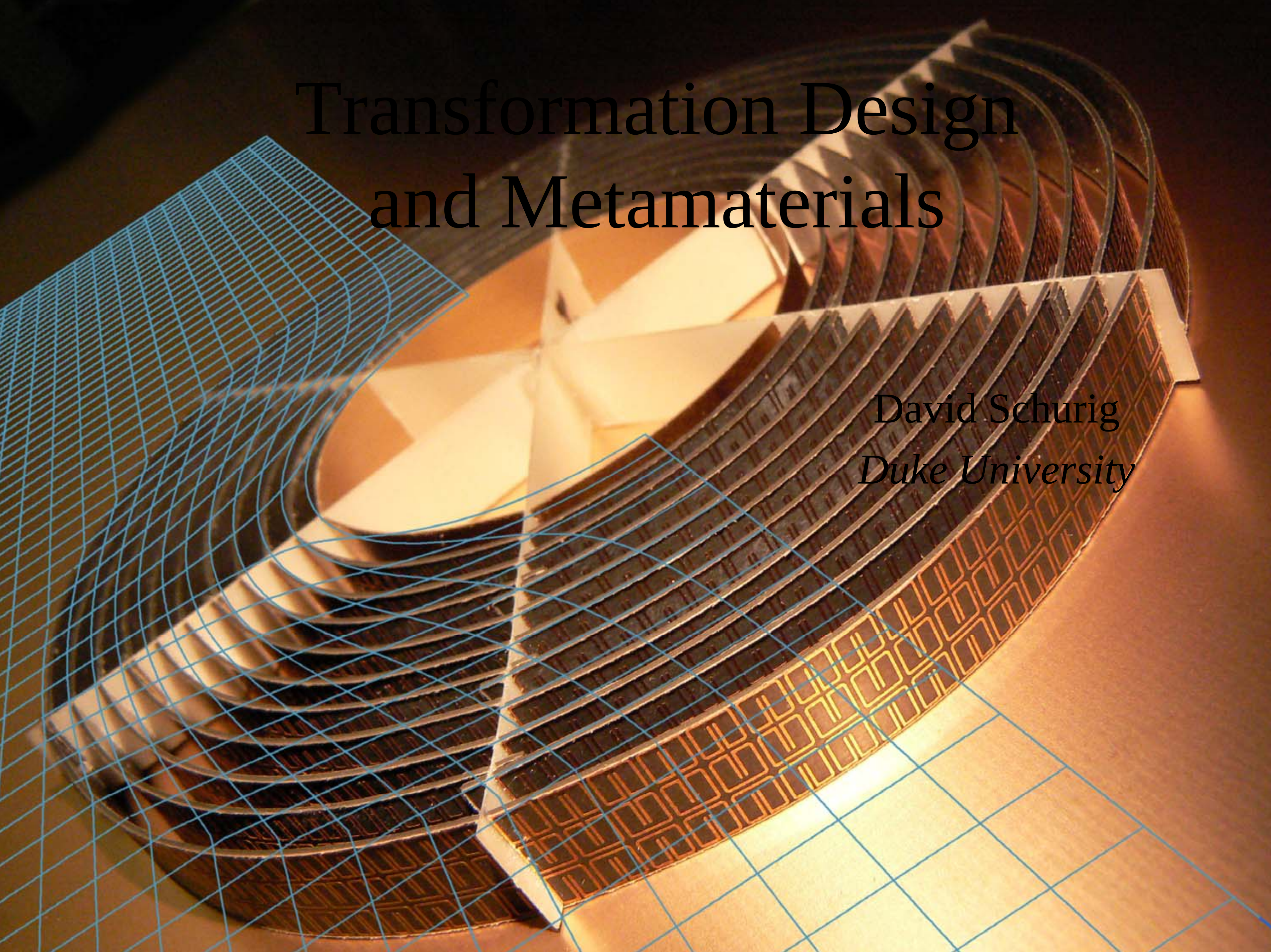


Transformation Design and Metamaterials

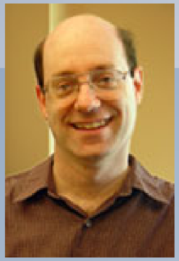
David Schurig
Duke University

The Transformation Design Group

John Pendry
Imperial College

Duke University

David Smith



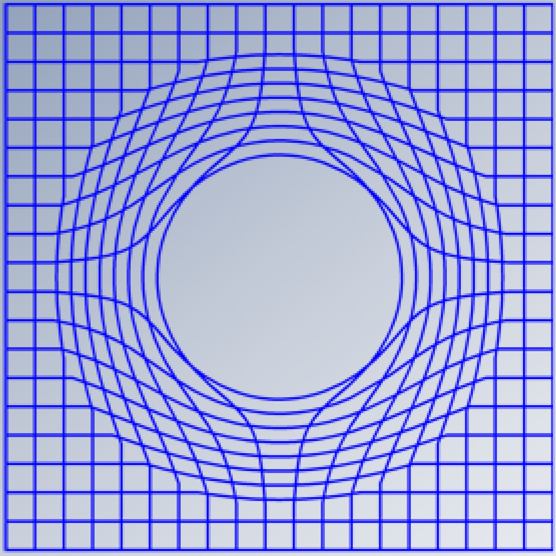
Steve Cummer

Jack Mock

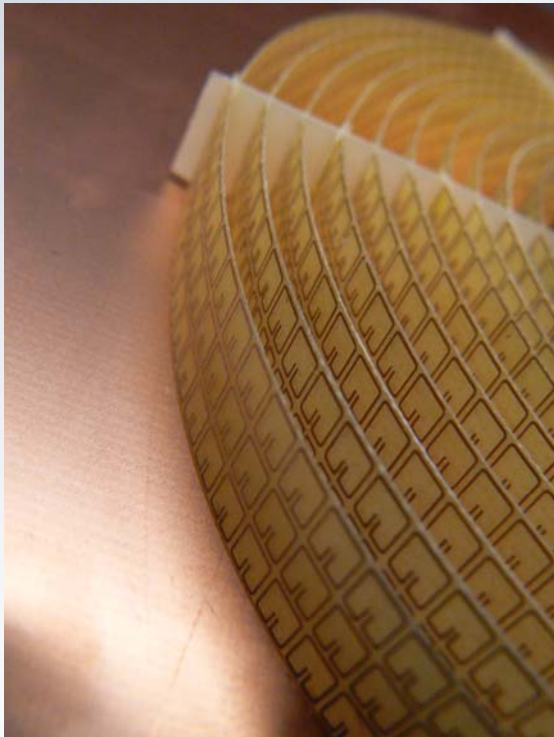
Bryan Justice

Tony Starr
SensorMetrix

Overview

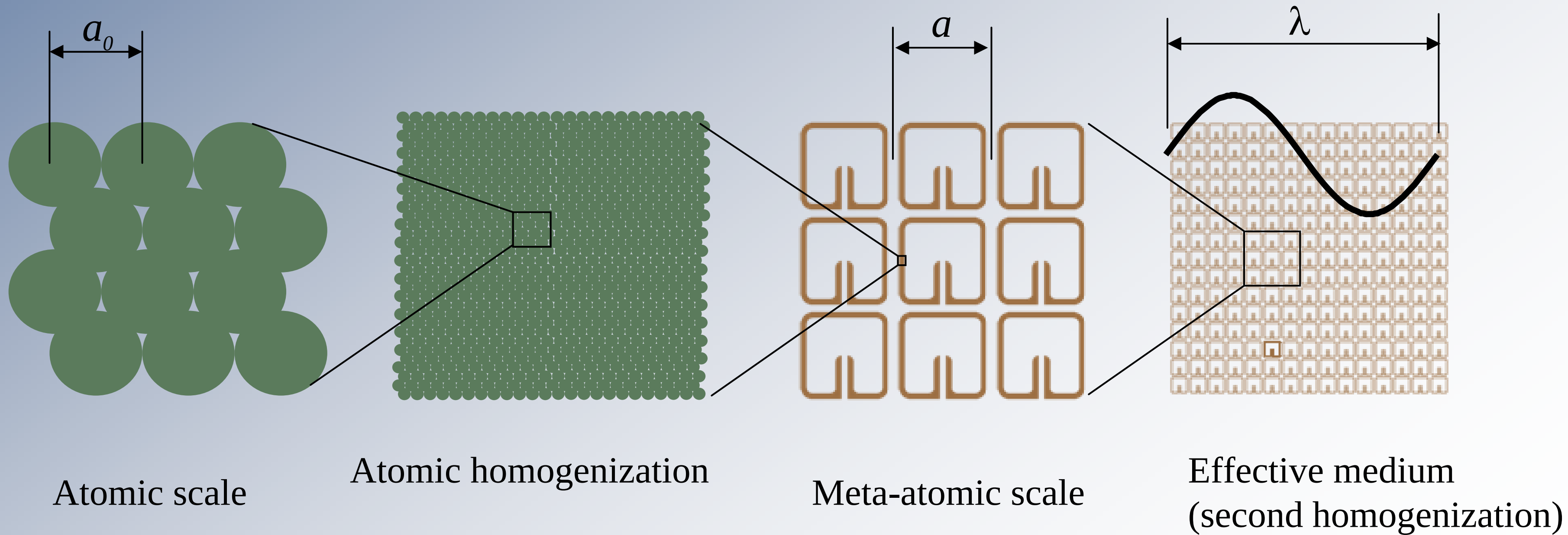


- Metamaterials
  - What are they?
  - What can you do with them?
- Transformation design
- 2D Microwave cloak



What are Metamaterials?

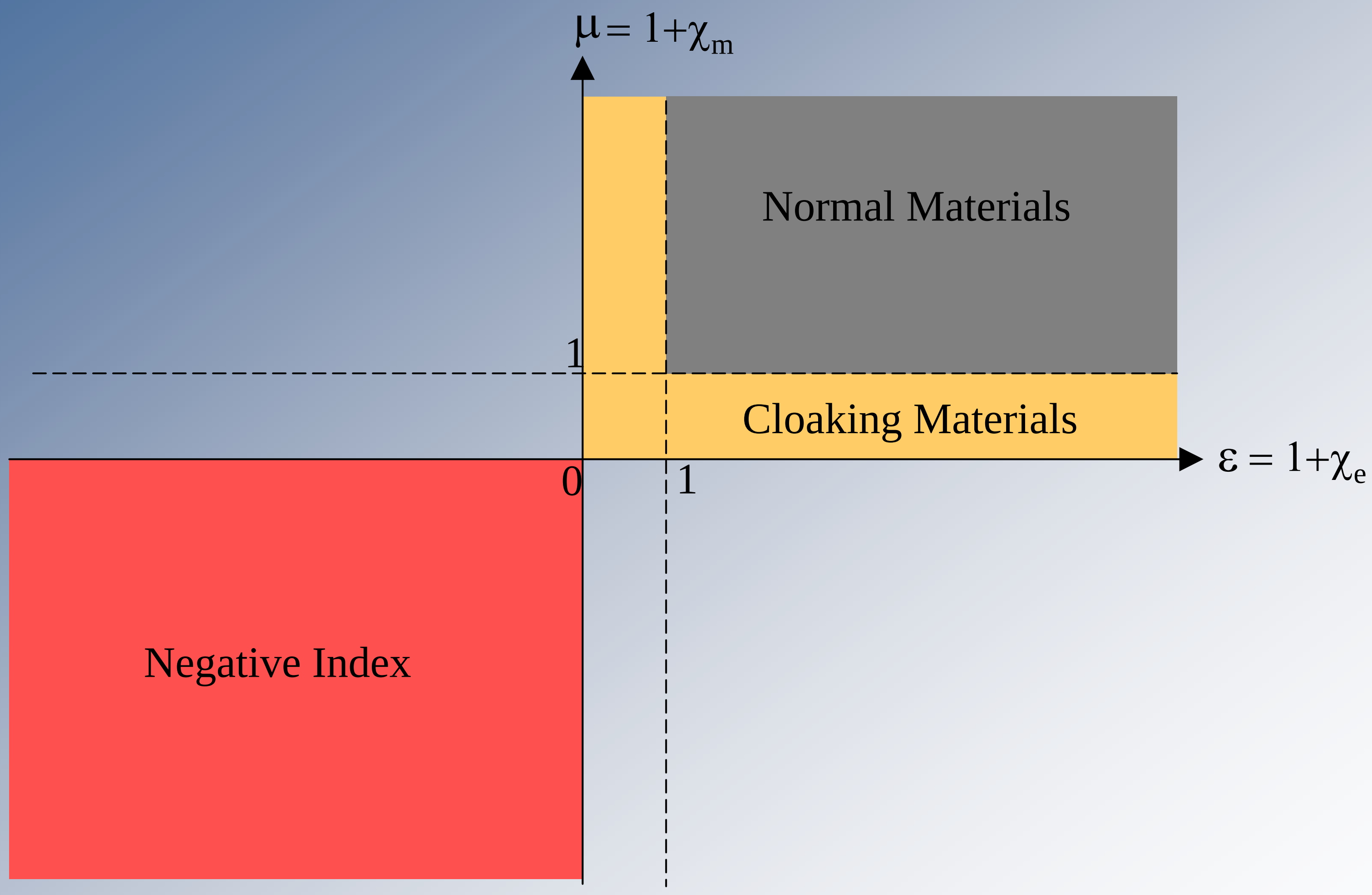
- Materials
  - Intrinsic properties
  - Described by a few parameters
- $a_0 << a << \lambda$
- Material scale order determines the properties



