Lattice QCD with Applications to B Physics

Lecture 3

Recapitulation

Strong Coupling and Confinement

Errors and Their Controls

CKM: f_K/f_π, B Mixing

Predictions

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Recapitulation of Lecture 2

Lattice scalar fields are a simple generalization of the quantum mechanics.

Gauge symmetry is reasonably straightforward to respect: basic variables are SU(3) matrices $U_\mu(x)$, one for each "link" of the lattice.

Fermions (*i.e.*, quarks) pose theoretical problems: it is difficult to retain full chiral symmetry (a là Ginsparg-Wilson); doing so is computationally much more demanding.

Fermions (even ultra-local ones) are computationally demanding (compared to gauge fields). The Dirac operator $\not{D} + m_q$ becomes a sparse matrix; the time for the quark sea, and for the valence quark propagators, grows as a power of

$$\frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} = \frac{\pi}{a}\frac{1}{m_q}.$$

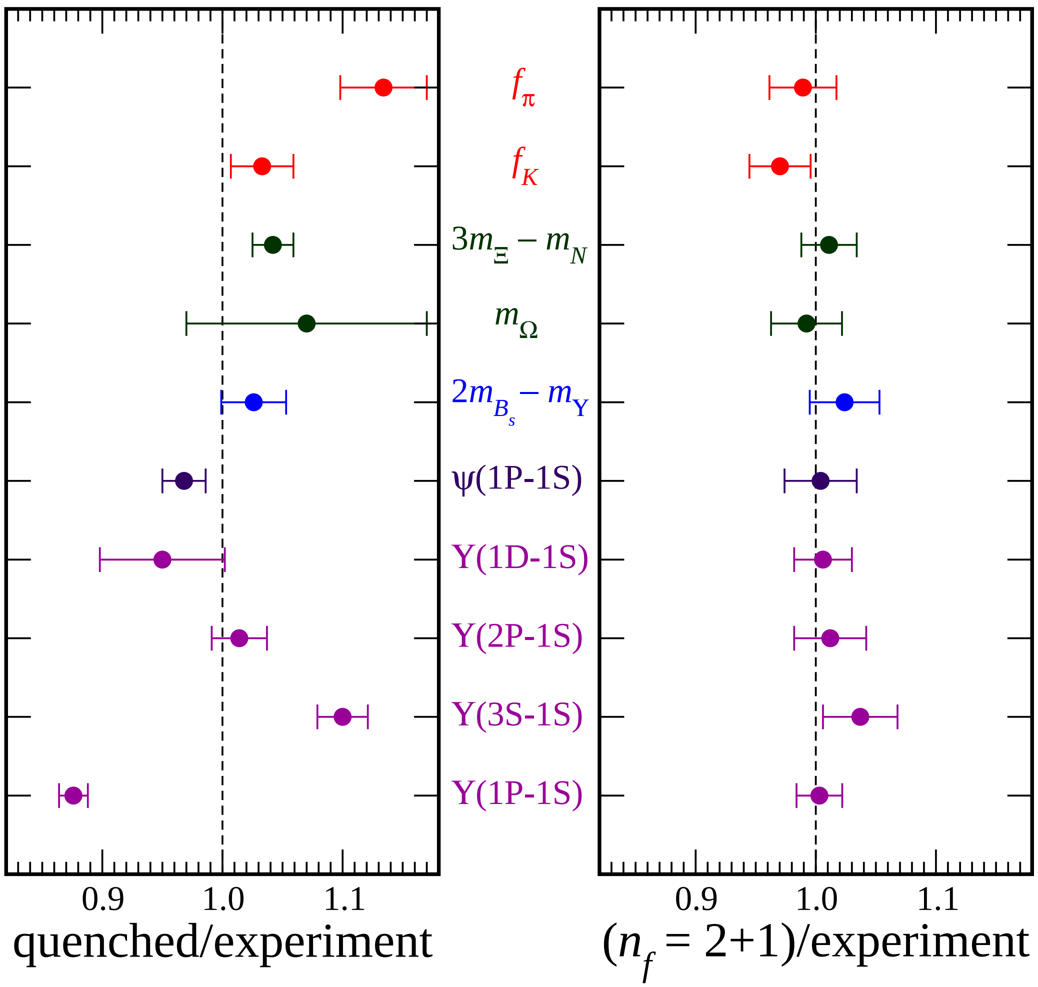
Consequently, no one has achieved masses as small as those of the up and down quarks. Chiral Extrapolation.

In 2003, however, 26 authors produced this plot: $\Longrightarrow$

Set $1+4$ free parameters with $1+4$ meson masses.

Quenched (on left) shows discrepancies as much as 10–15%.

Unquenched QCD (on right) shows discrepancies of a few %—within the error bars.



Davies *et al.*, hep-lat/0304004

The five fiducial quantities ($m_{\Upsilon(2S)} - m_{\Upsilon(1S)}$, m_π^2, m_K^2, m_{D_s}, and $m_{\Upsilon(1S)}$) and the nine shown are all, in a certain sense, "gold-plated."

The gold-plated class includes stable-particle masses and hadronic matrix elements with at most one hadron in the initial or final states

Unstable particles and non-leptonic decays inevitably entail multi-particle states—much more difficult (to be explained later).

This may seem like a disappointing restriction.

There are, however, gold-plated matrix elements for extracting *all* CKM elements $|V_{qq'}|$, except $|V_{tb}|$. (Top quark decays before hadronizing.)

It's not unrealistic to expect the theoretical uncertainty in the CKM matrix to be reduced to a few percent in the next few years.

Disclaimer

These results obtained with improved staggered quarks in the sea: 2+1 flavors.

Recall that staggered fermions come in four "tastes." The extra degrees of freedom are removed by using $[\det_4(\not{D}_{\text{stag}}+m)]^{1/4}$ instead of $\det_1(\not{D}+m)$.

