Physics and the Violin

Fermi Lab. Colloquium

May 23, 2007

W. B. Atwood

A Brief History of the Instrument

Violin Given its Final Form

Gasparo da Salo & Maggini (~1550)

The Amati Family - Cremona, Italy

Andrea (1535-1611) - Start of School

Heronymus & Antonio (1556-1630) - 2nd Generation

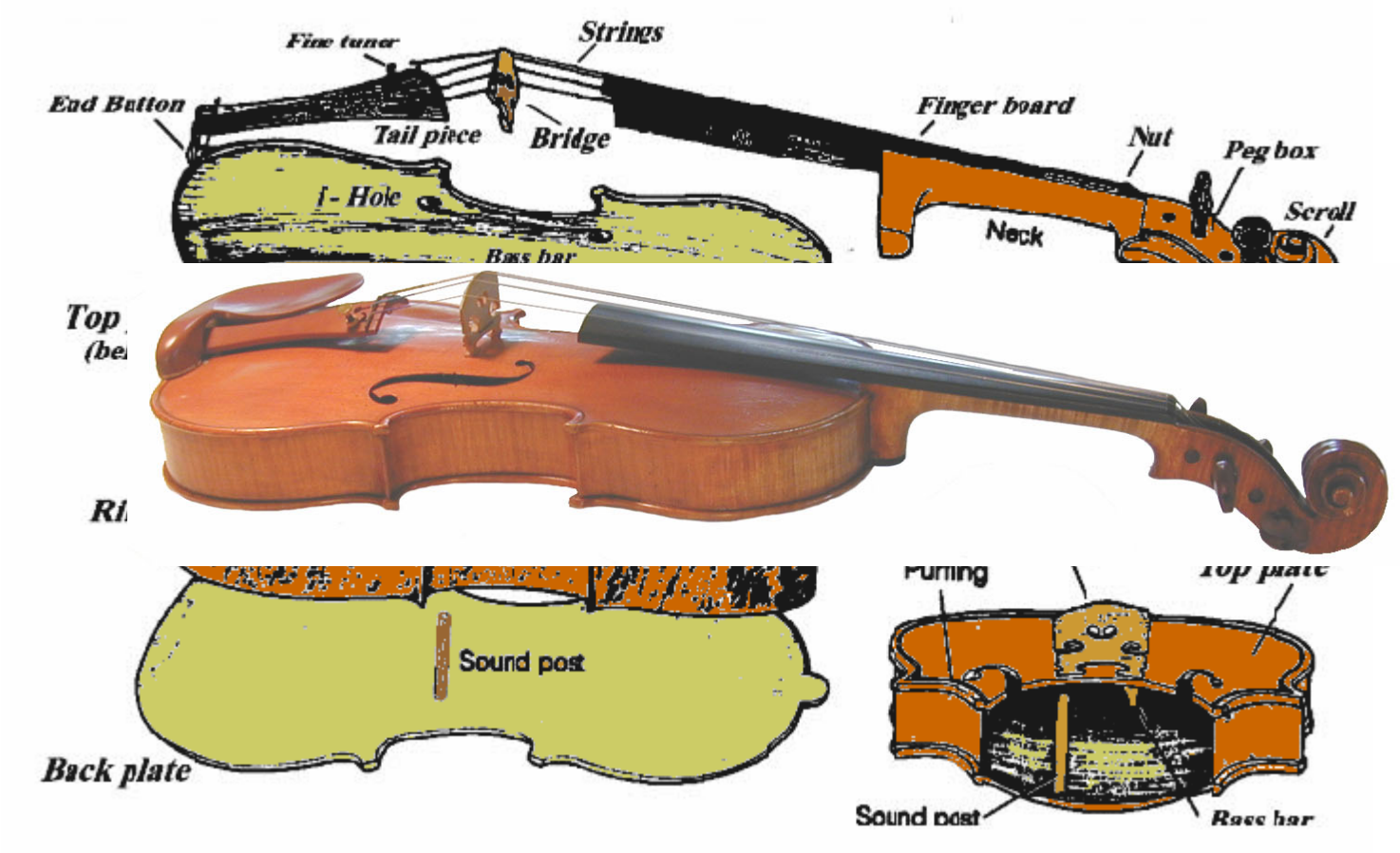
Nicola (1596-1684) – The Grand Amati's

Best Known

Antonio Stradivarius (1644-1737) - 1200 violins!

Giuseppe Guarnerius (1687-1742) - Preferrred?

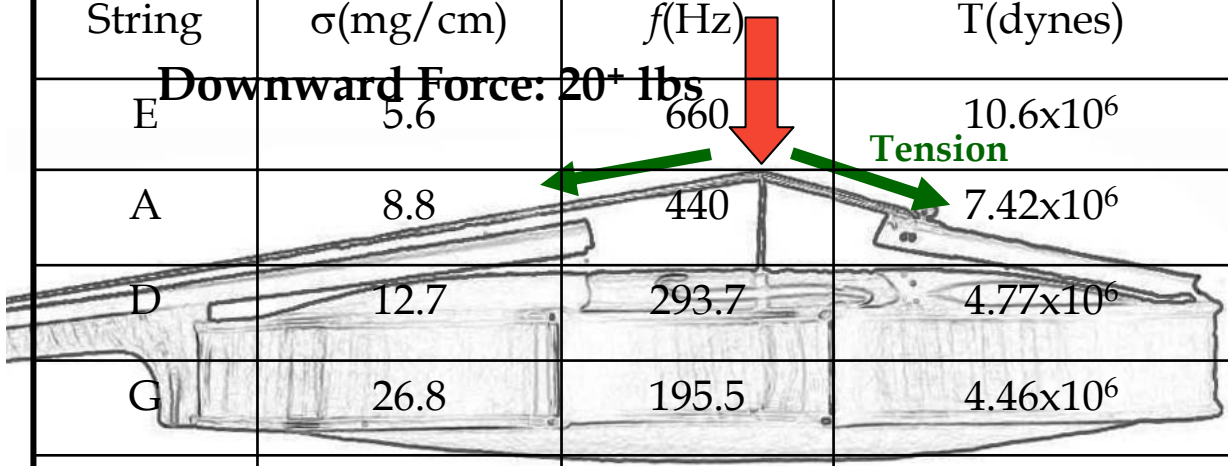
Details of the Sound Box



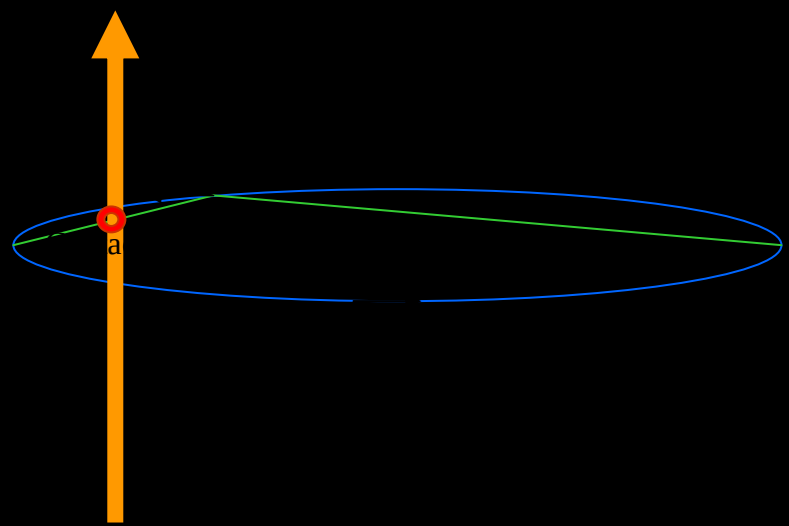
Overview of the Structure

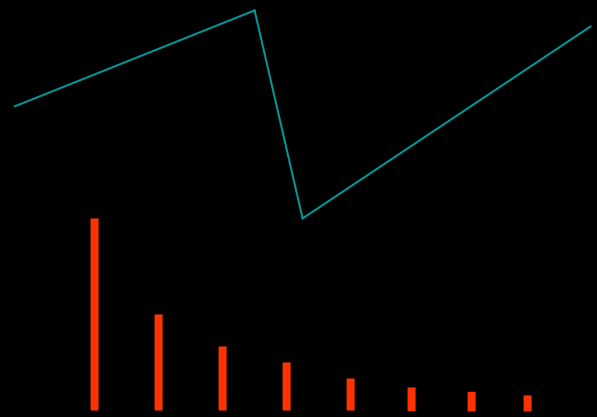
Mersenne's Law (1636): $$f = \frac{1}{2L}\sqrt{\frac{T}{\sigma}}$$

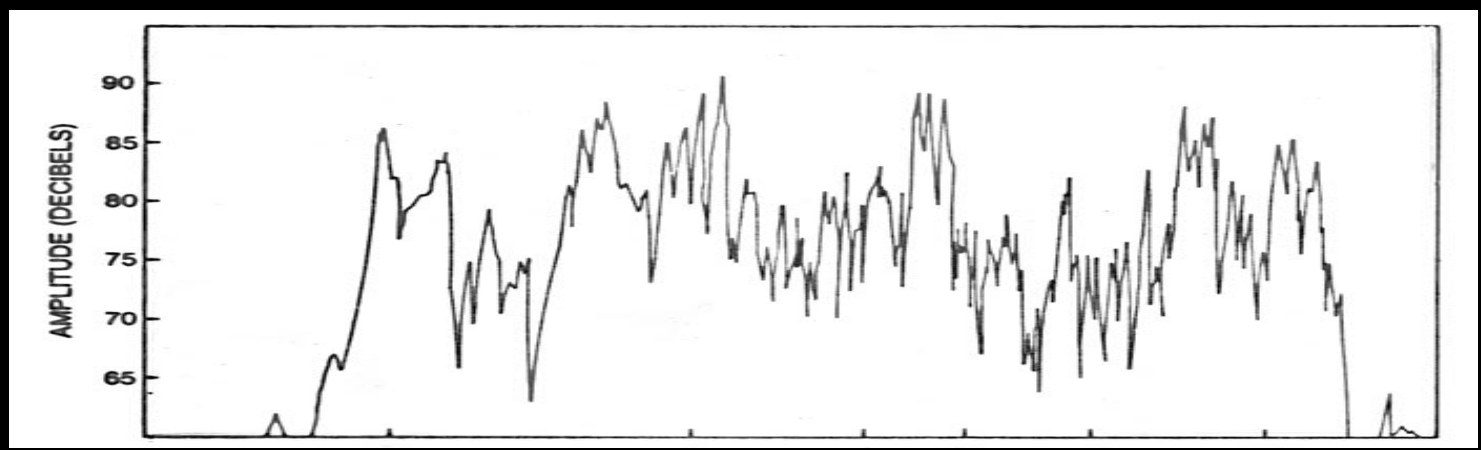
where ***f*** = Frequency; ***L*** = String Length; ***T*** = Tension; **σ** = Linear Mass Density



