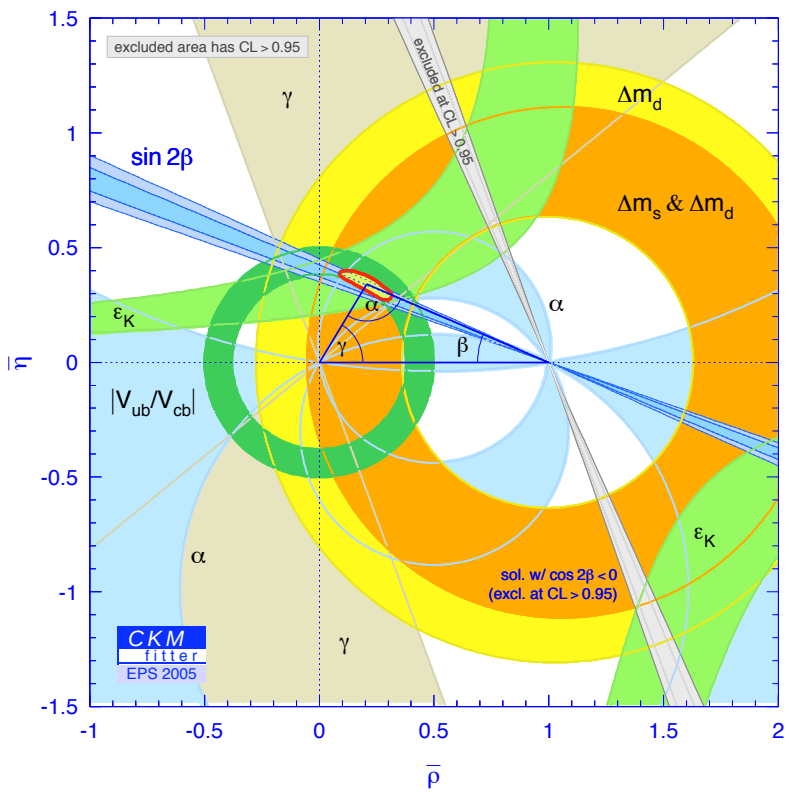
$$\sqrt{2}\epsilon' = i\,\underbrace{e^{i(\delta_2-\delta_0)}}_{\text{strong}}\,\underbrace{\frac{\text{Re } A_2}{\text{Re } A_0}}_{\sim 0.045}\,\underbrace{\left[\frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0}\right]}_{I=0,2 \text{ mismatch}}$$

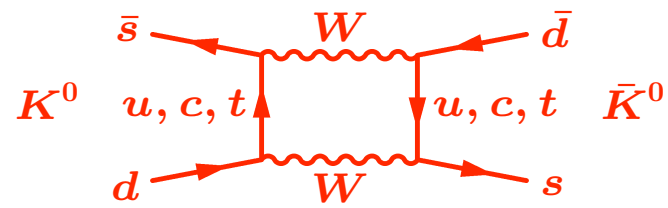
- **Box diagrams**

$$\tilde{\epsilon}_{box} \sim \text{Im}\ (V_{ts}^* V_{td}) F(m_t, m_c, \text{angles, QCD})$$
$$\sim \bar{\eta}\,[(1-\bar{\rho})F_1 + F_2]$$

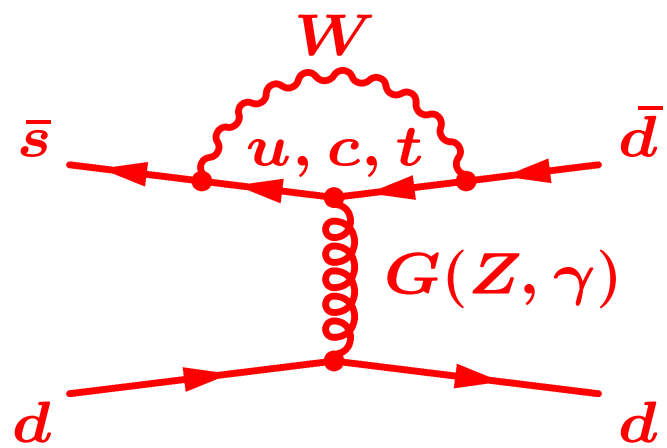


- **Hyperbola in $(\bar{\rho}, \bar{\eta})$ plane**





- ϵ' from direct CP violation (i.e., in decay amplitudes)
- Believed dominated by penguin diagrams (A_0 enhanced)
- $\epsilon'/\epsilon = 1.67(26)\times 10^{-3} \propto \bar{\eta}$
- Roughly consistent with SM, but large theory uncertainty



