# Introduction to Flavor Physics in and beyond the Standard Model

*Lecture 1/4*

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References:

The BaBar physics book, http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-r-504-TOC-pdf.pdf

B physics at the Tevatron: Run II and beyond, hep-ph/0201071

The Discovery potential of a Super B Factory, hep-ph/0503261

# Outline

- The Standard Model
- The Cabibbo-Kobayashi-Maskawa (CKM) matrix

# *The Standard Model*

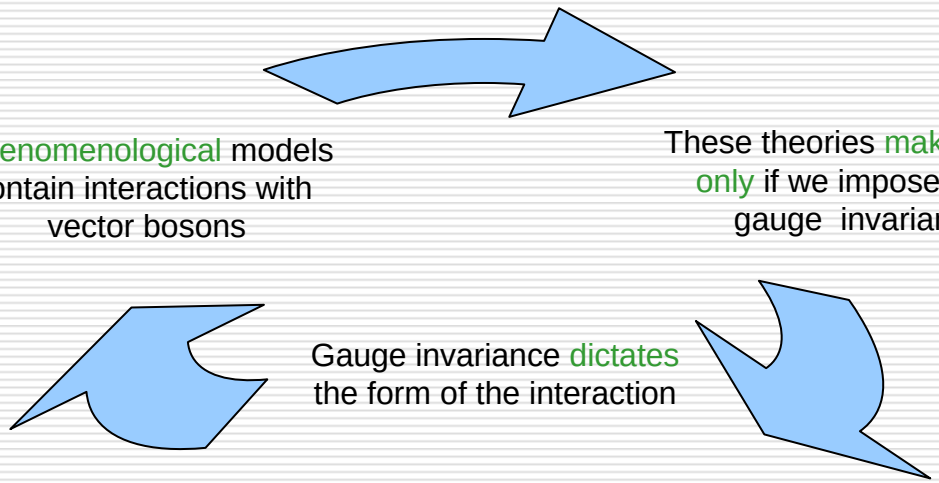
# The ingredients

- Quantum Field Theory *(special relativity + quantum mechanics)*

- Particle content: down-type quarks ($d,s,b$)
  up-type quarks ($u,c,t$)
  charged leptons ($e,\mu,\tau$)
  } left and right-handed fermions
  neutrinos ($\nu_e,\nu_\mu,\nu_\tau$) → left-handed fermions
  gauge bosons ($\gamma,W^\pm,Z,g$) → vectors
  Higgs boson ($h$) → scalar

- Local gauge invariance: $SU(3)_c \times SU(2)_I \times U(1)_Y$

$g_a$ $W_i$ $B^0$ ( $W_{1,2} \to W^\pm$, $[W_3,B^0]\to[\gamma, Z]$ )



# Example of gauge transformations

- The QED lagrangian is:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\gamma^\mu(i\partial_\mu - eQ_\psi A_\mu)\psi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ &\equiv \bar{\psi}(i\not{\partial} - eQ_\psi \not{A})\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

- This lagrangian has a U(1) local gauge symmetry:

$$\psi \to e^{i\,e\,Q_\psi\,\omega(x)}\,\psi \qquad A_\mu \to A_\mu - \partial_\mu\omega(x)$$

- Let's try to write a similar transformation involving two fermions:

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \to U\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = e^{i\,g\,\sigma^a\,\omega_a(x)}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \sigma^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$ Pauli matrices

- We need three gauge bosons:

$$\sigma^a W^a_\mu \to U(\sigma^a W^a_\mu)U^\dagger + \sigma^a\,\partial_\mu\omega_a(x)$$

# Gauge transformations of quarks and leptons

- The electroweak theory is *chiral*: left and right-handed fields transform differently under $SU(2)_I \times U(1)_Y$

- singlets

$$E_R = (e_R, \mu_R, \tau_R)\,, \qquad Y_E = -1$$
$$U_R = (u_R, c_R, t_R)\,, \qquad Y_U = 2/3$$
$$D_R = (d_R, s_R, b_R)\,, \qquad Y_D = -1/3$$

$$\mathcal{L}_R = \bar{E}_R(i\partial\!\!\!/ - g_1 Y_E B\!\!\!/)E_R + \bar{D}_R(iD\!\!\!/ - g_1 Y_D B\!\!\!/)D_R + \bar{U}_R(iD\!\!\!/ - g_1 Y_U B\!\!\!/)U_R$$

doublets

$$L_L = \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right\}, \qquad Y_L = -1/2$$
$$Q_L = \left\{ \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \right\}, \qquad Y_Q = 1/6$$

$$\mathcal{L}_L = \bar{L}_L(i\partial\!\!\!/ - g_1 Y_L B\!\!\!/ - g_2 W\!\!\!/)L_L + \bar{Q}_L(iD\!\!\!/ - g_1 Y_Q B\!\!\!/ - g_2 W\!\!\!/)Q_L$$