String	σ(mg/cm)	*f*(Hz)	T(dynes)
E	5.6	660	10.6×10^6
A	8.8	440	7.42×10^6
D	12.7	293.7	4.77×10^6
G	26.8	195.5	4.46×10^6
TOTAL			27.3×10^6 = 61.4 lbs

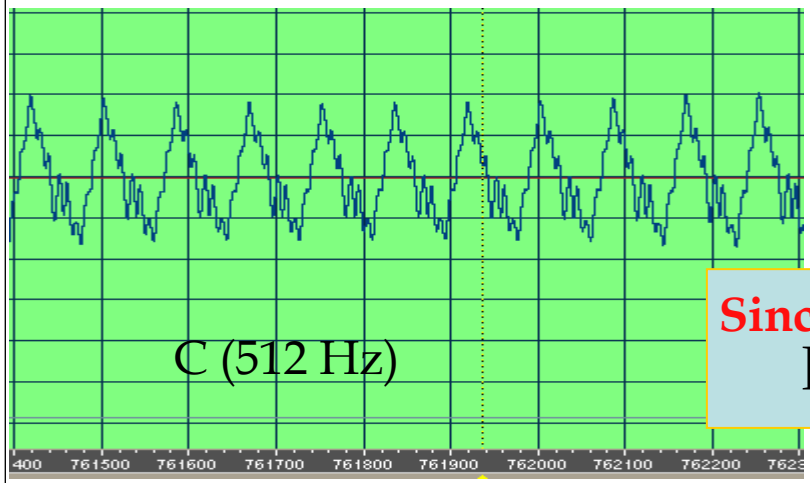

a

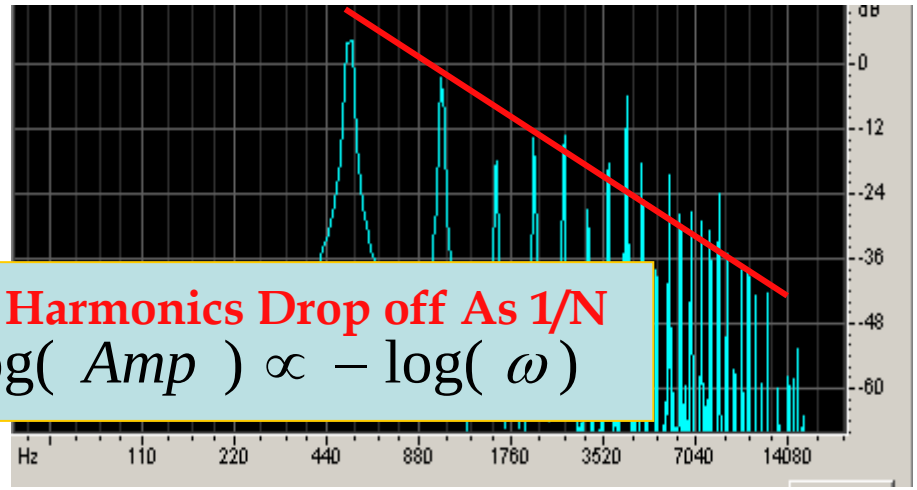



AMPLITUDE (DECIBELS)
90
85
80
75
70
65

Drive + Response

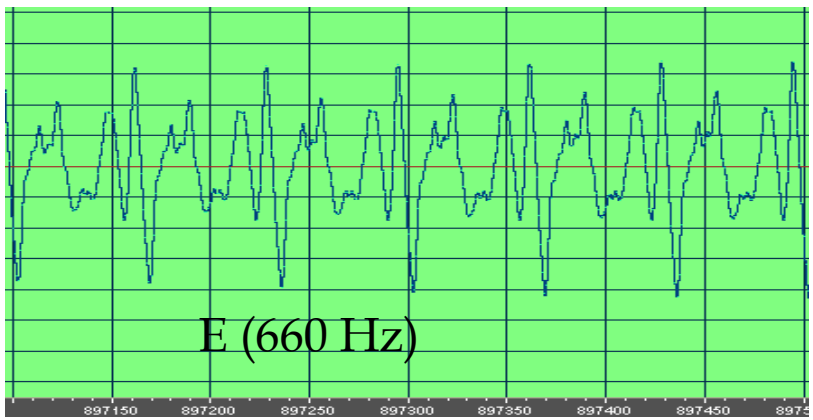
Different Notes Have Different Harmonic Content



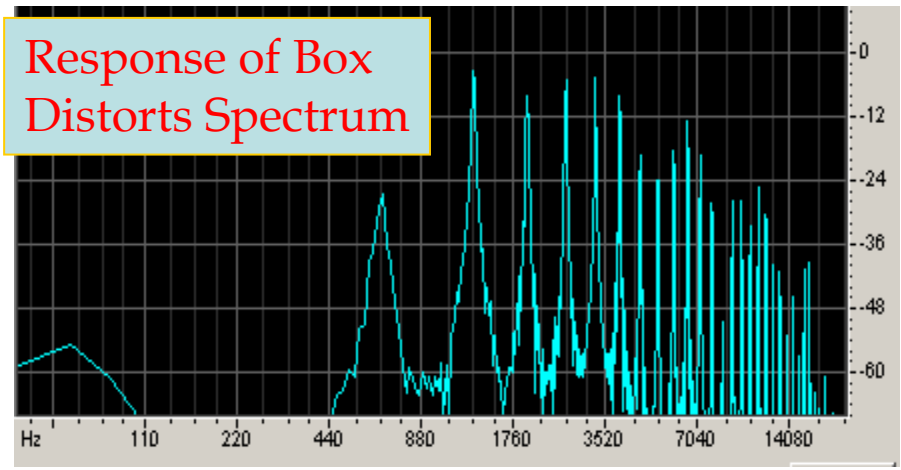


Since Harmonics Drop off As 1/N

$$\log(\ Amp\ ) \propto -\log(\ \omega\ )$$



Response of Box Distorts Spectrum



Acoustic Basics

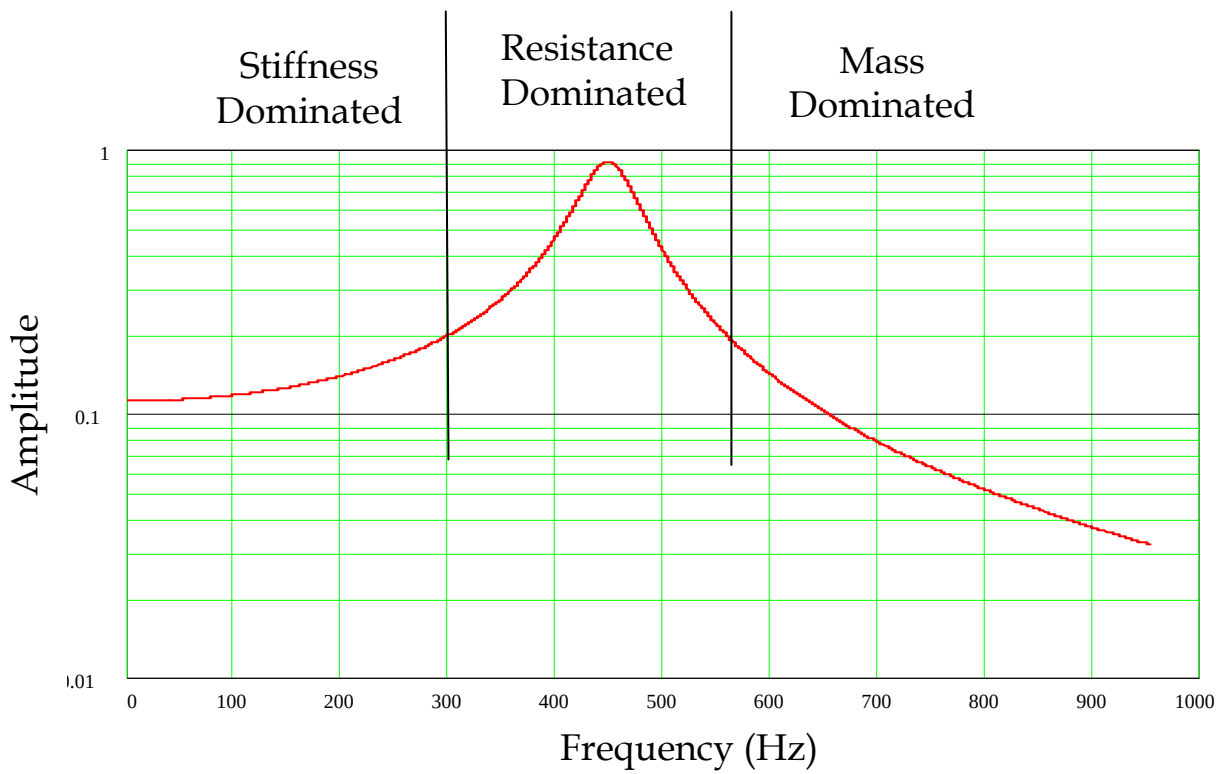
Mechanical Oscillators have the same mathematics as RLC circuits

Inductance L ⇔ Mass M
Resistance R ⇔ Mechanical Resistance R_m
Capacitance 1/C ⇔ Stiffness: K

} IMPEDANCE

$$Z = j\omega L + R - \frac{j}{\omega C}$$

$$Z_m = -i\omega m + R_m + \frac{iK}{\omega}$$

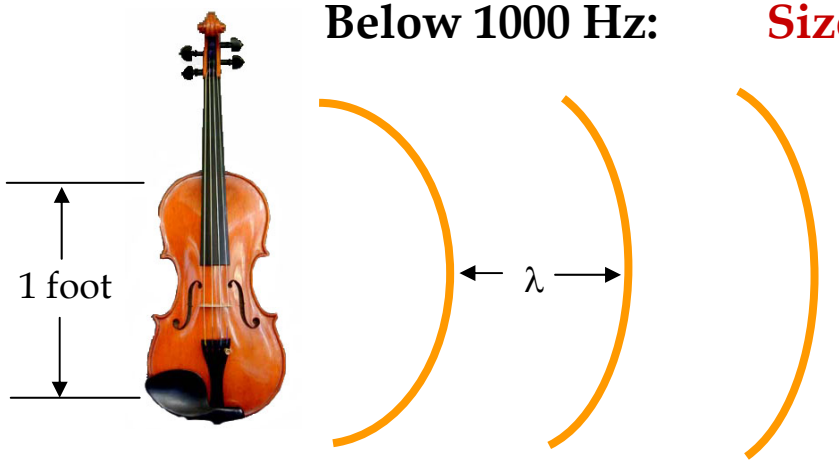


Violin:

$\sum$ COUPLED LINEAR OSCILLATORS

Regions of the Response Spectrum

Below 1000 Hz: **Size of violin sets scale**



Only modes in which the VOLUME of the Violin changes contribute:

- QUASI-MONOPOLAR MODES –

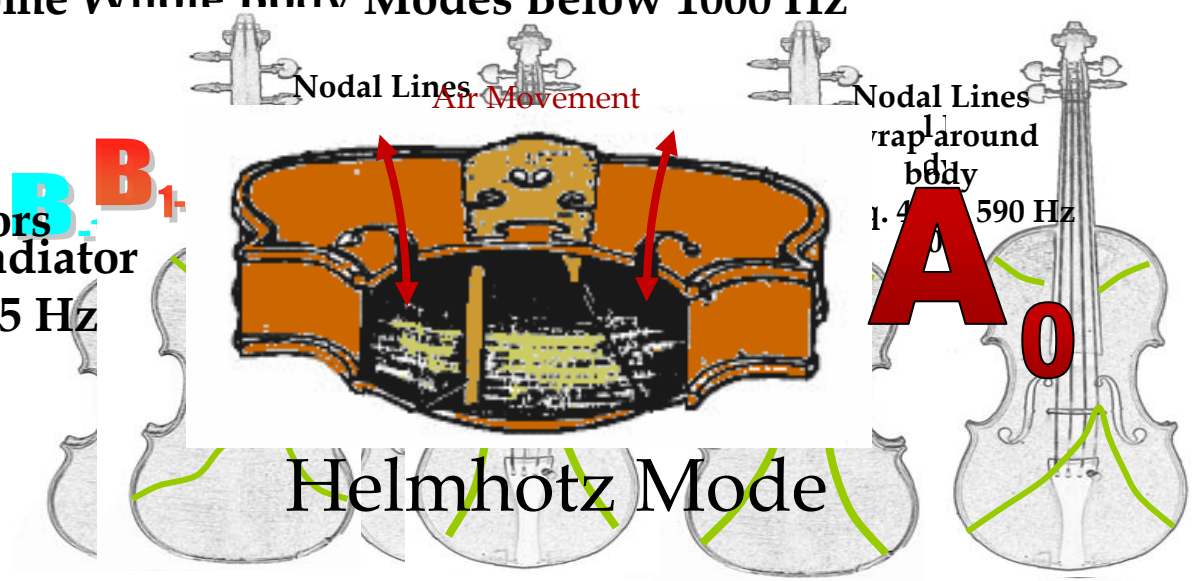
Names: A_0, B_{1-}, B_{1+}, etc.

Helmholtz Mode: A0 (265 -280 Hz)

Some Whole Body Modes Below 1000 Hz

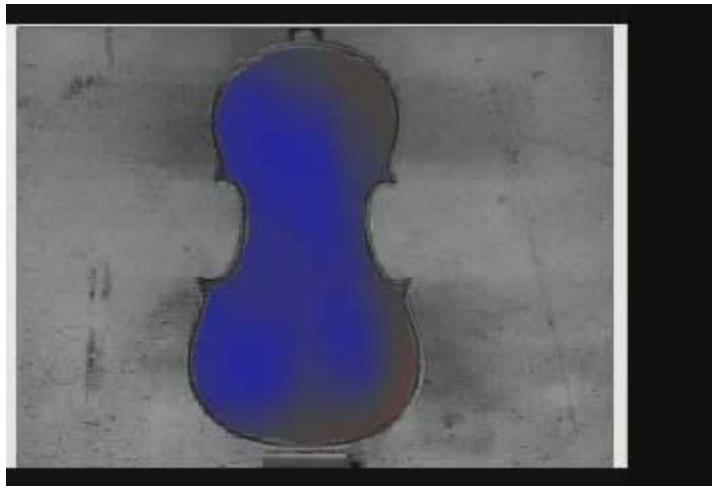
Two Poor Sound Radiators
Not truely monopolar
Large ΔV over a cycle
Small ΔV over a cycle

Two good sound Radiators
Excellent Sound Radiator
Frequency 260-285 Hz
Plates move together



The A_0 Mode in Motion

Per Gren et al, Meas. Sci. Tech., 17 (2006) 635-644



Simple Radiation Pattern

Approx. uniform in all directions

Simple mono-polar radiator

The body is expanding and contracting (pumping)

Region Above 1000 Hz

1000 Hz ⇨ 2500 Hz

2500 Hz ⇨

Top & Back Tend to Become Decoupled

Frequency Above First Bridge Resonance

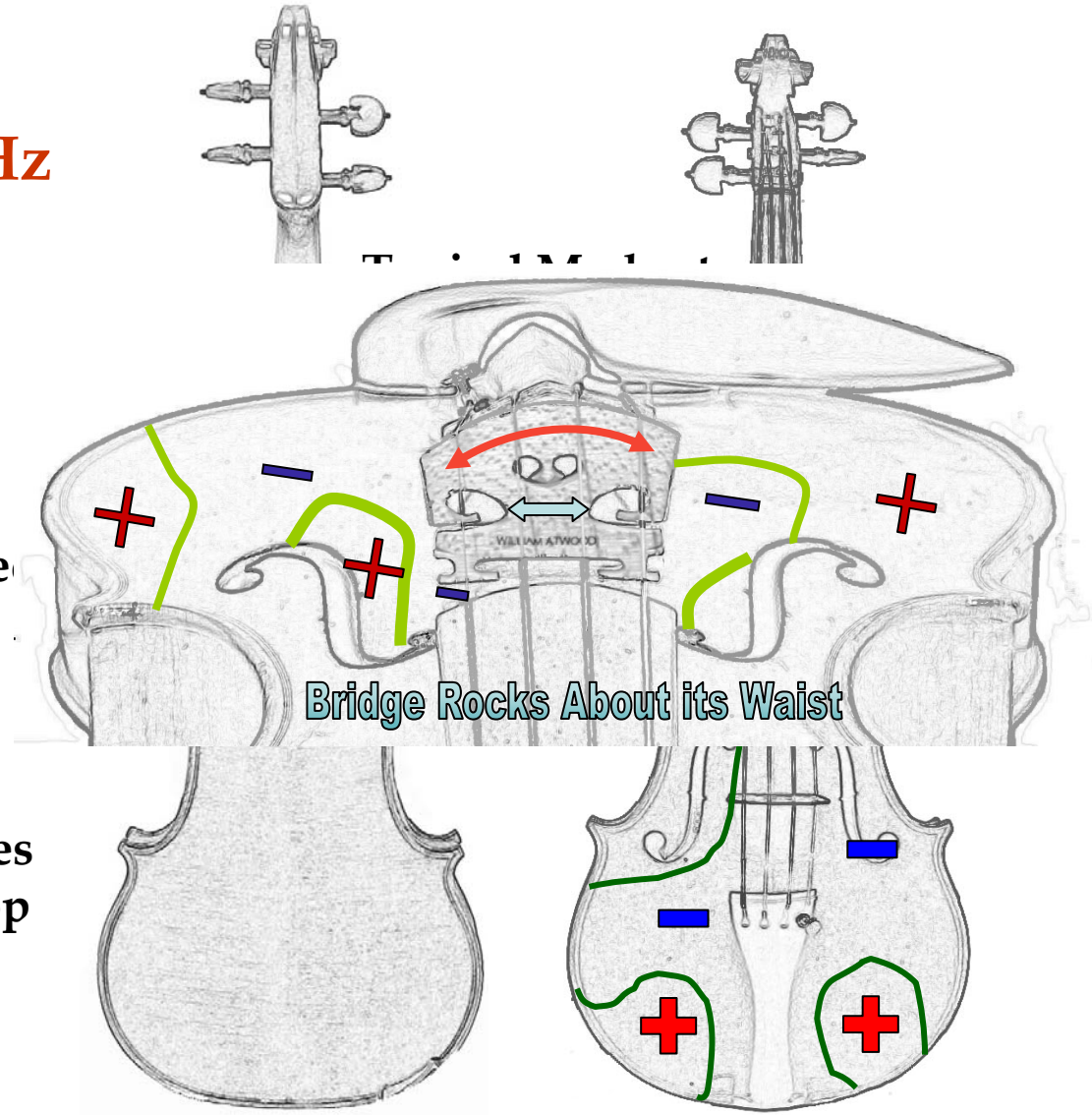
Modes Become "Blurred" – Overlapped and Not Easily Distinguishable

Modes even more blurred

Modes have many nodal/anti-nodal areas

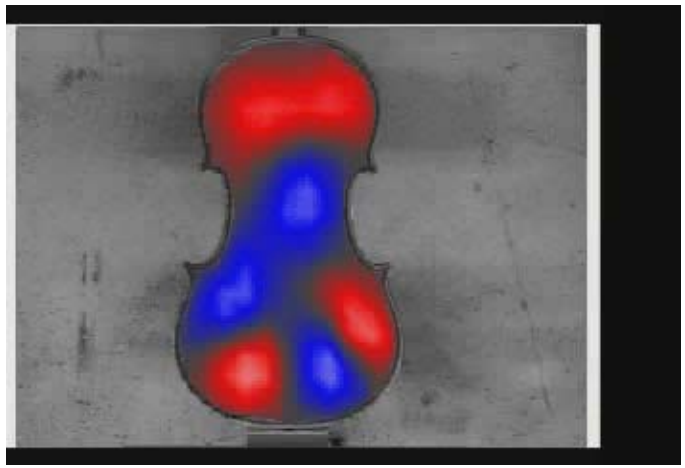
Bridge an active Radiator

Radiated Sound Comes Primarily from the Top



Motion at 1415 Hz (Several Modes Contribute)

