The Theory of Hadronic Collisions

William Kilgore
Brookhaven National Laboratory

Hadron Collider Physics Summer School
August, 2006

The Theory of Hadronic Collisions

In my first lecture, I discussed the origins of QCD, which were largely experimental in nature.

Both the Quark and Parton Models were phenomenologically driven. Feynman used a great deal of field theory intuition in developing the parton model, but the structure was set by the need to explain scaling in DIS.

Rigorous theoretical developments, like non-Abelian gauge theories, the anomaly in π^0 decay and Asymptotic Freedom came late in the game.

Quantum ChromoDynamics

Having established its experimental pedigree, I devote this lecture to perturbative QCD and its application to the study of hadronic collisions.

I will discuss:

- Infrared Safety.
- The Factorization Theorem and perturbative QCD.
- Methods of applying perturbation theory.
- Examples of fixed order calculations.

Quantum Chromodynamics

With the discovery of Asymptotic Freedom, QCD (SU(3) gauge theory) was proposed as a fundamental theory of the strong interactions.

All the prescriptions and hand-waving arguments of the parton model had to be made rigorous.

In particular, one needed to:

- Specify what can be calculated.
- Derive factorization in DIS and hadron scattering.
- Define the parton densities.
- Identify Rules for performing pQCD calculations.

Infrared Safety

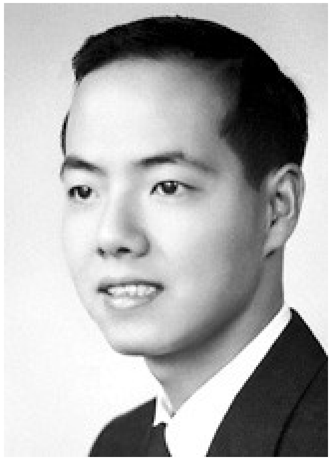
The guiding principle of perturbative QCD is Infrared Safety.

Infrared Safe quantities do not depend on the long-distance behavior of QCD. In particular, they are finite in the limit of vanishing masses so that

$$\sigma\left(\frac{s_{ij}}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu)\right) = \sigma\left(\frac{s_{ij}}{\mu^2}, 0, \alpha_s(\mu)\right)\left\{1 + O\left(\frac{m_i^2}{Q^2}\right)\right\}$$

where Q^2 is a scale characteristic of the larger s_{ij}. Renormalization Group invariance then says:

$$\sigma\left(\frac{s_{ij}}{\mu^2}, 0, \alpha_s(\mu)\right) = \sigma\left(\frac{s_{ij}}{Q^2}, 0, \alpha_s(Q)\right)$$



T.D. Lee

Infrared Safety

The proof of Infrared safety comes from the KLN theorem, which states that fully inclusive measurements, which sum over all degenerate initial and final states, are free from infrared divergences.

The short distance physics of parton scattering does not interfere with the long distance process that turns partons into hadrons.

This is why jet production is computed as simple parton scattering. The probability that partons will produce hadrons is unity.

Infrared Safety

What about less inclusive processes?

The KLN theorem can be extended to cover differential cross sections. The key is to understand the origin of infrared divergences.

Sterman showed that all infrared divergences are related to either soft or collinear momentum configurations.

As long as a measurement is "sufficiently inclusive", i.e. it sums over the soft and collinear configurations, it will be Infrared Safe and calculable in perturbative QCD.

Infrared Safety

For an operational definition of infrared safety, consider a higher order calculation:

$$\sigma^{(2)}(\mathcal{J}) = \int d\,\Omega_n \frac{d\,\sigma_n^{(2)}}{d\,\Omega_n} \mathcal{S}_n^{\mathcal{J}}(p_1,\ldots,p_n)$$
$$+ \int d\,\Omega_{n+1} \frac{d\,\sigma_{n+1}^{(1)}}{d\,\Omega_{n+1}} \mathcal{S}_{n+1}^{\mathcal{J}}(p_1,\ldots,p_{n+1})$$
$$+ \int d\,\Omega_{n+2} \frac{d\,\sigma_{n+2}^{(0)}}{d\,\Omega_{n+2}} \mathcal{S}_{n+2}^{\mathcal{J}}(p_1,\ldots,p_{n+2})$$