$B^0 - \bar{B}^0$ mixing

CLEO, LEP, Tevatron, BaBar (SLAC), BELLE (KEK)

- Strong interaction states

$$|B_d^0\rangle = |d\bar{b}\rangle \qquad |\bar{B}_d^0\rangle = |\bar{d}b\rangle = CP|B_d^0\rangle$$
$$|B_s^0\rangle = |s\bar{b}\rangle \qquad |\bar{B}_s^0\rangle = |\bar{s}b\rangle = CP|B_s^0\rangle$$

- Mixed by second order weak (box diagrams)

$$|B_{H_i}\rangle = p_i|B_i^0\rangle - q_i|\bar{B}_i^0\rangle, \qquad i = d, s$$
$$|B_{L_i}\rangle = p_i|B_i^0\rangle + q_i|\bar{B}_i^0\rangle$$

where $\frac{p_i}{q_i} = \frac{1-\tilde{\epsilon}_i}{1+\tilde{\epsilon}_i}$ and $\tilde{\epsilon}_i \sim$ Im $M_{12}^i \ll 1$

- Observe $\Delta m_i \equiv m_{H_i} - m_{L_i}$ by $B^0 - \bar{B}^0$ oscillations (cf ν oscillations). eg., suppose initial state tagged as B_i^0

$$|\psi(0)\rangle = |B_i^0\rangle = \frac{1}{2p_i}\left(|B_{L_i}\rangle + |B_{H_i}\rangle\right)$$

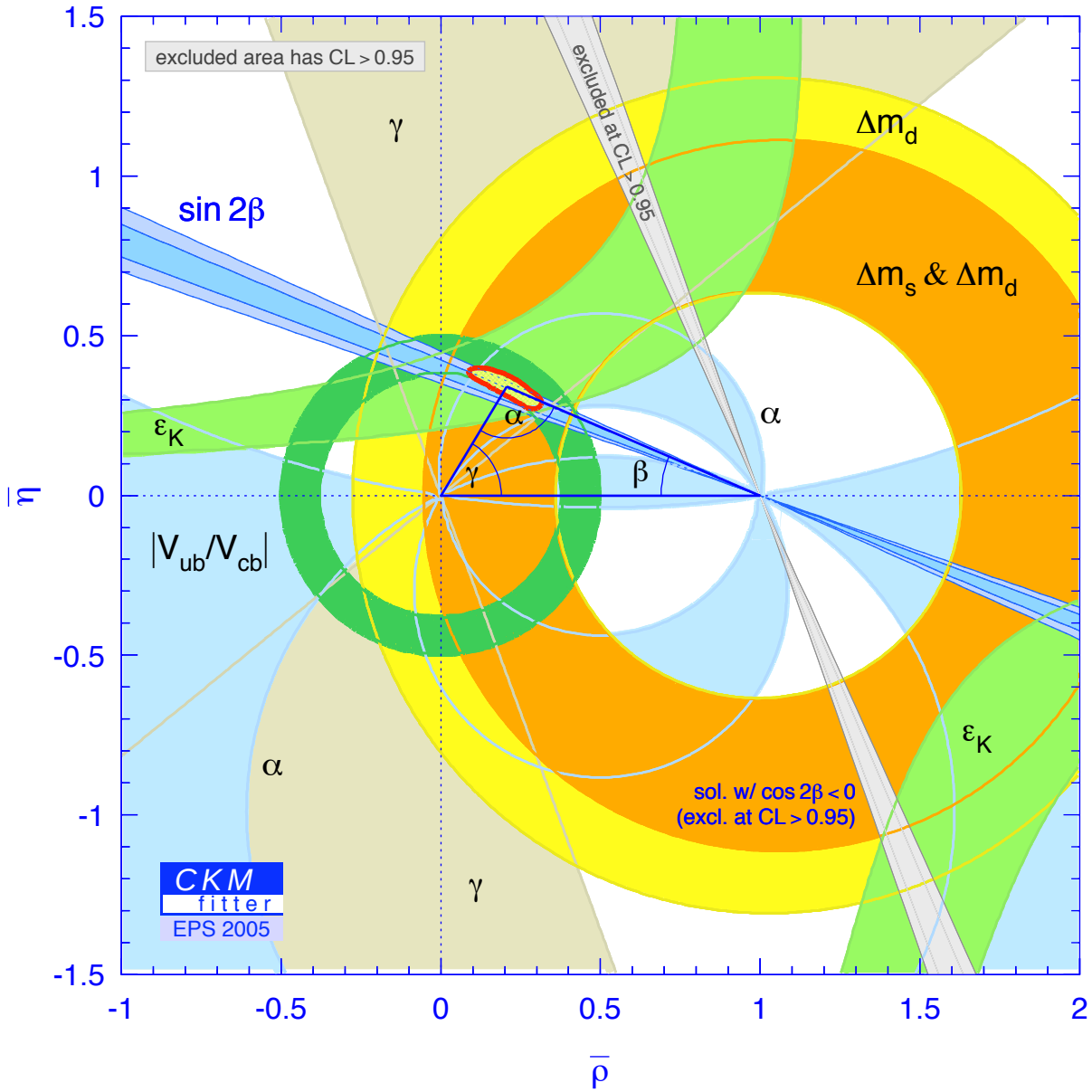
$$|\psi(t)\rangle \sim \frac{1}{2p_i}\left(|B_{L_i}\rangle e^{-im_{L_i}t} + |B_{H_i}\rangle e^{-im_{H_i}t}\right)$$