Per Gren et al, Meas. Sci. Tech., 17 (2006) 635-644

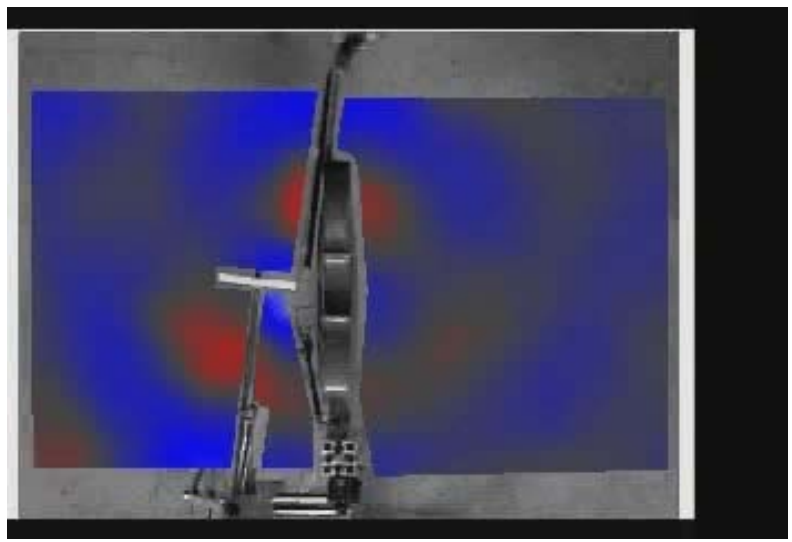


Complex Radiation Pattern

Presents different sound depending on direction which is constantly changing

"… as if the sound was coming for everywhere at once, dancing and sparkling in the listeners ear…"

G. Wienwreich



The Violin Maker's Role

Making the Minimal Structure

Objective: Making the most efficient radiator

or

Making the lightest structure consistent with the strength and resonance requirements

- Wood Selection
  - Grain Orientation
    - To Split or Not?
    - Book matching
  - Shaping
    - Arching Pattern
    - Thickness

Circuits $\neq$ Violins

CIRCUITS	VIOLINS
R, **L**, and **C**	**M**, R_m, and **K**
INDEPENDENT	COUPLED TOGETHER one piece of WOOD

Example: Stiffness of a beam: $K \propto H^3$
Mass of a beam: $M \propto H$
} Natural Freq. $f = \sqrt{\frac{K}{M}} \propto H$

More Complications: Wood is **NON-ISOTROPIC**
Strength properties described by Tensor

Diag. Elements = Young's Modulus | Off Diag. Elements = Shear Mod.

Over-Arching Physics Principles

Scaling laws: Material Properties ⇔ Frequency and Response Properties
(Originally due to John Schelleng, E.E.)

Overall Dimensions: ***H*** = Thickness & ***L*** = Length

Y = Young's Modulus (Elastic Modulus), ρ = Density, Mass ***M*** = ρHL^2

$\mathbf{c_s}$ = Speed of Sound= $\sqrt{\frac{Y}{\rho}}$ Stiffness (**K**) $\propto YH^3L^{-2}$ Freq. $f = c_s \frac{H}{L^2}$

Example: Impedance Z (keeping *f* & *L* fixed)

$$Z = \frac{F}{v} \sim \frac{1}{\dot{x}}(M\ddot{x}) \sim (\rho HL^2) f \sim (\frac{\rho}{c_s}) f^2 L^4$$

Want Low Impedance ➔ More Motion ⇔More Sound

Wood Properties

Selecting the Wood

"BEST" BACKS

BOSIAN MAPLE

r = .56 - .65 gr/cc

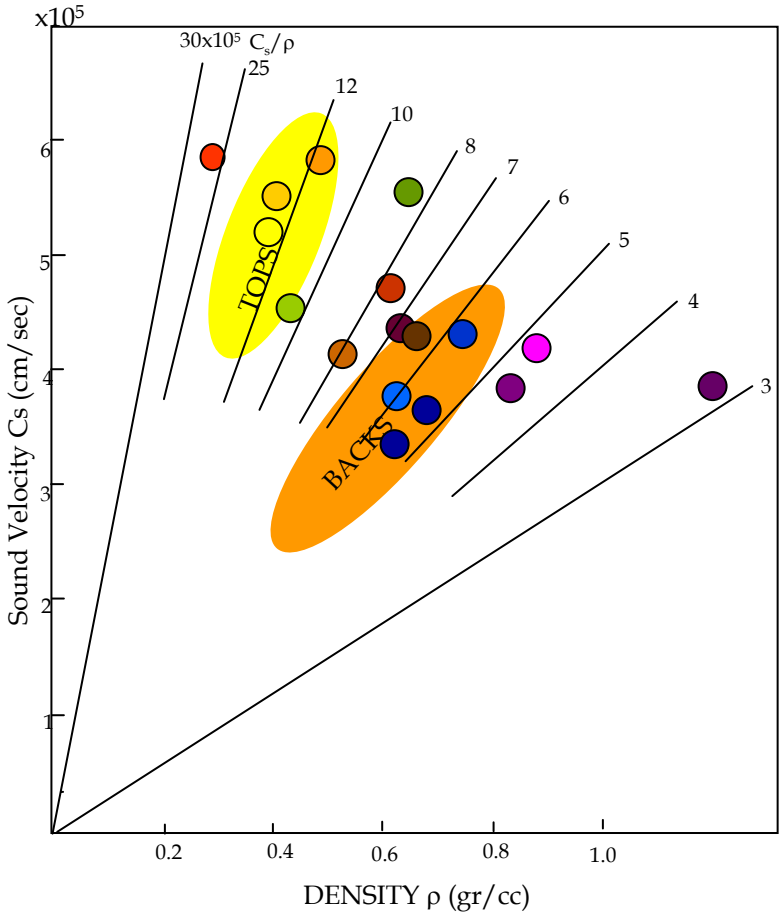
"BEST" TOPS

ITALIAN SPRUCE

r = .32 - .38 gr/cc

USE WOOD WITH THE HIGHEST Cs/ρ VALUE CONSISTANT WITH STRUCTURAL REQUIREMENTS

●	BALSA
●	POPLAR
●	NRW SPRUCE
●	LINDEN
●	PADOUK
●	WHITE FIR
●	PINE
●	LARCH
●	WALNUT
●	MAHOGANY
●	ASH
●	NRW MAPLE
●	SYC MAPLE
●	SCYMORE
●	OAK
●	PEAR
●	PERNAMBUCO

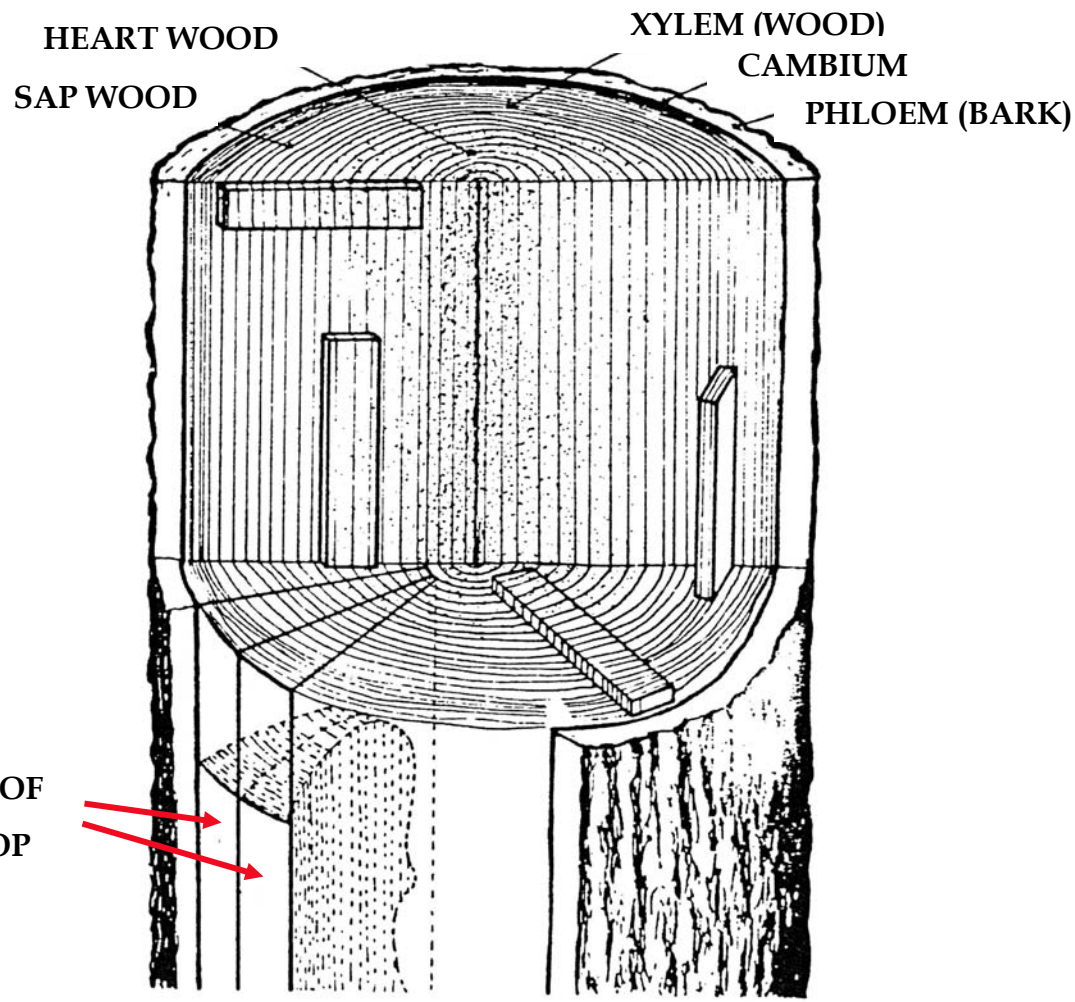


Grain Orientation

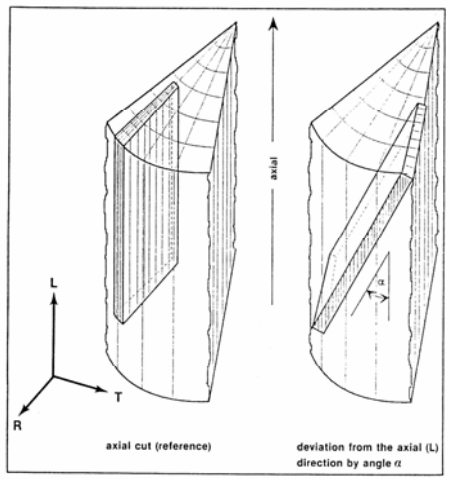
TOPS
Spruce:

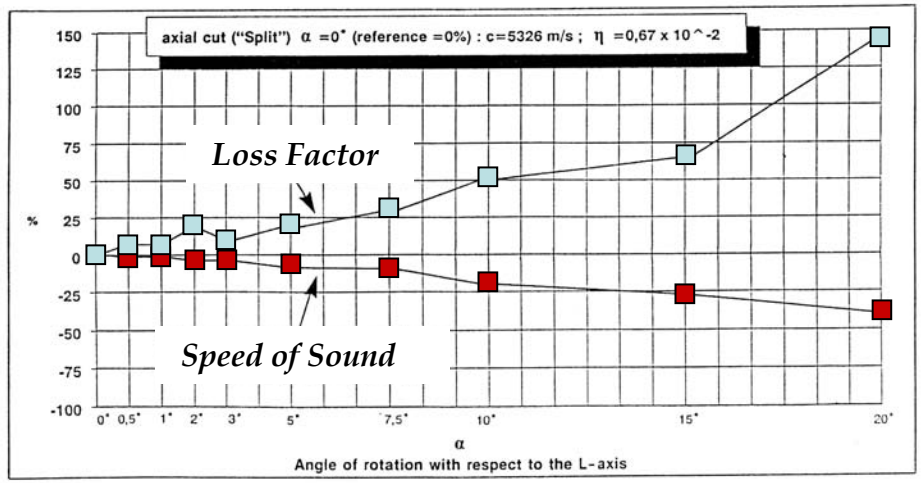
Uniform Growth
~ 1mm Annual Ring Spacing

Aging: Best if > 5 Years!



