QCD effects in B-decays: Lecture 2

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1. Heavy quark physics
   • Heavy-quark spin and flavor symmetry
     • Spectroscopic implications
   • Heavy Quark Effective Theory
     • $V_{cb}$ from exclusive semileptonic decay

2. Inclusive B-decays
   • Operator Product Expansion
   • Determination of $V_{ub}, V_{cb}$ from semileptonic decays
   • Radiative decays: test of FCNC interactions
   • Heavy hadron lifetimes

3. Exclusive radiative and hadronic B-decays
   • Factorization, Soft Collinear Effective Theory
Heavy-light meson spectrum

- To leading power, bottom and charm spectra are simply shifted by constant amount $m_b - m_c = 3.4$ GeV.
- $M_{B_1} - M_B = (455 \pm 4)$ MeV, $M_{D_1} - M_D = (555 \pm 1)$ MeV
- “Spin doublets” almost degenerate:
  - e.g. $M_{B^*} - M_B = 46$ MeV
Operator Product Expansion and Inclusive Weak Decays
Inclusive decays rates are much less sensitive to hadronization effects than exclusive decays.

Scale hierarchy

\[ \frac{1}{m_b} \ll \frac{1}{\Lambda_{QCD}} \]

b-quark decay hadronic effects
Inclusive $b$-decays

• Important class of decays, since rate can be calculated perturbatively $m_Q \rightarrow \infty$
• Semileptonic decay $B \rightarrow X_c \ell \nu$
  • Most precise determination of $|V_{cb}|$, $m_b$ and $m_c$.
• Semileptonic decay $B \rightarrow X_u \ell \nu$
  • Most precise determination of $|V_{ub}|$
• Radiative decays $B \rightarrow X_s \gamma$, $B \rightarrow X_s l^+l^-$
  • Sensitive probe of FCNC interactions
• Lifetime: $B \rightarrow X$
Operator Product Expansion

- Used this tool before, when integrating out heavy particles in the construction of the effective weak Hamiltonian.

![Diagram](image)

- At low energies $W$ is highly virtual. Propagates only very short distance. Expand around $x=y$.
- In momentum space this translates into expansion of $W$-propagator

$$\frac{1}{p^2 - m_W^2} = \frac{1}{-m_W^2} + \ldots$$
Optical theorem

- To apply the same technique to the inclusive B-decay, first use the optical theorem

\[ \Gamma(B \to X_c e \bar{\nu}) = \frac{1}{2M_B} \sum_X (2\pi)^4 \delta^4(p_B - p_{X_c} - p_e - p_{\nu}) |\langle X_c e \bar{\nu} | H_{SL} | B \rangle|^2 \]

\[ = \frac{1}{2M_B} 2 \text{Im} \langle B | i \int d^4 x \ T \{ H_{SL}^\dagger(x), H_{SL}(0) \} | B \rangle \]

forward-scattering amplitude
Operator product expansion

- Expand the product of operators into local ones

\[ i \int d^4x \, T \left\{ \mathcal{H}^\dagger_{\text{SL}}(x), \mathcal{H}_{\text{SL}}(0) \right\} = C_3 \bar{b}b + \frac{C_5}{m_b^2} \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b + \ldots \]

No dim. 4 operators: \( b i \not{D} b = m_b b b \)

- To evaluate the coefficients \( C_3 \) and \( C_5 \), we can use arbitrary external states

  - Use quark and gluon states and calculate the coefficients in perturbation theory!

- Then use HQET to evaluate the B-meson matrix elements of the operators \( \bar{b}b \) and \( \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \)

  - Will be given by HQET parameters \( \lambda_1, \lambda_2, \) etc.

- Q: Is the expansion well behaved? Are the higher order terms really suppressed by \( 1/m_b^2 \)?
Feynman Diagrams

\[ C_3 \bar{b} b \]

\[ C_5 \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \]
Matrix elements

- To calculate the matrix elements, we use HQET.

\[
\frac{1}{2M_B} \langle B| \bar{b} b | B \rangle = 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + \ldots
\]

No $1/m_b$ corrections!