$$\sim \cos\left(\frac{\Delta m_i t}{2}\right)|B_i^0\rangle + i\sin\left(\frac{\Delta m_i t}{2}\right)\frac{q_i}{p_i}|\bar{B}_i^0\rangle$$

- Measure Δm_i by time dependence (or integral) of decays, using $b \to c\ell^-\nu_\ell$ (from $\bar{B}^0$); $\bar{b} \to \bar{c}\ell^+\bar{\nu}_\ell$ (from B^0)

- $\Delta m_d = 0.502(6)$ ps$^{-1} \sim 3.3 \times 10^{-4}$ eV $\propto (1-\bar{\rho})^2 + \bar{\eta}^2$

- $\Delta m_s > 14.4$ ps$^{-1} \propto 1$ (QCD correlated $\longrightarrow$ Δm_d theory error reduced)
  - Measurement of Δm_s is major goal of Run II



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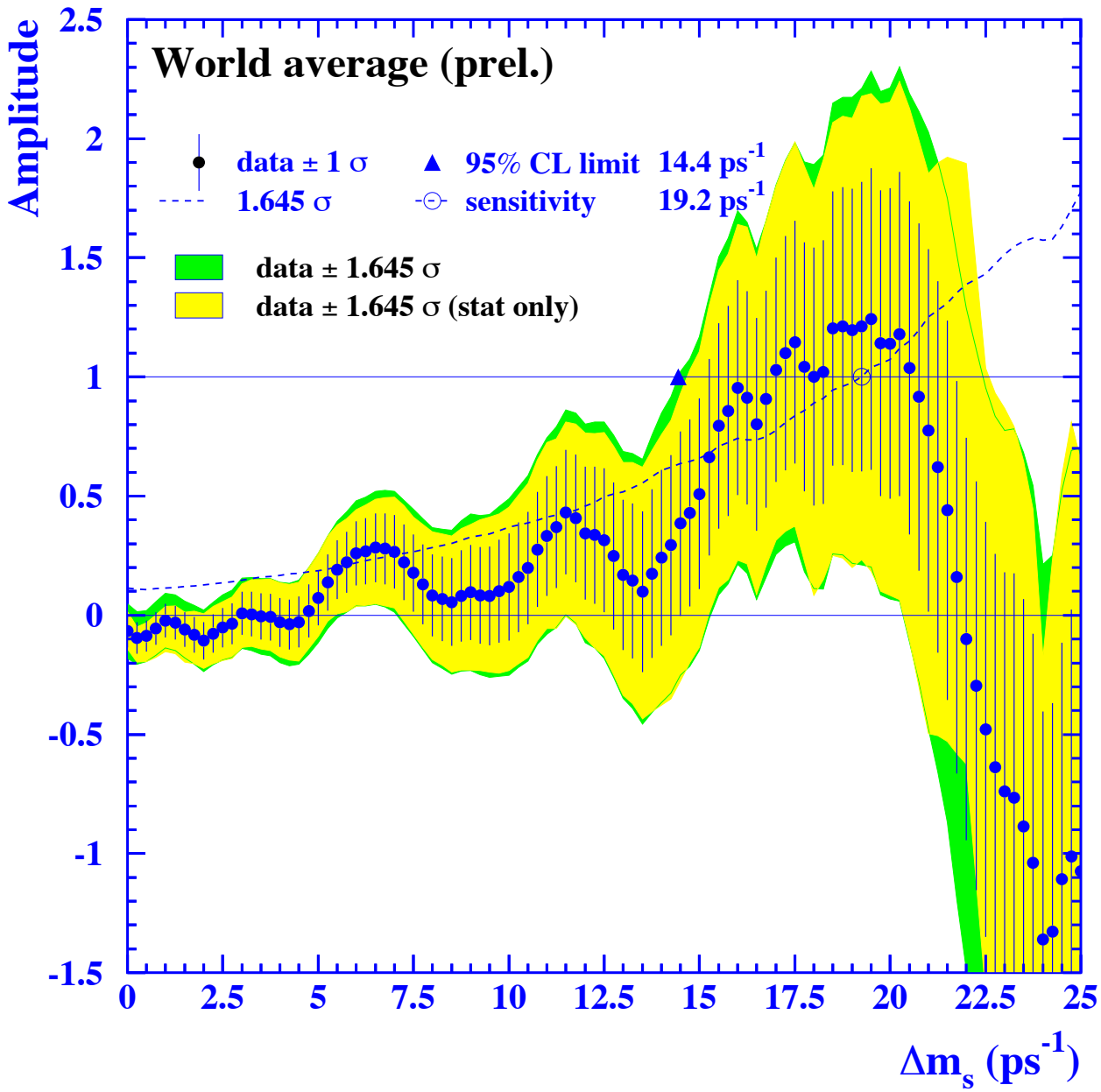

World average (prel.)
data ± 1 σ
1.645 σ
95% CL limit 14.4 ps-1
sensitivity 19.2 ps-1
data ± 1.645 σ
data ± 1.645 σ (stat only)
Amplitude
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
2.5
5
7.5
10
12.5
15
17.5
20
22.5
25
Δms (ps-1)

The CP asymmetries in B decays

- Many CP asymmetries to determine angles of unitarity triangle, but QCD uncertainties

- Cleanest: $B^0(\bar{B}^0) \to f$, where $f \sim CP$ eigenstate (up to CP breaking)

$$CP|f\rangle = \pm|f\rangle$$

  – e.g., $f = J/\psi K_S,\ \pi^+\pi^-,\ \rho K_S$

- Let $|B^0(t)\rangle$ be time-evolution of state tagged as $|B^0\rangle$ at $t = 0$, and similarly for $|\bar{B}^0(t)\rangle$

- CP-odd asymmetry

$$a_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$

$$= \frac{(1 - |\lambda|^2)\cos(\Delta m\, t) - 2\ \mathrm{Im}\lambda\ \sin(\Delta m\, t)}{(1 + |\lambda|^2)}$$

where

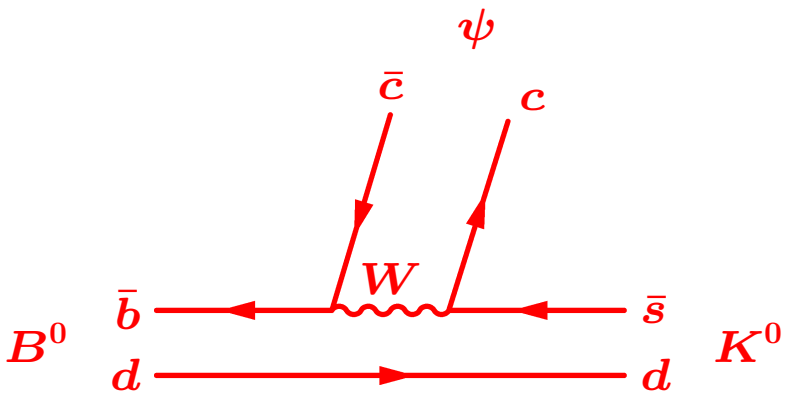
$$\lambda = \underbrace{\frac{p}{q}}_{\text{mixing}} \underbrace{\frac{\bar{A}}{A}}_{\text{decay}}$$