At non-zero lattice spacing, this prescription leads to violations of unitarity, observed in numerical data for the a_0 propagator.

Conjectured (based on plausibility arguments) to be manageable using "rooted staggered chiral perturbation theory."

No proof, however, and therefore remains controversial. A recent review of these issues deemed rooted staggered quarks to be "ugly" but likely viable in the continuum limit [S.R. Sharpe, hep-lat/0610094].

Other methods of treating the quark sea are 6–12 years behind.

Only simulations with staggered sea quarks have made close enough contact with chiral perturbation and, thus, reproduce a wide variety of masses, mass splitting, and decay constants.

The “fourth-root” prescription remains controversial.

Other fermion methods (improved Wilson, twisted mass, domain-wall) are starting to enter the chiral regime.

Quark Confinement

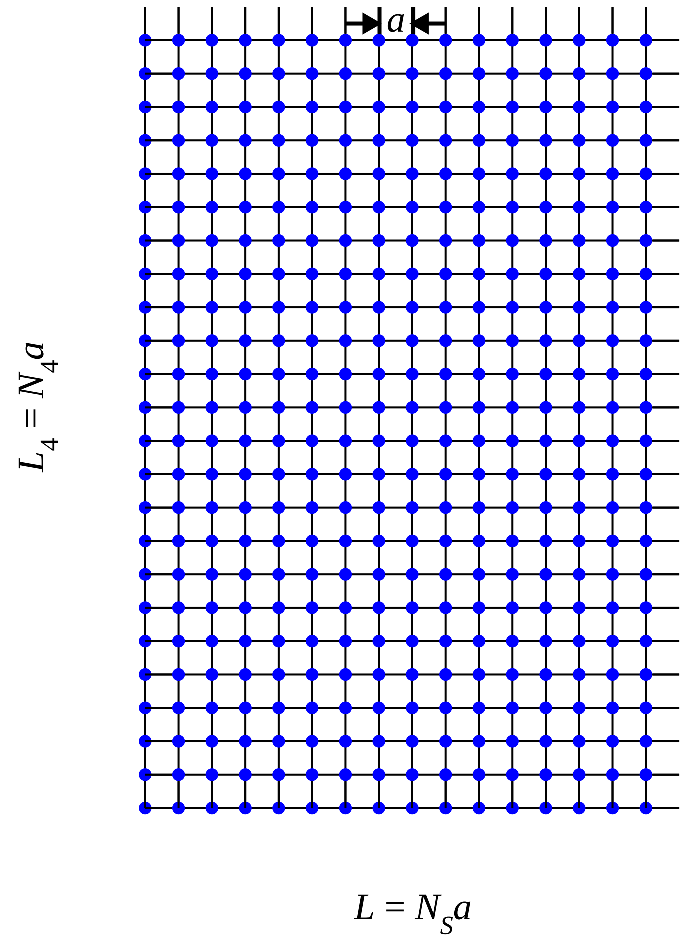
Consider the product of Us around a large $r \times t$ rectangle, $W(r,t)$.

Corresponds to a static quark and static anti-quarks, separating by distance r, propagating for time t.

$$\begin{aligned}\langle W(r,t)\rangle &= e^{-tV(r)} \\ &= \frac{1}{Z}\int \mathcal{D}U\, W(r,t) e^{-\beta\sum_{\mu,\nu,n} P_{\mu\nu}(n)} \\ &\propto \beta^{rt}\end{aligned}$$

Strong coupling expansion in small β.

$V(r) = \sigma r$, $\sigma \propto -\ln\beta$: confinement!



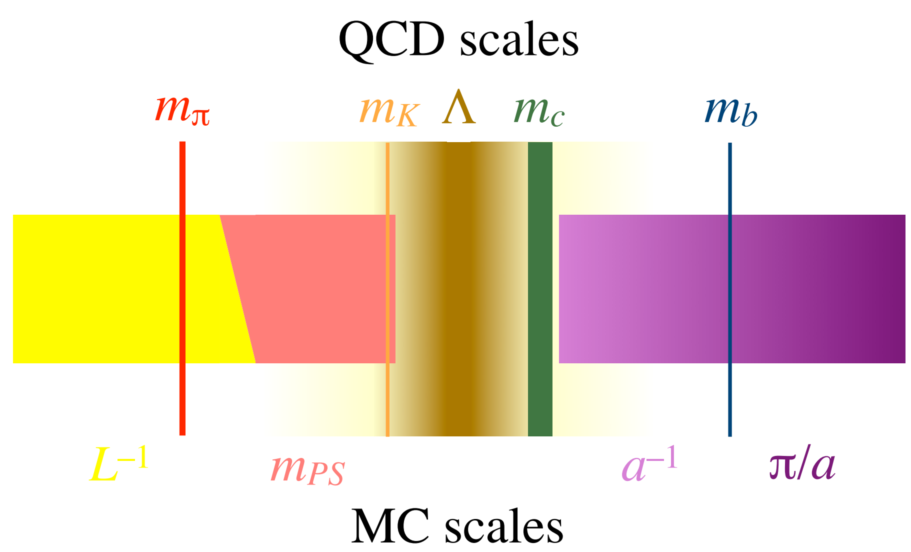
Sources of Uncertainty

source	control
"quenched" approx.*	just don't set $\det M \to 1$
$L < \infty$	finite-volume hadron EFT: e^{-mL}
$m_q > m_d$	chiral perturbation theory
$a > 0$	Symanzik effective field theory
$m_b \sim a^{-1}$	HQET ($\bar{q}Q$), NRQCD ($\bar{Q}Q$)

Most of the "extrapolations" needed are handled by fitting numerical data to a formula from an effective field theory.