# Gauge bosons

- The gauge bosons transforms as the adjoint of the respective group:

$$U(1)_Y \;\rightarrow\; B^\mu \quad \text{(singlet)}$$

$$SU(2)_I \;\rightarrow\; W^\mu = \sum_{i=1}^{3} W_i^\mu \, \sigma^i/2 \quad \text{(triplet)}$$

$$SU(3)_c \;\rightarrow\; g^\mu = \sum_{a=1}^{8} g_a^\mu \, T^a \quad \text{(octet)}$$

- Kinetic and self-interactions of gauge bosons:

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
F^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \, \epsilon^{ijk} \, W^j_\mu \, W^k_\nu \\
G^a_{\mu\nu} &= \partial_\mu g^a_\nu - \partial_\nu g^a_\mu + g_3 \, f^{abc} \, g^b_\mu \, g^c_\nu
\end{aligned}$$

- Structure constants:

$$SU(2): \quad [\sigma^i, \sigma^j] = \epsilon^{ijk} \, \sigma^k$$

$$SU(3): \quad [T^a, T^b] = f^{abc} \, T^c$$

# Fermion masses

- Mass terms for fermions (and gauge bosons) break gauge invariance:

$$\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

SU(2) singlet SU(2) doublet

- To construct a gauge invariant interaction between $\psi_L$ and $\psi_R$, we have to introduce a new field (Higgs):
  Lorentz invariance requires it to be a scalar
  SU(2) invariance requires it to be a doublet
  U(1) invariance fixes its hypercharge

- For the electron we can write the following Yukawa interaction:

$$\mathcal{L}_Y = -y_e\left(\bar{\ell}_L\,\phi\,e_R + \bar{e}_R\,\phi^\dagger\,\ell_L\right)\ ,$$

  where $Y_\phi = Y_L - Y_E = ½$ and, using $Q=Y+I_3$, we can write $\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$

- The scalar field $\phi^0$ has no color and no electric charge and can acquire a non-zero vacuum expectation value (vev) without breaking $SU(3)_c$, $U(1)_{em}$ and Lorentz invariance

# Spontaneous Symmetry Breaking (SSB)

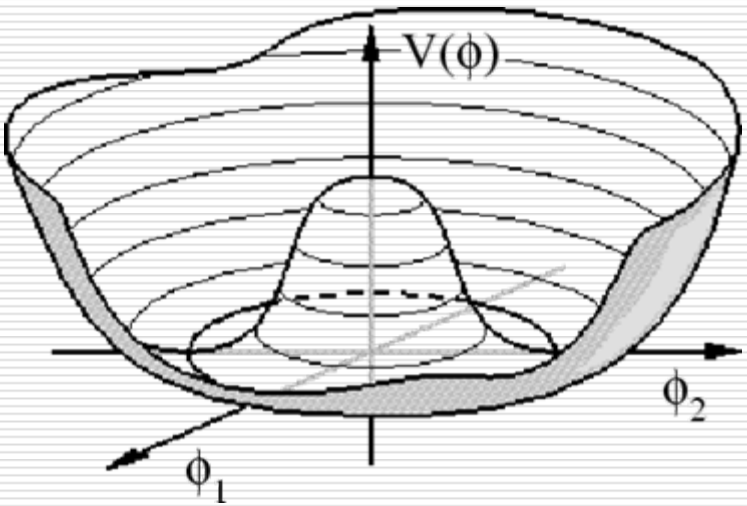
- *Reminder: the vacuum of the theory is the configuration that minimizes the energy*

- We cannot force a field to acquire a non-zero vev, it has to come from the minimization of the potential:

$$\mathcal{L}_{Higgs} = |D_\mu \phi|^2 - V(\phi) = |D_\mu \phi|^2 - \left(\lambda v^2 \, \phi^\dagger \phi + \lambda \, (\phi^\dagger \phi)^2\right)$$

$$D_\mu \phi = \left(\partial_\mu - g_1 Y_\phi B_\mu - g_2 W_\mu\right) \phi$$

- $\lambda$ has to be positive in order to bound from below the potential energy.
  If $v^2 < 0$ we have one miminum at $|\langle\phi\rangle| = 0$
  If $v^2 > 0$ we have a family of minima at $|\langle\phi\rangle| = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$