- Usually confused by sum over diagrams, with (CP-conserving) strong phases (final state interactions) and (CP-violating) CKM phases

$$A = \sum_\alpha |A_\alpha| e^{i\delta_\alpha} e^{i\phi_\alpha}$$

$$\bar{A} = \sum_\alpha |A_\alpha| \underbrace{e^{i\delta_\alpha}}_{\text{strong}} \underbrace{e^{-i\phi_\alpha}}_{\text{CKM}}$$

- **Golden mode, $B \to J/\psi K_S$** (only one significant diagram $\to$ no strong interaction uncertainty)



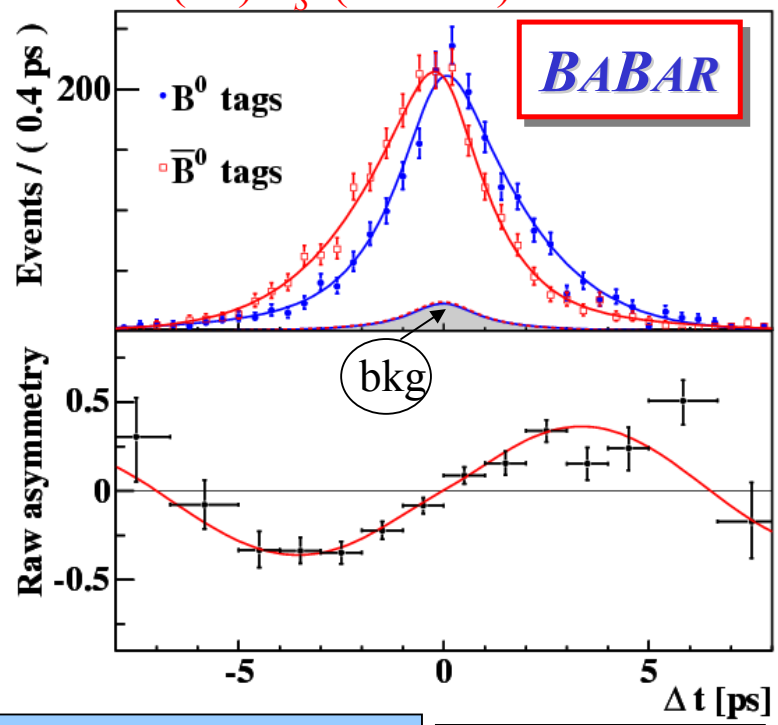
$$\lambda = \underbrace{\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)}_{B-\bar{B}\ \text{mixing}} \underbrace{\left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right)}_{\text{decay}} \underbrace{\left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right)}_{K-\bar{K}\ \text{mixing}}$$