The exception is the "quenched approximation" which means to omit the sea quarks, by setting the expensive $\det M$ to 1 in the MC weight.

* consigned to history; even most $n_f = 2$ projects now giving way to $2+1$.



$$m_\pi = 140\ \text{MeV} \quad m_K = 500\ \text{MeV}$$
$$\Lambda \sim 250 - 2500\ \text{MeV}$$
$$m_c = 1300\ \text{MeV} \quad m_b = 4200\ \text{MeV}$$

Remark on Heavy Quarks

There are some groups with a funny stance to HQET (and NRQCD).

They try to cope with $m_b \sim a^{-1}$ by taking a fictitiously small heavy quark mass m_Q, running with light quark methods and $m_Q a \lesssim 1$.

They boast of avoiding the “errors” of methods that take “effective theories” to heart.

Of course, they (ought to) use Symanzik effective theory to estimate their discretization errors, and they use HQET (in its most primitive form) to extrapolate $m_Q \to m_b$ (or $1/m_Q \to 1/m_b$).

These papers are easy to spot: they contain many pious statements but lack a full error budget.

Infrared 1: The Box

The effect of the finite volume on physics states is qualitatively different for 1-particle vs. multi-particle states.

It's useful (and correct) to think of every hadron as being surrounded by a cloud of other hadrons, first and foremost the pion (because it has the smallest mass).

For a 1-particle state, the issue is whether the rectangular shape distorts this cloud: such effects are suppressed by $e^{-cm_\pi L}$.

For 2- and multi-particle states one has to worry about rescattering. Hand-waving makes it clear; Lüscher formalism makes it possible.

A resonance (*e.g.*, ρ, Δ) or even a bound state near threshold (*e.g.*, ψ') fluctuates into two particles. Not impossible but less accurate.

We call quantities with one or zero hadron(s) in the initial or final state gold-plated, to emphasize that lattice QCD calculations of them are the most straightforward.

Infrared 2: The Pion

The pseudoscalars π, K, and η are "Goldstone bosons" of chiral symmetry breaking:

$$m_\pi^2 = (m_u + m_d)B, \quad m_K^2 = (m_s + m_l)B, \quad m_\eta^2 = [\tfrac{1}{6}(m_u + m_d) + \tfrac{2}{3}m_s]B,$$

where $B \approx 2.5$ GeV has come up before.

There is a well-developed effective theory for the dynamics (the cloud) of these Goldstone bosons, based on symmetry: chiral perturbation theory (χPT).

If, as has always been the case in practice, $(m_u, m_d) \to m_q > \frac{1}{2}(m_u + m_d)$, then the computer's hadrons live in an incorrect pion cloud.

Use χPT to subtract off the unphysical pion cloud and replace it with the physical one.

Chiral Perturbation Theory

Chiral perturbation theory (χPT) is a systematic method to compute the dependence of hadronic quantities on the masses of the light pseudoscalar mesons:

$$A = A_0 + A_1(\mu)\frac{m_\pi^2}{(4\pi f_\pi)^2} + A_\chi \frac{m_\pi^2}{(4\pi f_\pi)^2}\ln(m_\pi^2/\mu^2)$$

The last term is called a "chiral log". $f_\pi = 132$ MeV

Really the limiting behavior of the function obtained from 1-loop integrals.

Something non-analytic in $m_\pi^2 \propto m_q$ always appears; not always a log

e.g., $m_\pi^3 = (m_\pi^2)^{3/2}$ in masses of heavy hadrons.

Replace m_π with m_{PS}, the mass as calculated in the simulation, and fit.

Chiral symmetry constrains A_χ to something known or "knowable." It is not a completely free parameter.

Chiral Extrapolations of Decay Constants

$\pi^+ \to e^+\nu_e$: $\langle 0|A_\mu|\pi\rangle = -ip_\mu f_\pi$

Dots at 0.04 are experiment values.

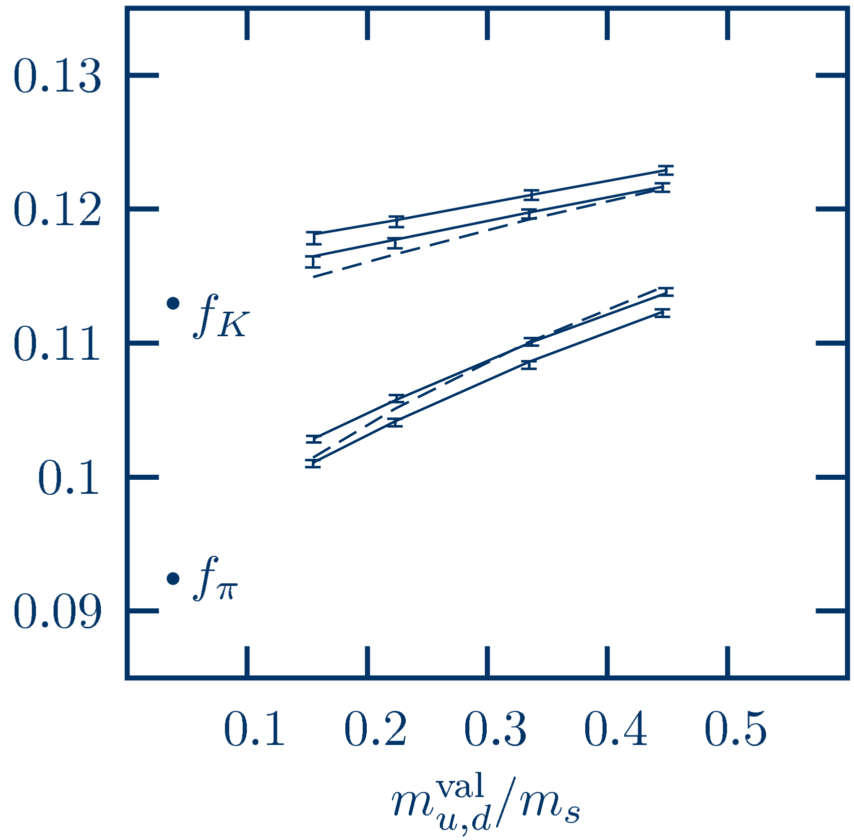
Error bars are lattice QCD.