# Generation of mass

- The Higgs doublet can be written as

$$\phi(x) = e^{i\sum_{a=1}^{3}\sigma_a\xi^a(x)/2v}\begin{pmatrix} 0 \\ [v+h(x)]/\sqrt{2} \end{pmatrix}$$

- Inserting this expression into the Yukawa interaction we obtain a mass term:

$$y_e\left(\bar{\ell}_L\,\phi\,e_R + \bar{e}_R\,\phi^\dagger\,\ell_L\right) \longrightarrow y_e\frac{v}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L) = m_e\,\bar{e}e$$

- The interactions of $\phi$ with the SM gauge bosons are completely determined by its SU(3)xSU(2)xU(1) quantum numbers:

$$\phi = (\ \mathbf{1}_{SU(3)}\ ,\ \mathbf{2}_{SU(2)}\ ,\ Y_\phi = 1/2\ )$$

$$D_\mu\phi \ = \ \left(\partial_\mu - g_2 W_\mu - g_1 Y_\phi B_\mu\right)\phi$$

- After exanding around the vev, the term $|D_\mu\phi|^2$ induces masses for the W and Z bosons and the term $-\lambda v^2\phi^\dagger\phi$ induces a mass for h:

$$\begin{aligned} m_W &= g_2\,v/2 \\ m_Z &= \sqrt{g_1^2+g_2^2}\,v/2 \\ m_h &= \sqrt{2\lambda}\,v \end{aligned}$$

$$\begin{aligned} v &= 246\ \text{GeV} \\ \lambda &= ? \end{aligned}$$

# The Yukawa matrices

- Yukawa interactions for leptons:

$$\mathcal{L}_{Y_e} = -\sum_{i,j=1}^{3} \hat{y}^e_{ij}\, \bar{L}^i_L\, \phi\, E^j_R + \text{h.c.} = -\underbrace{\bar{L}_L}_{Y_L=-\frac{1}{2}}\, \underbrace{\phi}_{Y_\phi=\frac{1}{2}}\, \hat{y}^e\, \underbrace{E_R}_{Y_E=-1} + \text{h.c.}$$

- Yukawa interactions for right-handed down quarks:

$$\mathcal{L}_{Y_d} = -\underbrace{\bar{Q}_L}_{Y_Q=\frac{1}{6}}\, \underbrace{\phi}_{Y_\phi=\frac{1}{2}}\, \hat{y}^d\, \underbrace{D_R}_{Y_D=-\frac{1}{3}} + \text{h.c.}$$

- Yukawa interactions for right-handed up quarks:

$$\mathcal{L}_{Y_u} = -\underbrace{\bar{Q}_L}_{Y_Q=\frac{1}{6}}\, \underbrace{\tilde{\phi}}_{Y_{\tilde{\phi}}=-\frac{1}{2}}\, \hat{y}^u\, \underbrace{U_R}_{Y_D=\frac{2}{3}} + \text{h.c.}$$

in the SM we can use $\tilde{\phi} \equiv i\sigma_2\phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$

# The complete Lagrangian

$$
\begin{aligned}
\mathcal{L}_{SM} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F^{i}_{\mu\nu}F^{i,\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu} + \theta_{\mathrm{QCD}}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^{a}_{\mu\nu}G^{a}_{\rho\sigma} \\
& + \bar{L}_L\, i\not{D}\, L_L + \bar{Q}_L\, i\not{D}\, Q_L + \bar{E}_R\, i\not{D}\, E_R + \bar{D}_R\, i\not{D}\, D_R + \bar{U}_R\, i\not{D}\, U_R \\
& + |D_\mu\phi|^2 - V(\phi) \\
& - \left[\bar{L}_L\, \phi\, \hat{y}^e\, E_R + \bar{Q}_L\, \phi\, \hat{y}^d\, D_R + \bar{Q}_L\, \tilde{\phi}\, \hat{y}^u\, U_R + \text{h.c.}\right]
\end{aligned}
$$

- Parameters:

| | | |
|---|---|---|
| $g_1,\ g_2,\ g_3,\ \theta_{\mathrm{QCD}}$ | 4 | 19 parameters |
| $\lambda,\ v$ | 2 | |
| $\hat{y}^e,\ \hat{y}^d,\ \hat{y}^u \;\rightarrow\; m_e,\ m_\mu,\ m_\tau,$ | | |
| $m_d,\ m_s,\ m_b,$ | 13 | |
| $m_u,\ m_c,\ m_t,$ | | |
| $V_{\mathrm{CKM}}\ (\lambda, A, \rho, \eta)$ | | |

# *The CKM matrix*

# Physical parameters in the Yukawas

A single electron example:

$$\mathcal{L} = \underbrace{\bar{e}_R \, i\not{D} \, e_R + \bar{\ell}_L \, i\not{D} \, \ell_L}_{\text{invariant under } e_R \to e^{i\phi_R} e_R \text{ and } \ell_L \to e^{i\phi_L} \ell_L} - \left( y_e e^{i\delta} \, \bar{\ell}_L \, \phi \, e_R + y_e e^{-i\delta} \, \bar{e}_R \, \phi^\dagger \, \ell_L \right) . \tag{1}$$

Let's define $e_R = e^{-i\delta} e'_R$ .