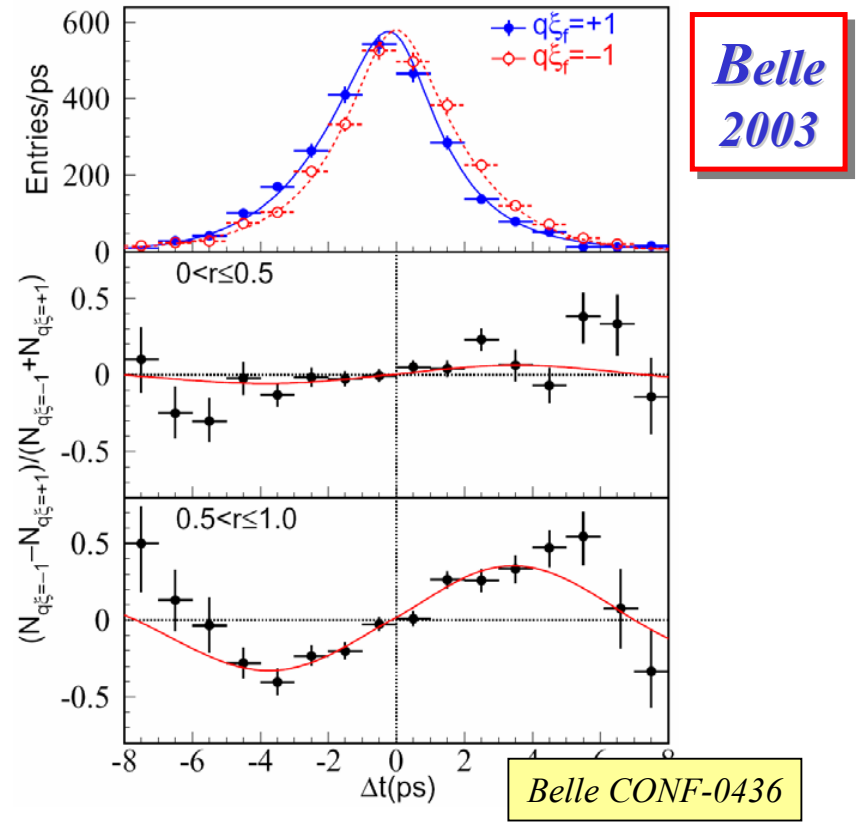
$$\text{Im}\ \lambda = -\sin 2\beta$$

- **BaBar, Belle, $\cdots$: $\sin 2\beta = 0.739(48)$**

$\sin 2\beta$ results from charmonium modes

$(c\bar{c})K_S^0$ (CP odd) modes





Update for ICHEP04

BABAR PUB-04/038

Belle CONF-0436

$\sin 2\beta = +0.722 \pm 0.040 \pm 0.023$

$|\lambda| = |\bar{A}/A| = 0.950 \pm 0.031 \pm 0.013$

$(c\bar{c})K_S^0 + (c\bar{c})K_L^0$

$\sin 2\beta = +0.728 \pm 0.056 \pm 0.023$