\[
\frac{1}{m_b^2} \frac{1}{2M_B} \langle B| \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle = \frac{6\lambda_2}{m_b^2} + \ldots
\]

\[
\lambda_2 = \frac{1}{4} \left( M_{B^*}^2 - M_B^2 \right) = 0.12\text{GeV}^2
\]
Result for the rate

\[ \Gamma(B \to X_c e \bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left\{ \left(1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2}\right) \left[f(\rho) + \frac{\alpha_s}{\pi} g(\rho)\right] - \frac{6\lambda_2}{m_b^2} (1 - \rho)^4 + \ldots \right\} \]

\[ f(\rho) = 1 - 8\rho - 12\rho^2 \log \rho + 8\rho^3 - \rho^4, \quad \rho = \frac{m_c^2}{m_b^2} \]

\[ g(\rho) = ”lengthy, known expression” \]

- Leading term in limit \( m_Q \to \infty \) is the free \( b \)-quark decay ("naive parton model")!
- Hadronization effects are suppressed as \( 1/m_b^2 \). Reduce the rate by \( \approx 4\% \)
- Values of \( m_b, m_c \) and \( \lambda_1 \)?
- Strictly speaking expansion is \( 1/(m_b - m_c) \approx 1/m_b \)
Side remark: $b$-quark mass

- We calculated in terms of the $b$-quark pole mass, i.e. the location of the pole in the heavy quark propagator.
  - Well defined in perturbation theory, but
  - does not make sense non-perturbatively because of confinement.

- The lack of a non-perturbative definition shows up via large higher-order perturbative corrections.
  - Upon relating the pole to the $\overline{\text{MS}}$ quark mass, one finds a badly divergent PT series.
  - Can resum this series, but prescription is not unique ("renormalon ambiguity").
Bad perturbative behavior also shows up in the decay rate, if it is expressed in the pole mass.

Eliminate pole mass in favor quark mass! Many mass schemes in the literature:

- MS mass (not suited for HQET)
- Kinetic mass (Urlatsev)
- Y(1S) mass (Hoang, Ligeti, Manohar)
- Potential subtracted (Beneke)
- Shape-function (Bosch, Lange, Neubert, Paz)

Also, better definition for \( \lambda_1, \lambda_2 \) are available in the kinetic and shape-function scheme.

The corresponding parameters are denoted by \( \mu_{\pi^2} (\equiv -\lambda_1) \) and \( \mu_{G^2} (\equiv \lambda_2) \)
Moments

• Calculate moments of the decay spectrum (with exp. cuts).

• Leptonic moments

\[ L_n = \frac{1}{\Gamma} \int dE_e (E_e)^n \frac{d\Gamma}{dE_e} \]

• Hadronic moments

\[ H_{ij} = \frac{1}{\Gamma} \int dM_X^2 dE_X (M_X^2)^i (E_X)^j \frac{d\Gamma}{dM_X^2 dE_X} \]

• Measurement of the moments and the rate determines \( V_{cb} \), \( m_b \), \( m_c \) and \( \lambda_1 \).
Moment measurements and global fit

**hadronic mass moments**

- **(a)**
- **(c)**
- **(e)**

**lepton energy moments**

- **(b)**
- **(d)**
- **(f)**

- Yellow band: exp. + th. uncertainty

Buchmüller and Flächer, hep-ph/0507253
Fit results

Moments included:
- Solid red: all
- Dashed blue: $B \to X_c e \nu$
- Dotted green: $B \to X_s \gamma$

Results of hep-ph/0507253, see also hep-ph/0408002

Most precise determination of 3 SM parameters in a single process!
News flash

• The “most precise determination” statement was true until last Tuesday:

High Energy Physics – Phenomenology

hep-ph new abstracts, Tue, 13 Feb 07 01:00:10 GMT
0702103 -- 0702123 received

hep-ph/0702103 [abs, ps, pdf, other] :
Title: Heavy Quark Masses from Sum Rules in Four-Loop Approximation
Authors: Johann H. Kuehn, Matthias Steinhauser, Christian Sturm
Comments: 29 pages

New data for the total cross section $\sigma(e^+e^-\to{\text{hadrons}})$ in the charm and bottom threshold region are combined with an improved theoretical analysis, which includes recent four-loop calculations, to determine the short distance $\bar{\text{MS}}$ charm and bottom quark masses. A detailed discussion of the theoretical and experimental uncertainties is presented. The final result for the $\bar{\text{MS}}$-masses, $m_c(3 \text{ GeV})=0.986(13)$ GeV and $m_b(10 \text{ GeV})=3.609(25)$ GeV, can be translated into $m_c(m_c)=1.286(13)$ GeV and $m_b(m_b)=4.164(25)$ GeV. This analysis is consistent with but significantly more precise than a similar previous study.
Future improvements of the moment analysis