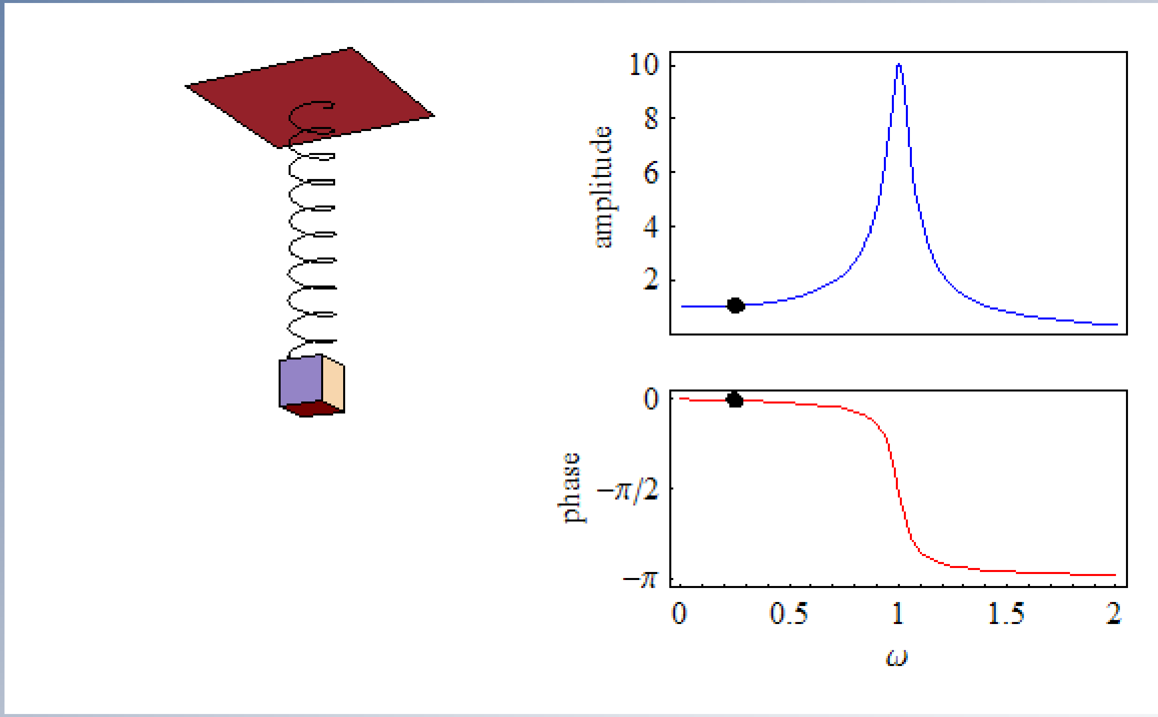
Why bother?

- Custom “meta-atoms”
- Complete control of “meta-atomic” order
- Not constrained to thermodynamic assembly
- Unique material properties not previously observed
  - Negative index
  - Large controlled gradients
  - Controlled Anisotropy

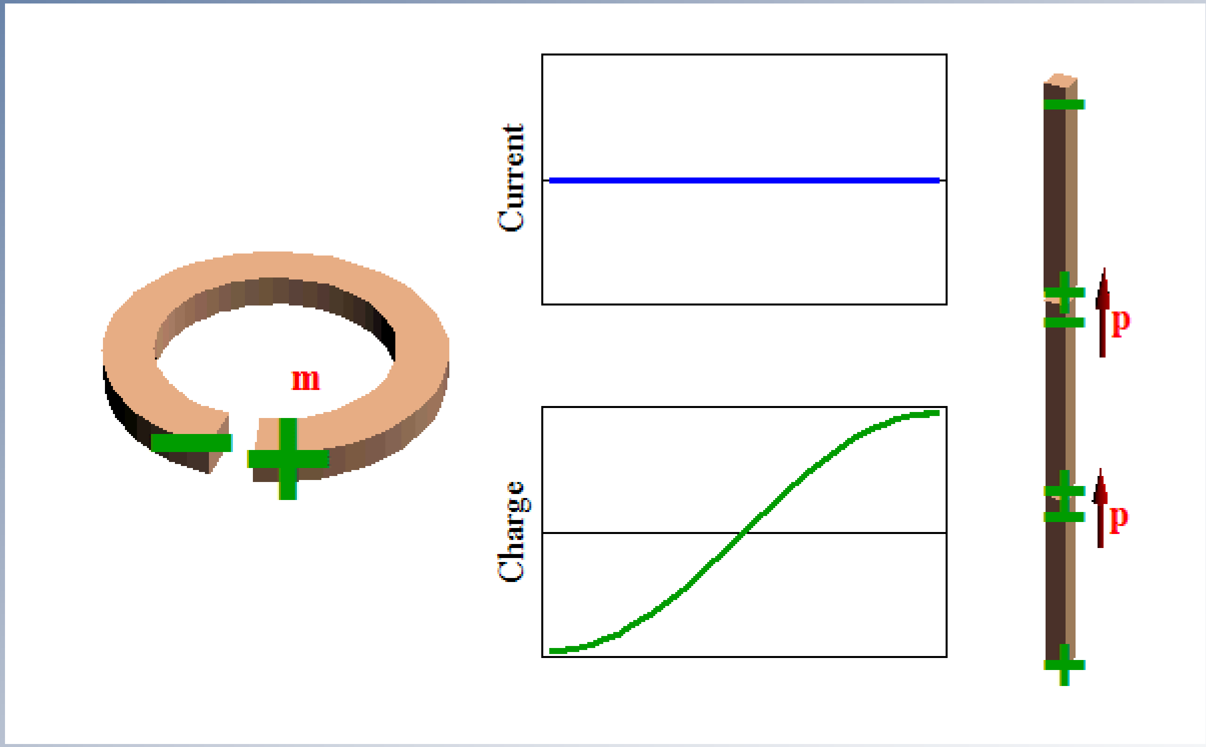
Electromagnetic Parameter Space



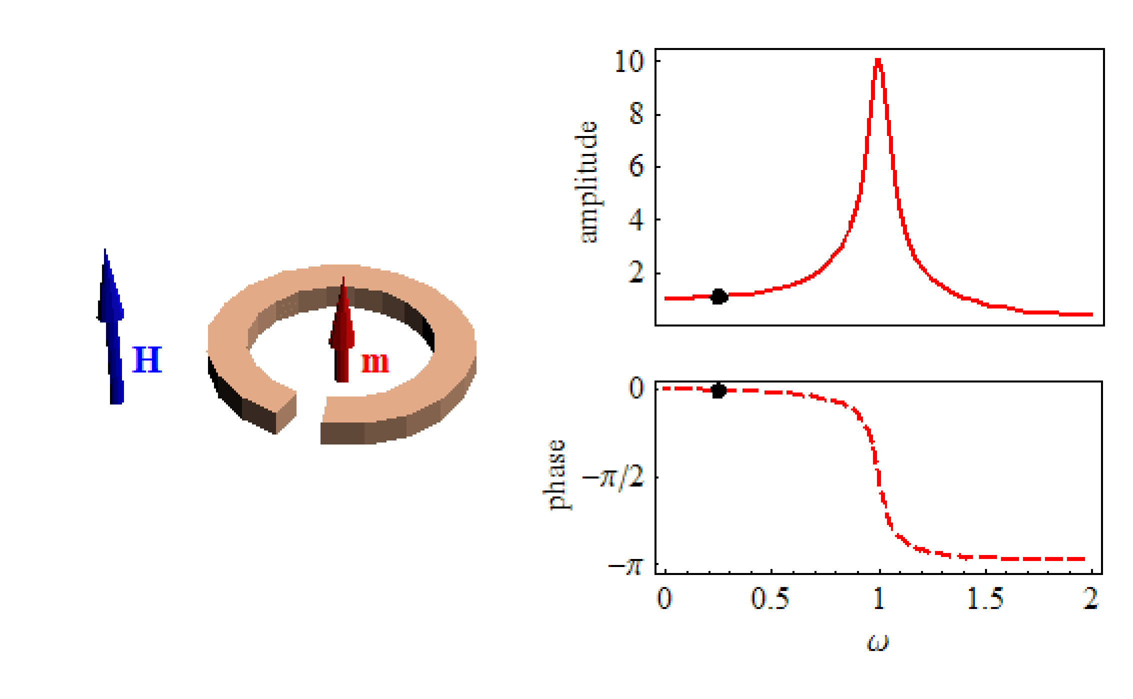
Resonance: Mechanical Analog



Ring and Cut Wire Resonators



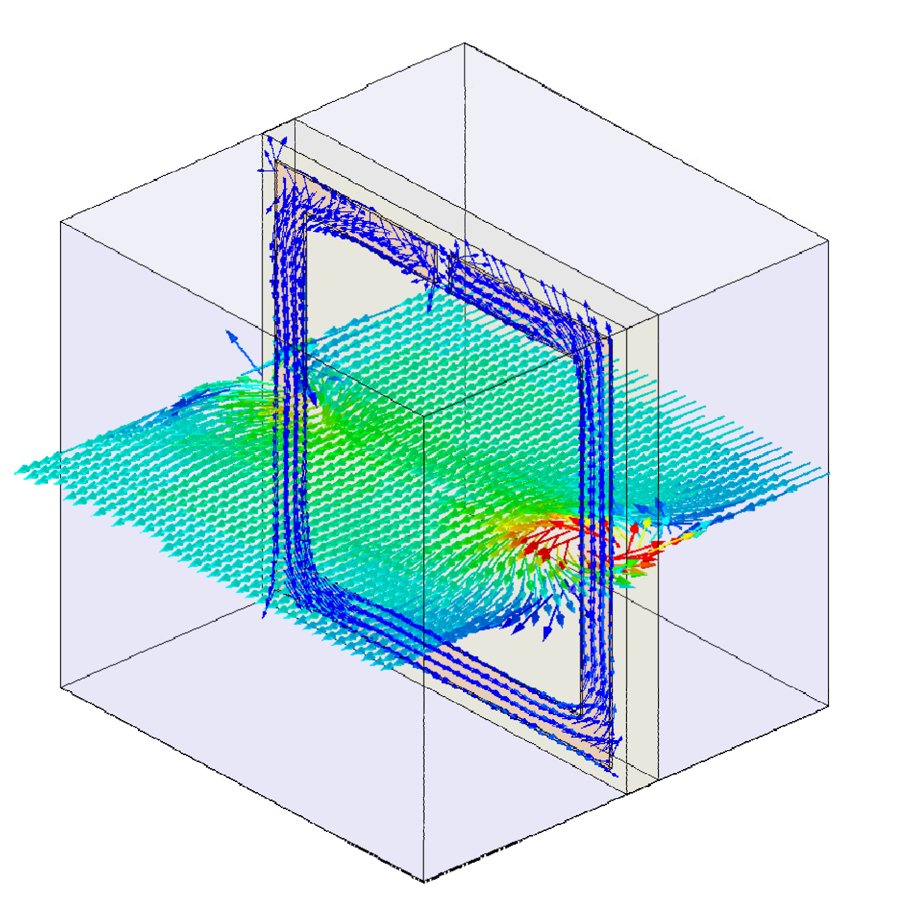
SRR Resonance



Microscale Simulations

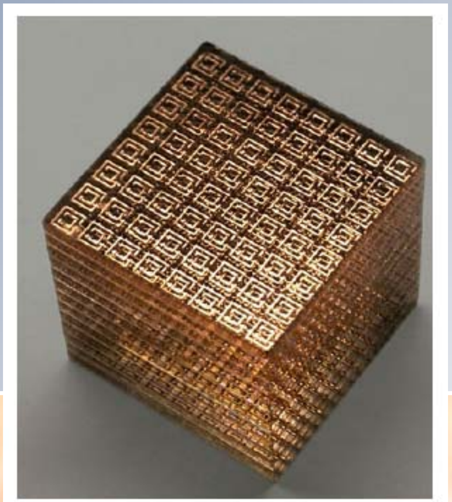
Commercial and custom codes can be used to find the local fields for any unit cell.