$|\lambda| = |\bar{A}/A| = 1.007 \pm 0.041 \pm 0.033$

Limit on direct CPV

$205\,fb^{-1}$ on peak or $227M$ $B\bar{B}$ pairs
7730 CP events (tagged signal)

$140\,fb^{-1}$ on peak or $152M$ $B\bar{B}$ pairs
4347 CP events (tagged signal)

ICHEP04-北京
August 20, 2004

Marcello A. Giorgi

M.Bruinsma, T. Higuchi, CP-3

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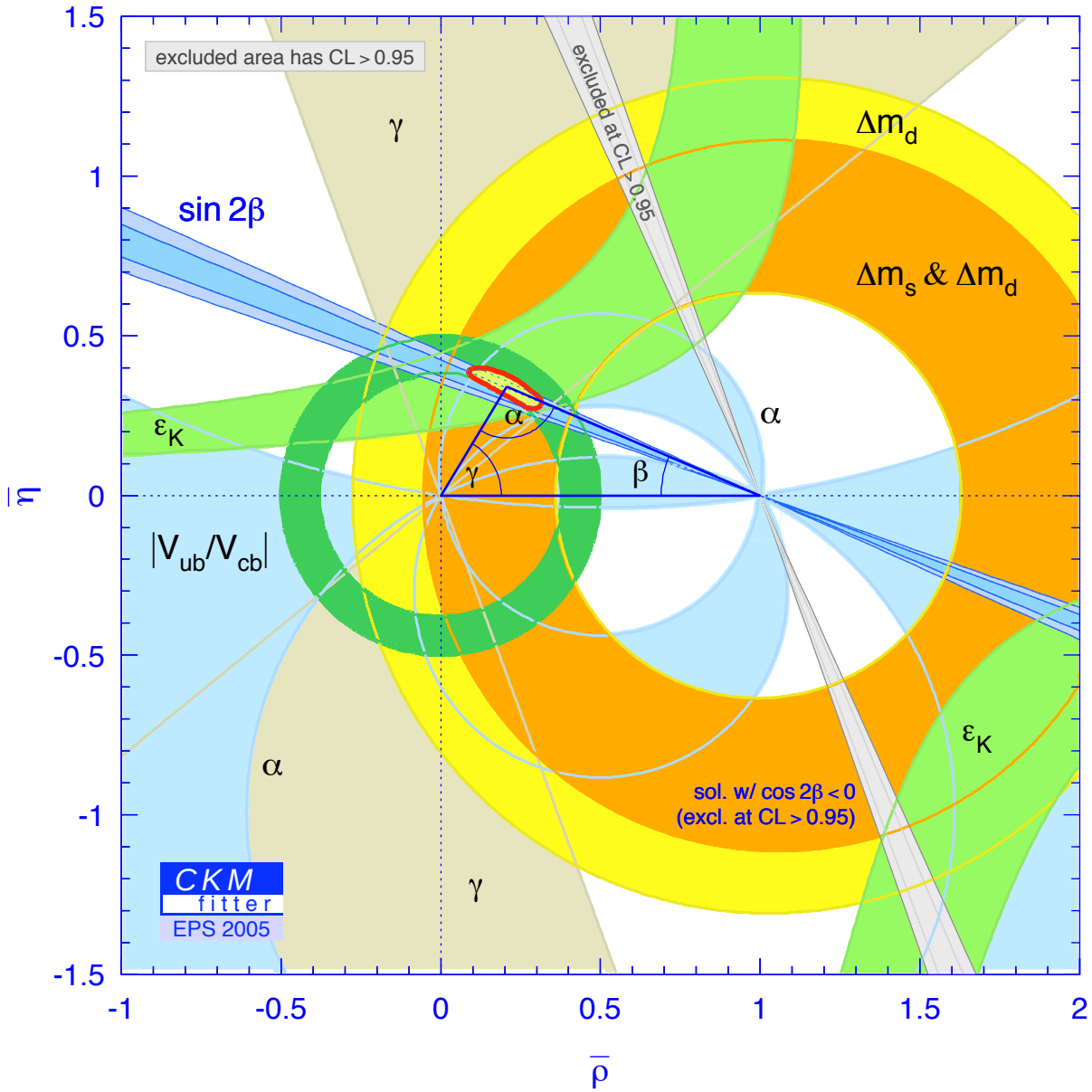

