Diffractive Scattering and Gauge/String Duality

March 28, 2007, FermiLab

Different faces of QCD:
  scale invariance vs confinement,
  Diffraction,
  Gauge/String Duality
  confinement: Glueball Spectrum
  scalar invariance: Hard Scattering
  Pomeron as Massive Graviton
  Beyond Pomeron
I. Many Faces of QCD
I. Many Faces of QCD

QCD Theory Space!

- String/Gravity
  - $\mathcal{N} = 0$
  - $\mathcal{N} = 1$, $n_f = 1$

- Flux Tubes/Spectra
  - (IR/Long Distances)

- Asymptotically Free
  - (UV/Short Distances)

- Chiral Restored
  - (High Temp)

- Color Supercond
  - (Dense quarks)

- $g^2$
- $1/g^2$
- $kT$
- $\mu_B$
I-a. Scale Dependence in QCD

Asymptotic Freedom

\[ \alpha_s(q) \equiv \frac{\bar{g}(q)^2}{4\pi} = \frac{c}{\ln(q/\Lambda)} + \ldots \]

Confinement

Force at Long Distance--Constant Tension/Linear Potential, Coupling increasing, Quarks and Gluons strongly bound \(\Leftarrow\Rightarrow\) “Stringy Behavior”
Examples:

• Perturbative:
  – Fixed-Angle exclusive scattering, jet production, DIS, etc.

• Non-Perturbative:
  – Meson spectrum, near-forward scattering, etc.
At WIDE ANGLES QCD exhibits power law behavior:

\[ A_{qcd}(s, t) \sim \left( \frac{1}{\sqrt{\alpha'_q c d s}} \right)^{n-4} \]

where \( n = \sum_i n_i \) is the number of \``partons'' in external lines.

The OPE gives

\[ n = \sum_i \tau_i = \sum_i (d_i - s_i) \]

in terms of the lowest twist \( \tau_i \).

*Actually QCD is only conformal up to small asymptotic freedom logs.*
Nonperturbative Confinement-Deconfinement

Probing 5th Dimension via lattice Simulation
$\alpha(t) \simeq \alpha' t + \alpha_0$

where $\alpha(t = M^2) = J$

**FIG. 1.** Meson ($\rho$, $K^*$ and $a$) Regge trajectories constructed from recent tabulated data (dark circles and error bars, PDG 2000). Boxes are model TDA predictions for the $\rho$ trajectory.
Regge Behavior and Regge Trajectory

\[ A \sim s^J(t) = s^{\alpha(0) + \alpha't} \]
I-b. Diffractive Scattering
Total Cross Sections

\[ A \sim s^J(t) = s^{\alpha(0) + \alpha'}t \]

\[ \sigma_{total} \sim A(s, 0)/s \sim s^{J(0)-1} \sim s^{\alpha(0)-1} \]

\[ \alpha(0) > 1 \]
CDF Diffractive Events at Fermilab Tevatron pbar-p Collider

http://www-cdf.fnal.gov/~terashi/evd/diff_evd.html
Diffraction at DØ

Andrew Brandt
University of Texas, Arlington

- Intro and Run I Hard Diffraction Results
- Run II and Forward Proton Detector

Physics Colloquium
November 5, 2003
UTA
Why study Diffractive W Boson?

The pomeron (IP) structure is not yet understood which motivates a study that will better clarify the quark/gluon composition involved. This is found in the diffractive W, which to leading order can only happen based on a quark component in the pomeron.¹

\[ a) \text{ LO: } q\bar{q} \to W \quad b) \text{ NLO: } qg \to q + W \]

Diffractive process (a) probes the quark content of the pomeron.

¹ (Bruni & Ingelman, Phys. Lett. B311(1993)318)
Double Diffractive Dijet
Exclusive Dijet
BFKL Pomeron, High Gluon Density
Small-x in DIS: Perturbative or Non-perturbative?