The Lagrangian becomes:

$$\mathcal{L} = \bar{e}'_R \, i\not{D} \, e'_R + \bar{\ell}_L \, i\not{D} \, \ell_L - y_e \left( \bar{\ell}_L \, \phi \, e'_R + \bar{e}'_R \, \phi^\dagger \, \ell_L \right) . \tag{2}$$

*This means that physical observables calculated using the lagrangian in (1) will not depend on $\delta$.*

# Physical parameters in the Yukawas *(cont'd)*

The three leptons case:

$$\mathcal{L} \;=\; \underbrace{\bar{E}_R\, i\not{D}\, E_R + \bar{L}_L\, i\not{D}\, L_L}_{\text{invariant under } E_R \to R_e\, E_R \text{ and } L_L \to S_e\, L_L \text{ where } S_e\,,\, R_e \in U(3)} - \left(\bar{L}_L\, \phi\, \hat{y}^e E_R + \text{h.c.}\right). \tag{1}$$

Let's define $E_R = R_e E'_R$ and $L_L = S_e L'_L$

The Lagrangian becomes:

$$\mathcal{L} \;=\; \bar{E}'_R\, i\not{D}\, E'_R + \bar{L}'_L\, i\not{D}\, L'_L - \left(\bar{L}'_L\, \phi\, \underbrace{S_e^\dagger \hat{y}^e R_e}_{\equiv\, y^e}\, E'_R + \text{h.c.}\right). \tag{2}$$

With proper choice of $S_e$ and $R_e$ , $y^e$ can be made diagonal, real and non-negative.

*This means that physical observables calculated using the lagrangian in (1) will depend only on three real parameters (no CP violation).*

# Physical parameters in the Yukawas *(cont'd)*

The six quarks case:

$$\mathcal{L} = \underbrace{\bar{U}_R\, i\not{D}\, U_R + \bar{D}_R\, i\not{D}\, D_R + \bar{Q}_L\, i\not{D}\, Q_L}_{} - \left(\bar{Q}_L\, \tilde{\phi}\, \hat{y}^u U_R + \bar{Q}_L\, \phi\, \hat{y}^d D_R + \text{h.c.}\right).$$

invariant under $U_R \to R_u\, U_R$ , $D_R \to R_d\, D_R$ and $Q_L \to S_u\, Q_L$

where $R_u$ , $R_d$ , $S_u \in U(3)$

Let's define $U_R = R_u U'_R$ , $D_R = R_d D'_R$ and $Q_L = S_u Q'_L$

The Lagrangian becomes:

$$\begin{aligned}\mathcal{L} \;=\; & \bar{U}'_R\, i\not{D}\, U'_R + \bar{D}'_R\, i\not{D}\, D'_R + \bar{Q}'_L\, i\not{D}\, Q'_L \\ & -\left(\bar{Q}'_L\, \tilde{\phi}\, \underbrace{S_u^\dagger \hat{y}^u R_u}_{\equiv\, y^u}\, U'_R + \bar{Q}'_L\, \phi\, \underbrace{S_u^\dagger \hat{y}^d R_d}_{\equiv\, S_u^\dagger S_d\, y^d \,\equiv\, V\, y^d}\, D'_R + \text{h.c.}\right).\end{aligned}$$

CKM matrix

- The quarks mass matrices are:

$$m_u = \frac{v}{\sqrt{2}} y_u, \quad m_d = \frac{v}{\sqrt{2}} y_d$$

# Mass eigenstate basis

- In phenomenological applications is more convenient to work in a basis in which the mass terms are diagonal:

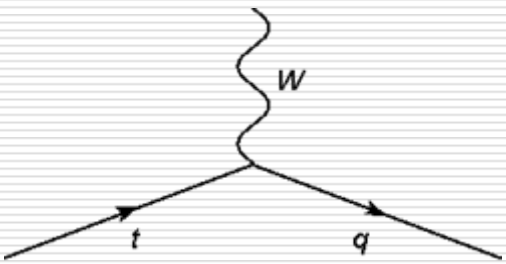
$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \to \begin{pmatrix} U_L \\ V D_L \end{pmatrix}$$

$$-\bar{U}_L m^u U_R - \bar{D}_L V m^d D_R + \text{h.c.} \to -\bar{U}_L m^u U_R - \bar{D}_L m^d D_R + \text{h.c.}$$