Linear extrapolation (by eye) gets close.

A chiral log fit gets closer.

Correcting for $O(a^2)$ gets even closer. (On the ratio plot.)

An even more remarkable analysis [Aubin & Bernard] follows.



Davies *et al.*, hep-lat/0304004

χPT for Taste-Symmetry Violation

WARNING: this gets complicated!

For 4 species the taste symmetry group should be SU(4)$\times$SU(4).

Discretization breaks it to $\Gamma_4 \times$U(1), leading to more non-analytic contributions in χPT.

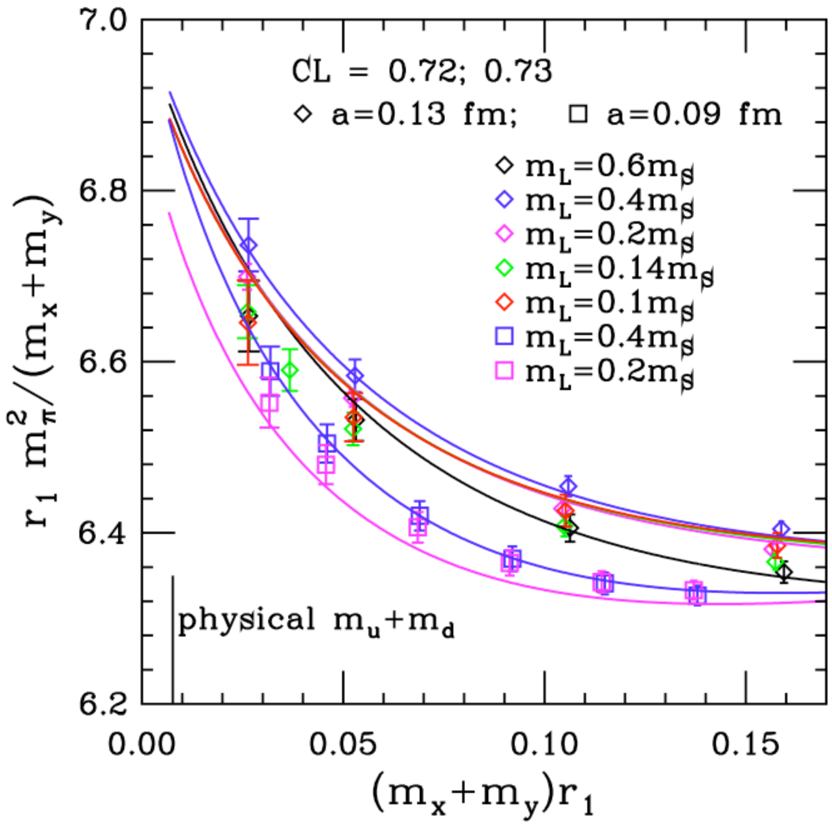
Also possible to account for $(\det M)^{n_f/M}$ in χPT: SU$(4|4-n_f)\times$SU$(4|4-n_f)$.

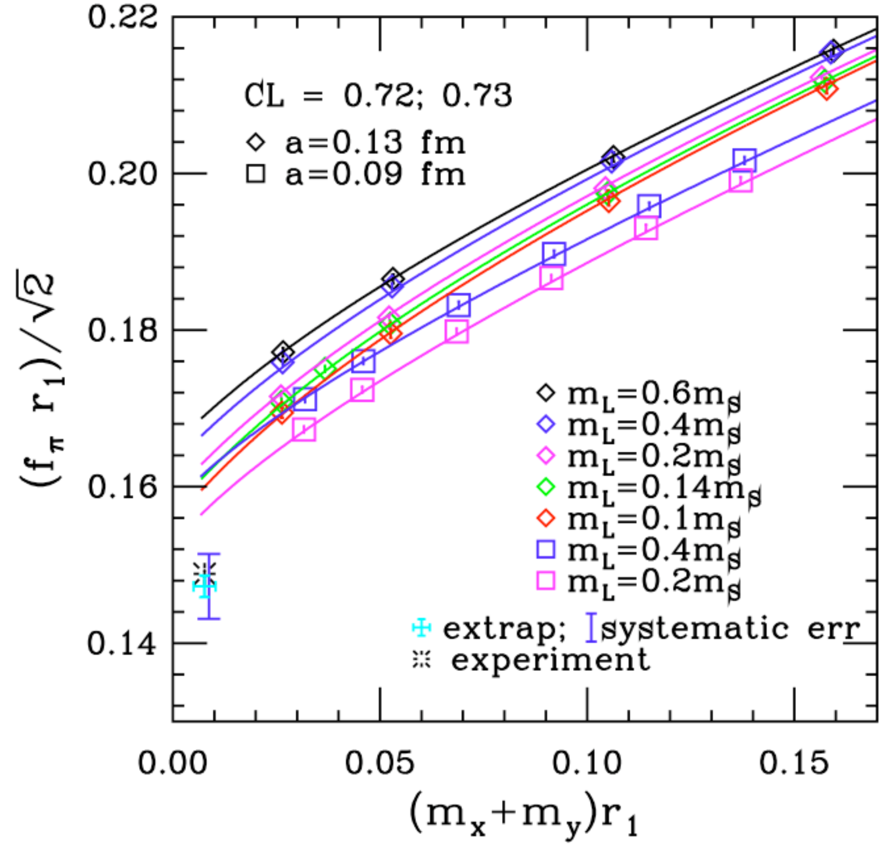
And even possible to account for $m_q^{\text{valence}} \neq m_q^{\text{sea}}$.

