Introduction to Flavor Physics in and beyond the Standard Model

Lecture 2/4

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Fermilab

References:

The BaBar physics book, http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-r-504-TOC-pdf.pdf

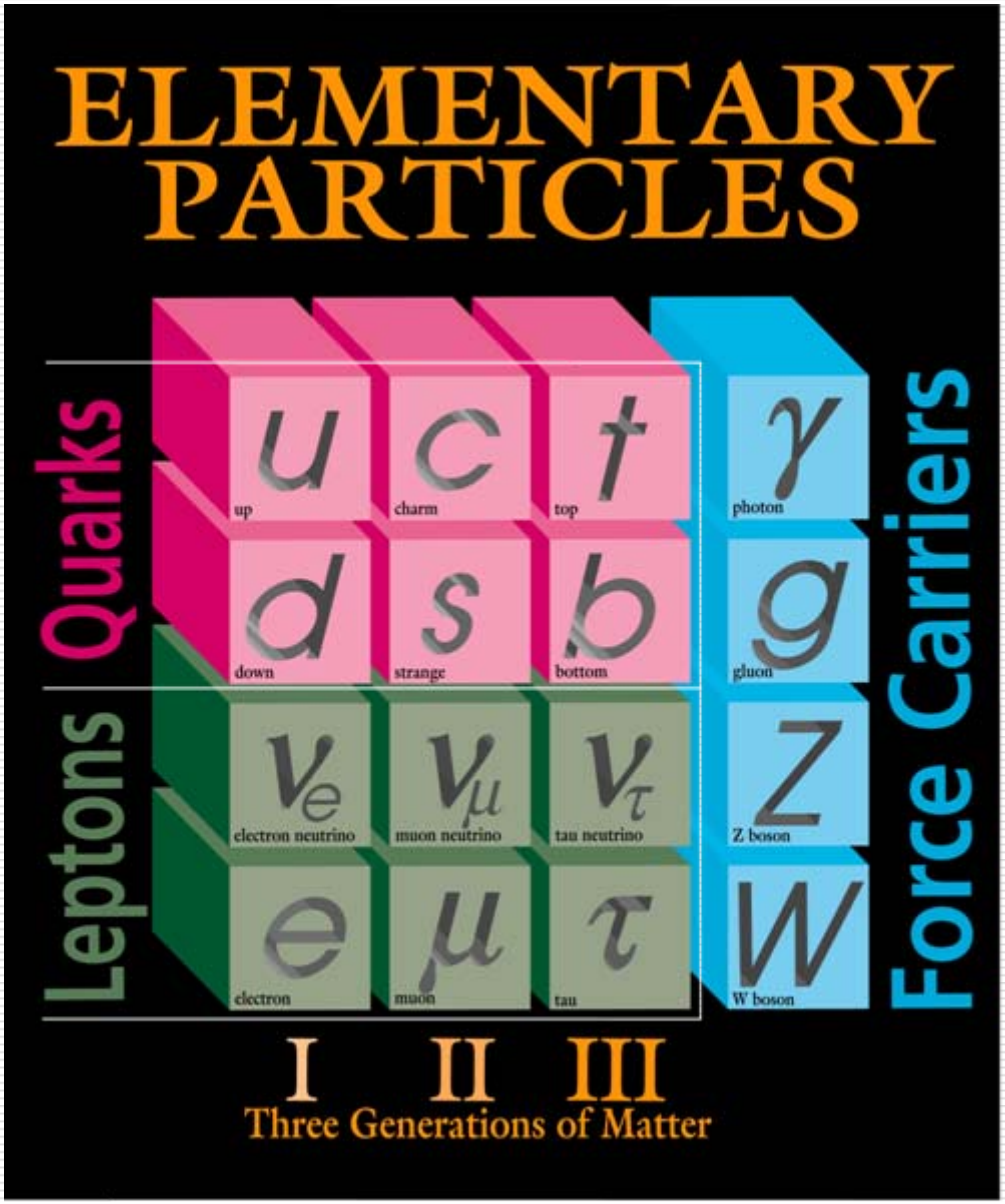
B physics at the Tevatron: Run II and beyond, hep-ph/0201071

The Discovery potential of a Super B Factory, hep-ph/0503261

Outline

- Summary of the previous lecture
- Theoretical tools (effective Hamiltonian approach)