• To go to next higher level in theoretical precision, we’ll need
  • Tree-level OPE to $1/m_b^3$. Already included.
    • Has recently even been calculated up to $1/m_b^4$, hep-ph/0611168.
  • Perturbative corrections to the leading power corrections, terms
    \[ \alpha_s(m_b) \frac{\mu_\pi^2}{2m_b} \quad \alpha_s(m_b) \frac{\mu_G^2}{2m_b} \]
    • doable, but nontrivial 1-loop calculation
  • Two-loop corrections to the leading power rate
    • Possible with new numerical techniques. Muon decay has been calculated, hep-ph/0505069 (same kinematics, but QED instead of QCD corrections)
Are there pieces that we are missing when calculating the rate using the OPE?

It is often stated that the OPE calculation “assumes quark hadron duality”, since we calculated the coefficients $C_3$ and $C_5$ with quarks instead of hadrons.

More precisely, we have expanded in the rate $1/m_b$, $\alpha_s(m_b)$ and $1/m_W^2$. Upon expanding, we lose non-analytic terms, such as

\[
\begin{align*}
\text{pert. expansion} & : e^{-1/\alpha_s} , \\
\text{OPE integrating out } W & : e^{-a^2/m_W^2} , \\
\text{OPE for incl. B-decay} & : \frac{1}{m_b^n} \sin\left(\frac{m_b}{b}\right) .
\end{align*}
\]

$W$ cannot go on-shell (Euclidean OPE)\qquad quark, gluons can be on shell (Minkowskian OPE)

Models give $n=8$ suppression compared to leading order in SL decay, see hep-ph/0009131. Hopefully, these effects are tiny.
Example of oscillatory behavior

\[ R(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons}, s)}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-, s)} \]

- Oscillatory behavior is not captured by OPE calculation.
- In inclusive quantities, the oscillations average out.
Heavy hadron lifetimes and the $\Lambda_b$ (ex-)puzzle
Heavy hadron lifetimes

- Same OPE technique can also be used to calculate the hadron lifetimes \( \tau = 1/\Gamma \)

\[
\Gamma(H_b) = \Gamma(H_b \to X) = \frac{1}{2M_B} \sum_X (2\pi)^4 \delta^4(p_B - p_X) |\langle X | H_{\Delta B=1} | B \rangle|^2
\]

\[
= \frac{1}{2M_B} 2 \text{Im} \langle B | i \int d^4x \ T \{ H_{\Delta B=1}(x), H_{\Delta B=1}(0) \} | B \rangle
\]

- Complete \( \Delta B=1 \) eff. Hamiltonian:

\[
H_{\Delta B=1} = \frac{4G_F}{\sqrt{2}} V_{cb} \left\{ c_1(m_b) \left[ \bar{d}'_L \gamma_\mu u_L \bar{c}_L \gamma^\mu b_L + \bar{s}'_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L \right] + c_2(m_b) \left[ \bar{c}_L \gamma_\mu u_L \bar{d}'_L \gamma^\mu b_L + \bar{c}_L \gamma_\mu c_L \bar{s}'_L \gamma^\mu b_L \right] \right.
\]

\[
+ \sum_{\ell=e,\mu,\tau} \bar{\ell}_L \gamma_\mu \nu_\ell \bar{c}_L \gamma^\mu b_L \left\} + \text{h.c. } ,
\]

hadronic decays

semileptonic decays
Result for the rate has the same structure as we had before.

Only small lifetime differences \( \sim 1-2\% \) to \( \mathcal{O}(\Lambda^2/m_b^2) \). Arise because \( \lambda_1, \lambda_2 \) are slightly different for different hadrons.