Aubin and Bernard put this all together to obtain unpresentable formulas.

Statistical precision of MILC is good enough to fit them.

χPT with Violations of Taste Symmetry





both fits represent a single fit to m_{sea}, m_{val} and a dependence

Physics Output

These fits yield a wealth of interesting information.

The decay constants f_π and f_K. In the ratio f_K/f_π some uncertainties cancel (*e.g.*, most of the statistical error). Marciano (for example) proposed using MILC's and f_K/f_π and measurements of $\Gamma_{K\to e\nu}/\Gamma_{\pi\to e\nu}$ to determine $|V_{us}/V_{ud}| = \tan\theta_c$.

The light quark masses [hep-lat/0609053]:

$$\begin{aligned} \bar{m}_s(2\ \text{GeV}) &= 90\ (0)(5)(4)(0)\ \text{MeV} \\ m_s/\hat{m} &= 27.2(0)(4)(0)(0), \quad 2\hat{m} = m_u + m_d \\ m_u/m_d &= 0.42(0)(1)(0)(4) \neq 0(!) \end{aligned}$$

where errors are statistics, other systematics, matching to $\overline{\text{MS}}$, EM effects.

Some couplings of the NLO chiral effective Lagrangian, called Gasser-Leutwyler coefficients or low-energy constants. Useful in light hadron phenomenology.

CKM Matrix

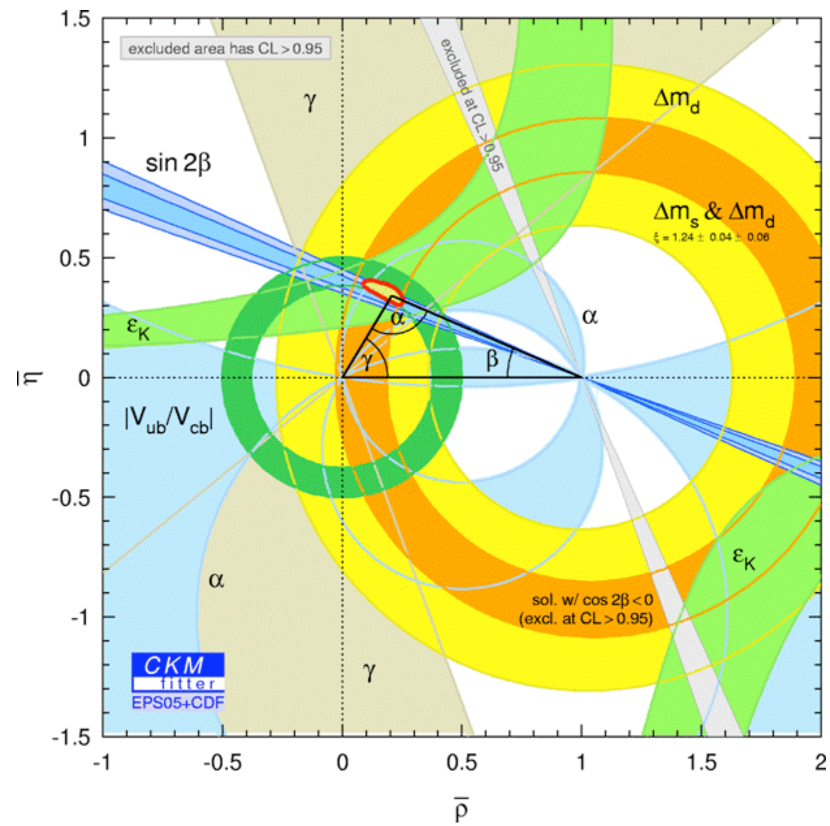
$|V_{cb}|$ from $B \to D^{(*)} l\nu$.

$|V_{ub}|$ from $B \to \pi l\nu$.

$|V_{td}|$ from B^0-$\bar{B}^0$ mixing frequency Δm_d.

$|V_{ts}|$ from B^0_s-$\bar{B}^0_s$ mixing frequency Δm_s.

$\bar{\eta}(1-\bar{\rho})$ from K^0-$\bar{K}^0$ mixing parameter ε_K.



All of these constraints can be obtained by combining a calculation of a gold-plated matrix element with an experimental measurement.

B_q^0-$\bar{B}_q^0$ Mixing

In the Standard Model, the theoretical expression for the oscillation frequency is

$$\Delta m_q = \left(\frac{G_F^2 m_W^2 S_0}{16\pi^2 m_{B_q}}\right) |V_{tq}^* V_{tb}|^2 \eta_B \mathcal{M}_q,$$

where $q \in \{d, s\}$, S_0 is an Inami-Lim function, η_B is a short-distance QCD correction.

$\mathcal{M}_q$ is the hadronic matrix element for $B_q^0 \leftrightarrow \bar{B}_q^0$ transitions:

$$\mathcal{M}_q = \langle \bar{B}_q^0 | [\bar{b}\gamma^\mu(1-\gamma^5)q][\bar{b}\gamma_\mu(1-\gamma^5)q] | B_q^0 \rangle$$

For historical reasons one usually writes

$$\mathcal{M}_q = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}$$

and focuses on the decay constants f_{B_q} and the bag parameters B_{B_q}.

The decay constant: $\langle 0 | \bar{b}\gamma_\mu\gamma^5 q | B_q^0 \rangle = -ip_\mu f_{B_q}$.

All quantities have the usual sources of uncertainty: Monte Carlo statistics, discretization effects, matching LGT to QCD. . . .