Anomalous Dimension of Leading twist operator

\[ \text{tr}(F_{+\mu} D_{+}^{-2} F_{+\mu}) \]

Regge Behavior

\[ \mathcal{A} \sim s^J(t) = s^{2+\alpha' t/2} \]
II. Gauge/String Duality
QCD, Gauge/String Duality, and High Energy Collisions

3+1 Yang Mills

3+1+1 AdS$^5$ like String Theory
Maldacena’s Gauge/String Duality

<table>
<thead>
<tr>
<th>Weak/Strong String/gravity ↔ Strong/Weak YM</th>
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<tbody>
<tr>
<td>e.g. Strings in $AdS^{d+1} \times X$ ↔ Conformal (SUSY) SU(N) YM in $d$</td>
</tr>
</tbody>
</table>

Anti-De Sitter metrics, $AdS^{d+1} \times X$, ($\mu = 1, \ldots, d$) are solution to low energy closed strings (e.g. gravity).

$$ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + ds_X^2$$

Open String $\leftrightarrow$ Closed String
duality

YM Wilson Loops $\leftrightarrow$ Expectation Values in Bulk!
Gauge-String Duality

- gauge coupling $\longleftrightarrow$ $1/$gauge coupling

- dimensions: $4 \longleftrightarrow 4+1 + (5)$

- Degrees of freedom:
  - Gauge theory: quarks and gluons
  - String Dual: metric fluctuations about AdS, etc.
Gravity vs Y.M. on Brane

11-d Super Gravity:

\[ S = -\frac{1}{2\kappa_{11}} \int d^{11}x \sqrt{-g_{11}} (R_{11} - |F_4|^2) + \frac{1}{12\kappa_{11}} \int A_3 \wedge F_4 \wedge F_4 + \text{fermions} \]

Born-Infeld dynamics on 3 brane and 4 brane

\[ S = \int d^4x \det [G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_0 F \wedge F + C_2 \wedge F + C_4) , \]

\[ S = \int d^5x \det [G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_1 F \wedge F + C_3 \wedge F + C_5) \]

Viewing the gravity fields as coupling constants to the gauge fields we can identify quantum number for them.
Scale Invariance and AdS

What is the curved space?

Maldacena: UV (large $r$) is (almost) an $AdS_5 \times X$ space

$$ds^2 = r^2 dx_\mu dx^{\mu} + \frac{dr^2}{r^2} + ds^2_X$$

Captures QCD’s approximate UV conformal invariance

$$x \rightarrow \zeta x, \quad r \rightarrow \frac{r}{\zeta} \quad \text{(recall } r \sim \mu)$$

Confinement: IR (small $r$) is cut off in some way

$$r \sim \mu > r_{min} \sim \Lambda_{QCD}$$

For Pomeron: *string theory* on cut-off $AdS_5$ ($X$ plays no role)
Scale Invariance and the 5th dimension

pt defects at $r \equiv 1/z = 1/\rho \rightarrow *$

$\Leftrightarrow$ Instanton radius $\rho$

Strings (Gravity) in AdS$^5 \equiv$ (SUSY) Yang Mills
Cutoff AdS$_5$

Large Sizes

pt defects at $r \equiv 1/z = 1/\rho \rightarrow *$

$\Leftrightarrow$ Instanton radius $\rho$

Add Confinement

IR wall!

String/Glueball

$x_1, x_2, x_3, x_4$

$r = \infty$ (UV)
Gauge/String Dual: with Confinement

- Identify model independent features
- Use models to provide concrete mathematical realization of Gauge/String for QCD
- For simplicity, mostly use Hard-Wall Model
II-a. Early Years

Failures of (flat space) String for QCD

(i) ZERO MASS STATE (gauge/graviton)
(ii) EXTRA SUPER SYMMETRY
(iii) EXTRA DIMENSION \(4+6 = 10\)
(iv) NO HARD PROCESSES! (totally wrong dynamics)

Wide angle is ridiculous: 
\[ A(s, t) \to \exp[-\alpha'(s \ln s + t \ln t)] \]