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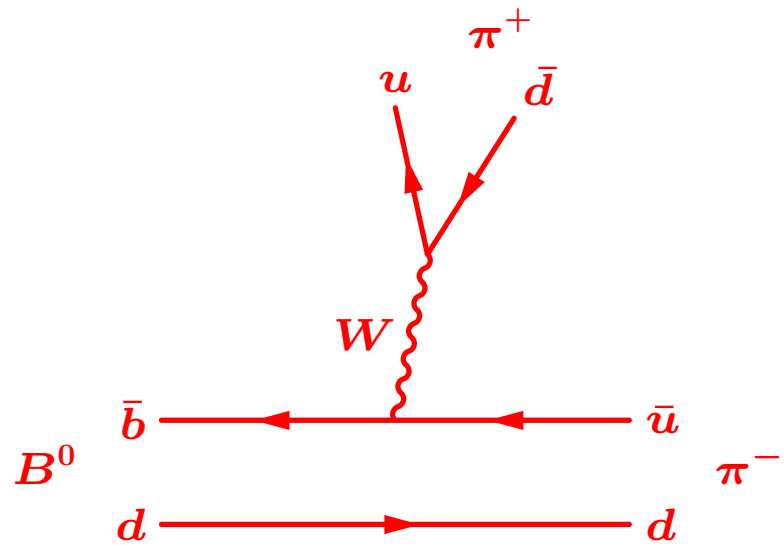
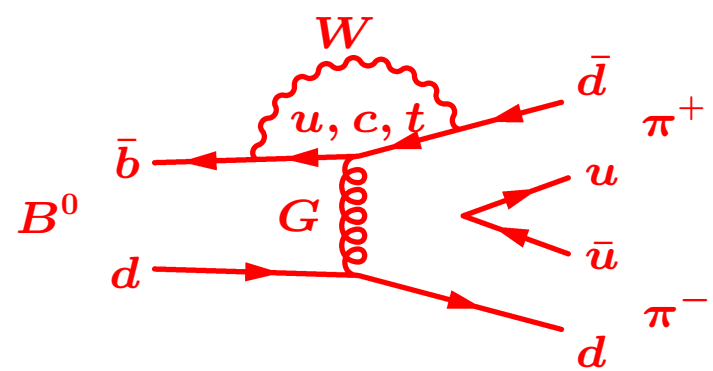
- Many other modes studied