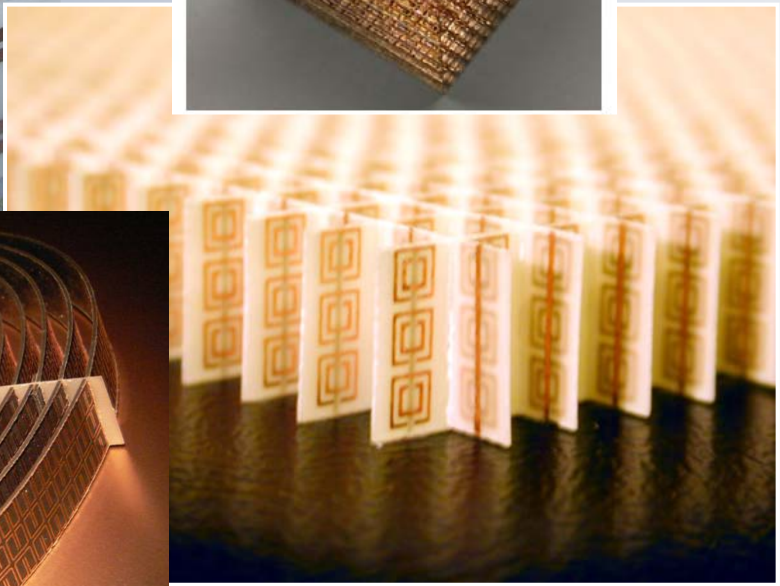
Comsol Multiphysics

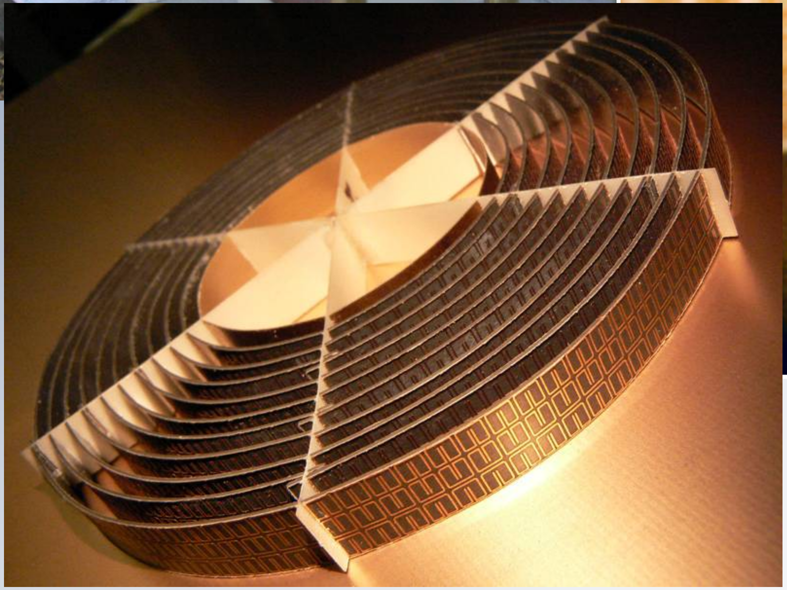


A *new* approach to *numerical* effective medium retrieval arises. Two examples are *field averaging* and *S-parameters retrieval*.

Microwave Metamaterial Samples







Transformation Design

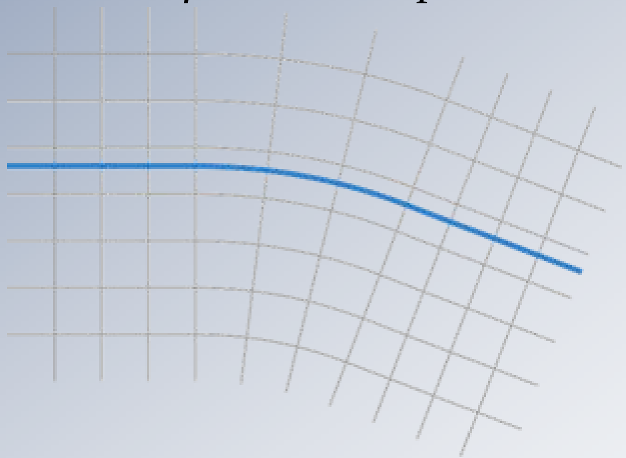
Transformation Design

- Why a new design paradigm?
- Metamaterials provide unprecedented control over electromagnetic properties.
- Not obvious how to take advantage of this flexibility.

Controlling Electromagnetic Fields

Traditional

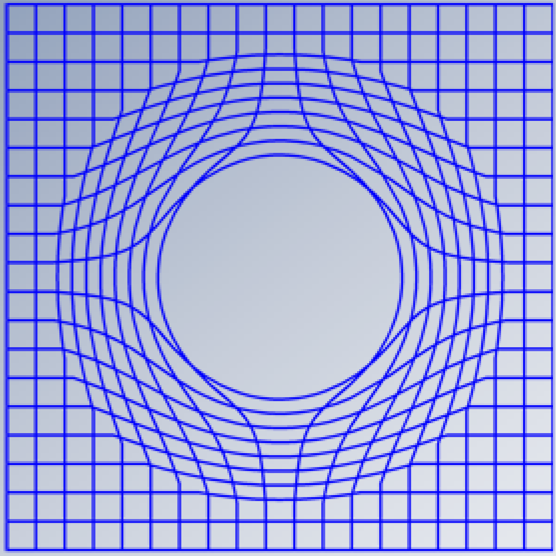
Transformation Optics



- Index and shape
- Simple materials

- Coordinate transform
- Complex materials
- Simultaneous geometric and wave design
- Relfectionless design

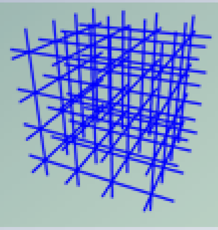
Transformation Design



- Imagine an interesting space
- Describe it with a coordinate transformation
- Calculate the material specification

Space I

- Flat space
  - 3D Cartesian : (x, y, z) $\qquad (g_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$



- Objects
  - electromagnetic fields: $\mathbf{E} \quad \mathbf{B} \quad \mathbf{D} \quad \mathbf{H}$
  - sources: $\rho \quad \mathbf{J}$
  - medium: $\boldsymbol{\varepsilon} \quad \boldsymbol{\mu} \quad \boldsymbol{\xi} \quad \boldsymbol{\zeta}$

- Physical Laws
  - Maxwell's Equations: $\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \qquad \nabla \cdot \mathbf{D} = \rho \qquad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$
  - Constitutive Equations: $\mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E} + \boldsymbol{\xi}\mathbf{H} \qquad \mathbf{B} = \boldsymbol{\zeta}\mathbf{E} + \boldsymbol{\mu}\mathbf{H}$

James Maxwell

Space II

Hermann Minkowski

- Flat space
  - Minkowski space: (ct, x, y, z) $\quad (g_{ij}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

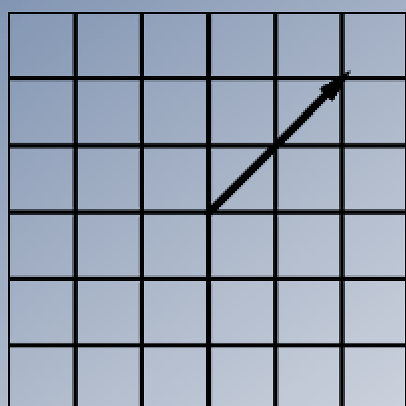
- Objects
  - electromagnetic fields: $(F_{\alpha\beta}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -cB_z & cB_y \\ -E_y & cB_z & 0 & -cB_x \\ -E_z & -cB_y & cB_x & 0 \end{pmatrix}$ $\quad (G^{\alpha\beta}) = \begin{pmatrix} 0 & -cD_x & -cD_y & -cD_z \\ cD_x & 0 & -H_z & H_y \\ cD_y & H_z & 0 & -H_x \\ cD_z & -H_y & H_x & 0 \end{pmatrix}$
  - sources: $(J^{\alpha}) = \begin{pmatrix} c\rho & J_x & J_y & J_z \end{pmatrix}$
  - medium: $c^{\alpha\beta\mu\nu}(\boldsymbol{\varepsilon}, \boldsymbol{\mu}, \boldsymbol{\xi}, \boldsymbol{\zeta})$

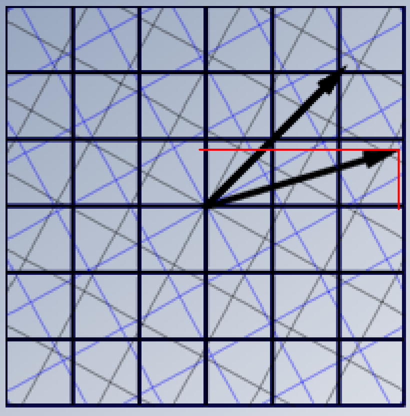
- Physical Laws
  - Maxwell's Equations $F_{\alpha\beta,\mu} + F_{\beta\mu,\alpha} + F_{\mu\alpha,\beta} = 0 \qquad G^{\alpha\beta}_{,\alpha} = J^{\beta}$
  - Constitutive Equations $G^{\alpha\beta} = \frac{1}{2} c^{\alpha\beta\mu\nu} F_{\mu\nu}$