Strings are too soft: 
\[ \langle X_\perp^2 \rangle \simeq \alpha' \log[N_{\text{modes}}] \]

Form Factors do not exist: 
\[ F[q^2] \simeq \exp[-q^2 \log(\infty)] \]

No longitudinal modes on the Flux tube, etc.
Need to give mass to

- $t=0$
- $t>0$
- $t<0$

\[ J \]

Regge $\alpha(t)$

\[ 2^{++} \] Graviton

Closed String

Open String

\[ 1^{--} \] Photon/Gluon

$t<0$  $t=0$  $t>0$

Need Hard Collisions

- Fixed-Angle
- DIS
- Etc.

Need to Confront

Exp: HE Collisions, Spectrum, etc.
II-b: How to give mass to Graviton turning into $2^{++}$Glueball?

Regge $\alpha(t)$

Closed String

Open String

Maldacena: “Solution put 10-d (super) strings in curved space”

first example: $\text{AdS}^5 \times S^5$ string $\equiv \mathcal{N} = 4$ Super Conformal YM in 4-d
4-Dim Massive Graviton

5-Dim Massless Mode:

\[ 0 = E^2 - (p_1^2 + p_2^2 + p_3^2 + p_r^2) \]

If, due to Curvature in fifth-dim, \( p_r^2 \neq 0 \),

Four-Dimensional Mass:

\[ E^2 = (p_1^2 + p_2^2 + p_3^2) + M^2 \]
III. Gauge/String Duality

with confinement

- Scale Invariance: Hard Scattering, DIS, etc.
- Confinement: Particle Spectrum, etc.
- Pomeron as Massive Graviton
IIIa. Spectrum
IIIa. Glueballs at $g^2 N_c = \infty$

Strong coupling Dual to Gravity

\[
\alpha'_{qcd} = \frac{R^2_{ads}}{r^2_{min}} \alpha'_{s} = \frac{1}{\Lambda^2_{qcd} \sqrt{g^2 N_c}} \rightarrow 0
\]
Gravity vs Y.M. on Brane

11-d Super Gravity:

\[ S = -\frac{1}{2\kappa_{11}} \int d^{11}x \sqrt{-g_{11}} \left( R_{11} - |F_4|^2 \right) + \frac{1}{12\kappa_{11}} \int A_3 \wedge F_4 \wedge F_4 + \text{fermions} \]

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\[ S = \int d^4x \det[G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_0 F \wedge F + C_2 \wedge F + C_4), \]

\[ S = \int d^5x \det[G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_1 F \wedge F + C_3 \wedge F + C_5) \]

Viewing the gravity fields as coupling constants to the gauge fields we can identify quantum number for them.
The $AdS^7$ glueball spectrum for $QCD_4$ in strong coupling (left) compared with the Morningstar/Peardon lattice spectrum for pure $SU(3)$ QCD (right) with $1/r_0 = 410$ Mev.

R. Brower, S. Mathur, and C-I Tan, hep-th/0003115, “Glueball Spectrum of QCD from AdS Supergravity Duality”.
Comparison with MIT Bag Calculation