Dominant contribution to lifetime differences from the four-quark operators suppressed by \( \mathcal{O}(\Lambda/m_b)^3 \). Enhanced by a large numerical prefactor \( 4\pi^2 \), they are \( \mathcal{O}(5-10\%) \).
Situation a few years ago

Latest numbers:

Theory, hep-ph/0612176

\[ \frac{\tau(B^+)}{\tau(B^0)} \]_{NLO} = 1.063 \pm 0.027

Experiment

\[ \frac{\tau(B^+)}{\tau(B^0)} \] = 1.071 \pm 0.009

\[ \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01 \]

hep-ph/9906031

\[ \frac{\tau(\Lambda_b)}{\tau(\Lambda_c)} = \begin{cases} 0.91(1) & \text{for } am_\pi = 0.74(4) \\ 0.93(1) & \text{for } am_\pi = 0.52(3) \end{cases} \]

a^{-1} = 1.1 \text{GeV}

- New CDF result, hep-ex/0609021

\[ \frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 1.041 \pm 0.057 \text{ (stat. + syst.).} \]

DZero: \[ \frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 0.87^{+0.17}_{-0.14} \text{ (stat) } \pm 0.03 \text{ (syst),} \]

3.2\sigma higher than world average, with comparable precision!
$B \rightarrow X_u e\nu$

Experimental cuts, shape function and the extraction of $V_{ub}$
Interesting tension between $|V_{ub}|$ and $\sin(2\beta)$ measurements

- $\sin(2\beta)$: loop process in SM
  - sensitive to new physics
- $|V_{ub}|$: tree level weak decay
  - insensitive to new physics,
  - but extraction is sensitive to QCD effects!
It is trivial to obtain the $B \to X_u e \nu$ rate from our expression for $B \to X_c e \nu$.

Set $m_c=0$, replace $V_{cb} \to V_{ub}$.

However, experimentally, it is impossible to measure the total $B \to X_u e \nu$ rate.

$B \to X_c e \nu$ signal is much larger.

Need kinematical cuts to eliminate $B \to X_c e \nu$.

- e.g. $M_X < M_D$ or $E_e > \frac{M_B^2 - M_D^2}{2M_B}$
$E_e$ spectrum

- $S/B \sim 1/15$ for $E_e > 2$ GeV. Background subtraction challenging!
The cuts reduce the small $b \to u$ signal even farther!

Are a theoretical challenge

- Reduce available phase space enforce small $M_X$.
- OPE breaks down! Terms $\frac{\Lambda_{QCD} E_X}{M_X^2}$ in OPE are no longer suppressed.
\( p_B = m_b v + k \)

- **\( u \)-quark propagator denominator**
  \[
  \frac{1}{(m_b v + k - q)^2} = \frac{1}{(m_b v - q)^2 - 2(m_b v - q)k + k^2}
  \]

- **Total rate: (\( p_X = m_b \, v - q \))**
  \[
  p_X^2 \sim m_b^2, \quad p_X \cdot k \sim m_b \Lambda_{QCD}, \quad k^2 \sim \Lambda_{QCD}^2
  \]

- **After cut to eliminate \( B \rightarrow X_c \, e \, \nu \)**
  \[
  p_X^2 \sim m_b \Lambda_{QCD}, \quad p_X \cdot k \sim m_b \Lambda_{QCD}, \quad k^2 \sim \Lambda_{QCD}^2
  \]
\[ p_B = m_{b\nu} + k \]

- Without cut (or modest cuts)

\[
\frac{1}{(p_X + k)^2} = \frac{1}{p_X^2} \left[ 1 + \frac{2 p_X \cdot k}{p_X^2} + \ldots \right]
\]

- Hadronic part: local operators with derivatives

\[
h_v(k) \rightarrow h_v(x) h_v(x),
\]

\[
h_v(k) k_\mu h_v(k) \rightarrow h_v(x) iD_\mu h_v(x) \text{ etc.}
\]

- With cut to eliminate \( B \rightarrow X_c e \nu \)

\[
\frac{1}{(m_{b\nu} + k - q)^2} = \frac{1}{p_X \cdot (p_X - k)} \left[ 1 + \frac{k^2}{p_X \cdot (p_X - k)} + \ldots \right]
\]

Function of \( p_X \cdot k \) ! Nonlocal object in position space. Matrix element is “Shape function”
Total rate (or mild cuts)

Two relevant scales: \( m_b \gg \Lambda_{\text{QCD}} \)
Small $M_X$: “shape function region”

**Three different scales:** $m_b \gg M_X \gg \Lambda_{QCD}$

**Double expansion:** $M_X/m_b$ and $\Lambda_{QCD}/M_X$

Can use soft-collinear effective theory (SCET) to perform expansion in a systematic way
QCD