Coordinate Transformations

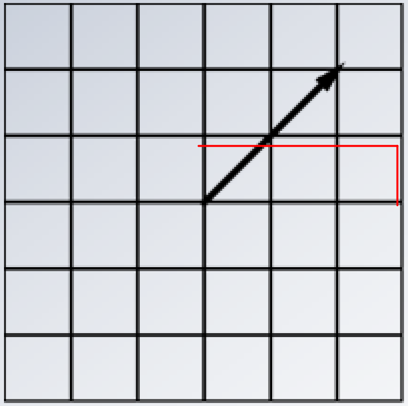
$$x' = x\cos\psi - y\sin\psi$$

$$y' = x\sin\psi + y\cos\psi$$





Passive



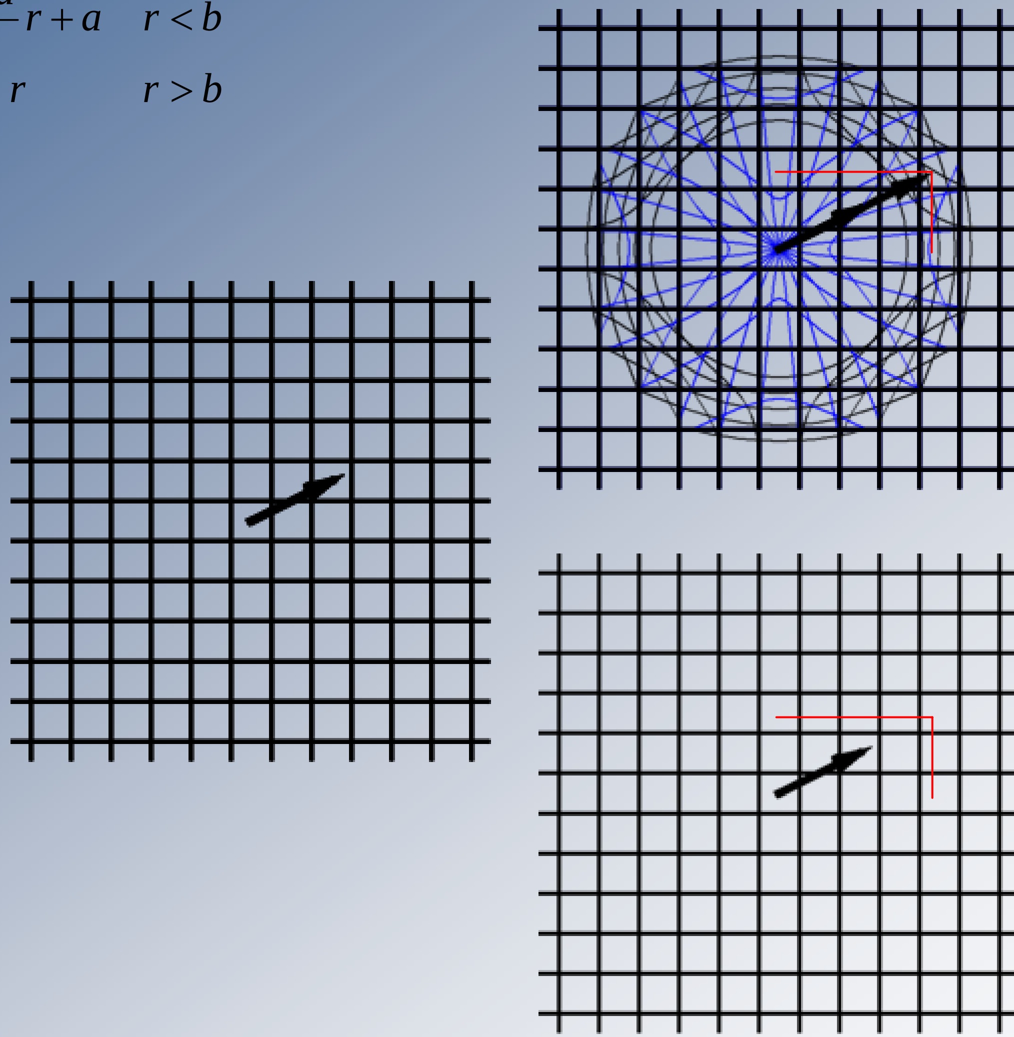
Active

Coordinate Transformations

Cloak of Invisibility

$$r' = \begin{cases} \dfrac{b-a}{b} r + a & r < b \\ r & r > b \end{cases}$$

$$\phi' = \phi$$



Passive

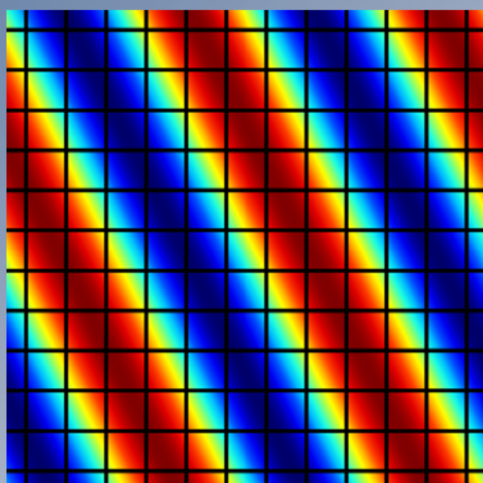
Active

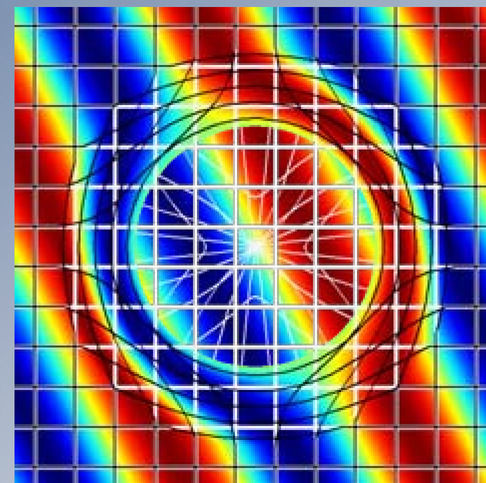
Coordinate Transformations

Cloak of Invisibility

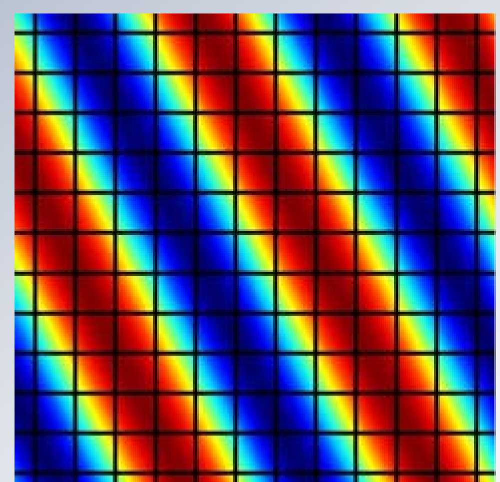
$$r' = \begin{cases} \dfrac{b-a}{b} r + a & r < b \\ r & r > b \end{cases}$$

$$\phi' = \phi$$





Passive



Active

Space II

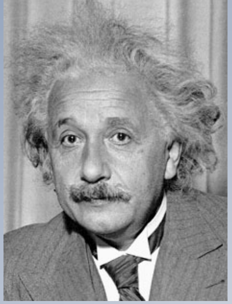
Hermann Minkowski

- Flat space
  - Minkowski space: (ct, x, y, z) $\quad (g_{ij}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- Objects
  - electromagnetic fields: $(F_{\alpha\beta}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -cB_z & cB_y \\ -E_y & cB_z & 0 & -cB_x \\ -E_z & -cB_y & cB_x & 0 \end{pmatrix}$ $\quad (G^{\alpha\beta}) = \begin{pmatrix} 0 & -cD_x & -cD_y & -cD_z \\ cD_x & 0 & -H_z & H_y \\ cD_y & H_z & 0 & -H_x \\ cD_z & -H_y & H_x & 0 \end{pmatrix}$
  - sources: $(J^\alpha) = (c\rho \quad J_x \quad J_y \quad J_z)$
  - medium: $c^{\alpha\beta\mu\nu}(\boldsymbol{\varepsilon}, \boldsymbol{\mu}, \boldsymbol{\xi}, \boldsymbol{\zeta})$

- Physical Laws
  - Maxwell's Equations $\quad F_{\alpha\beta,\mu} + F_{\beta\mu,\alpha} + F_{\mu\alpha,\beta} = 0 \qquad G^{\alpha\beta}_{,\alpha} = J^\beta$
  - Constitutive Equations $\quad G^{\alpha\beta} = \frac{1}{2} c^{\alpha\beta\mu\nu} F_{\mu\nu}$

Form Invariance



Hermann
Weyl

Transformation Properties

$$F_{\alpha'\beta'} = \Lambda^{\alpha}_{\alpha'}\Lambda^{\beta}_{\beta'}F_{\alpha\beta}$$

$$G^{\alpha'\beta'} = \left|\det(\Lambda)\right|^{-1}\Lambda^{\alpha'}_{\alpha}\Lambda^{\beta'}_{\beta}G^{\alpha\beta}$$

$$c^{\alpha'\beta'\mu'\nu'} = \left|\det(\Lambda)\right|^{-1}\Lambda^{\alpha'}_{\alpha}\Lambda^{\beta'}_{\beta}\Lambda^{\mu'}_{\mu}\Lambda^{\nu'}_{\nu}c^{\alpha\beta\mu\nu}$$

$$x^{\alpha'} = x^{\alpha'}\left(x^{\alpha}\right)$$
$$x^{\alpha} = x^{\alpha}\left(x^{\alpha'}\right)$$

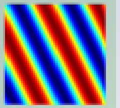
$$\Lambda^{\alpha'}_{\alpha} = \frac{\partial x^{\alpha'}}{\partial x^{\alpha}}$$

Electromagnetic Equations

$$F_{\alpha\beta,\mu} + F_{\beta\mu,\alpha} + F_{\mu\alpha,\beta} = 0$$

$$G^{\alpha\beta}_{,\alpha} = 0$$

$$G^{\alpha\beta} = \frac{1}{2}c^{\alpha\beta\mu\nu}F_{\mu\nu}$$



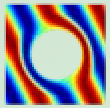
holonomic

Electromagnetic Equations
Transformed

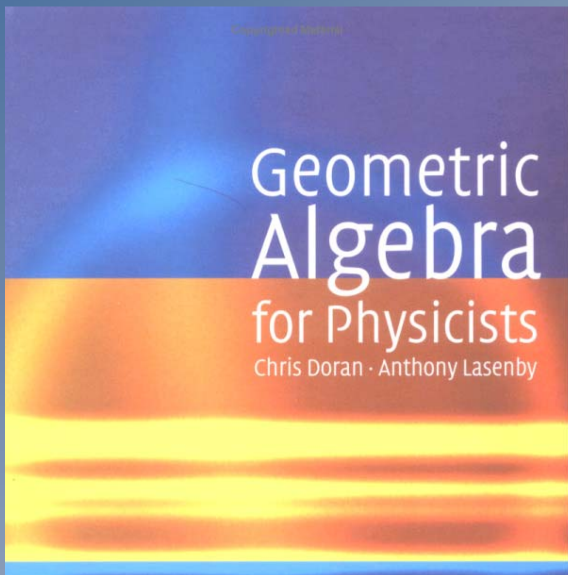
$$F_{\alpha'\beta',\mu'} + F_{\beta'\mu',\alpha'} + F_{\mu'\alpha',\beta'} = 0$$

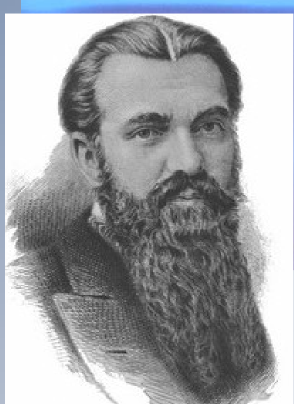
$$G^{\alpha'\beta'}_{,\alpha'} = 0$$

$$G^{\alpha'\beta'} = \frac{1}{2}c^{\alpha'\beta'\mu'\nu'}F_{\mu'\nu'}$$

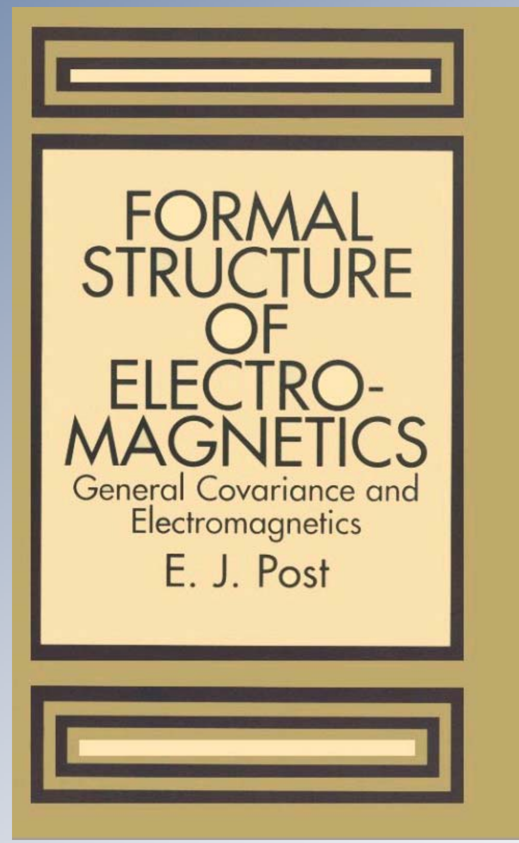


Resources





Clifford



WIKIPEDIA

Maxwell’s Equations

Spatial Transformations

$$c^{\alpha'\beta'\mu'\nu'} = \left|\det(\Lambda)\right|^{-1} \Lambda^{\alpha'}_{\alpha} \Lambda^{\beta'}_{\beta} \Lambda^{\mu'}_{\mu} \Lambda^{\nu'}_{\nu} c^{\alpha\beta\mu\nu}$$