- Flavor changing charged currents:

$$-\frac{g_2}{\sqrt{2}}\left[\bar{U}_L \not{W}^+ D_L + \bar{D}_L \not{W}^- U_L\right] \to -\frac{g_2}{\sqrt{2}}\left[\bar{U}_L \not{W}^+ V D_L + \bar{D}_L V^\dagger \not{W}^- U_L\right]$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



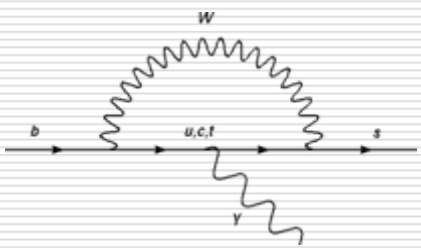
# Neutral currents

- Tree-level neutral currents remain flavor diagonal:

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \to \begin{pmatrix} U_L \\ V D_L \end{pmatrix}$$

$$a_R\, \bar{D}_R \not{Z} D_R + a_L\, \bar{D}_L \not{Z} D_L \to a_R\, \bar{D}_R \not{Z} D_R + a_L\, \bar{D}_L \underbrace{V^\dagger \not{Z} V}_{\not{Z}} D_L$$

- What happens at the 1-loop level?




$\sim 10^{-4}$ $\sim 4$

$$= V_{ub}^* V_{us} f(\frac{m_u^2}{m_W^2}) + V_{cb}^* V_{cs} f(\frac{m_c^2}{m_W^2}) + V_{tb}^* V_{ts} f(\frac{m_t^2}{m_W^2})$$

$$= V_{tb}^* V_{ts} \left( f(\frac{m_t^2}{m_W^2}) - f(\frac{m_u^2}{m_W^2}) \right) + V_{cb}^* V_{cs} \left( f(\frac{m_c^2}{m_W^2}) - f(\frac{m_u^2}{m_W^2}) \right)$$

$$\simeq V_{tb}^* V_{ts} \left( f(\frac{m_t^2}{m_W^2}) - f(0) \right) \propto V_{tb}^* V_{ts} \frac{m_t^2}{m_W^2}$$

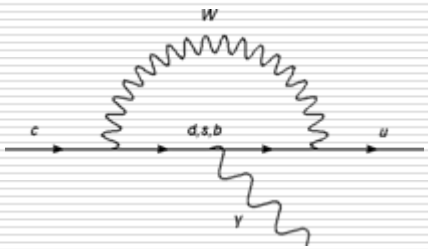
Unitarity: $V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us} = (V^\dagger V)_{32} = 0$

# Neutral currents *(cont'd)*

- FCNC in the down-quark sector:

$$\sim \text{(combination of CKM entries)}\ \frac{m_t^2}{m_W^2}$$

- FCNC in the up-quark sector:




$$\begin{aligned}
&= V_{ud}^* V_{cd} f\left(\frac{m_d^2}{m_W^2}\right) + V_{us}^* V_{cs} \underbrace{f\left(\frac{m_s^2}{m_W^2}\right)}_{\sim \times 10^{-6}} + V_{ub}^* V_{cb} \underbrace{f\left(\frac{m_b^2}{m_W^2}\right)}_{\sim 4 \times 10^{-3}} \\
&= V_{ub}^* V_{cb} \left( f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) + V_{us}^* V_{cs} \left( f\left(\frac{m_s^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) \\
&\simeq V_{ub}^* V_{cb} \left( f\left(\frac{m_b^2}{m_W^2}\right) - f(0) \right) \propto V_{ub}^* V_{cb} \frac{m_b^2}{m_W^2} \\
&\sim \text{(combination of CKM entries)}\ \frac{m_b^2}{m_W^2}
\end{aligned}$$

# Physical parameters in the CKM matrix

- We started with two arbitrary complex matrices ($\widehat{y}^u$, $\widehat{y}^d$): $4n_f^2$
- We used three unitary matrices: $3n_f^2$

  $S_u^\dagger \widehat{y}^u R_u = y^u \quad , \quad S_u^\dagger \widehat{y}^d R_d = V y^d$
- The matrices $e^{i\varphi} S_u$ , $e^{i\varphi} R_u$ , $e^{i\varphi} R_d$ achive the same structure: $3n_f^2 - 1$
- Total number of parameters: $4n_f^2 - (3n_f^2 - 1) = \underset{y^u}{n_f} + \underset{y^d}{n_f} + \underset{V}{(n_f - 1)^2}$
- $(n_f - 1)^2 = \underbrace{n_f(n_f - 1)/2}_{\text{real}} + \underbrace{(n_f - 1)(n_f - 2)/2}_{\text{imaginary}}$