Grain Orientation

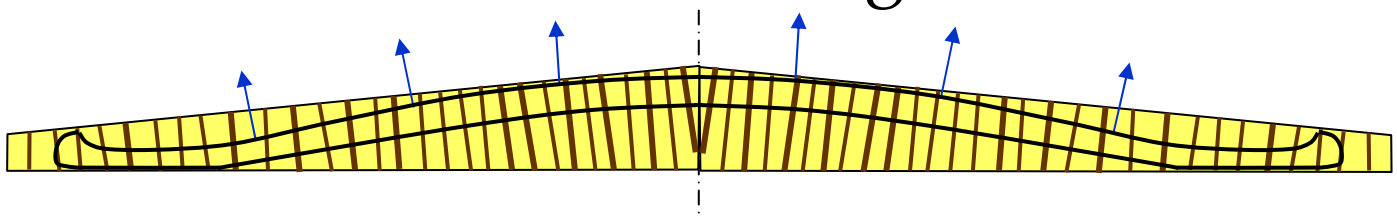




Martin Schleske, "Speed of Sound and Damping...", CAS Vol 1, No 6 (Series II), 1990

Investigation of speed of sound (C_s) and damping (loss factor) as a function of cut angle: "run-out"

"Book Ending"



Notice the "V" Pattern
Sets grain lines ~ parallel to local normals

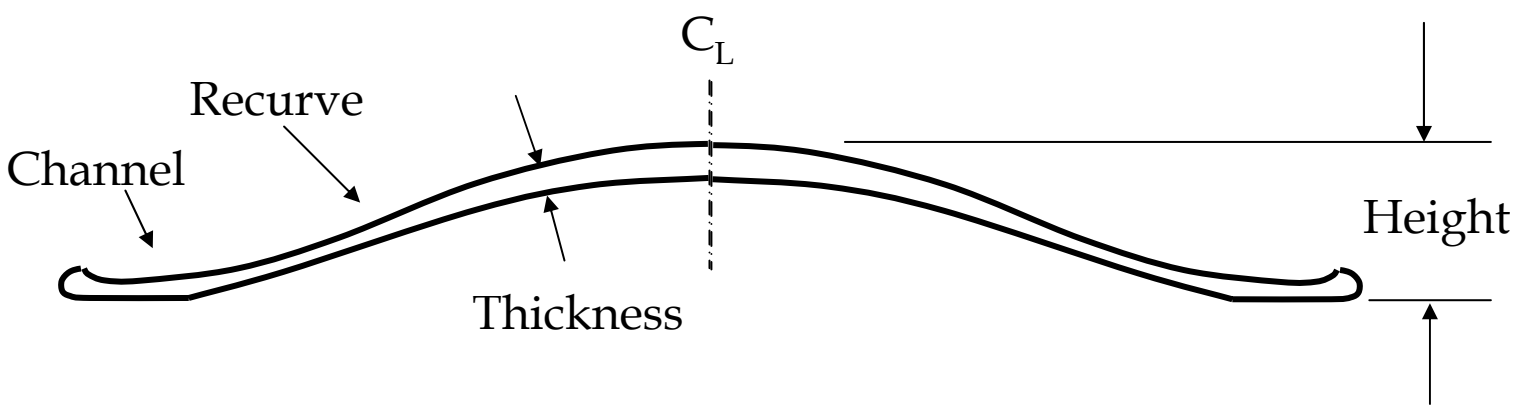
Shaping the Plates: Part I

Overall Shape (as seen for the outside): Arch or Arching Pattern

Thickness Pattern (Inside Arch): Graduations

"Arching is sculpting in sound!" Tom Croen, 1996

Cross Section of the Back of a Violin



Flat Arch: (Height 12-14 mm)- not strong – REQUIRES GOOD WOOD

Characterizing the Arching

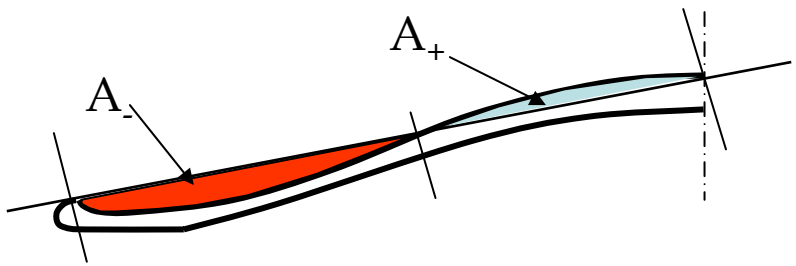
$A_s = (A_+ - A_-) / (A_+ + A_-)$

A_s has a range from -1 to +1

When $A_s = 0$ the curve is "balanced"

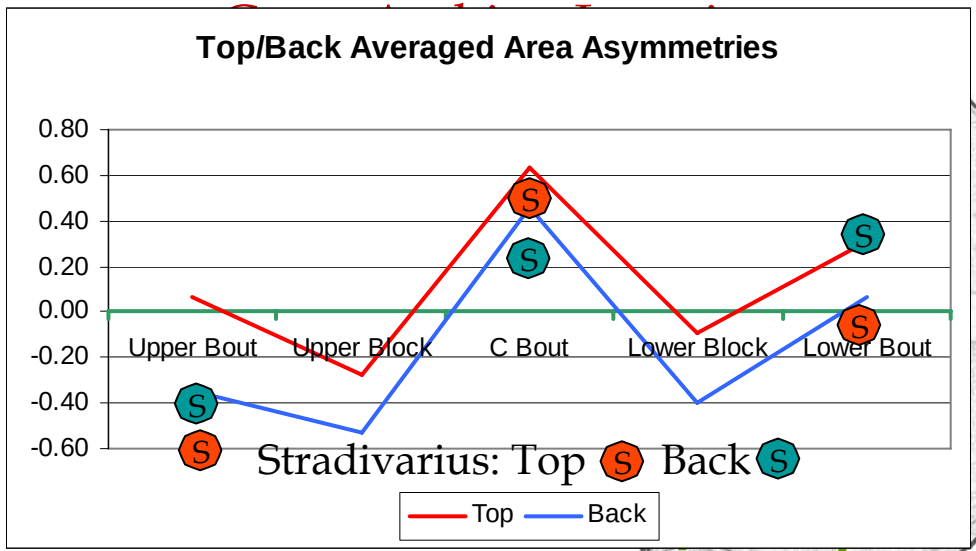
$A_s > 0$ the curve is "full"

$A_s < 0$ the curve is "swoopy"



Results of a study on 25 Guarneri Violins.

Data extracted from
"Guiseppe Guarneri del Gesu"
Peter Biddulph, London, 1990



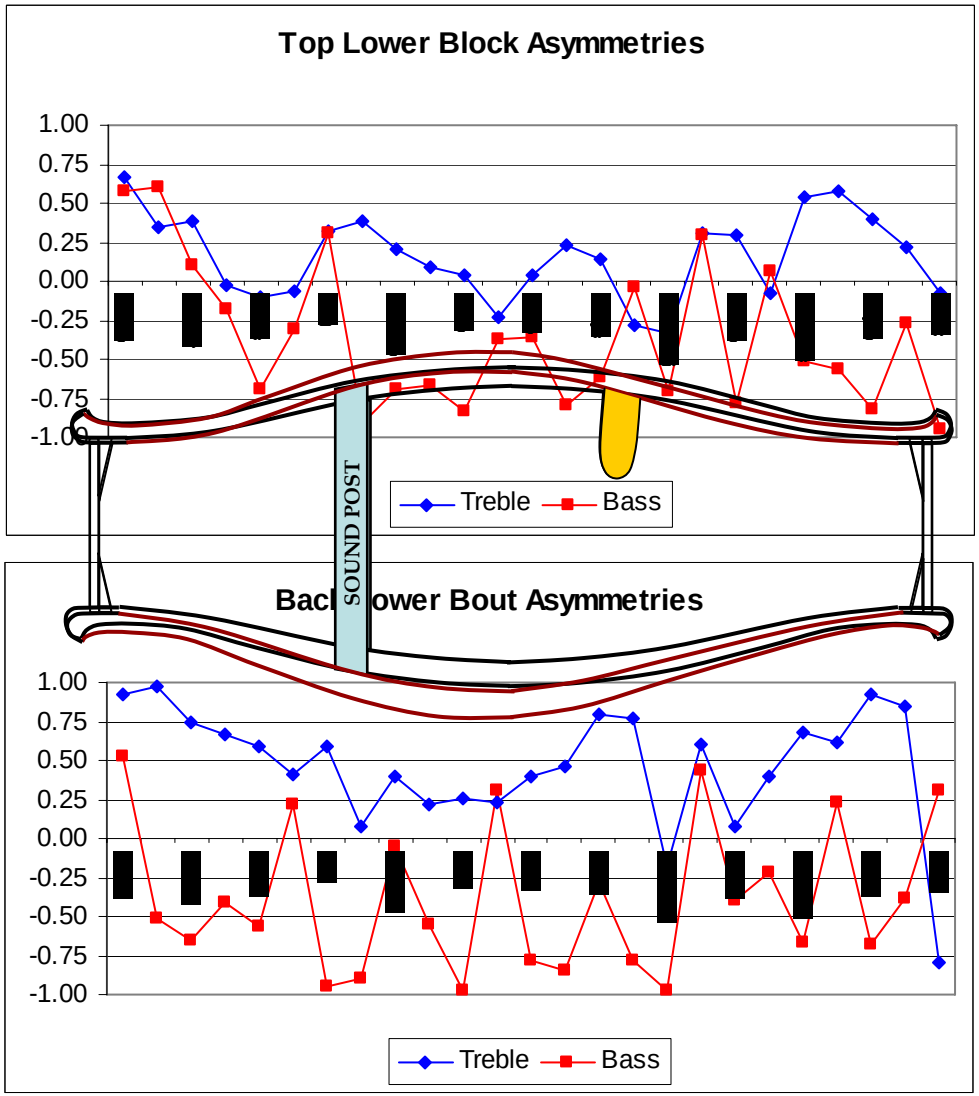
Time Works Wonders?