Time invariance

$$\varepsilon^{i'j'} = \left|\det(\Lambda)\right|^{-1} \Lambda^{i'}_{i} \Lambda^{j'}_{j} \varepsilon^{ij}$$

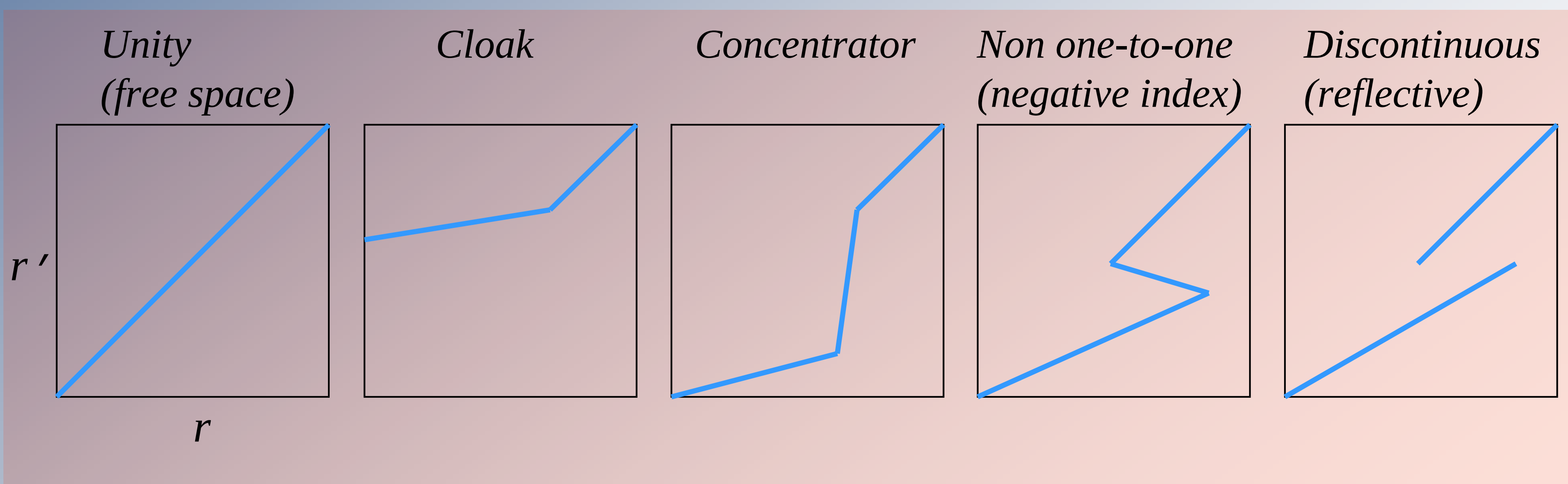
$$\mu^{i'j'} = \left|\det(\Lambda)\right|^{-1} \Lambda^{i'}_{i} \Lambda^{j'}_{j} \mu^{ij}$$

$$x^{i'} = x^{i'}\left(x^{i}\right)$$

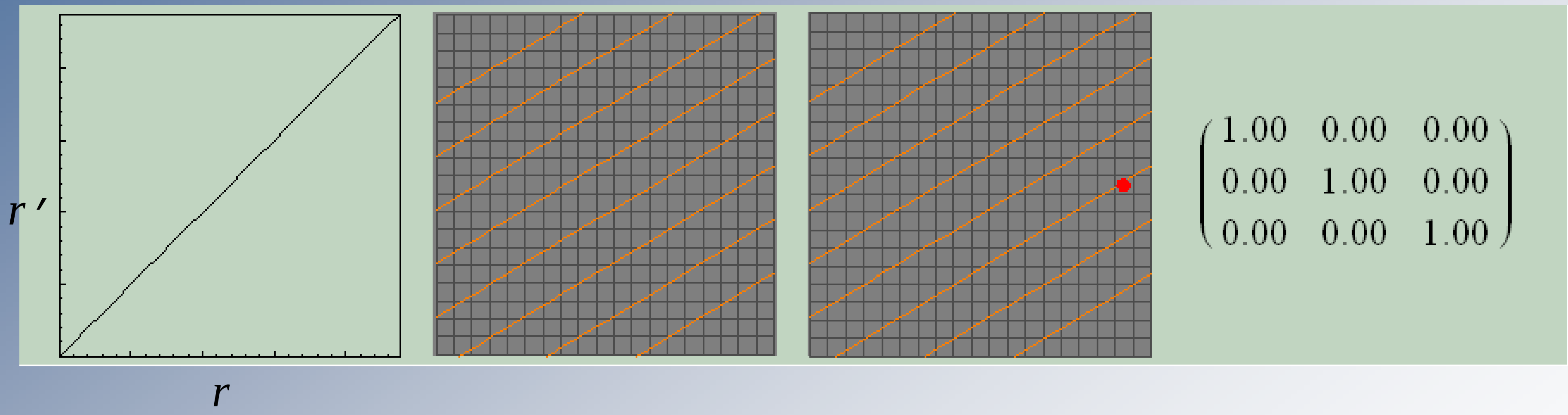
$$\Lambda^{i'}_{i} = \frac{\partial x^{i'}}{\partial x^{i}}$$

Radial Transforms

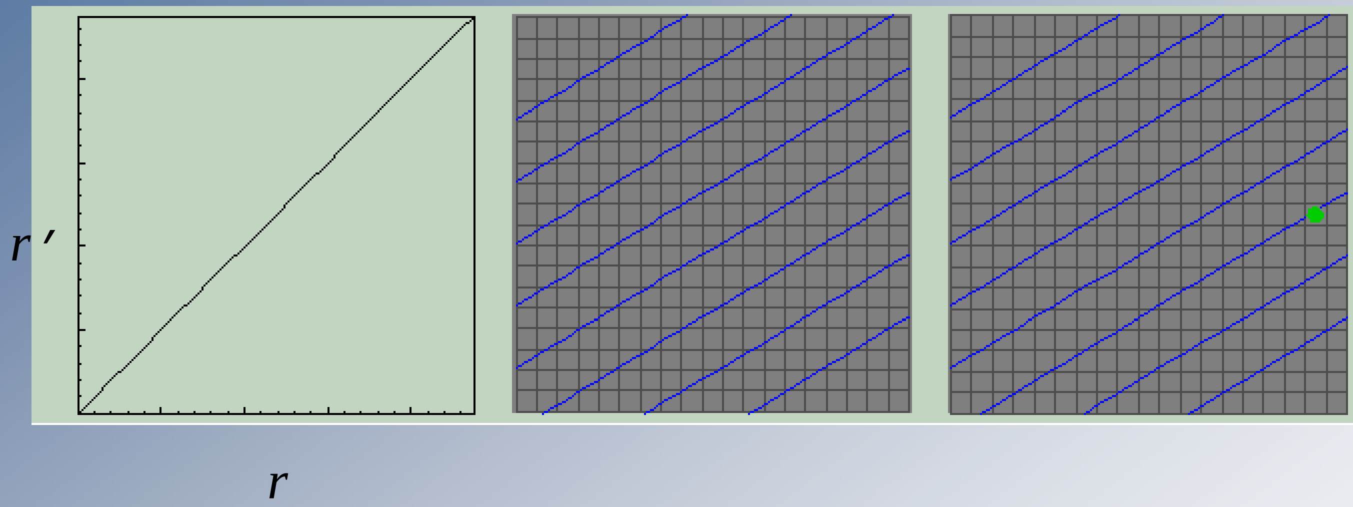
spherical	*cylindrical*
$r' = f(r)$	$r' = f(r)$
$\theta' = \theta$	$\theta' = \theta$
$\phi' = \phi$	$z' = z$



Radial Transforms: Cloak



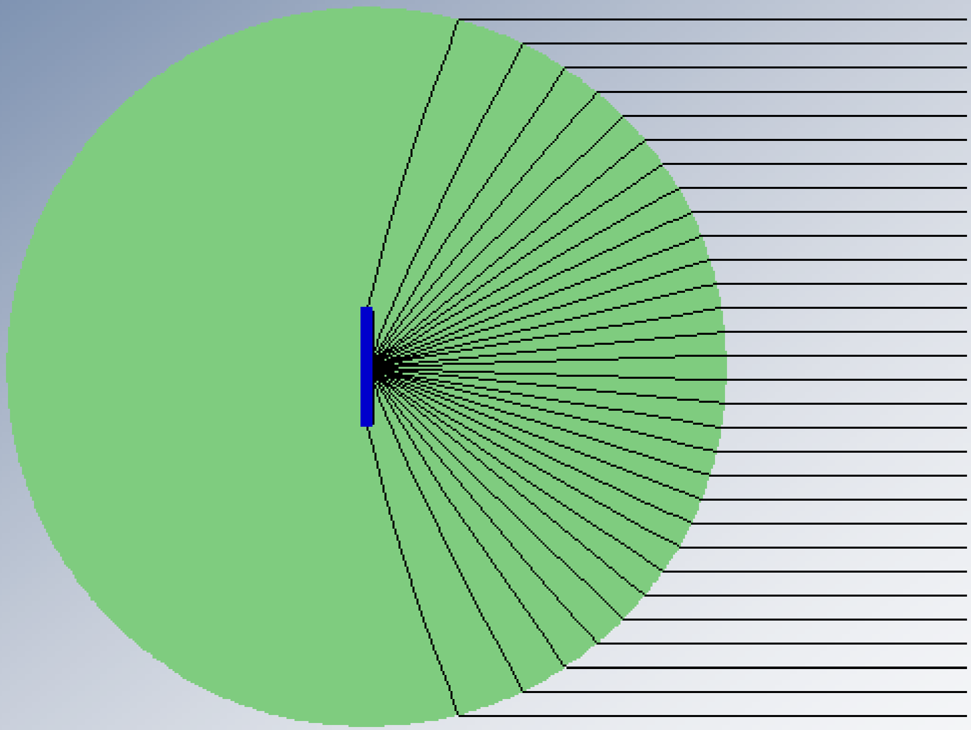
Radial Transforms: Concentrator



$$\begin{pmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{pmatrix}$$

Cloaking on a Ground Plane

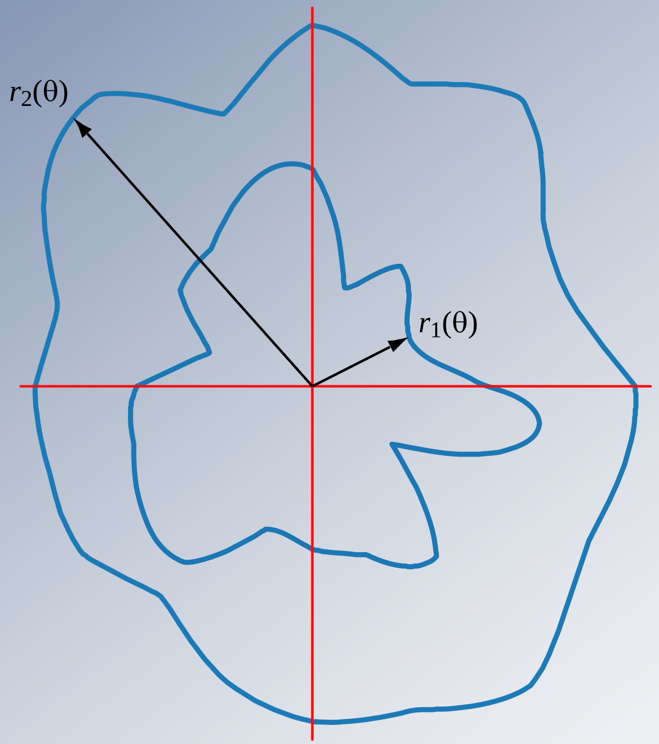
Concentrator

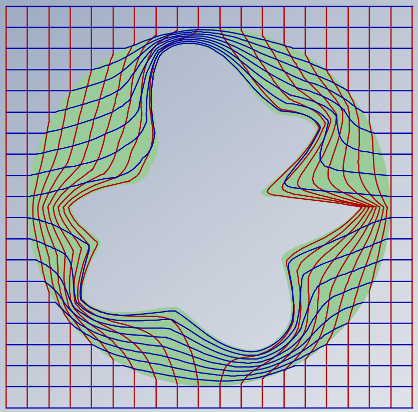


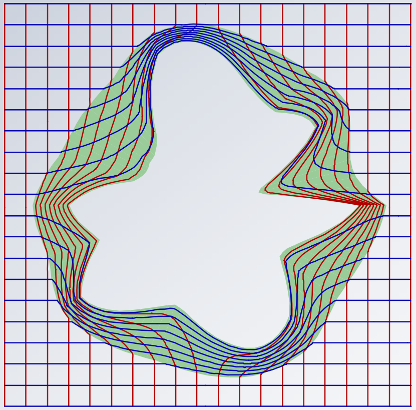
Cloaking Arbitrary Shapes

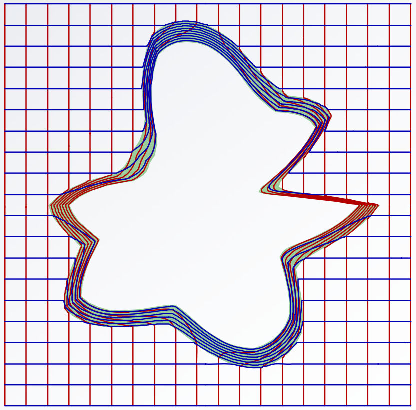
$$r = \frac{r_2(\theta')}{r_2(\theta') - r_1(\theta')}\left[r' - r_1(\theta')\right]$$