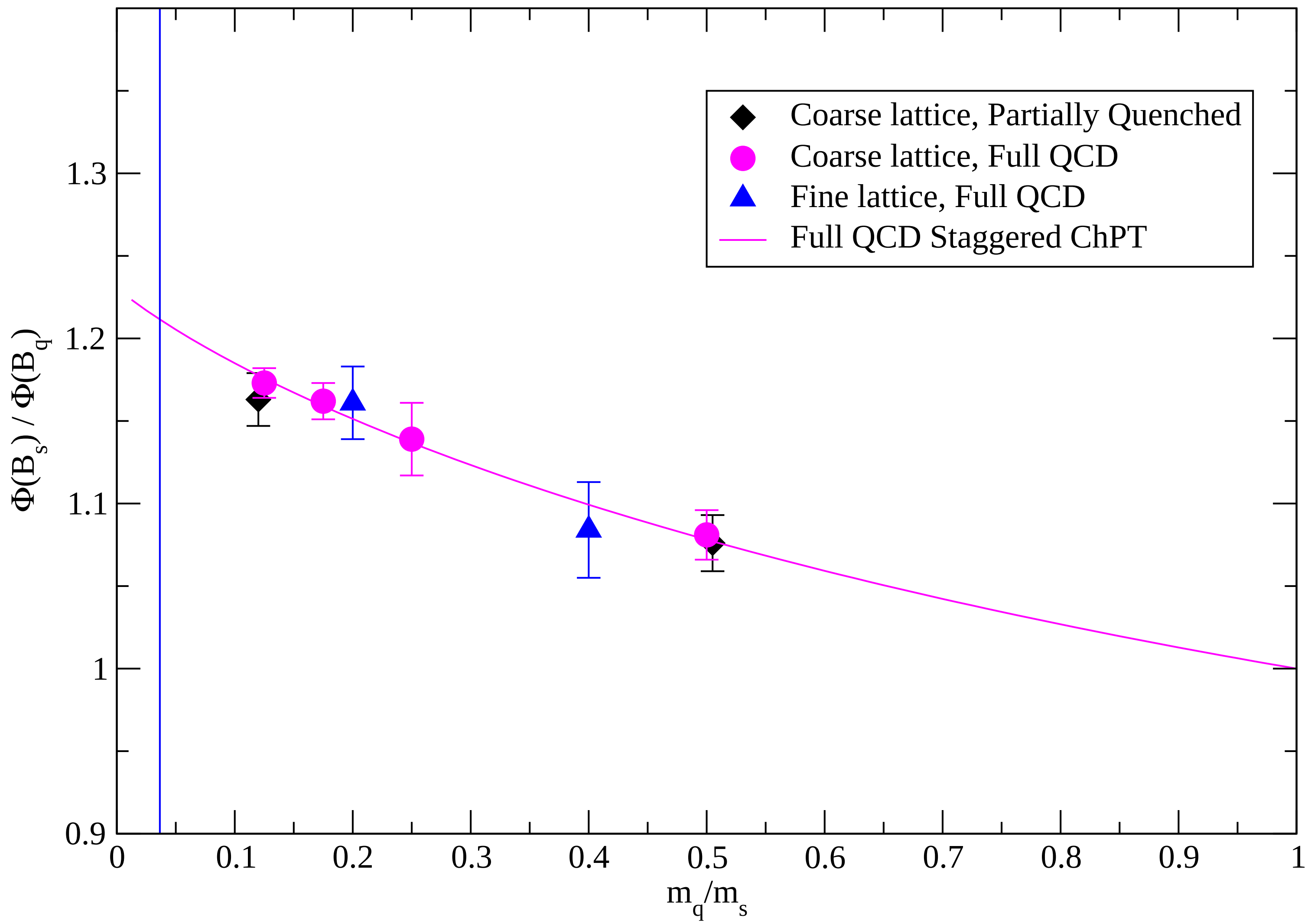
The B_s matrix elements do not have much sensitivity to the chiral extrapolation. The only light quarks are sea quarks, and once the sea is light enough, the dependence on m_{sea} is weak.

The B_d matrix elements are very sensitive to the mass of the light valence quark: this is where the chiral logarithm comes from. Sketch.

Therefore, you should take from lattice QCD both the B_s and ratios:

$$
\begin{array}{ccc}
f_{B_s}, & \sqrt{B_{B_s}}, & f_{B_s}\sqrt{B_{B_s}}, \\
\dfrac{f_{B_s}}{f_{B_d}}, & \dfrac{\sqrt{B_{B_s}}}{\sqrt{B_{B_d}}}, & \dfrac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} =: \xi
\end{array}
$$

If you take B_d quantities and ratios, the chiral extrapolation error is double counted.



Gray et al., hep-lat/0507015

Outlook

The recent results with improved staggered quarks are very promising.