Violins start life ~ **symmetrically**

Over time the body **distorts**

Due to the **unbalanced forces** generated by the bridge acting on the sound post and bass bar.

This may contribute to **asymmetries** in modes below 1000 Hz.



Shaping the Plates: Part II

(Graduations – Thicknessing)

Goals: **To promote Resonance** (Enhance Q)

Set Resonances in Assembled Instruments

at **Particular Frequencies**.

Tradition & Intuition:

Use resonance properties of components as

a guide -

Plate tuning:

Tap tones

Chaldini Patterns

Eigenmode Plate Tuning

Where to remove wood:

Mode 5:

Traditional Receipt:

Hold Plate at

Sensitivity ~ .5

Tap Plate at

Desired Note ~ F-F# (350 – 370 Hz)

Mode 2:

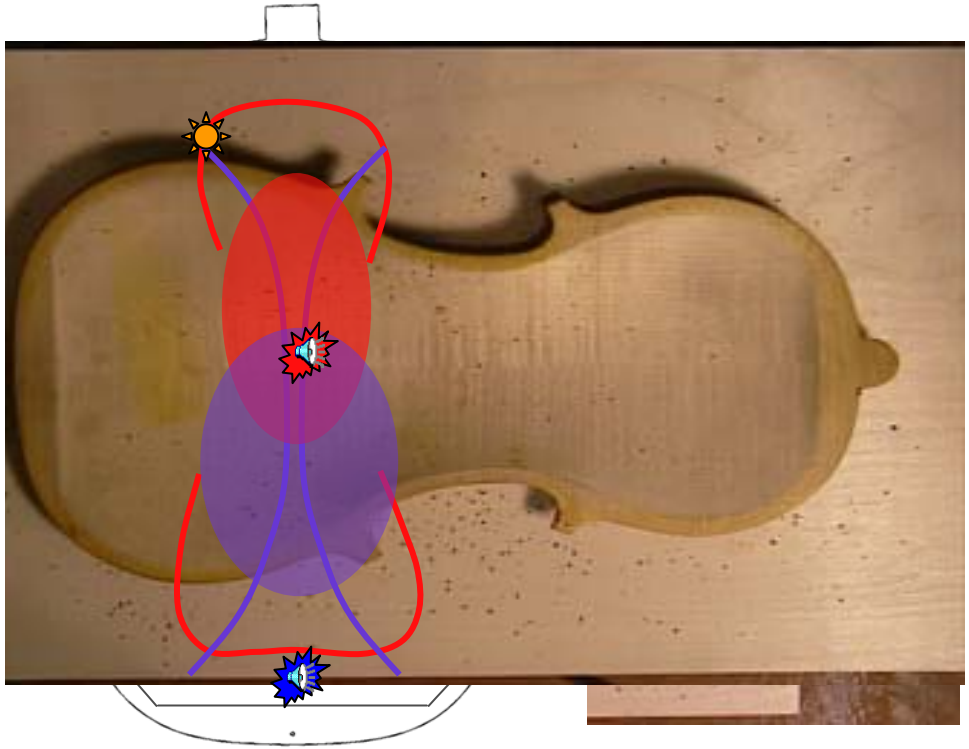
Tap Plate at

Desired Note ~ an octave lower

(150 – 180 Hz)

Sensitivity ~ 1.

$$\textbf{Sensitivity} = \frac{\partial f / f}{\partial m / m}$$



Chladini Patterns

Same Modes Found in Top Plate

Bass Bar Profile

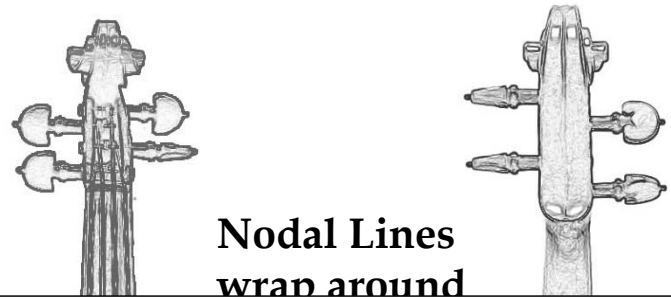
Where to Remove Wood

Plate ~ uniform thickness

Bass Bar Profile and Angle Controls Modes

Free Plate to Assembled Instrument

B_{1+} Mode



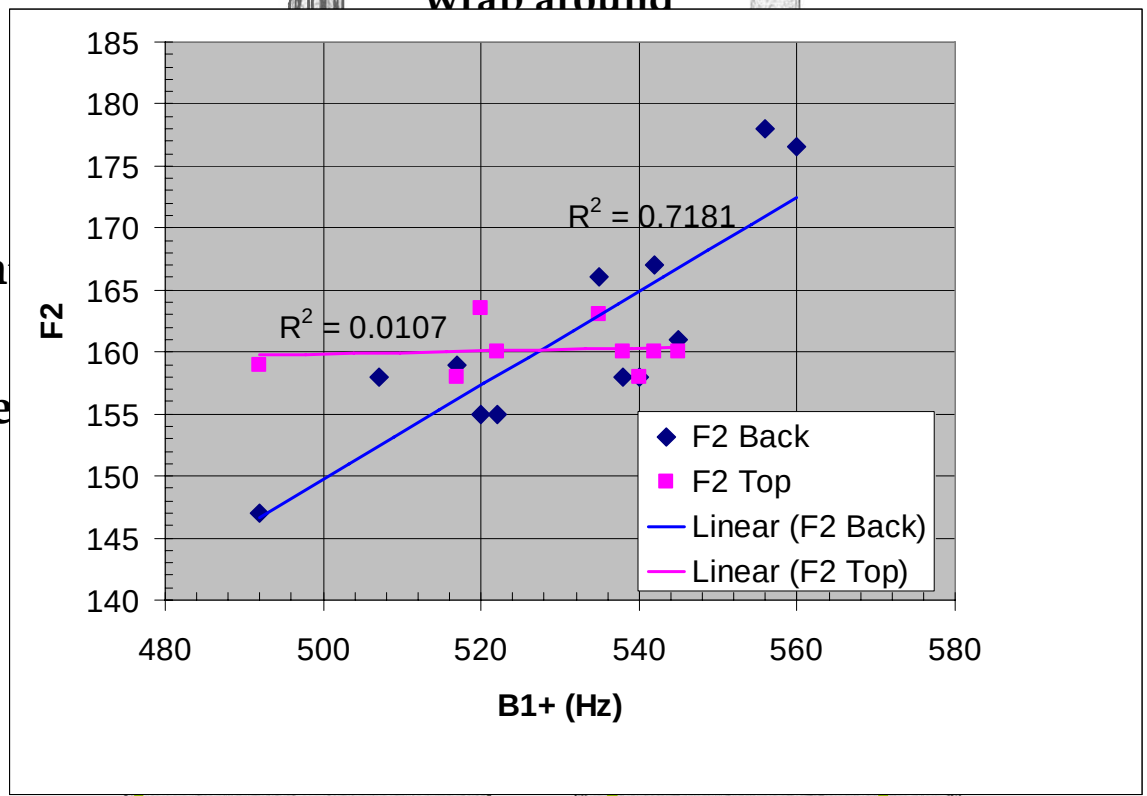
Nodal Lines wrap around

What matters are the Eigenmodes in the Assembled Instrument

Relationship to Free Plate Eigenmodes not obvious!

Establish Relationship by Correlation Studies

Allows Maker to Build in Specific Frequencies

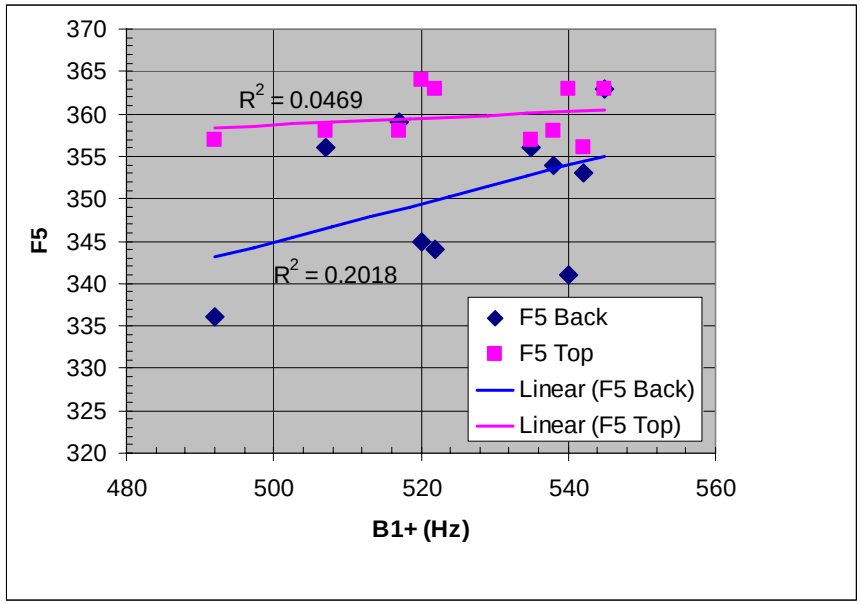


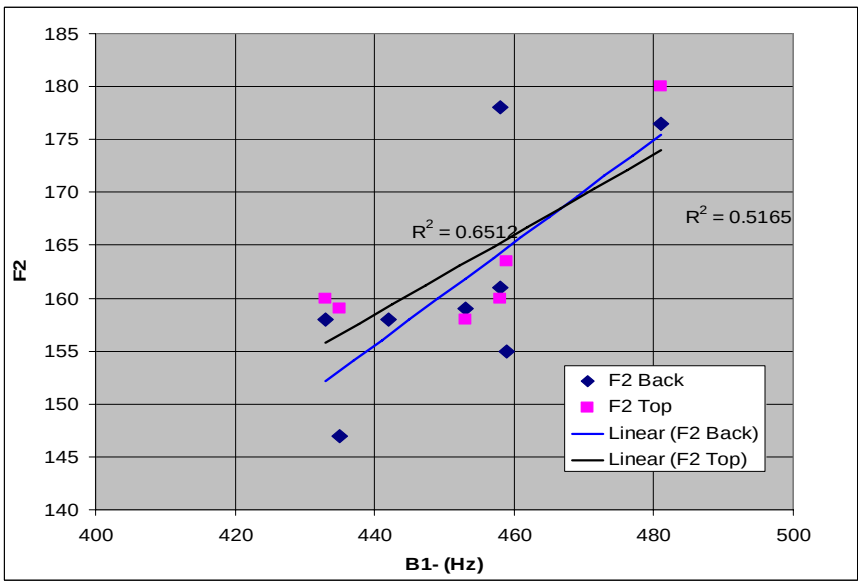
More Mode Correlations

B_{1+} & Mode 5

Small if any Correlation to $\mathbf{B_{1+}}$

Recall that Mode Freq. go as plate thickness Creates false Correlation!