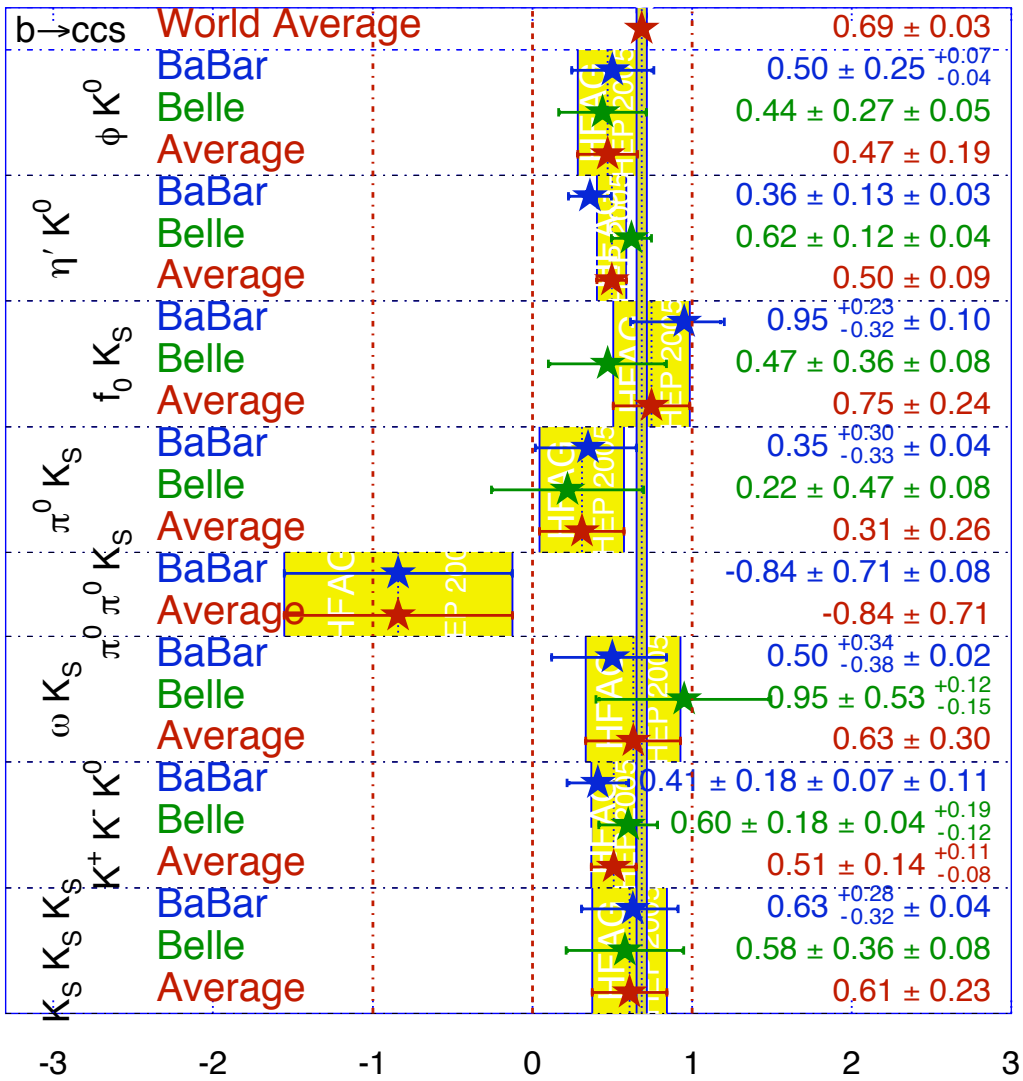
- $B \to \pi^+\pi^-$
  - Would yield $\lambda = \sin 2\alpha$ if tree-level only
  - However, competition from significant penguin $\to$ large uncertainty



- Charmless decays (penguin-dominated), $b \to s\bar{s}s$, e.g., $B \to \phi K_S$ (hint of discrepancy?)
  - SM loops; may be significant new physics from tree (FCNC Z', extended Higgs) or enhanced supersymmetric loops (large $\tan\beta$)



$\sin(2\beta^{eff})/\sin(2\phi_1^{eff})$



(**HFAG:** `http://www.slac.stanford.edu/xorg/hfag/`)

Conclusions

- Standard model, supplemented with neutrino mass (Dirac or Majorana):

$$SU(3)\times SU(2)\times U(1)\times \text{classical relativity}$$

- Mathematically consistent field theory of strong, weak, electromagnetic interactions
- Gauge interactions correct to first approximation to 10^{-16} cm
- Complicated, free parameters, fine tunings $\Rightarrow$ must be new physics
- Promising theoretical ideas (e.g., string theory)
- Exciting times ahead