$$\theta = \theta'$$





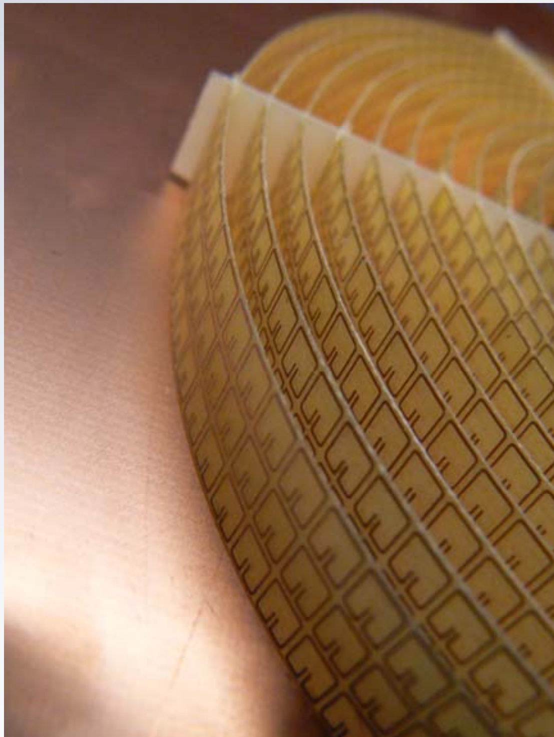




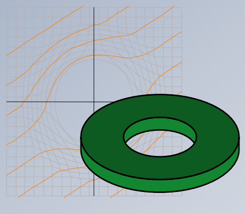
2D Microwave Cloak

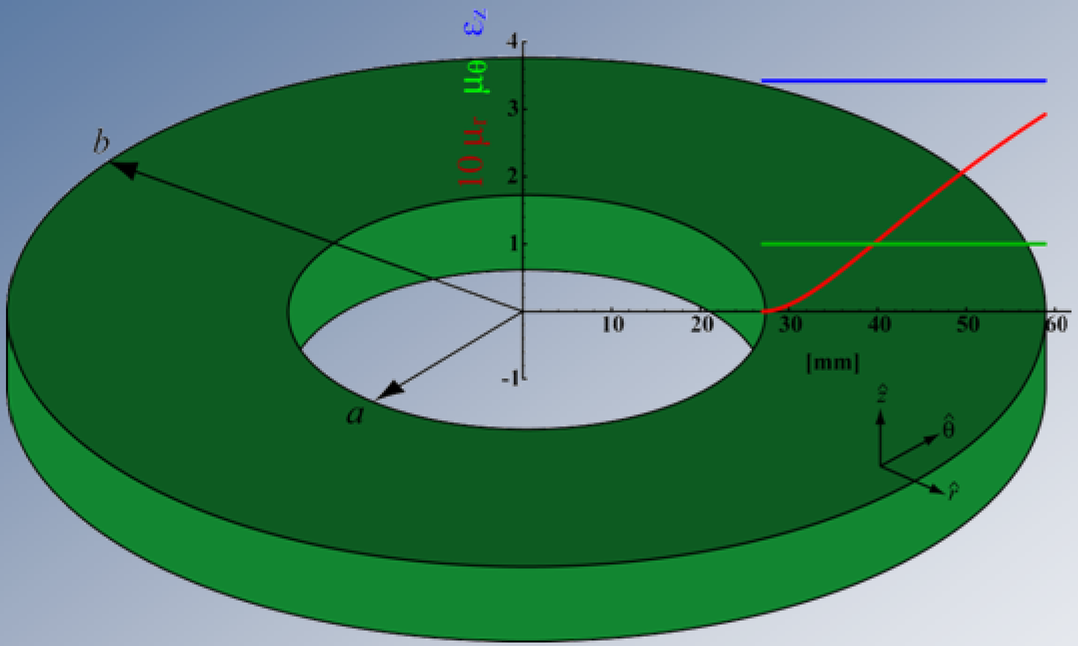
Metamaterial Implementation

- Design the unit cell
- Design the unit cell layout
- Fabrication: patterning, cutting, assembly
- Measurement



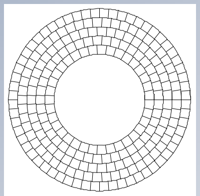
Cloak Design: Theory

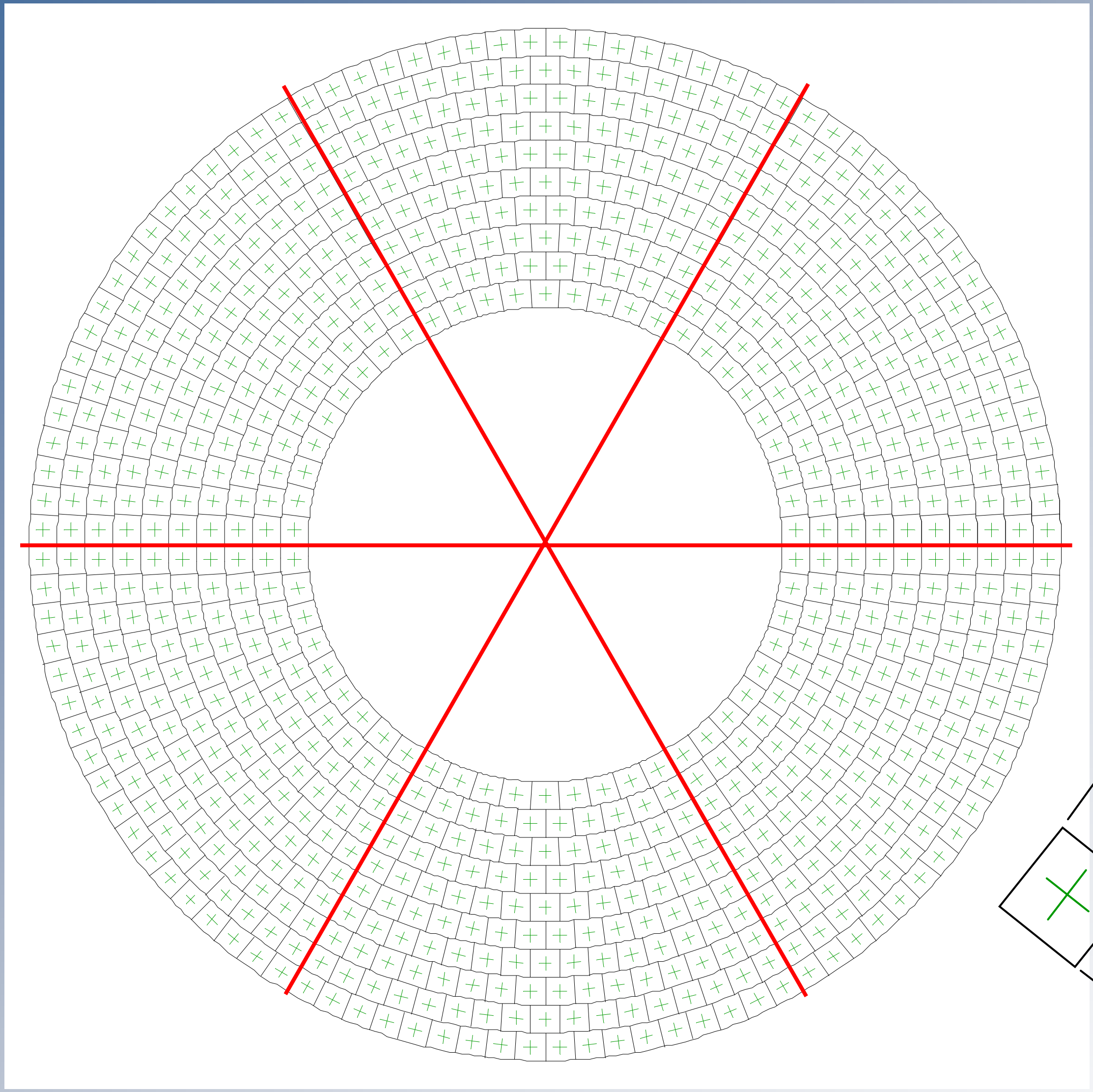




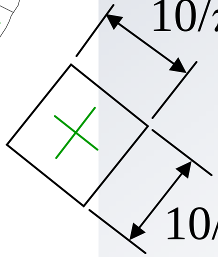
	Transform Media	*Reduced Media*
•	$\mu_r = \dfrac{r-a}{r}$	$\left(\dfrac{r-a}{r}\right)^2$
•	$\mu_\theta = \dfrac{r}{r-a}$	1
•	$\varepsilon_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r}$	$\left(\dfrac{b}{b-a}\right)^2$

Cloak Design: Layout

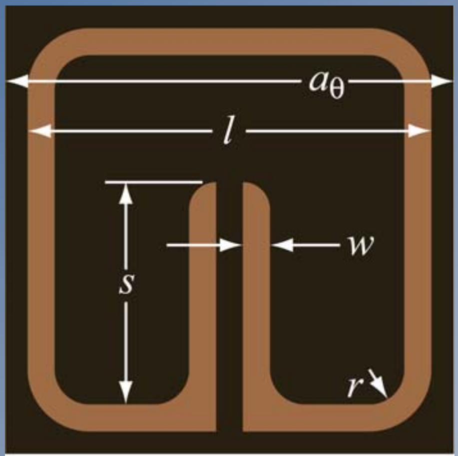




- Space Filling
- but not crystalline
- Multiple environments
- Six fold symmetry