SU(3) Glueball Spectrum
Wilson action data and gluon modes

$J^{PC}$ continuum limit extrapolations
Tensor Glueball/Graviton Wave functions

n=0

n=1

n=3

n=8

r_{min} Potential

Randall-Sundram graviton
## Bag Classification of States

<table>
<thead>
<tr>
<th>Dimension</th>
<th>State</th>
<th>Operator</th>
<th>Supergravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>d=4</td>
<td>0++</td>
<td>$\text{Tr}(FF) = E^a \cdot E^a - B^a \cdot B^a$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>d=4</td>
<td>2++</td>
<td>$T_{ij} = E^{a_i} \cdot E^{a_j} + B^{a_i} \cdot B^{a_j} - \text{trace}$</td>
<td>$G_{ij}$</td>
</tr>
<tr>
<td>d=4</td>
<td>0+</td>
<td>$\text{Tr}(F^*F) = E^a \cdot B^a$</td>
<td>$C_\tau$</td>
</tr>
<tr>
<td>d=4</td>
<td>0++</td>
<td>$2T_{00} = E^a \cdot E^a + B^a \cdot B^a$</td>
<td>$G_{\tau\tau}$</td>
</tr>
<tr>
<td>d=4</td>
<td>2+</td>
<td>$E^{a_i} \cdot B^{a_j} + B^{a_i} \cdot E^{a_j} - \text{trace}$</td>
<td>absent</td>
</tr>
<tr>
<td>d=4</td>
<td>2++</td>
<td>$E^{a_i} \cdot E^{a_j} - B^{a_i} \cdot B^{a_j} - \text{trace}$</td>
<td>absent</td>
</tr>
<tr>
<td>d=6</td>
<td>(1,2,3)++</td>
<td>$\text{Tr}(F_{\mu\nu}[F_{\rho\sigma},F_{\lambda\eta}]) \sim d^{abc}F^a F^b F^c$</td>
<td>$B_{ij}$, $C_{ij}$</td>
</tr>
<tr>
<td>d=6</td>
<td>(1,2,3)++</td>
<td>$\text{Tr}(F_{\mu\nu}[F_{\rho\sigma},F_{\lambda\eta}]) \sim f^{abc}F^a F^b F^c$</td>
<td>absent</td>
</tr>
</tbody>
</table>

These are all the local $d=4$ and $d=6$ operators: See Jaffe, Johnson, Ryzak (JJR), “Qualitative Features of the Glueball Spectrum”, Ann. Phys. 168 334 (1986)
Approx. Scale Invariance and the 5th dimension

\[ \Phi(r) \]

Hadron Glueball

Massive

Current

\[ r = r_{\text{mi}} - \Delta \]

\[ M R^2 / \sqrt{g^2 N_c} \]

\[ q R^2 \]

\[ r^\Delta - 4 \]

\[ r^- \]

\[ ds^2 = \frac{r^2}{R^2} \left[ dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 \right] + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \]
References:

Witten: hep-th/9803131,

Jevicki, et al., hep-th/9806125,

Ooguri, et al., hep-th/9806021,

Brower, Mathur, Tan, hep-th/0003115,

Pando Zayas, et al., hep-th/0311190,

.........
At WIDE ANGLES QCD exhibits power law behavior:

\[ A_{qcd}(s, t) \sim \left( \frac{1}{\sqrt{\alpha'_qcds}} \right)^{n-4} \]

where \( n = \sum_i n_i \) is the number of ``partons'' in external lines.

The OPE gives

\[ n = \sum_i \tau_i = \sum_i (d_i - s_i) \]

in terms of the lowest twist \( \tau_i \).

_Actually QCD is only conformal up to small asymptotic freedom logs._
Wide Angle Scattering

The 2-to-m glueball scattering amplitude $T(p_1, p_2, \cdots, p_{m+2})$ for plane wave glueball:

$$
\phi_j(r, X) \exp\left[i x^\mu_j p^\mu_j\right]
$$

scatter via the string(M-theory) amplitude: $A(p_i, r_i, X_i)$ in the 10-d (or 11-d) bulk space $(x, r, Y)$:

$$
T(p_i) = \prod_j \int dr_j dY_j \sqrt{-g_i} \; \phi_j(r_j, Y_j) \times A(p_1, r_1, Y_1, p_2, r_2, Y_2 \cdots)
$$

We now discuss two different approaches to the QCD string that both give the correct parton scaling formula.

- $\text{AdS}^5 \times X$ with IR cut-off on $r > r_{\text{min}}$ or 10-d IIB string theory
- $\text{AdS}^7 \times S^4$ Black Hole with horizon $r = r_{\text{min}}$ or 11-d M-theory.