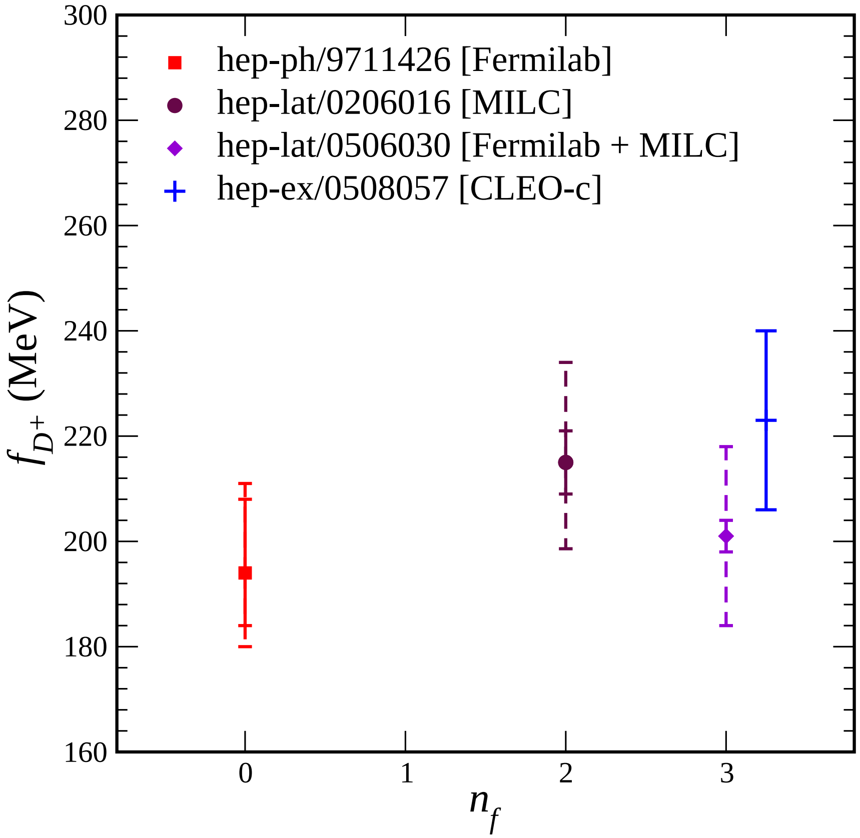
The goals are, however, extremely ambitious: uncertainties not merely small but robust enough to support a claim of new phenomena in B physics (if indeed it's there).

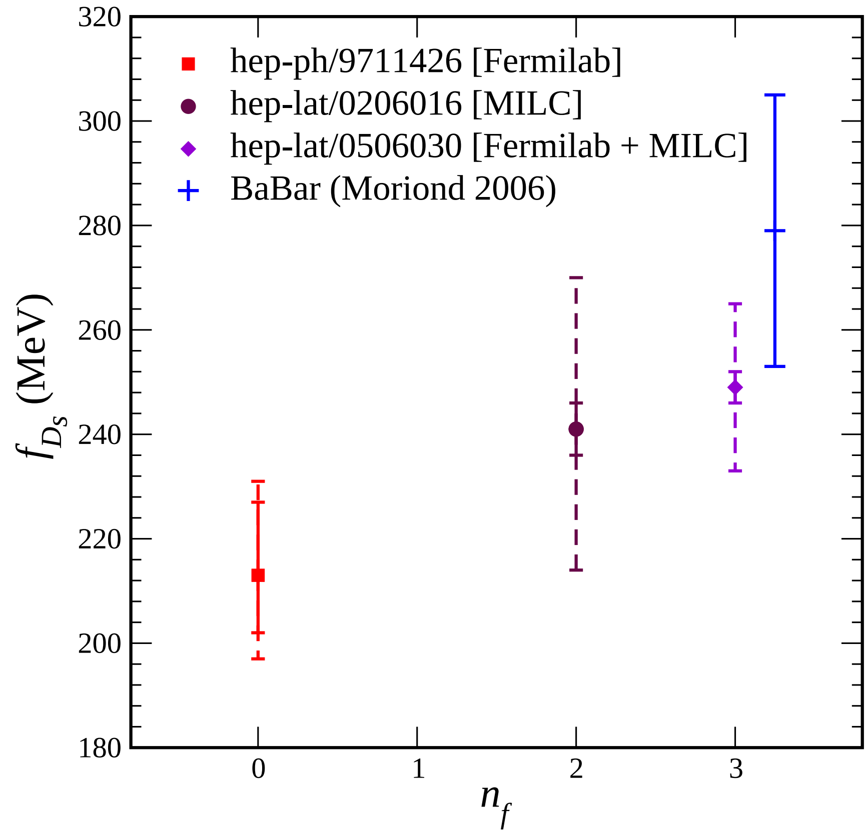
Any numerical simulation is, in the end, fairly inscrutable to outsiders. Are there any predictions? Any tests?

In 2003 we noticed several things that should be easy for us, and, though unmeasured or poorly measured, would be measured well soon.

Decay constants of D and D_s mesons (from CLEO-c, BaBar); shape of semileptonic $D \to Kl\nu$ form factors (FOCUS, Belle, CLEO-c); the mass of the B_c (CDF).