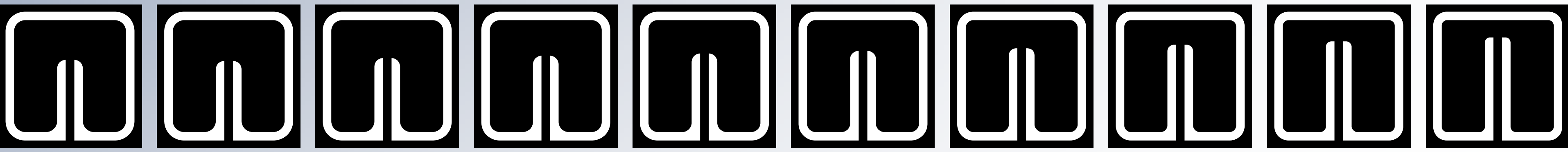


Cloak Design: Unit Cells

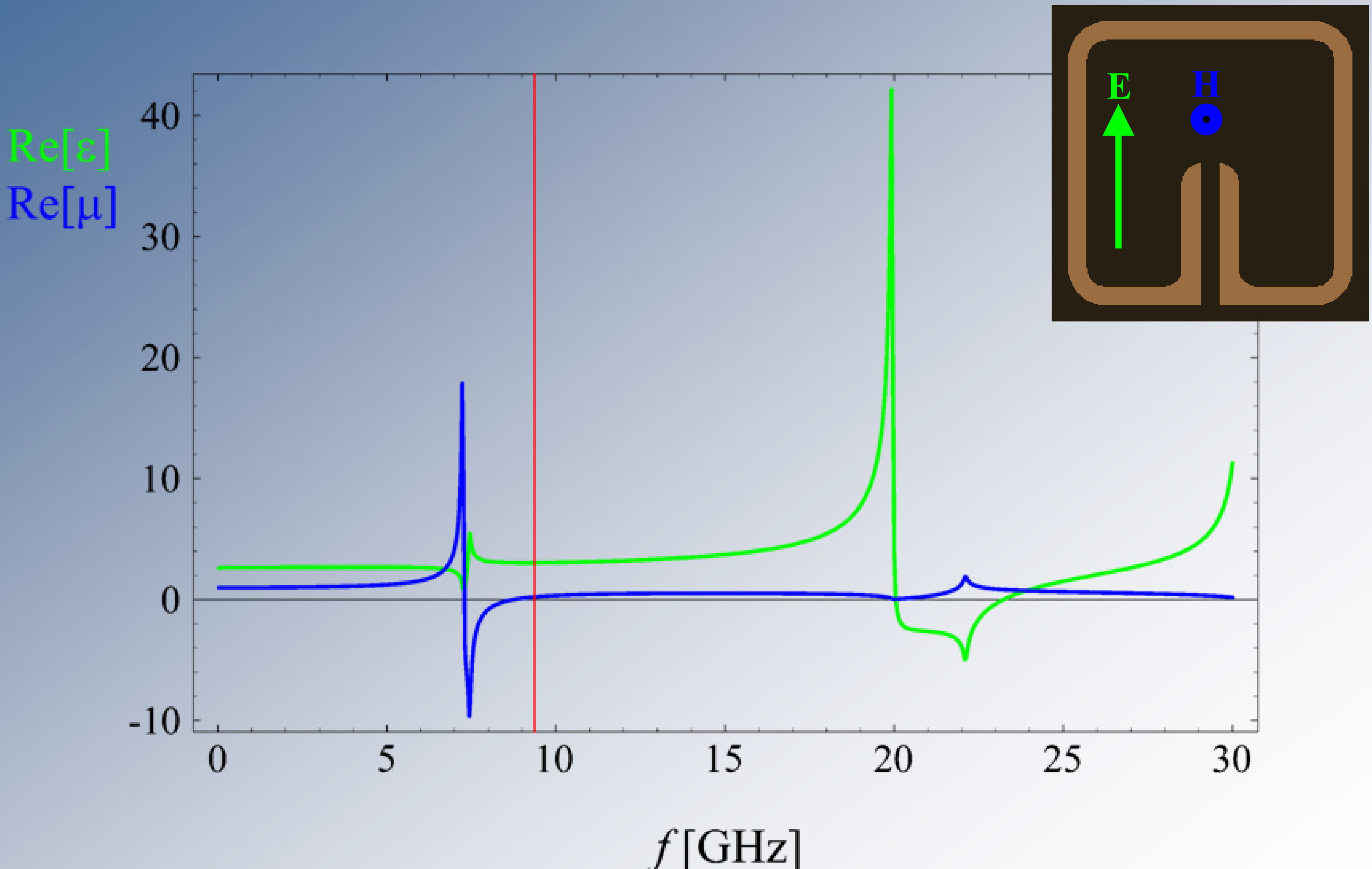


cyl.	r	s	μ_r
1	0.260	1.654	0.003
2	0.254	1.677	0.023
3	0.245	1.718	0.052
4	0.230	1.771	0.085
5	0.208	1.825	0.120
6	0.190	1.886	0.154
7	0.173	1.951	0.188
8	0.148	2.027	0.220
9	0.129	2.110	0.250
10	0.116	2.199	0.279

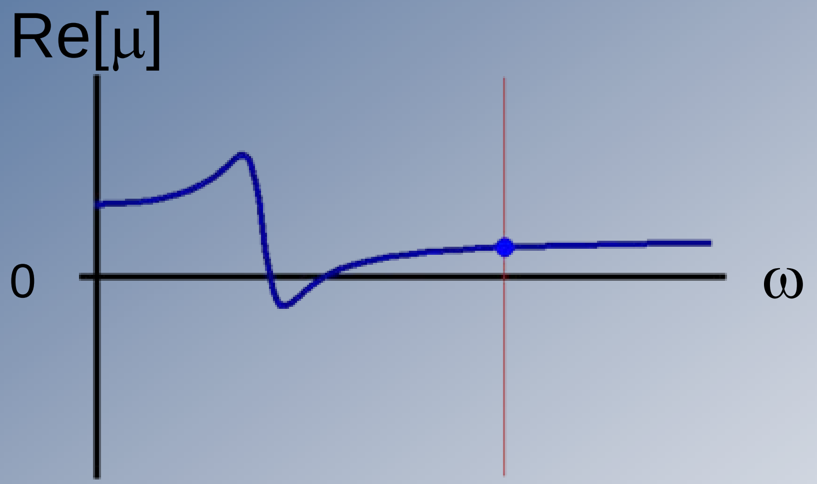
- Simulate rectangular unit cells
- Copper on FR4 or Duroid
- Magnetic and electric response from SRR
- Tune ε with *r* and μ with *s*



Simulated Material Properties

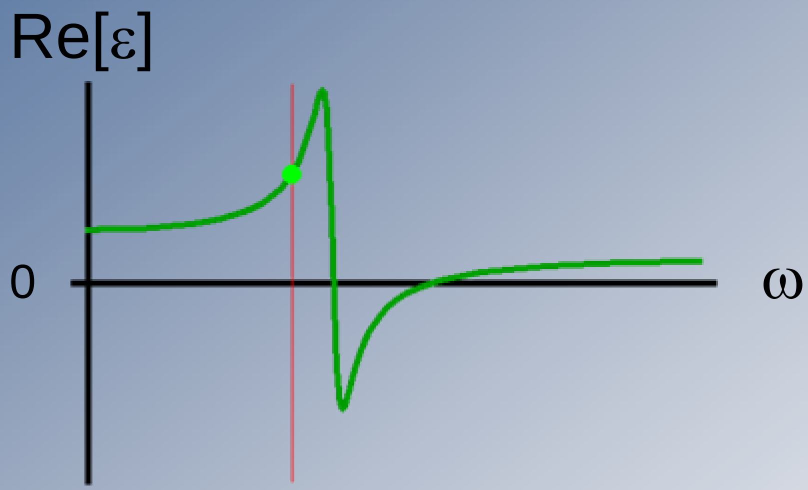


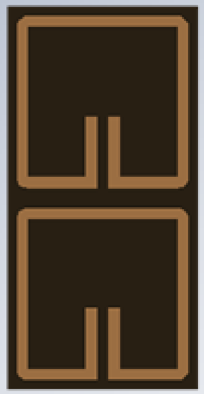
Tuning the Magnetic Resonance



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

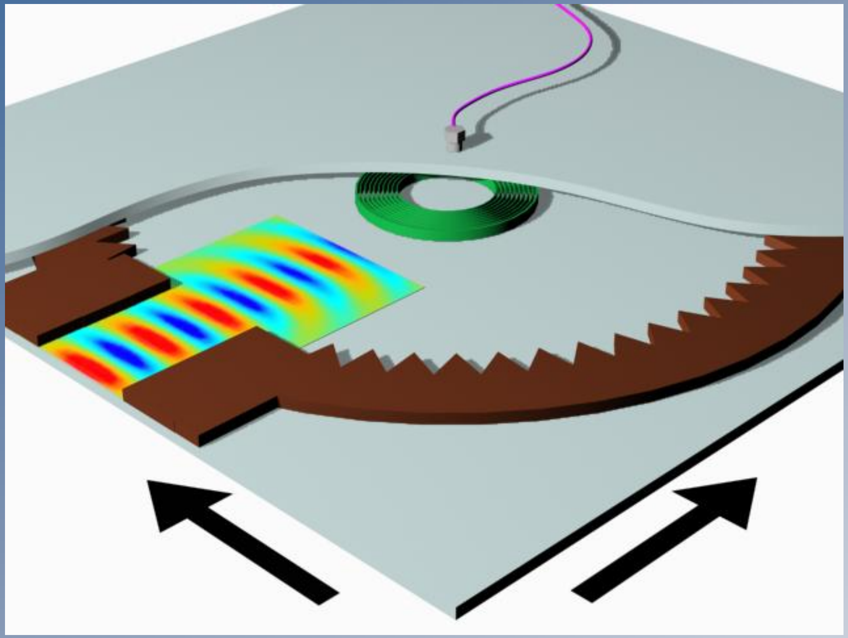
Tuning the Electric Resonance

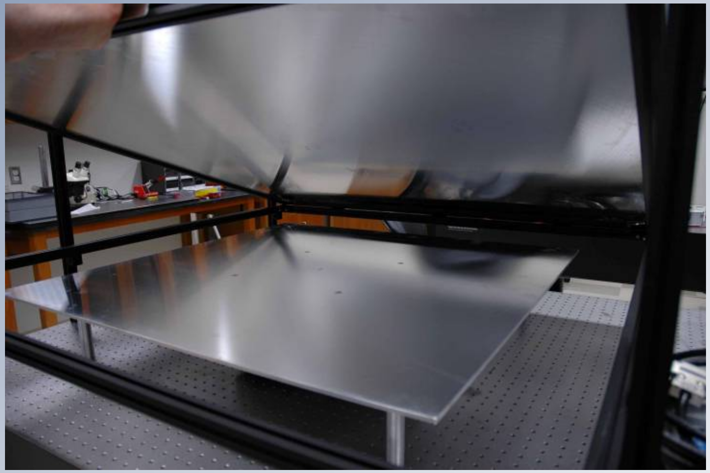




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

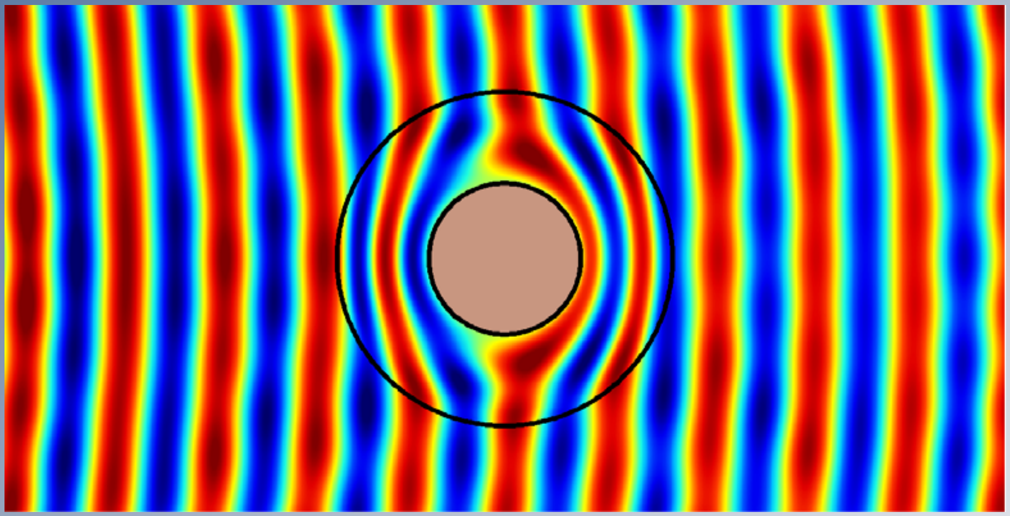
Cloak Design: Measurement



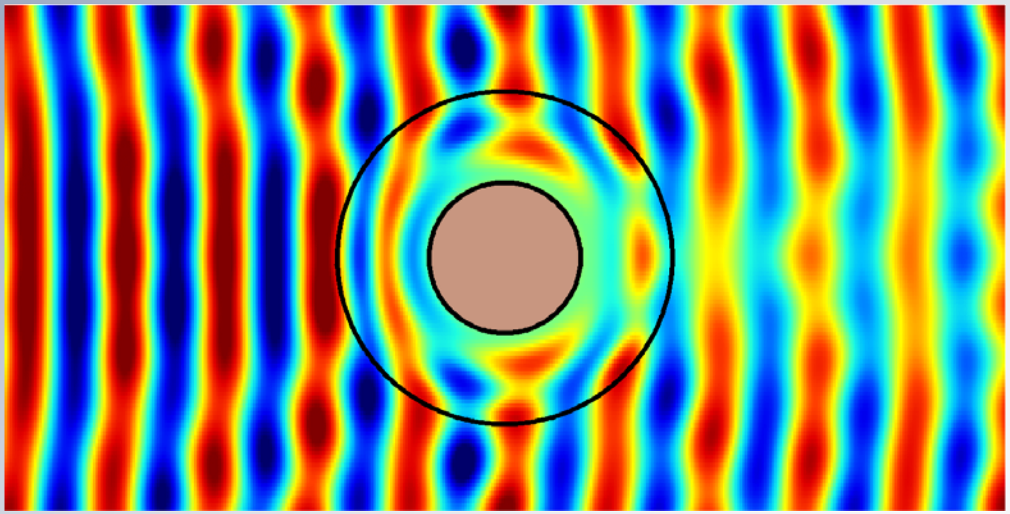


- Scan range: 20cm x 20cm
- Frequency range: X-band
- 8.5 GHz (λ/unit cell > 10)