B_{1-} & Mode 2

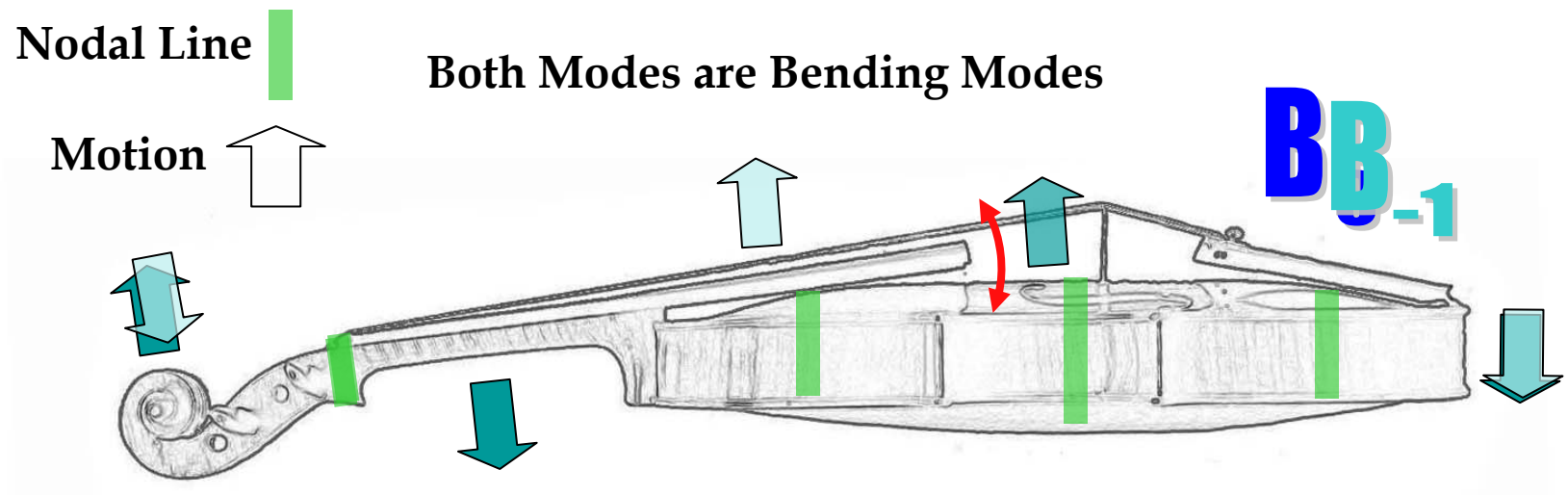
Both Mode 2 in Top & Back Appear to be Correlated

Mode 5 again shows little Correlation

Mode Adjustment in Assembled Instruments

Frequencies of Certain Modes can be moved in assembled instruments

Example: B_0 & B_{-1} (poor sound radiators)

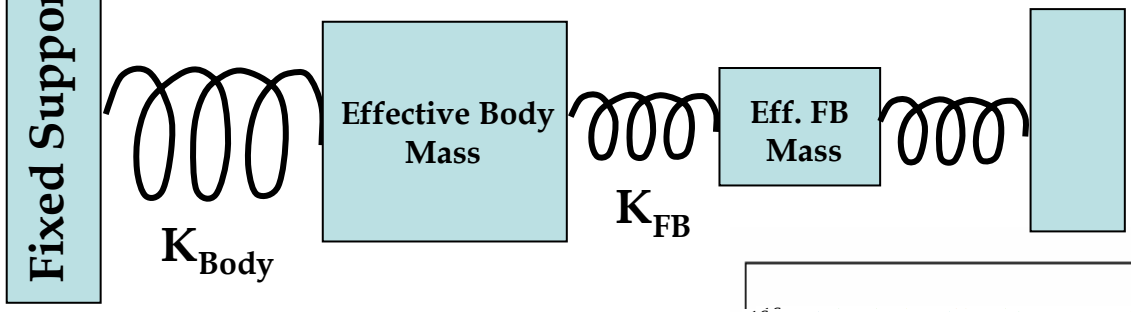


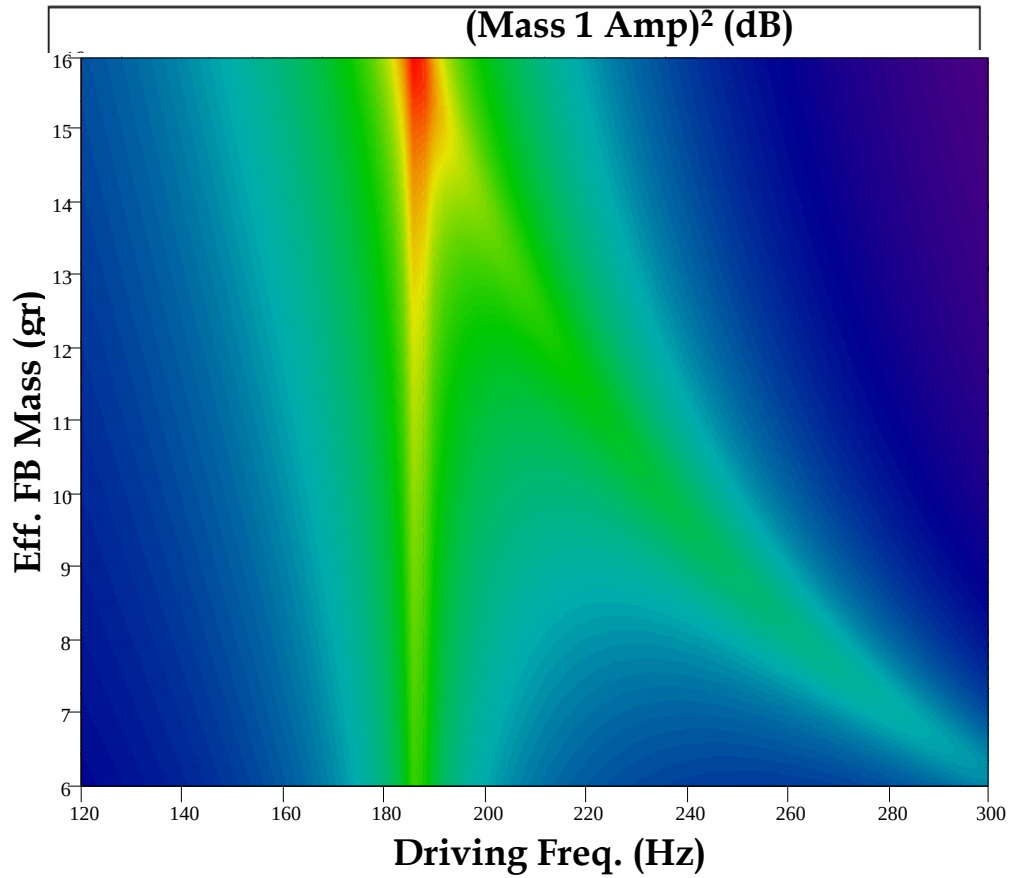
Finger Board acts as a coupled Oscillator

Adding Weight to end of Finger Board Lowers Frequencies

Coupled Linear Oscillators

A Simple Model





Two Modes

Masses move in Phase

(Lower Frequency)

Masses move with opposite Phase

(Higher Frequency)

How Well does the Model Fit?

Data

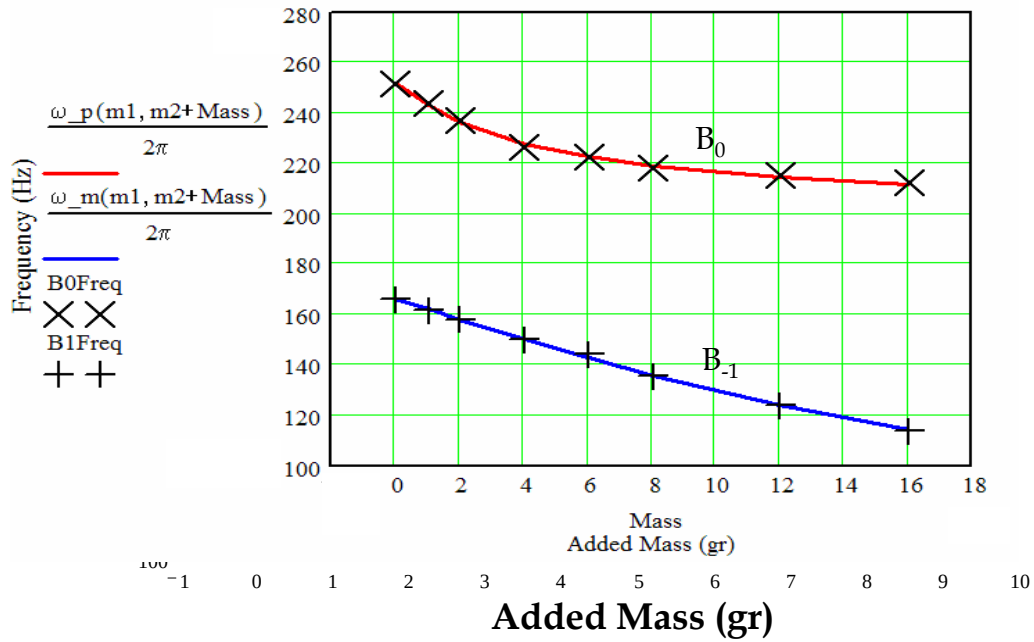
Masses: 0 gr – 8 gr added to end of FB

Recorded B_0 & B_{-1}

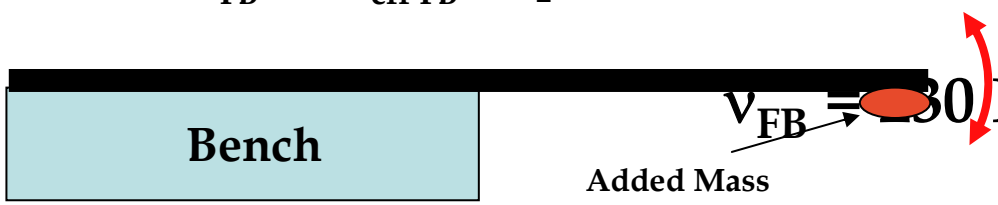
Adjusting Parameter

$M_{eff\ FB}$ = 11 gr

ν_{FB} = 221 Hz



Test: What are ν_{FB} & $M_{eff\ FB}$ Separate from the Violin?