Summary of lecture 1



Fermilab 95-759

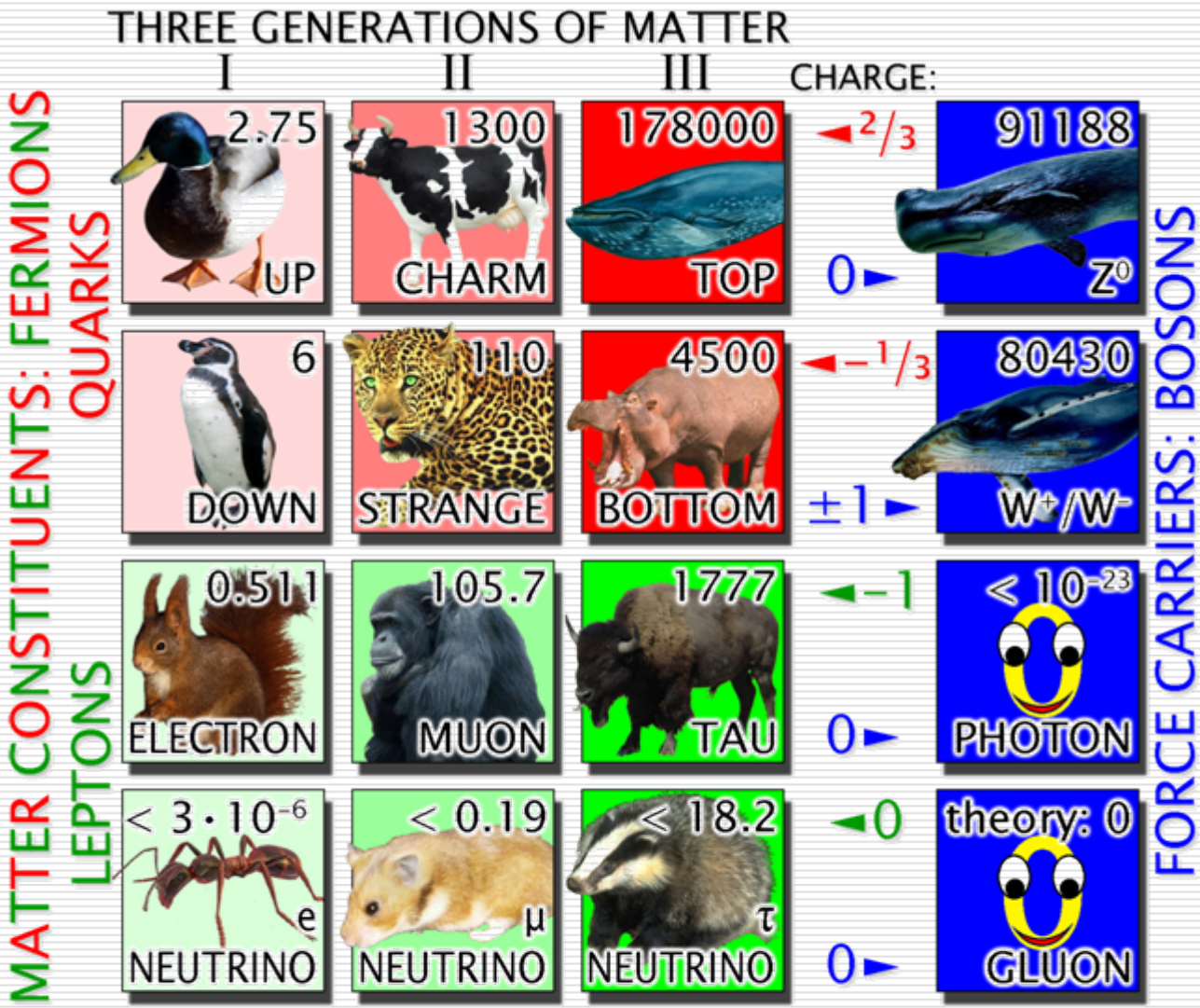
$$SU(3)_C \times SU(2)_I \times U(1)_Y$$

$$E_R = (1, 1, -1)$$
$$L_L = (1, 2, -1/2)$$

$$U_R = (3, 1, +2/3)$$
$$D_R = (3, 1, -1/3)$$
$$Q_L = (3, 2, +1/2)$$

$$g = (8, 1, 0)$$
$$W = (1, 3, 0)$$
$$B = (1, 1, 0)$$

$$\phi = (1, 2, +1/2)$$



ALL MASSES IN MEV;
ANIMAL MASSES
SCALE WITH
PARTICLE MASSES

The Standard Model fundamental particle zoo

Spontaneous Symmetry Breaking

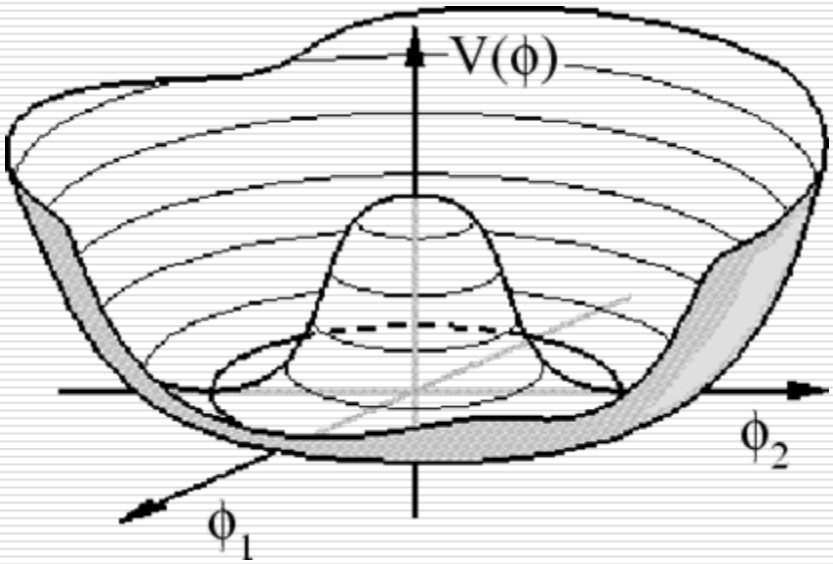
- Mass terms are forbidden by Gauge invariance:

$$\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

SU(2) singlet SU(2) doublet

- We introduce an Higgs doublet to dodge the problem

- Choosing a proper Higgs potential we induce a non-vanishing vacuum expecation value and masses for fermions and gauge bosons.



Generation of mass

- Masses for the fermions come from Yukawa interactions:

$$y_e \left(\bar{\ell}_L \, \phi \, e_R + \bar{e}_R \, \phi^\dagger \, \ell_L\right) \longrightarrow y_e \frac{v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) = m_e \, \bar{e} e$$

- The interactions of ϕ with the SM gauge bosons are completely determined by its SU(3)xSU(2)xU(1) quantum numbers:

$$\phi = (\ \mathbf{1}_{SU(3)}\ ,\ \mathbf{2}_{SU(2)}\ ,\ Y_\phi = 1/2\ )$$

$$D_\mu \phi \ = \ \left(\partial_\mu - g_2 W_\mu - g_1 Y_\phi B_\mu\right) \phi$$

- After exanding around the vev, the term $|D_\mu \phi|^2$ induces masses for the W and Z bosons and the term $-\lambda v^2 \phi^\dagger \phi$ induces a mass for h:

$$\begin{aligned} m_W &= g_2 \, v/2 \\ m_Z &= \sqrt{g_1^2 + g_2^2} \, v/2 \\ m_h &= \sqrt{2\lambda} \, v \end{aligned}$$