Cloak Simulations

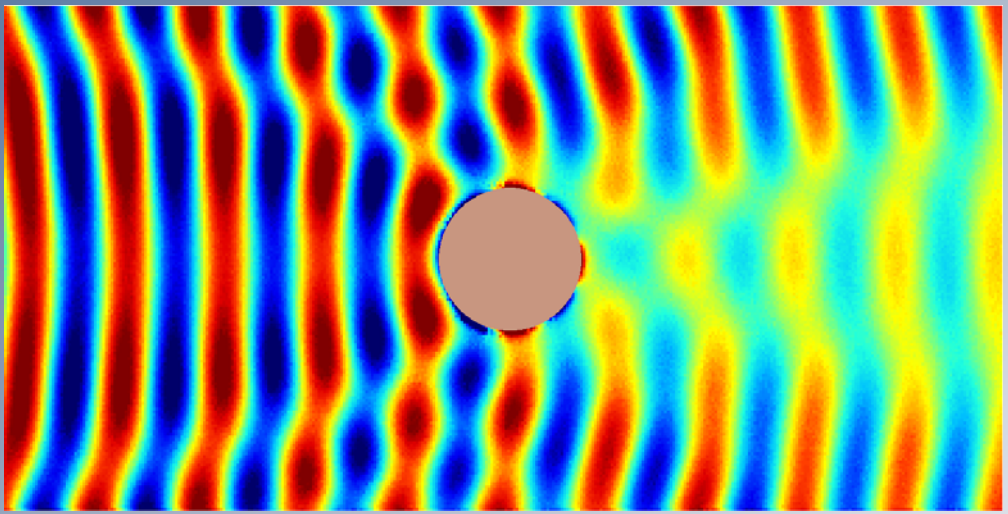


Ideal Cloak

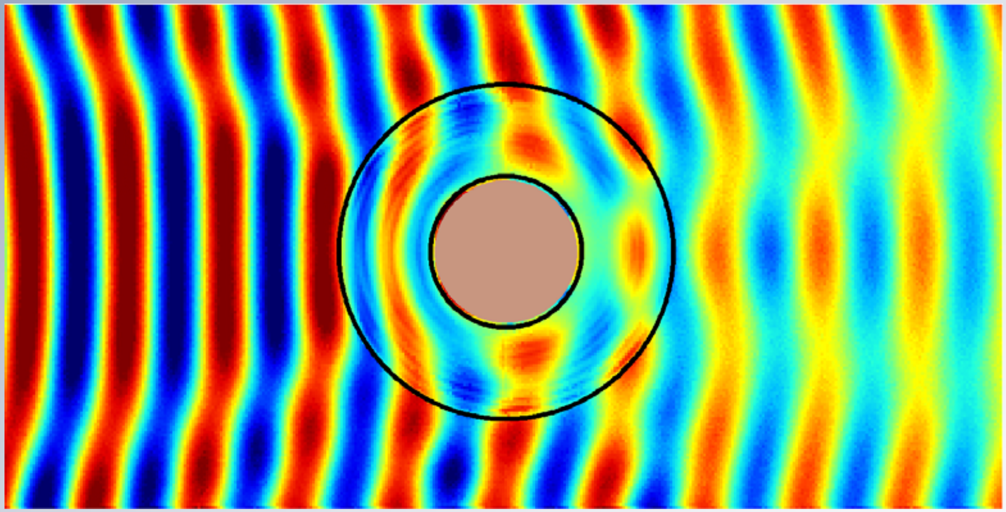


Reduced parameter set

Cloak Measurements

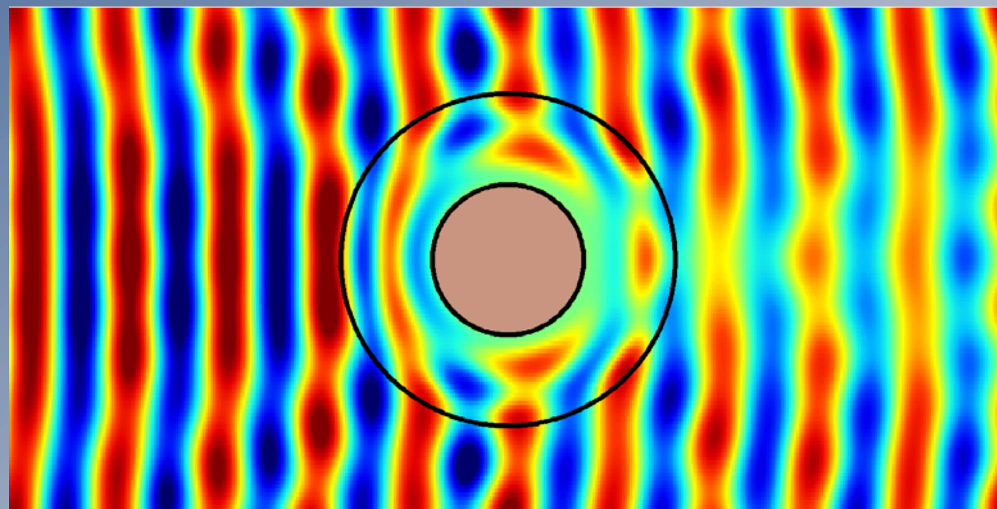


Bare scatterer measurement

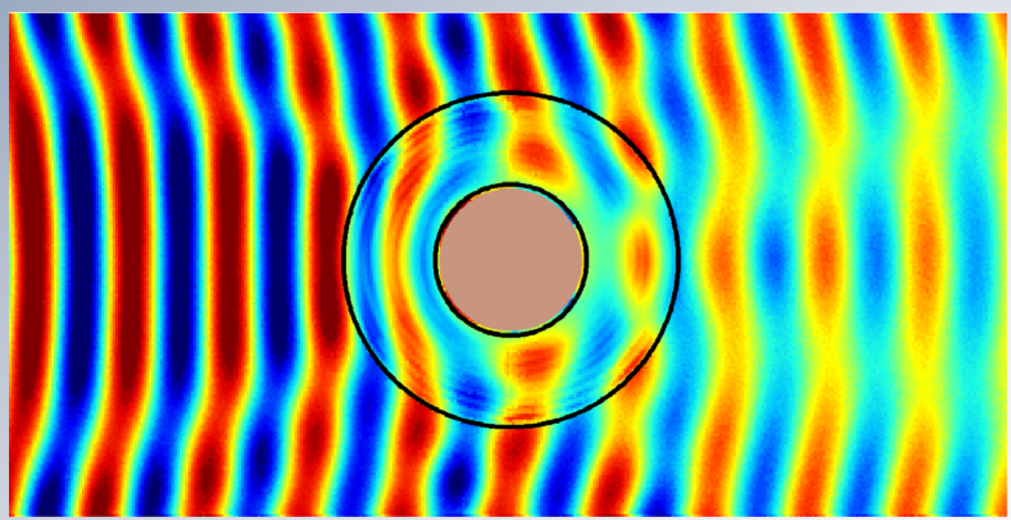


Cloaked scatterer measurement

Simulation Measurement Comparison



Simulation



Measurement

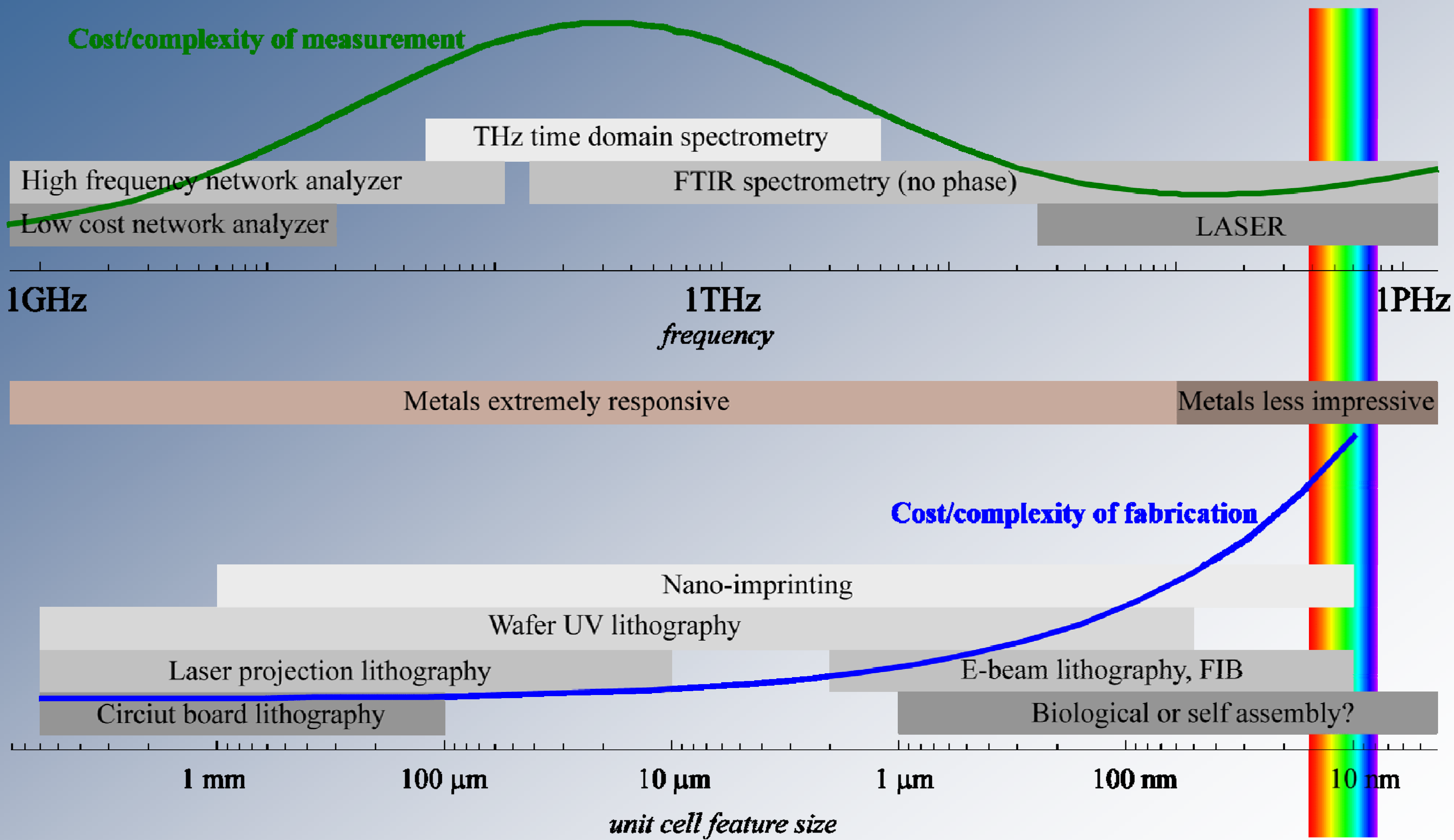
Cloak Applications

- Concealing structures, vehicles, personnel
- Removing obstructions
- Shielding
- Nefarious

The Future

- Better cloaks
- Other transformation designed devices
- Other wave phenomena
- Higher frequency

Metamaterials across the electromagnetic spectrum



Thank You