This is a check on the underlining universality of Maldacena's duality conjecture.
Approx. Scale Invariance and the 5th dimension

\[ \Phi(r) \]

Hadron Glueball

Massive

Current

\[ r = r_{\text{mi}} \]

\[ r \rightarrow \]

\[ M R^2 / \sqrt{g^2 N_c} \quad q R^2 \]

\[ ds^2 = \frac{r^2}{R^2} [dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2] + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \]
Summary on Hard Scattering

(1) AdS$^5$ Hard Scattering (PolchinskiStrassler):

$$\Delta \sigma_{2 \rightarrow m} \approx \frac{1}{s} f\left(\frac{p_i \cdot p_j}{s}\right) \left(\frac{\sqrt{g^2 N}}{N^{2m}}\right)^m \prod_i \left(\frac{\sqrt{g^2 N} \Lambda_{qcd}^2}{s}\right)^{n_i-1}$$

WHY is it same QCD perturbative result with $g^2 N \rightarrow (g^2 N)^2$?

(2) AdS$^7$ Hard Scattering (Brower-Tan):

$$\Delta \sigma_{2 \rightarrow m} \approx \frac{1}{s} f\left(\frac{p_i \cdot p_j}{s}\right) \frac{1}{N^{2m}} \prod_i \left(\frac{1}{\alpha'_qcd s}\right)^{n_i-1}$$

WHY does this only depend on the string tension?

(3) Compared with lowest order perturbative results:

$$\Delta \sigma_{2 \rightarrow m} \approx \frac{1}{s} f\left(\frac{p_i \cdot p_j}{s}\right) \left(\frac{g^2 N}{N^{2m}}\right)^m \prod_i \left(\frac{g^2 N \Lambda_{qcd}^2}{s}\right)^{n_i-1}$$
IV: The Pomeron
The Pomeron

Will show that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously from tree-level string scattering amplitudes.

Pomeron can be identified as Massive Graviton.

Both the IR Pomeron and the UV Pomeron are dealt in a unified single step.
<table>
<thead>
<tr>
<th>BFKL vs Soft Pomeron</th>
<th></th>
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<tbody>
<tr>
<td><strong>Perturbative QCD</strong></td>
<td><strong>Non-Perturbative</strong></td>
</tr>
<tr>
<td><strong>Short-Distance</strong></td>
<td><strong>Long-distance: Confinement</strong></td>
</tr>
<tr>
<td>( \alpha_{\text{BFKL}} (0) \sim 1.4 )</td>
<td>( \alpha_{\text{P}}(0) \sim 1.08 )</td>
</tr>
<tr>
<td><strong>Increasing Virtuality</strong></td>
<td><strong>Fixed trans. Momenta</strong></td>
</tr>
<tr>
<td><strong>No Shrinkage of elastic peak</strong></td>
<td><strong>Shrinkage of Elastic Peak:</strong> ( &lt;</td>
</tr>
<tr>
<td><strong>Fixed-cut in ( t )</strong></td>
<td>( \alpha'(0) \sim 0.3 \text{ Gev}^{-2} )</td>
</tr>
<tr>
<td><strong>Diffusion in Virtuality</strong></td>
<td><strong>Diffusion in impact space</strong></td>
</tr>
</tbody>
</table>

**Unified treatment in terms of diffusion in AdS with confinement deformation**

Brower, Polchinski, Strassler, CIT (hep-th/0603115)
BFKL (Balitsky-Lipatov-Fadin-Kuraev)

BFKL Summation: Scale Invariance

- Weak perturbation theory: 1st order in $\alpha_s$ and all orders $(\alpha_s \log s)^n$
- Implies “planar” diagrams (e.g. $N_c = \infty$) and conformal scaling
- BFKL is essentially a large $N_c$ CFT results!