where $\mathcal{S}^{\mathcal{J}}_n$ is a measurement function for observable $\mathcal{J}$. Infrared safety requires that:

$$\mathcal{S}_{n+1}^{\mathcal{J}}(p_1,\ldots,(1-\lambda)\,p_n,\lambda\,p_n)=\mathcal{S}_n^{\mathcal{J}}(p_1,\ldots,p_n), \qquad 0\leqslant\lambda\leqslant 1$$
$$\mathcal{S}_{n+1}^{\mathcal{J}}(p_1,\ldots,p_n,0)=\mathcal{S}_n^{\mathcal{J}}(p_1,\ldots,p_n).$$

The Factorization Theorem in DIS

The Factorization Theorem (Collins, Soper, Sterman) is the field theory realization of the parton model. For DIS, it states that:

$$F_{1,3}(x,Q^2) = \sum_{i=f,\bar{f},g} \int_0^1 \frac{d\xi}{\xi} C^{(i)}_{1,3}(x/\xi, Q^2/\mu^2, \mu_f^2/\mu^2, \alpha_s(\mu))\, \phi_{i/p}(\xi, \mu_f, \mu)$$

$$F_2(x,Q^2) = \sum_{i=f,\bar{f},g} \int_0^1 d\xi\, C^{(i)}_{2}(x/\xi, Q^2/\mu^2, \mu_f^2/\mu^2, \alpha_s(\mu))\, \phi_{i/p}(\xi, \mu_f, \mu)$$

The $C^{(i)}_a$ are hard scattering functions. They are IR safe and calculated from Feynman diagrams.

The $\phi_{i/p}$ are parton distributions and contain all of the infrared sensitivity of the cross section. However, they are universal to all $C^{(i)}_a$.

Factorization in Hadron-Hadron Collisions

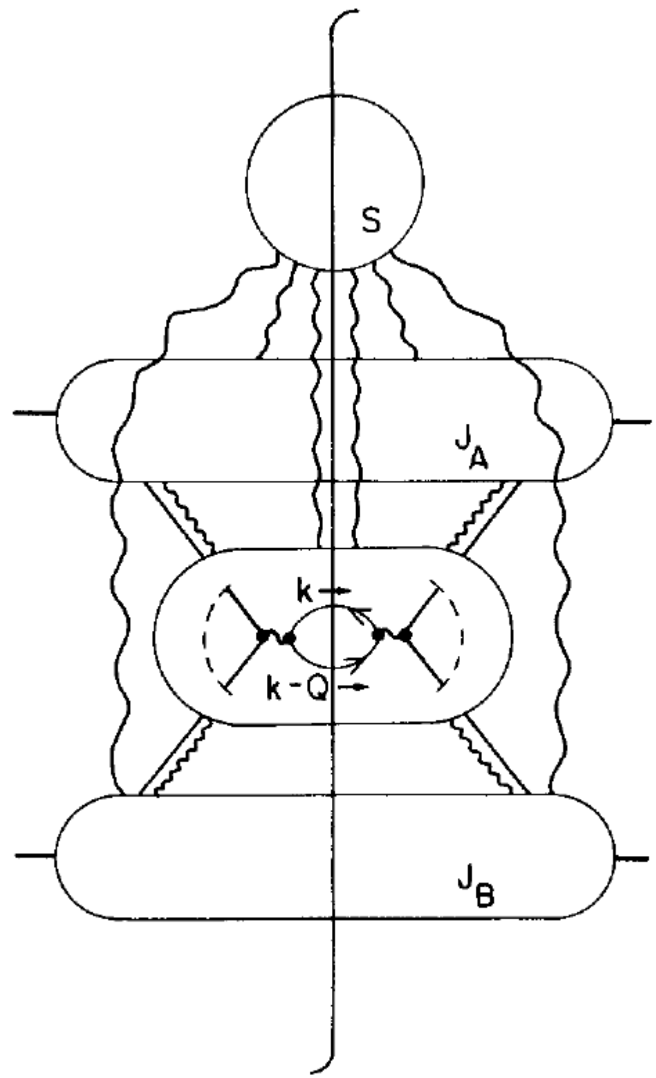
The factorization theorem also justifies the extension of the parton model to hadron-hadron collisions. Here it states:

$$\sigma(A+B\to \jmath) = \sum_{a,b=q,\bar{q},g} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \, \hat{\sigma}_{ab\to \jmath}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, \mu, \alpha_s, \ldots\right) \phi_{a/A}(\xi_A,\mu)\, \phi_{b/B}(\xi_B,\mu) + O(1/Q^2)$$