# A global look at the SM

- The gauge invariant part of the Lagrangian depends on 4 parameters

- The Yukawa sector depends on 54 parameters ( $\widehat{y}^e,\ \widehat{y}^d,\ \widehat{y}^u$ ) out of which only 13 are observables.

$$U(3)_{L_L} \otimes U(3)_{E_R} \otimes U(3)_{Q_L} \otimes U(3)_{U_R} \otimes U(3)_{D_R}$$

$$\Big\downarrow \ \widehat{y}^e,\ \widehat{y}^d,\ \widehat{y}^u$$

$$\underbrace{U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau}_{\text{Lepton numbers}} \otimes \underset{\downarrow}{U(1)_B}$$

baryon number

- Homework. How many physical parameters are there if we introduce a second Higgs doublet? How many phases? Where do they show up?

$$\sum_{i=1}^{2} \left( \bar{L}_L\, \phi_i\, \widehat{y}_i^e E_R + \bar{Q}_L\, \tilde{\phi}_i\, \widehat{y}_i^u U_R + \bar{Q}_L\, \phi_i\, \widehat{y}_i^d D_R \right)$$

# Parametrizations of the CKM matrix

- Standard parametrization:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$

1) $s_{12} \simeq |V_{us}|$ , $s_{23} \simeq |V_{cb}|$ , $s_{13} \simeq |V_{ub}|$

2) The CP phase is always multiplied by the very small $s_{13}$

- Wolfenstein parametrization:

$\lambda \equiv s_{12}$ , $A \equiv s_{23}/\lambda^2$ , $\rho + i\eta \equiv s_{13}e^{i\delta}/A\lambda^3$

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

# The Jarlskog invariant

- It can be shown that, in the SM, CPV effects are present if and only if

$$F_u \, F_d \, J \neq 0$$

where:

$$\begin{aligned} F_u &= (m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_u^2) \\ F_d &= (m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_d^2) \\ J &= \mathrm{Im}[V_{us} V_{cd} V_{cs}^* V_{ub}^*] \\ &= c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta \\ &= A^2 \lambda^6 \eta \end{aligned}$$

- If there is a degeneracy either in the up or down quark mass matrices we have more freedom to choose the unitary transformations $S_u$, $R_d$ and $R_u$ and we can eliminate the CPV phase $\delta$

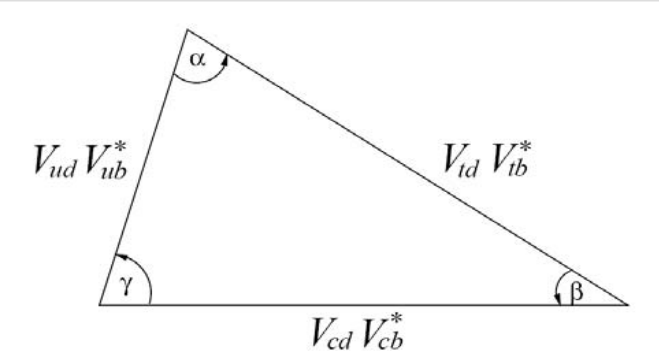
- All CPV effects are proportional to J, hence they are small even if the phase $\delta$ is large

# Direct measurement of CKM entries

- $|V_{ud}|=0.97377\pm0.00027$: nuclear $\beta$ decay
- $|V_{us}|=0.2257\pm0.0021$: $K^+\to\pi^0 l\nu$, $k^0\to\pi^- l\nu$
- $|V_{cd}|=0.230\pm0.011$: $\nu_\mu N\to\mu^- X$ vs $\nu_\mu N\to\mu^-\mu^+\nu_\mu X$, $D\to Kl\nu$, $D\to\pi l\nu$
- $|V_{cs}|=0.957\pm0.017\pm0.093$: $D\to Kl\nu$, $D\to\pi l\nu$, $W\to cs$
- $|V_{cb}|=(41.6\pm0.6)\ 10^{-3}$: $B\to X_c l\nu$, $B\to D^{(*)} l\nu$
- $|V_{ub}|=(4.31\pm0.30)\ 10^{-3}$: $B\to X_u l\nu$, $B\to\pi l\nu$
- $|V_{tb}|=1.3\pm0.2$: single top production (!)
- $V_{ts}$ and $V_{td}$: no direct measurements