$$A(s, t = 0) \sim \int \frac{dk_{\perp}}{k_{\perp}} \int \frac{dk'_{\perp}}{k'_{\perp}} \Phi_1(k_{\perp}) K(s; k_{\perp}, k'_{\perp}) \Phi_2(k'_{\perp})$$

$$K(s, k_{\perp}, k'_{\perp}) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-\left[(\ln k'_{\perp} - \ln k_{\perp})^2 / 4D \ln s\right]}$$

Weak Coupling:

$$\alpha(0) = 1 + \ln(2) g^2 N / \pi^2$$

$$D = \frac{14 \zeta(3)}{\pi} g^2 N / 4\pi^2.$$
Regge Behavior in AdS$_5$

\[ A \sim s^J(t) = s^{\alpha(0)+\alpha't} \]

\[ t \leftrightarrow - \nabla^2 \]
String amplitudes $\rightarrow$ Regge behavior $\mathcal{A} \sim \sum_i s^{J_i(t)}$

$[J(t) = \alpha(t) = \alpha_0 + \alpha't]$

- $t$ negative; Fourier transform momentum space $\rightarrow$ position space

$J(t) \sim \alpha_0 + \alpha't \Rightarrow \mathcal{A} \sim s^{\alpha_0} \exp \left[ -\frac{|\vec{x}|^2}{\alpha' \ln s} \right] \frac{\sqrt{\ln s}}{\sqrt{\ln s}}$

Strings grow: $\langle |\vec{x}|^2 \rangle \sim \ln s$

(random-walk diffusion, with $\tau \sim \ln s$)
Regge in Curved Space

\[ \mathcal{A} \sim s^{J(t)} = s^{2+\alpha't/2} \quad \text{(flat space)} \]
\[ \rightarrow s^{2+\alpha'\nabla^2/2} \quad \text{(curved space)} \]
\[ = s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau} \]

where \( \tau \propto \ln s \) is again a diffusion time, and

\[ H \propto -\nabla^2 = -\frac{1}{r^2} \nabla_{3+1} - \nabla^2_r + 0 = -\partial^2_u + (4 - e^{-2u}t/t_0) \]

where \( u = \ln r \)

A Schrödinger operator with potential \( V(u; t) = 4 - e^{-2u}t/t_0 \)
Effective Hamiltonian at $t=0$

\[ \mathcal{A} \sim s^J(t) = s^{2+\alpha't/2} \quad \text{(flat space)} \]
\[ \rightarrow s^{2+\alpha'\nabla^2/2} \quad \text{(curved space)} \]
\[ = s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau} \]

where $\tau \propto \ln s$ is again a diffusion time, and for $t = 0$,

\[ H \propto -\nabla^2 = -\frac{1}{r^2} \nabla_{3+1} - \nabla_r^2 + 0 = -\partial_u^2 + 4 \]

where $u = \ln r$

A Schrödinger operator with potential $V(u; t) = 4$

\[ \mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-D\tau[-\partial_u^2]} \], \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}}, \quad D = \frac{1}{2\sqrt{\lambda}} \]
**Diffusion in AdS:** \( u = \log r \)

\[
\mathcal{A} \sim s^2 e^{-H \tau} \sim s^{j_0} e^{-\mathcal{D} \tau [-\partial_u^2]} , \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}} , \quad \mathcal{D} = \frac{1}{2\sqrt{\lambda}}
\]

Sandwiching this differential operator between the two scattering hadrons, writing the kernel explicitly, and recalling \( \tau \propto \ln s, \ u = \ln r \),

\[
\mathcal{A} \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \ \Phi_1 (r) \ s^{j_0} e^{-\frac{(\ln[r'/r])^2}{4\mathcal{D} \ln s}} \ \frac{\Phi_2 (u')}{\sqrt{4\pi \mathcal{D} \ln s}}
\]

\[
\dot{j}_0 = 2 - \frac{2}{\sqrt{\lambda}} , \quad \mathcal{D} = \frac{1}{2\sqrt{\lambda}}
\]