ν_{FB} = 230 Hz

$$\frac{\partial \omega}{\partial m} = -\frac{\omega}{2m}$$

The Player's Domain: Controlling the Helmholtz String Motion

Main Components

Speed

Dictates String Amplitude

Pressure

Too little – no stick

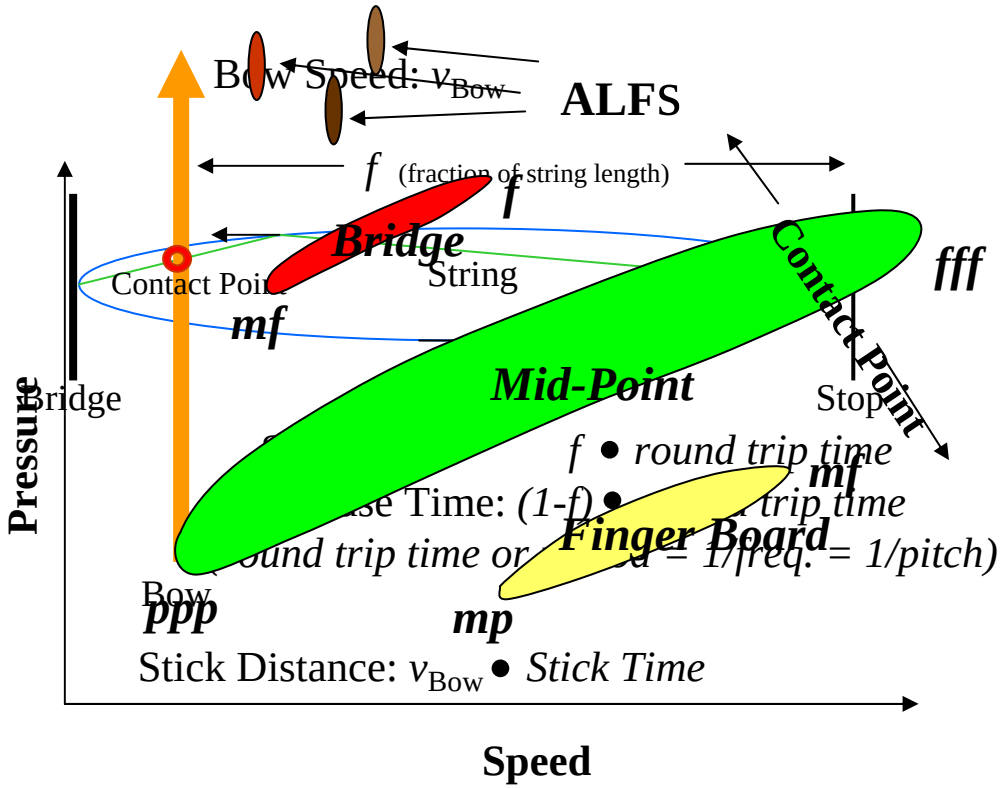
Too much – no release

In between - changes harmonic content

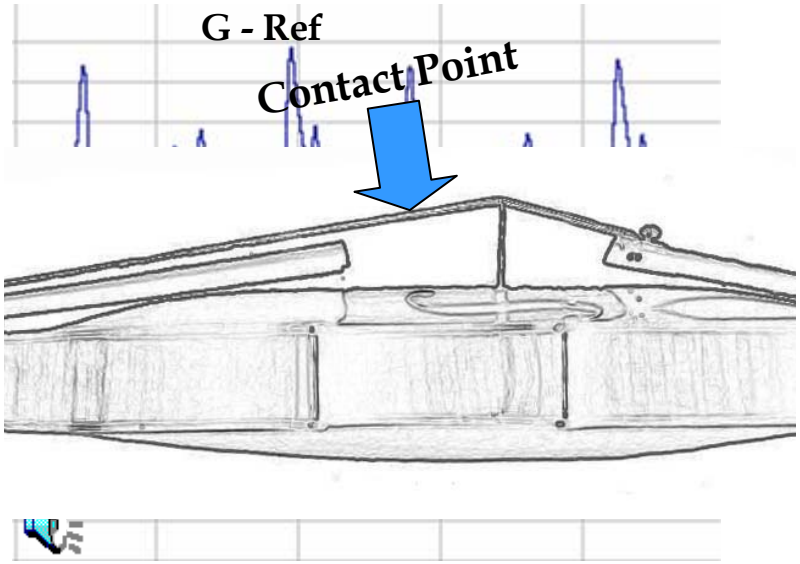
Contact Point

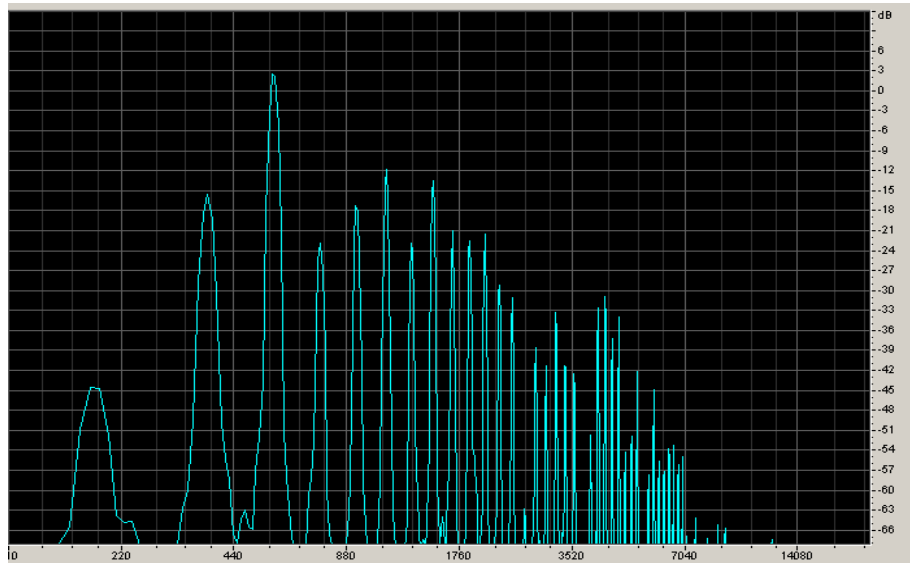
Close to Bridge – increased harmonics

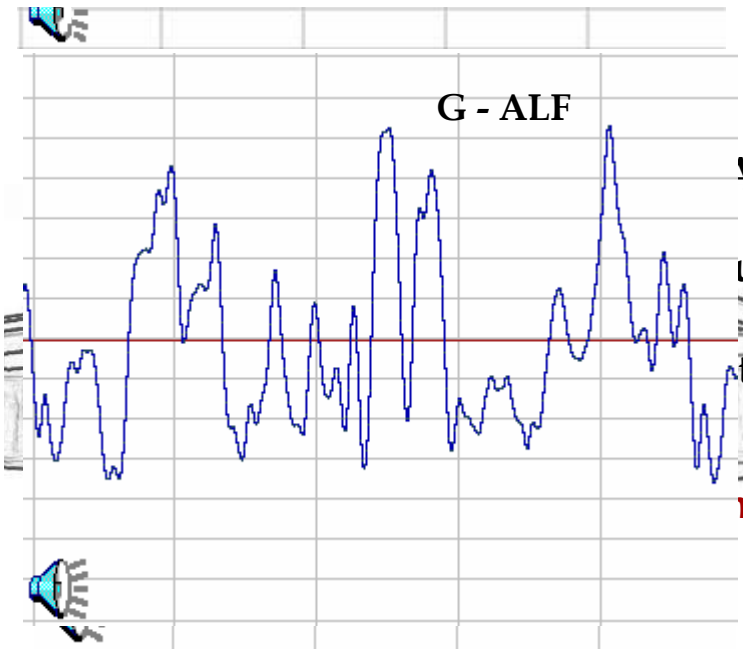
Close to Finger Board – decrease harmonics

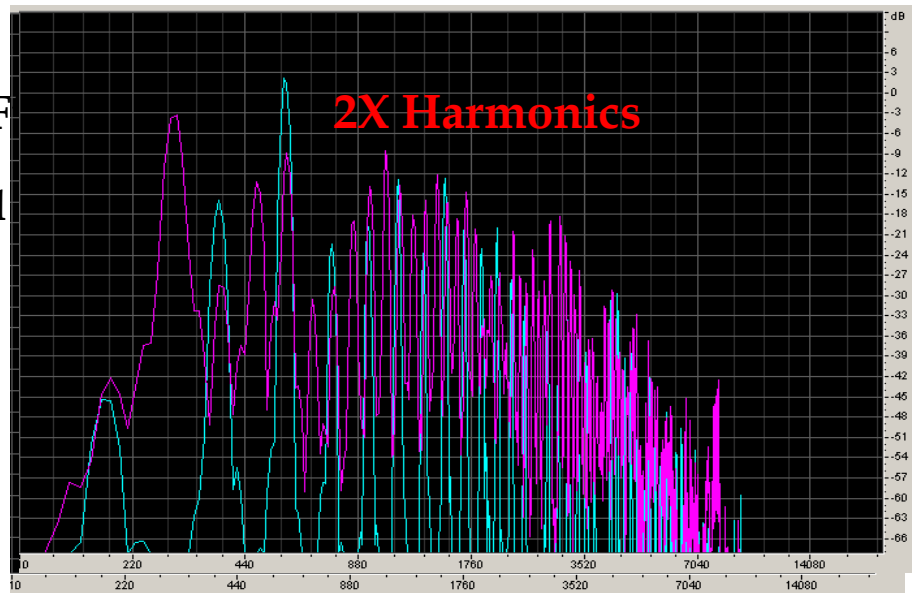


Sound Comparisons









Conclusions

- Many of the Traditional Methods of Making are found to be well based on Physics Principles.
  - The Violin is a complex, interlocking system of coupled oscillators.
    - Disentangling Material Properties, Shaping, and Thicknessing is challenging.
      - Beginning to understand how to quantifiably control results.
        - Both Maker and Player have significant roles in the resulting sound.

The Future

- Non-traditional materials

  Balsa wood, Composites (carbon fiber, etc.)

- Electric Violins

- More complete understanding of mode structure

- But… the renaissance in traditional making is over. No longer economically viable.