The SM lagrangian

$$
\begin{aligned}
\mathcal{L}_{SM} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F^{i}_{\mu\nu}F^{i,\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu} + \theta_{\mathrm{QCD}}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^{a}_{\mu\nu}G^{a}_{\rho\sigma} \\
& + \bar{L}_L\, i\not{D}\, L_L + \bar{Q}_L\, i\not{D}\, Q_L + \bar{E}_R\, i\not{D}\, E_R + \bar{D}_R\, i\not{D}\, D_R + \bar{U}_R\, i\not{D}\, U_R \\
& + |D_\mu\phi|^2 - V(\phi) \\
& - \left[\bar{L}_L\, \phi\, \hat{y}^e\, E_R + \bar{Q}_L\, \phi\, \hat{y}^d\, D_R + \bar{Q}_L\, \tilde{\phi}\, \hat{y}^u\, U_R + \text{h.c.}\right]
\end{aligned}
$$

$g_1,\ g_2,\ g_3,\ \theta_{\mathrm{QCD}}$	4	19 parameters
$\lambda,\ v$	2	
$\hat{y}^e,\ \hat{y}^d,\ \hat{y}^u \;\rightarrow\; m_e,\ m_\mu,\ m_\tau,$		
$m_d,\ m_s,\ m_b,$	13	
$m_u,\ m_c,\ m_t,$		
$V_{\mathrm{CKM}}\ (\lambda, A, \rho, \eta)$		

The quark Yukawas

$$\mathcal{L} = \underbrace{\bar{U}_R\, i\not{D}\, U_R + \bar{D}_R\, i\not{D}\, D_R + \bar{Q}_L\, i\not{D}\, Q_L}_{\text{invariant under } U(3)_{U_R} \times U(3)_{D_R} \times U(3)_{Q_L}} - \left(\bar{Q}_L\, \tilde{\phi}\, \hat{y}^u U_R + \bar{Q}_L\, \phi\, \hat{y}^d D_R + \text{h.c.}\right)$$

$$\begin{aligned}\mathcal{L} \;=\; & \bar{U}'_R\, i\not{D}\, U'_R + \bar{D}'_R\, i\not{D}\, D'_R + \bar{Q}'_L\, i\not{D}\, Q'_L \\ & -\Big(\bar{Q}'_L\, \tilde{\phi}\, \underbrace{S_u^\dagger \hat{y}^u R_u}_{\equiv\, y^u}\, U'_R + \bar{Q}'_L\, \phi\, \underbrace{S_u^\dagger \hat{y}^d R_d}_{\equiv\, S_u^\dagger S_d\, y^d \,\equiv\, V\, y^d}\, D'_R + \text{h.c.}\Big)\,.\end{aligned}$$

CKM matrix

- Proper counting of the parameters shows that the CKM matrix depends on three angles and one phase.

Flavor changing currents

- In phenomenological applications is more convenient to work in a basis in which the mass terms are diagonal:

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \to \begin{pmatrix} U_L \\ V D_L \end{pmatrix}$$

$$-\bar{U}_L m^u U_R - \bar{D}_L V m^d D_R + \text{h.c.} \to -\bar{U}_L m^u U_R - \bar{D}_L m^d D_R + \text{h.c.}$$

$$-\frac{g_2}{\sqrt{2}}\left[\bar{U}_L \not{W}^+ D_L + \bar{D}_L \not{W}^- U_L\right] \to -\frac{g_2}{\sqrt{2}}\left[\bar{U}_L \not{W}^+ V D_L + \bar{D}_L V^\dagger \not{W}^- U_L\right]$$

$$a_R\, \bar{D}_R \not{Z} D_R + a_L\, \bar{D}_L \not{Z} D_L \to a_R\, \bar{D}_R \not{Z} D_R + a_L\, \bar{D}_L \underbrace{V^\dagger \not{Z} V}_{\not{Z}} D_L$$

- GIM suppression:

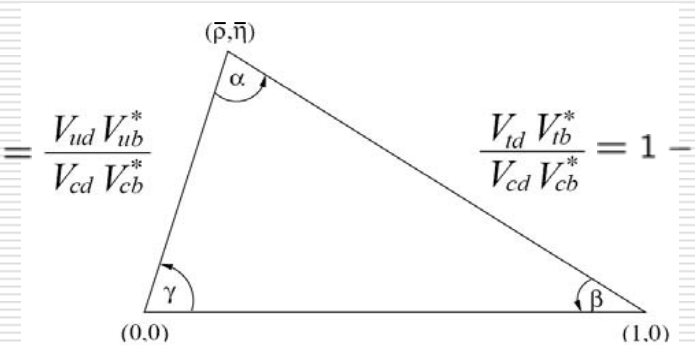
$$\text{FCNC in the down sector} \sim (\text{combination of CKM entries})\, \frac{m_t^2}{m_W^2}$$

$$\text{FCNC in the up sector} \sim (\text{combination of CKM entries})\, \frac{m_b^2}{m_W^2}$$

Unitarity triangles

- B_d triangle: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

$$\bar{\rho} + i\bar{\eta} = \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



$$\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 1 - \bar{\rho} - i\bar{\eta}$$

$$\alpha = \phi_2 = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$$

$$\beta = \phi_1 = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

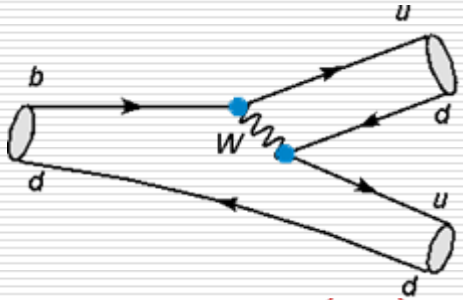
$$\gamma = \phi_3 = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

- B_s triangle: $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

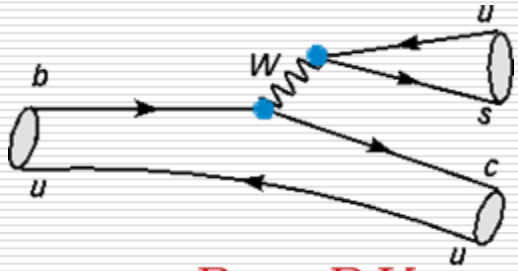
$A\lambda^4\sqrt{\rho^2+\eta^2}$ $\qquad$ $A\lambda^2$

$$\beta_s = \phi_s = \arg\left[-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right] = \lambda^2\eta + O(\lambda^4)$$