# Unitarity triangle

- $B_d$ triangle: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

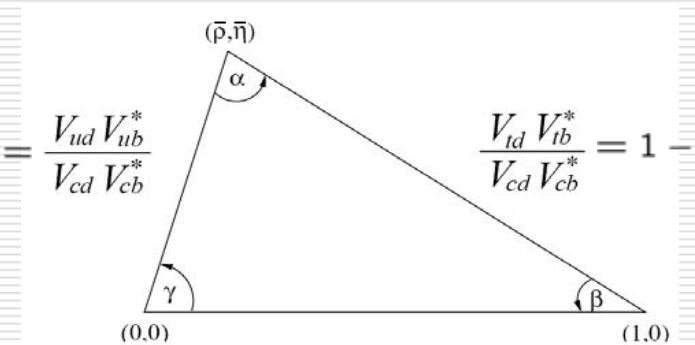



$$\alpha = \phi_2 = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$$

$$\beta = \phi_1 = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \phi_3 = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

$$\bar\rho + i\bar\eta = \frac{V_{ud}\,V_{ub}^*}{V_{cd}\,V_{cb}^*}$$




$$\left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{\bar\rho^2 + \bar\eta^2} + O(\lambda^6)$$

$$\left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{(1-\bar\rho)^2 + \bar\eta^2} + O(\lambda^6)$$