Factorization theorem

\[ \Gamma \sim H^2 J \otimes S \]

hard jet soft shape function

Korchemsky, Sterman '94
Shape function $S(\omega)$

- At this point things look bleak: Even in the limit $m_b \to \infty$, we need a nonperturbative shape function $S(\omega)$ as input!
- Similar to hadron collider physics, where we need non-perturbative parton distributions to make predictions.
- Know a few properties: once we integrate over the spectrum, we obtain usual OPE expression.
- Moments are given in terms of HQET parameters, such as $m_b$, $\lambda_1$. 
\( B \rightarrow X_s \gamma \) to the rescue

- Fortunately, the same shape function \( S(\omega) \) appears in the calculation of the \( B \rightarrow X_s \gamma \) photon energy spectrum.

\[
\frac{d\Gamma}{dE_\gamma} = H_\gamma J \otimes S
\]

- Only hard function differs from \( B \rightarrow X_u l \nu \).

- Two strategies to extract \( |V_{ub}| \):
  - Make ansatz for \( S(\omega) \), depending on a number of parameters. Constrain with \( B \rightarrow X_s \gamma \) spectrum and \( B \rightarrow X_c l \nu \) moments.
  - Use relations between the \( B \rightarrow X_s \gamma \) and \( B \rightarrow X_u l \nu \) spectra in which shape function drops out.
Analysis also includes subleading shape functions.

Uses $B \to X_s \gamma$ and $B \to X_c \ell \nu$ to constrain shape functions.

Uses different parameterizations to estimate dependence on functional form.
**\( V_{ub} \) without shape function**

- Babar \( |V_{ub}| \) values from weighted integrals over the two spectra appeared very recently in hep-ph/0702072.

- They use 3 different theoretical evaluations of the weight function.

| Method    | \( |V_{ub}| \cdot 10^3 \)         |
|-----------|----------------------------------|
| LLR [3, 4]| \( 4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28 \) |
| Neubert [6]| \( 4.01 \pm 0.27 \pm 0.29 \pm 0.32 \pm 0.27 \) |
| BLNP [7, 8]| \( 4.40 \pm 0.30 \pm 0.41 \pm 0.23 \) |

Uncertainties: \( b \to u \), \( b \to s \), theory, \( V_{ts} \)
$B \rightarrow X_s \gamma$

Chasing New Physics with 4-loops
A sensitive probe of New Physics

- FCNC process. Loop suppressed in the SM
- e.g. strong constraint on the MSSM

\[ b V_{tb} V_{ts}^* s \]

\[ b \rightarrow \tilde{t} \tilde{W} \]

\[ b \rightarrow \tilde{b} \tilde{g} \]
Elements of the NNLO calculation

• After large theory effort over the last years we have obtained the rate at NNLO level.
• $O(20)$ papers with necessary calculations.
• Needs all of the following at NNLO:
  1. Effective weak Hamiltonian
     a. Matching at high scale (2- and 3-loop)
     b. RG evolution to low scale (3- and 4-loop)
  2. Calculation of rate at NNLO
     a. OPE for total rate (← so far only estimate)
     b. Effect of the photon energy cut
        $E_\gamma > E_0 \approx 1.9 \text{GeV}$. 
Match and run, match and run...

- Many energy scales. Use different effective theories to treat each of them in turn.
- $O(10^5)$ diagrams along the way...

$M_W, m_t, ...$

$SM \rightarrow$ Fermi Theory $\rightarrow$ SCET $\rightarrow$ HQET

$M_X$ after photon energy cut
NNLO result

- Experimental average (HFAG)
  \[ \text{Br}(\bar{B} \to X_s \gamma) = (3.55 \pm 0.24 \pm 0.09 \pm 0.10 \pm 0.03) \cdot 10^{-4} \]

- for cut $E_\gamma > E_0 = 1.6$ GeV
- stat.+syst., extrapolation to low $E_0$, $b \to \gamma d$ subtr.

- Theory @ NNLO (hep-ph/0610067 with hep-ph/0609232)
  \[ \text{Br}(\bar{B} \to X_s \gamma) = (2.98 \pm 0.26) \cdot 10^{-4} \]

- $+4^-6%$ perturbative, $4%$ parametric, $5%$ power corrections, $3%$ interpolation in $m_c$.

- $1.4\sigma$ below exp. value. $1$-$2\sigma$ below NLO value. (Gambino Misiak ‘01 found $\text{BR} = (3.6\pm0.3) \times 10^{-4}$ at NLO.)