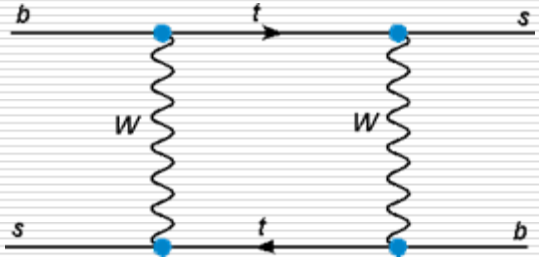
$$V_{td} = |V_{td}|e^{-i\beta}$$
$$V_{ub} = |V_{ub}|e^{-i\gamma}$$
$$V_{ts} = |V_{ts}|e^{-i\beta_s}$$



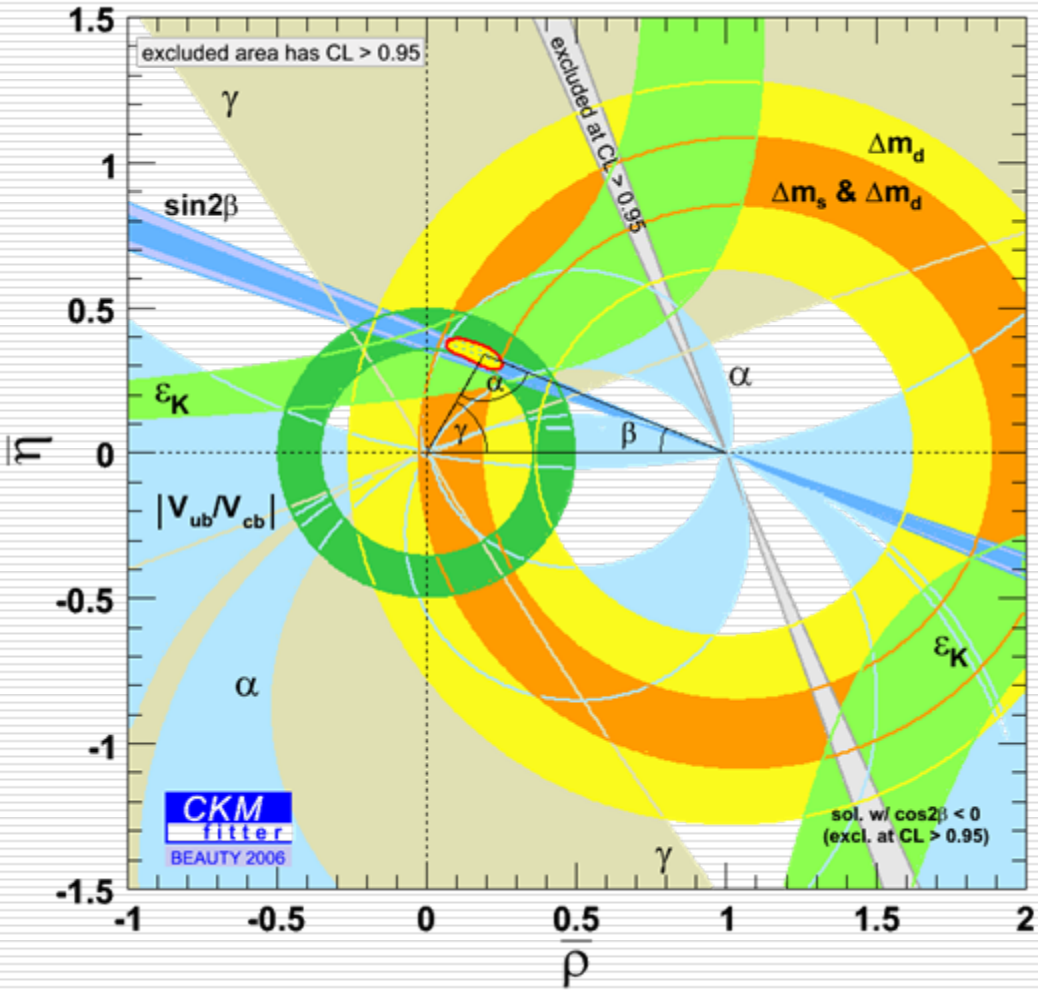
$\alpha: \; B \to \pi\pi(\rho\rho)$

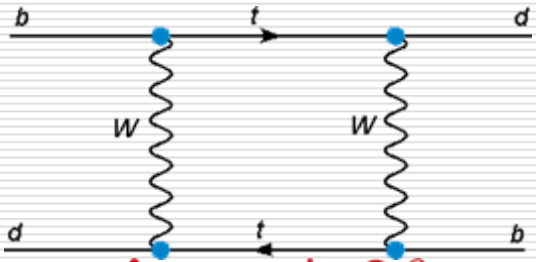


$\gamma: \; B \to DK$

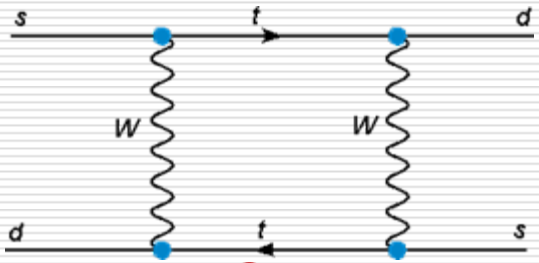


Δm_s





$\Delta m_d, \; \sin 2\beta$



ε_K

$$
\begin{aligned}
V_{td} &= |V_{td}| e^{-i\beta} \\
V_{ub} &= |V_{ub}| e^{-i\gamma} \\
V_{ts} &= |V_{ts}| e^{-i\beta_s}
\end{aligned}
$$