$$(\bar\rho, \bar\eta) \equiv (\rho, \eta)(1 - \lambda^2/2)$$

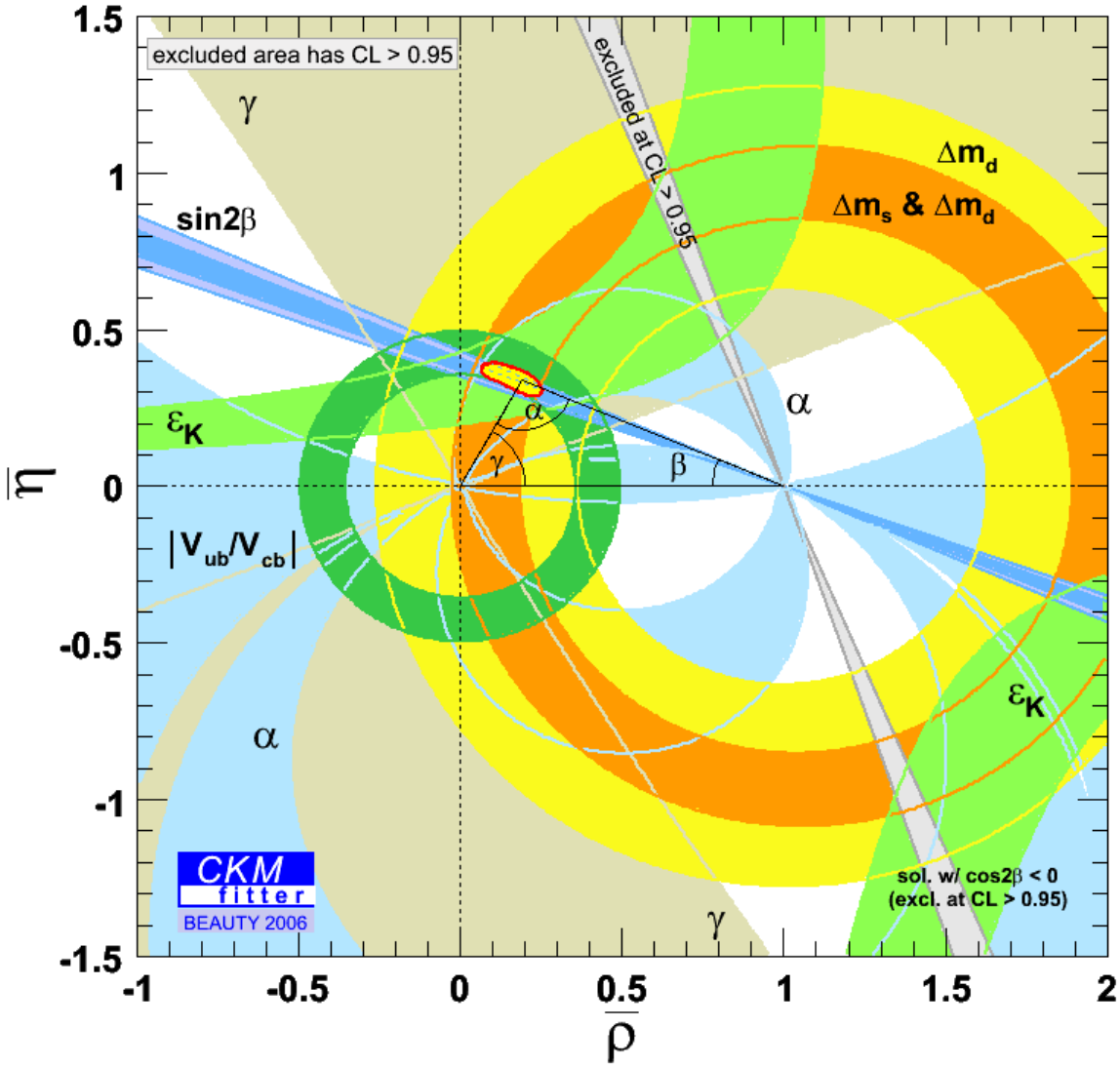
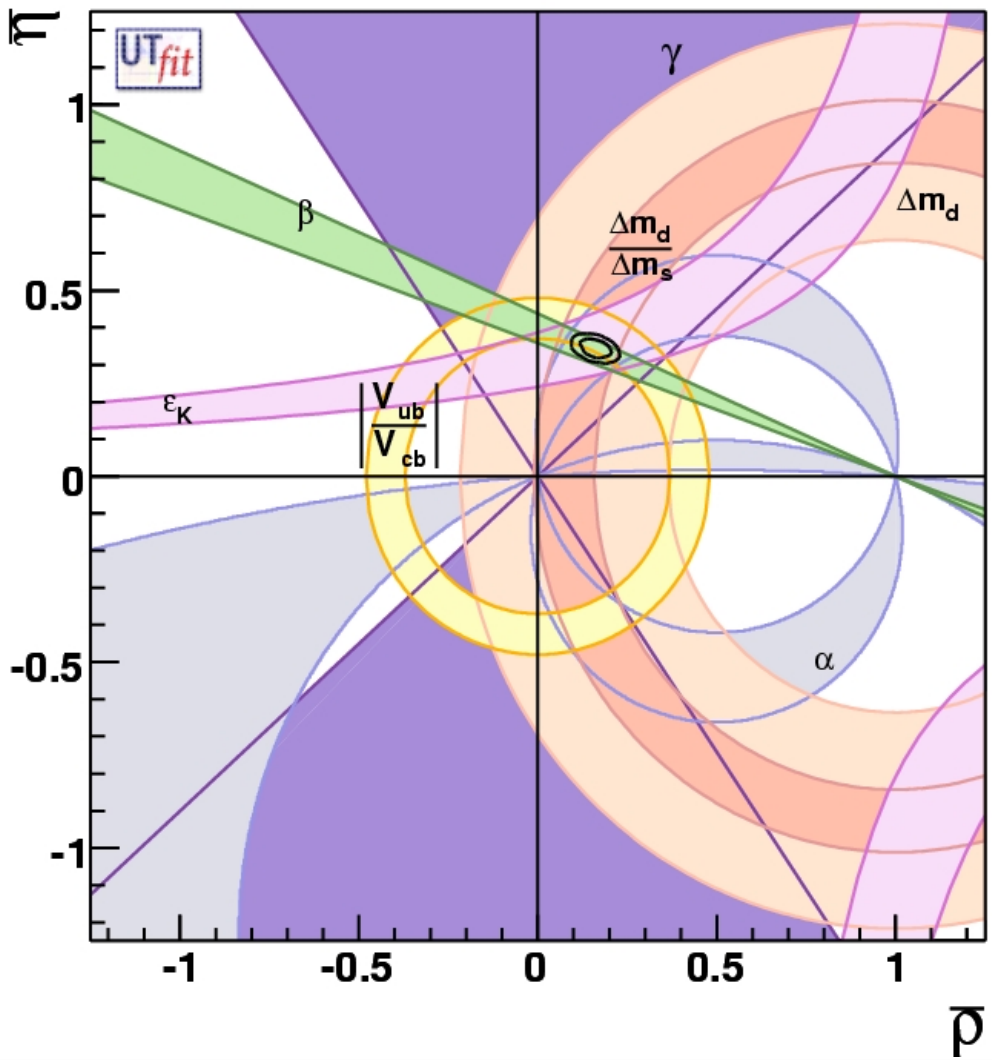
# Unitarity triangle *(cont'd)*

- $B_s$ triangle: $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

$A\lambda^4\sqrt{\rho^2+\eta^2}$ $\qquad$ $A\lambda^2$

$$\beta_s = \arg\left[-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right] = \lambda^2\eta + O(\lambda^4)$$

# Unitarity triangle fit (CKMfitter)



# Unitarity triangle fit (UTfit)