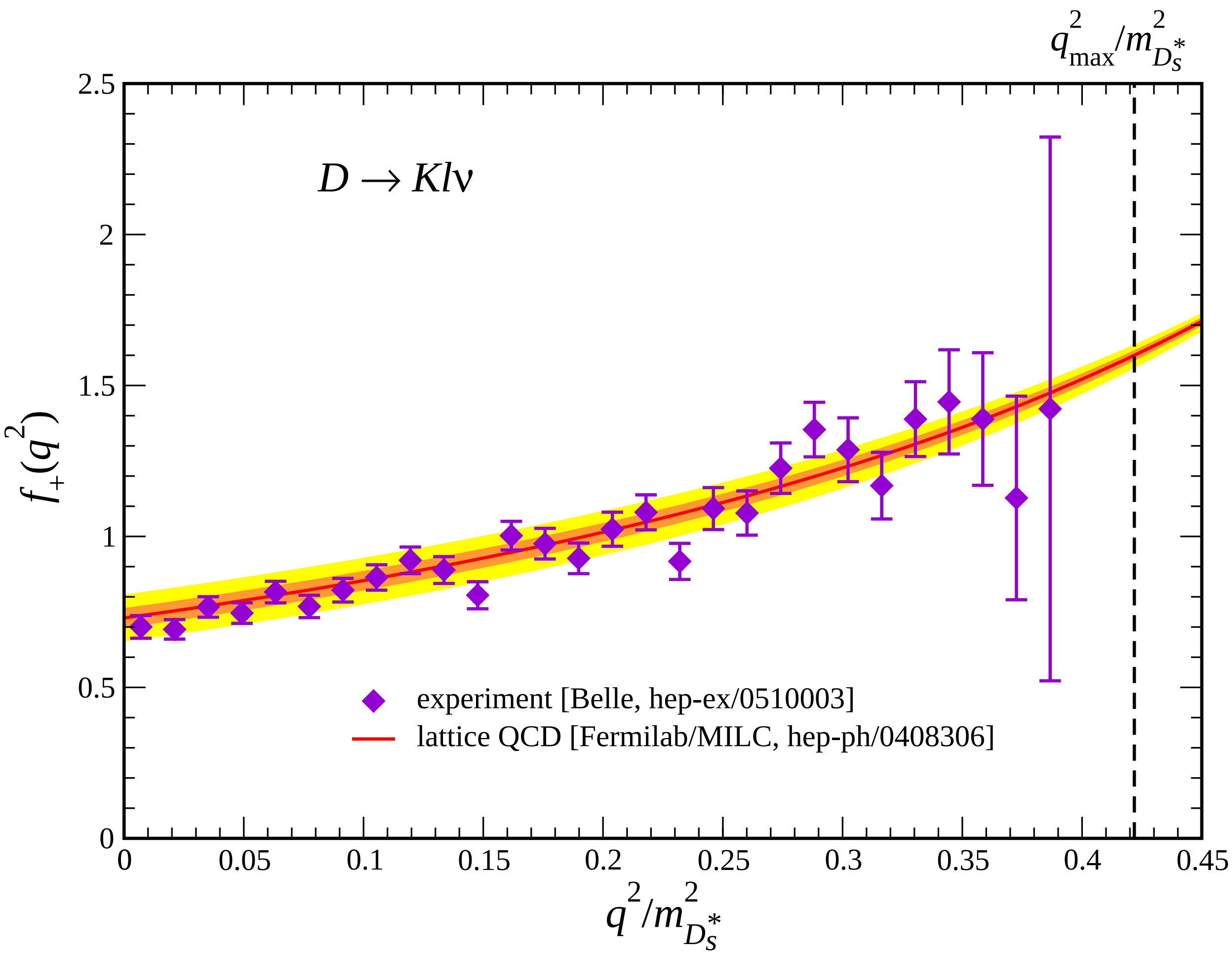
Leptonic D Decay





ratio: $\sqrt{m_{D^+}} f_{D^+} / \sqrt{m_{D_s}} f_{D_s} = 0.786(42)\ [\text{lat}] = 0.779(93)[\text{expt}]$

Semileptonic D Decay



B_c Mass

$$m_{B_c}^{n_f=0} = 6386 \pm 9 \pm 15 \pm 98 \text{ MeV}$$

[Phys. Lett. B 453, 289 (1999)]

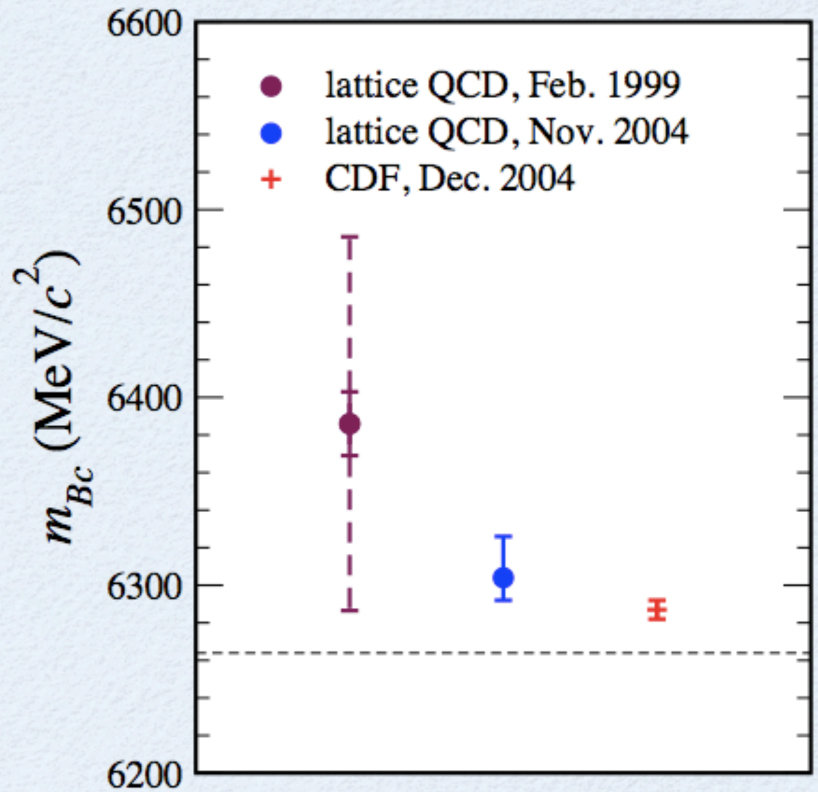
$$m_{B_c}^{2+1} = 6304 \pm 4 \pm 11^{+18}_{-0} \text{ MeV}$$

[`hep-lat/0411027` → PRL]

$$m_{B_c}^{\text{expt}} = 6287 \pm 5 \text{ MeV}$$

[CDF, W&C seminar, 12/3/2004]

hep-ex/0505076



Outlook

With several tests and predictions, the stage is set for a full suite of CKM-relevant lattice calculations.

The decay constants, bag parameters, and heavy-to-light form factors are being calculated with more than one heavy-quark technique.

The heavy-to-heavy form factor is being calculated only with the "Fermilab method" for heavy quarks (by us).

Cross-checks with different sea quarks will come over the next few years, once ensembles with theoretically better quarks are analyzed for a wide variety of masses, mass splittings, and matrix elements.