Same form as the BFKL kernel for \( t = 0 \):
Comparison of Diffusion in AdS and BFKL

\[ A = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) \quad s^{j_0} e^{-\left[ \frac{\ln(k'_\perp/k_\perp)}{4\mathcal{D} \ln s} \right]^2} \] \frac{\sqrt{4\pi \mathcal{D} \ln s}}{\Phi_2(k'_\perp)} \]

\[ j_0 = 1 + \frac{4 \ln 2}{\pi} \alpha N, \quad \mathcal{D} = \frac{7 \zeta(3)}{\pi} \alpha N. \]

\[ A \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \Phi_1(r) \quad s^{j_0} e^{-\left[ \frac{\ln(r'/r)}{4\mathcal{D} \ln s} \right]^2} \] \frac{\sqrt{4\pi \mathcal{D} \ln s}}{\Phi_2(u')} \]

\[ j_0 = 2 - \frac{2}{\sqrt{\lambda}}, \quad \mathcal{D} = \frac{1}{2\sqrt{\lambda}} \]
BFKL is Regge in AdS

In this string calculation, the exchanged Pomeron — the graviton trajectory — propagates in the curved 5th dimension!

- BFKL result = Regge behavior in 5 curved dims.
- Amplitude takes BFKL form, with $k_\perp \to r$.
- BFKL diffusion is Regge diffusion (space = $\ln r$, time = $\ln s$).
- Coefficients differ (as expected; $\lambda$ is different)
- Form of answer follows from conformal invariance.
The argument given on the previous slide was heuristic.

_The result is not._

A detailed derivation from string theory has been obtained, but requires some small technical developments in string theory.
IVb. Synthesis of Hard (BFKL) and Soft (Confinement) Pomeron
IV-b. Synthesis of Hard (BFKL) & Soft (Regge) Pomeron

Analytic Structure of Pomeron Propagator

Conformal UV, Confining IR (large $N$)
Pomeron in Confined AdS Deformation

Confinement----> Regge trajectory, Resonances, etc.

Simplest Model: Hard-Wall

\[ \frac{1}{2\sqrt{g^2 N}} \left[ -\frac{d}{du} te^{-2u} \right] \psi(u, J) = (2 - J - \frac{2}{\sqrt{g^2 N}}) \psi(u, J) \]

- \( V(u) = -t e^{-u} \), \( 0 < u < \infty \)
- Attractive for \( t > 0 \), Regge Pole + BFKL cut
- \( t < 0 \) only scattering state for BFKL

Hard Wall at \( r = r_{\text{min}} \)
Hardwall Regge Spectrum and Cut

\[ j - 2 \]
Running UV, Confining IR (large $N$)

The hadronic spectrum is little changed, as expected.
The BFKL cut turns into a set of poles, as expected.
Summary: QCD String/Gauge Duality in AdS
Summary

- **Gauge/String Duality** allows us to compute **Pomeron propagator** in gauge theories with large ‘t Hooft coupling $\lambda$ and large $N$.

- Expansion in $\lambda^{-1/2}$

- Obtain for all $t$:
  - Gluballs and their trajectories at $t > 0$,
  - The Soft Pomeron at $|t| < \Lambda$, *(4d Regge physics)*
  - The BFKL hard Pomeron at large negative $t$, *(5d Regge physics)*

- See BFKL cut and the Regge trajectories in theories with a conformal UV and IR confinement.
• Adding a running coupling, we see the BFKL poles moving continuously as a function of \( t \) and becoming the Regge trajectories at \( t > 0 \)

• A Pomeron vertex operator is introduced into string theory and proves useful for the computations.

• Clarify relation between DGLAP and BFKL operators, e.g., \( \Delta(j) \), its \( \lambda \) dependence, and its appearance in string theory.

  ▪ Many applications remain to explore; many questions remain to be answered.
Lesson from AdS/CFT dual description of Diffraction

Here $\lambda \equiv R^4/\alpha'^2 = g^2_{YM}N = 4\pi\alpha N$ in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory — the numerical coefficient can differ in other theories but the proportionality always holds — so large $\lambda$ is large ’t Hooft coupling.