Theoretical tools

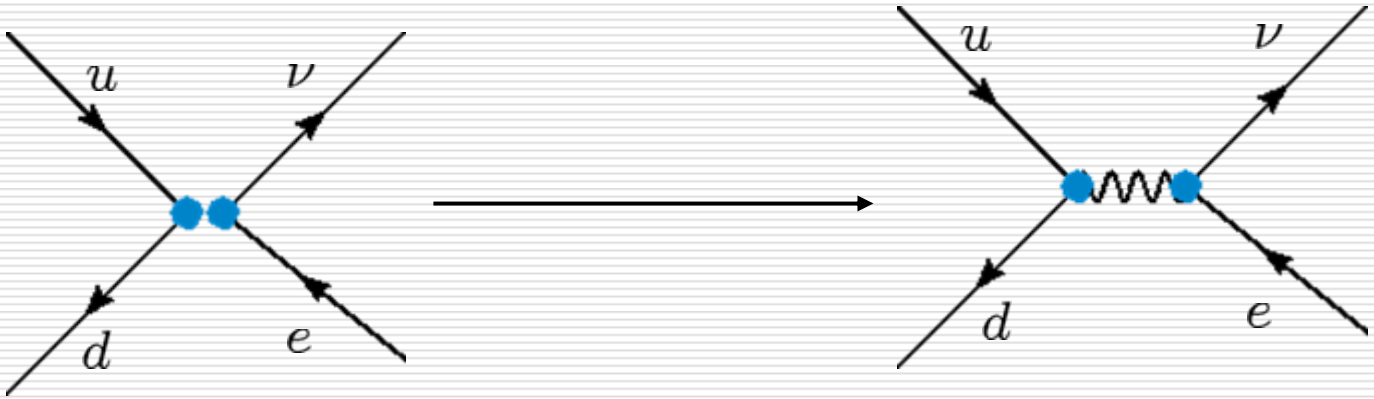
Short distance, Long distance

- There is a hierarchy between the W/Z/t scales and the energies/masses of the external particles we are interested in:

$$m_W, m_Z, m_t \longleftrightarrow m_b, m_c, m_s$$

- This suggests that the physics at these two scales can be treated independently

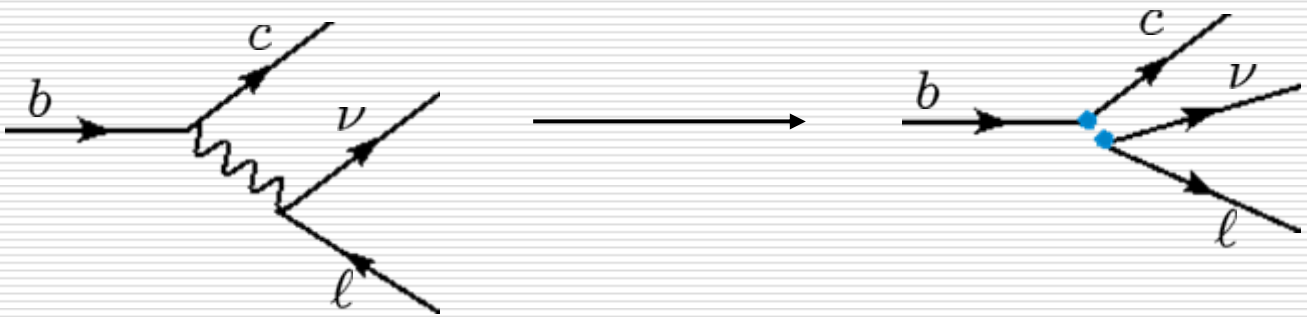
- Historically this is what happened for the β-decay



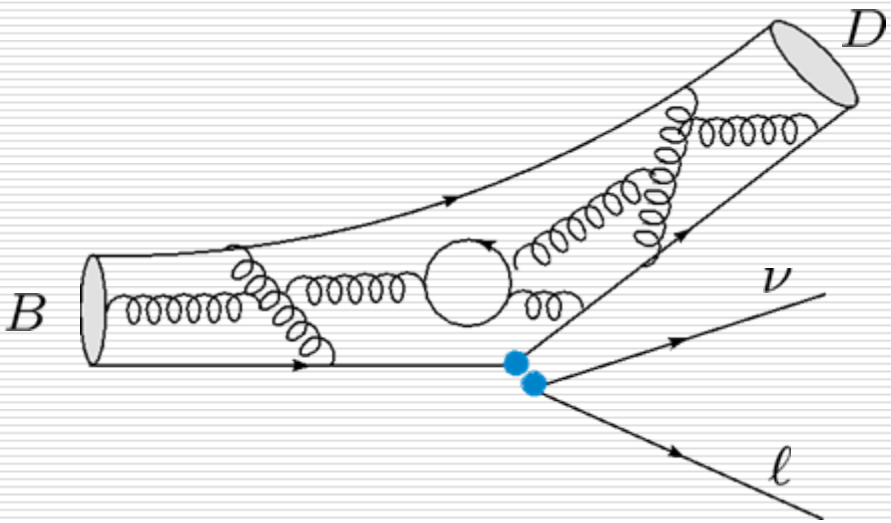
$$\frac{G_F}{\sqrt{2}} \qquad\qquad \frac{g_2^2}{8m_W^2}$$

Example: $B \to D\ell\nu$

- The first step is to identify the "core" short distance interaction:

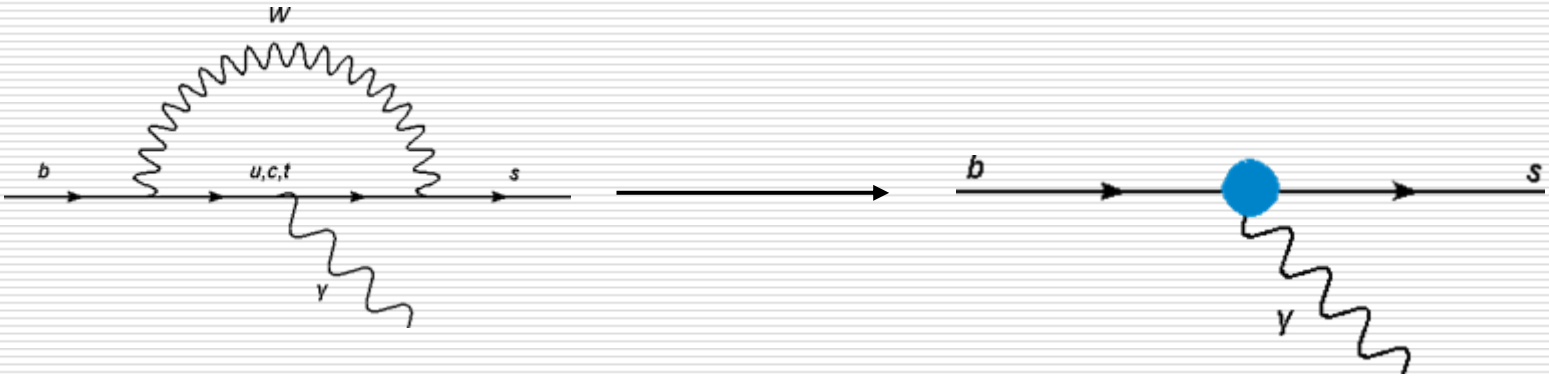


- The second step is to relate the *b* and *c* quarks to the particles we observe:



Example: $B \to X_s \gamma$

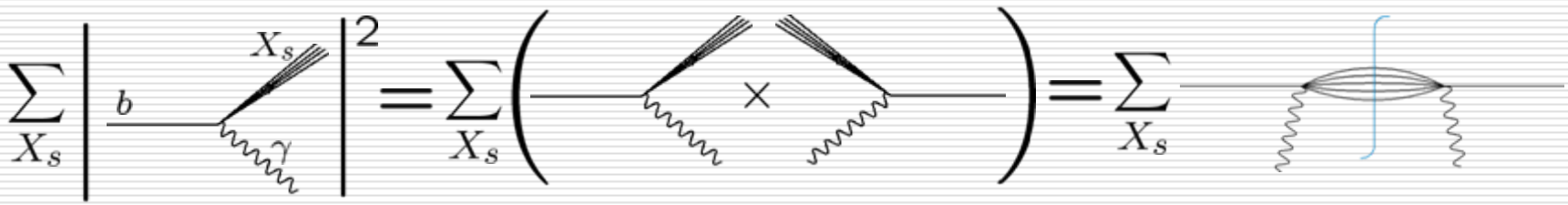
- The first step is to identify the "core" short distance interaction:



- Quark-hadron duality:

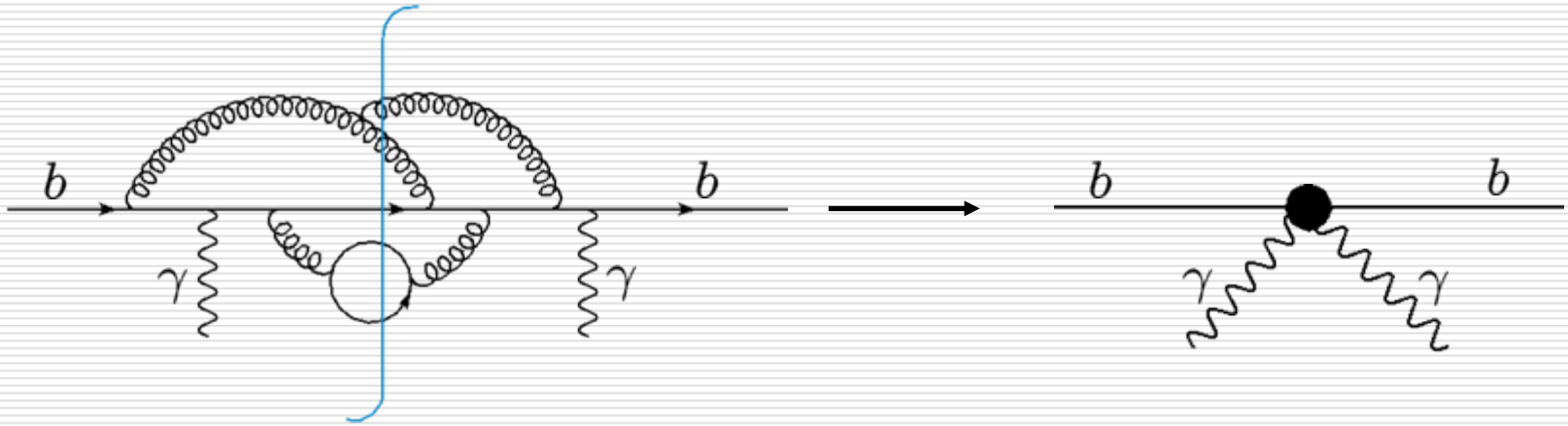
$$\sum_{X_s \in \text{hadrons}} \Gamma(b \to X_s \gamma) = \sum_{X_s \in \{\text{quarks,gluons}\}} \Gamma(b \to X_s \gamma)$$

- Optical theorem:

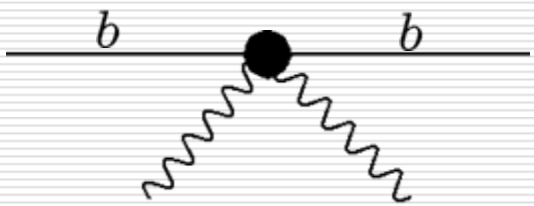


Example: $B \to X_s \gamma$

- This part of the calculation is perturbative (hopefully):



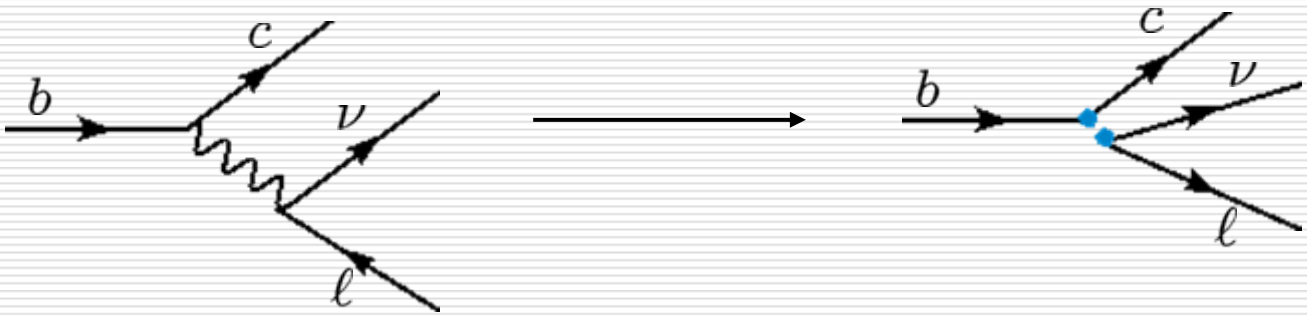
- The b quark has *non-perturbative* overlap with the B meson:

$\langle B|$  $|B\rangle = \Gamma(B \to X_s \gamma)$

Effective Hamiltonian

- The strategy described in the previous slides allows to decouple the problem of calculating short-distance effects from the complications of non-perturbative QCD

- Note that after "integrating out" the heavy field, we are left with new contact interactions:



$$\frac{g_2^2}{8}\frac{1}{m_W^2 - p_W^2}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\nu}_L\gamma_\mu \ell_L\right) \qquad \frac{G_F}{\sqrt{2}}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\nu}_L\gamma_\mu \ell_L\right) + O\left(\frac{p_W^2}{m_W^2}\right)$$

- We construct an effective Hamiltonian that does not contain the W, Z and t anymore, but has new operators.

Effective Hamiltonian *cont'd*

- $\mathcal{L}_{SM} \longrightarrow \mathcal{L}_{eff} \equiv [\mathcal{L}_{SM}]_{\text{no W,Z,t}} - \sum C_i O_i$

Wilson coefficients

- For example:

$$b \to c\ell\nu \Rightarrow \begin{cases} C = \frac{G_F}{\sqrt{2}} \\ O = (\bar{c}_L \gamma^\mu b_L)\ (\bar{\nu}_L \gamma_\mu \ell_L) \end{cases}$$

$$b \to s\gamma \Rightarrow \begin{cases} C = 4\frac{G_F}{\sqrt{2}}\ V_{tb}^* V_{ts}\ f(\frac{m_t^2}{m_W^2}) \\ O = \frac{e m_b}{16\pi^2}\ (\bar{s}_L \sigma^{\mu\nu} b_R)\ F_{\mu\nu} \end{cases}$$

- One advantage is that a given operator contributes to many different processes

Convergence of perturbation theory

- Let us consider the perturbative expansion of a given amplitude:

$$A(i \to f) \sim 1 + \alpha_s(1+L) + \alpha_s^2(1+L+L^2) + O(\alpha_s^3)$$

where $L = \log \frac{m_W^2}{p_{\text{ext}}^2}$, $\alpha_s = g_s^2/(4\pi)$ and p_{ext} is a generic external momentum

- In general at order α_s^n we will find terms proportional to $\alpha_s^n L^n$, $\alpha_s^n L^{n-1}$, ...

- If $p_{\text{ext}}^2 \ll m_W^2$, $\alpha_s L \sim O(1)$ and the convergence of the perturbative expansion is spoiled

- We need to find a way to get rid of these logs

Convergence of perturbation theory *cont'd*

- The effective Hamiltonian approach is precisely the framework that will allow us to resum these logs at all orders in perturbation theory:

$$A(i \to f) \sim 1 + \alpha_s \log \frac{m_W^2}{p_{\text{ext}}^2} + \cdots$$

$$= 1 + \underbrace{\alpha_s \log \frac{m_W^2}{\mu^2}}_{\text{in the WC}} + \underbrace{\alpha_s \log \frac{\mu^2}{p_{\text{ext}}^2}}_{\text{in the matrix element of the eff. operator}} \cdots$$

$$A(i \to f) = C(\mu) \langle f | O(\mu) | i \rangle$$

- Unfortunately the problem is not solved yet: we can not choose a value of μ that eliminates at the same time the large logs in the Wilson coefficient and in the operator matrix element

RG-Improved perturbation theory

- Using the fact that $A(i \to f) = C(\mu)\,\langle f\,|O(\mu)|\,i\rangle$ has to be μ independent we can write a differential equation for C :

$$\frac{\mathrm{d}C(\mu)}{\mathrm{d}\log\mu} = \gamma\, C(\mu)$$

anomalous dimension

Renormalization Group Equation

- - the anomalous dimension has an expansion in α_s
  - α_s depends on μ (running coupling constant)

$$\begin{cases} \dfrac{\mathrm{d}C(\mu)}{\mathrm{d}\log\mu} = \dfrac{\alpha_s}{4\pi}\gamma_0\, C(\mu) \\ \dfrac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\log\mu} = -2\beta_0\dfrac{\alpha_s^2}{4\pi} \end{cases} \Rightarrow C(\mu) = C(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma_0}{2\beta_0}}$$

- We are now free to choose $\mu^2 \sim O(p_{\mathrm{ext}}^2)$

Invitation: effective theories

- If a physical problem contains widely separated scales, it is almost always worthwhile to pursue an effective theory strategy

m_W

perturbative → integrate out

$$A(B \to X) = \sum C_i(\mu) \langle X | O_i(\mu) | B \rangle + O\left(\frac{m_b^2}{m_W^2}\right)$$

m_b

dependence on external states (possibly non-perturbative)

Invitation: effective theories below m_b

m_b

still perturbative → integrate out
but $p_{ext} \sim m_b$!!

$\sqrt{\Lambda_{QCD} m_b}$

$$\langle X | O_i(\mu) | B \rangle = C \, J * \Phi + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

Λ_{QCD}

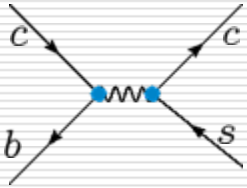
non-perturbative

Some effective Hamiltonians: b→s transitions

$$H_{\text{eff}} = -4\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\left[\sum_{i=1}^{10} C_i O_i + \sum_{i=3}^{6} C_{iQ} O_{iQ}\right]$$

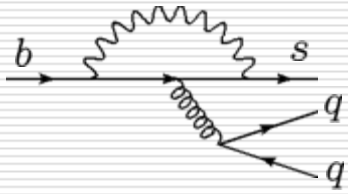
$$\left.\begin{aligned} O_1 &= (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L) \\ O_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L) \end{aligned}\right\}$$

current-current:



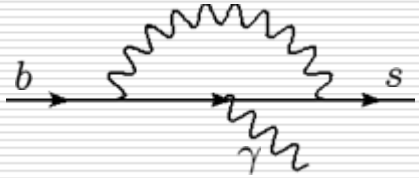
$$\left.\begin{aligned} O_3 &= (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q) \\ O_4 &= (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q) \\ O_5 &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L)\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q) \\ O_6 &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L)\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q) \end{aligned}\right\}$$

QCD penguin:



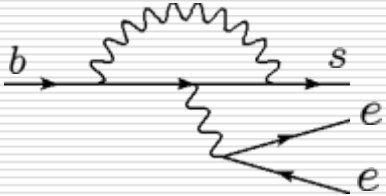
$$\left.\begin{aligned} O_7 &= \frac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu} \\ O_8 &= \frac{g_s}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu} \end{aligned}\right\}$$

magnetic moment:



$$\left.\begin{aligned} O_9 &= (\bar{s}_L\gamma_\mu b_L)\sum(\bar{\ell}\gamma^\mu \ell) \\ O_{10} &= (\bar{s}_L\gamma_\mu b_L)\sum(\bar{\ell}\gamma^\mu\gamma_5 \ell) \end{aligned}\right\}$$

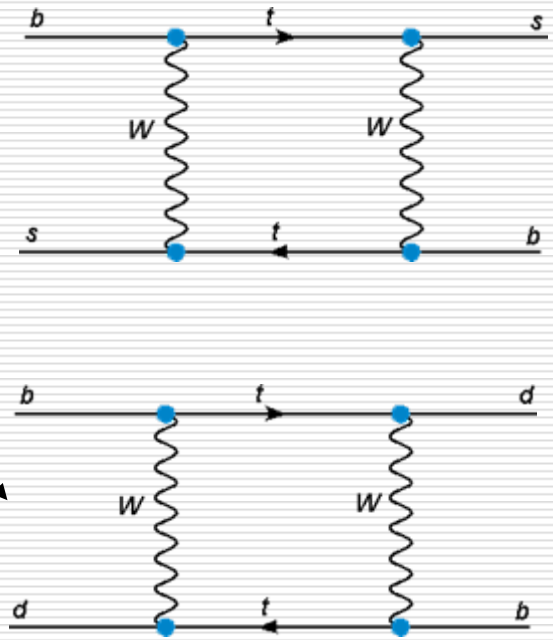
semileptonic:



Some effective Hamiltonians: B mixing

$$H_{\text{eff}} = \frac{G_F^2 m_W^2}{16\pi^2}(V_{tb}^* V_{tq})^2 \sum_{i=1}^{8} C_i O_i$$

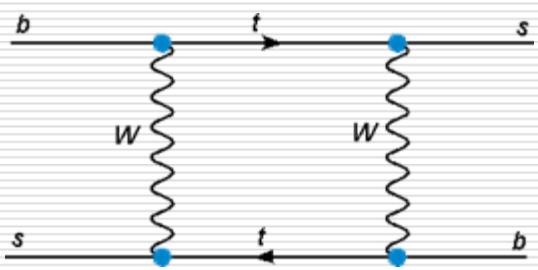
$$\begin{aligned}
O^{VLL} &= (\bar{b}_L \gamma_\mu q_L)(\bar{b}_L \gamma^\mu q_L) \\
O^{VRR} &= (\bar{b}_R \gamma_\mu q_R)(\bar{b}_R \gamma^\mu q_R) \\
O_1^{LR} &= (\bar{b}_L \gamma_\mu q_L)(\bar{b}_R \gamma^\mu q_R) \\
O_2^{LR} &= (\bar{b}_R q_L)(\bar{b}_L q_R) \\
O_1^{SLL} &= (\bar{b}_R q_L)(\bar{b}_R q_L) \\
O_1^{SRR} &= (\bar{b}_L q_R)(\bar{b}_L q_R) \\
O_2^{SLL} &= (\bar{b}_R \sigma_{\mu\nu} q_L)(\bar{b}_R \sigma^{\mu\nu} q_L) \\
O_2^{SRR} &= (\bar{b}_L \sigma_{\mu\nu} q_R)(\bar{b}_L \sigma^{\mu\nu} q_R)
\end{aligned}$$



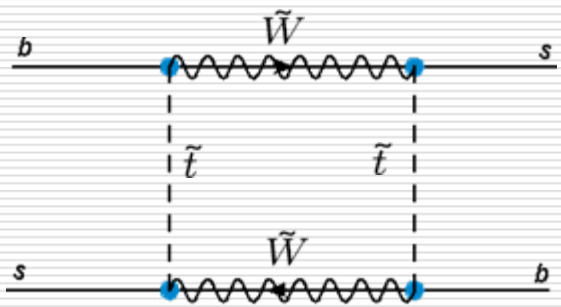
Effective Hamiltonian and New Physics

- A great advantage of the effective Hamiltonian approach is the extreme transparency to new physics

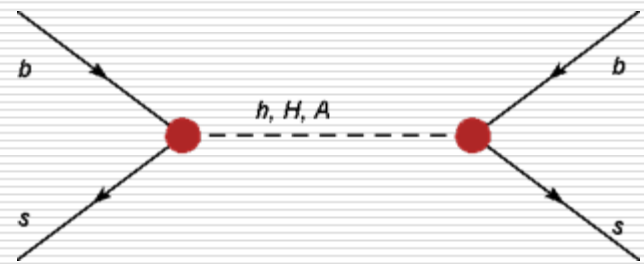
- Since the matrix elements of the various operators are dominated by large distance physics, new physics can enter only by:
  1. modifying the Wilson coefficients
  2. inducing new operators



SM



Supersymmetry



SUSY at large tan(β)

$$O^{VLL} = (\bar{b}_L \gamma_\mu q_L)(\bar{b}_L \gamma^\mu q_L)$$

$$O_2^{LR} = (\bar{b}_R q_L)(\bar{b}_L q_R)$$
$$O_1^{SLL} = (\bar{b}_R q_L)(\bar{b}_R q_L)$$