The identification of $r$ and $k_\perp$ has its source in the UV/IR correspondence and has been suggested in numerous contexts, but here appears as a nontrivial and precise match. The effective diffusion time, $\ln s$, holds for both the BFKL and the Regge diffusions, at both large and small $\lambda$.

General form depends on Conformal Symmetry.
V. Beyond Pomeron

• Exclusive Production:
  – Diffractive Vector meson production at LHC, Higgs production, etc.

• Inclusive Production:
  – Diffraction Dissociation, Double DD, etc.
  – DIS, etc.

• Eikonal Summation:
  – Summing Witten Diagrams—(in progress)

• Heavy Ion Physics:
  – viscosity, “black hole” production, etc.
Small-x in DIS: Perturbative or Non-perturbative?

Anomalous Dimension of Leading twist operator

\[ \text{tr}(F_{+\mu} D_{+}^{J-2} F_{+\mu}) \]

Regge Behavior

\[ A \sim s^J(t) = s^{2+\alpha' t/2} \]
Spin vs Dimension

The physical state conditions $L_0 = \tilde{L}_0 = 1$ determine $\zeta$ and hence $\Delta$ as a function of $j$

$$L_0 = j/2 + (1/4\sqrt{\lambda})(\nu^2 + 4)$$

$$1 = L_0 = \frac{j}{2} - \frac{1}{4\sqrt{\lambda}}(\Delta - 2)^2 + \frac{1}{\sqrt{\lambda}}.$$
Unified Treatment of Spin vs Dimension

(4,2) and (0,2) have zero anomalous dimension

\[ \lambda = 0, \text{ BFKL} \]

\[ \Delta \rightarrow 4 - \Delta \]
Near $j=1$, $\Delta=3$:

\[(\Delta - 2 - j)(\Delta - 2 + j)(j - 1) = 0\]

\[
\begin{align*}
\Delta - 2 - j &= a(\Delta, j)g^2 \\
j - 1 &= 1 + \frac{ag^2}{2(\Delta - j - 2)} = 1 + \frac{ag^2}{2(\Delta - 3)} + O(g^4)
\end{align*}
\]

\[
\Delta = 2 + j + \frac{ag^2}{2(j - 1)}.
\]

\[a = -N/\pi^2\]
Summing over Graviton Loops
(in progress)

• Using Witten Diagrams
• AdS Propagator
• Reduction from AdS5 to AdS3 at high energies
• Pomeron Calculus
• ..........
Small-x Saturation:

Small $x$ Behavior of Parton Distributions from the Observed Froissart Energy Dependence of the Deep Inelastic Scattering Cross Sections

Small $x$ Behavior of Parton Distributions—imposing Analyticity and Unitarity Constraints on Deep Inelastic Scattering

E. Berger, M. Block, CIT
Structure Function and Unitarity

FIG. 1: Fits to the proton structure function data, $F_2^p(x, Q^2)$ vs. $x$, for 13 values of $Q^2$. The data are from the ZEUS collaboration [3]. The curves show 13 of our 28 global
FIG. 2: A plot of the derivative $\frac{dF}{dx} (x, Q^2) \times \frac{1}{\text{Im} \, T(Q^2)}$ vs. $Q^2$. H1 collaboration [4]. The exterior lines for $z = 0.0008$ are...
Extrapolation to Small-x

FIG. 3: A plot of $F_2^p(x, Q^2)$ vs. $x$ at $Q^2 = 25$ GeV$^2$ and $Q^2 = M_W^2$, where $M_W$ is the mass of the $W$ boson, along with results based on the CTEQ6.5M parton distribution functions [9].
VI. Further Developments

- Odderon --
- ................
- ................
- Diffractive Higgs Production at LHC
- ................
- Pomeron Calculus in AdS-3 Space,
- RHIC --- high density physics, BK equation, etc.
- ................
- Black hole production, etc.
- .........
More on the way!