The key departure from the simple parton model picture is that factorization works only to leading order in Q^2. At low Q^2, caveat emptor!

Factorization for Drell-Yan

A crucial piece of the factorization theorem is that soft exchanges between the incoming hadrons cancel at the leading power of $1/Q^2$.



Power corrections at low Q^2 explain why early Drell-Yan measurements did not support the parton model .

The Factorization Theorem

The fundamental aspect of the factorization theorem is the separation of long-distance and short-distance effects. The factorization scale μ is arbitrary.

$$\sigma(A+B\to \jmath) = \sum_{a,b=q,\bar{q},g} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \, \hat{\sigma}_{ab\to \jmath}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, \mu, \alpha_s, \ldots\right) \phi_{a/A}(\xi_A,\mu)\, \phi_{b/B}(\xi_B,\mu) + O(1/Q^2)$$

All long-distance initial-state physics is contained in $\phi_{a/A}$, $\phi_{b/B}$. Short-distance physics is in $\hat{\sigma}$ and is computed in perturbation theory.

Parton Distributions

We would like to define parton density functions like those in the parton model. That is, for instance, $\phi_{u/p}(x)$ representing the probability of finding a u-quark in the proton with momentum fraction between x and x+dx.

Since we are now working within a fundamental theory where one can calculate radiative correction, however, we must demand a rigorous definition.

Parton Distributions

Parton Distribution Functions are defined in terms of matrix elements of renormalized operators in QCD. For a hadron h with momentum p,

$$\phi_{q_j/h}(x,\mu) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p|\overline{\psi}_j(0,y^-,\mathbf{0}_T)\gamma^+ \boldsymbol{W}(y^-,0)\psi_j(0)|p\rangle_R$$

$$\phi_{\bar{q}_j/h}(x,\mu) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p|\psi_j(0,y^-,\mathbf{0}_T)\gamma^+ \boldsymbol{W}(y^-,0)\overline{\psi}_j(0)|p\rangle_R$$

$$\phi_{g/h}(x,\mu) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p|F_a^{+\nu}(0,y^-,\mathbf{0}_T)\gamma^+ \boldsymbol{W}(y^-,0)F_{a\nu}{}^{+}(0)|p\rangle_R$$

Where W is a Wilson line,

$$\boldsymbol{W}(y^-,0) = \boldsymbol{P}\ \exp[ig\int_0^{y^-} ds^- A_a^+(0,s^-,\mathbf{0}_T)t^a]$$

Parton Distributions

Observations:

1) PDFs are non-perturbative.

The matrix elements involve the proton wave function. They must be extracted from measurements.

2) PDFs are Ultraviolet Singular.

Renormalization spoils the interpretation as number densities. Treated as distributions, they still satisfy sum rules.

3) PDFs are renormalized.

They obey renormalization group equations (the DGLAP equations), and evolve in Q^2.

4) PDFs are universal.

They are process independent. PDFs determined in DIS can be used in hadron-hadron collisions.

Parton Evolution

Unlike the parton densities of the parton model, PDFs evolve in Q^2 according to DGLAP equations:

$$\mu^2 \frac{d}{d\mu^2}\phi_{a/p}(x) = (P_{ab} \otimes \phi_{b/p})(x),$$

where

$$(f \otimes g)(x) = \int_0^1 dy\, dz\; f(y)\, g(z)\, \delta(x - yz) = \int_x^1 \frac{dz}{z} f(x/z)\, g(z)$$

$$P_{ab}(x) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^{n+1} P_{ab}^{(n)}(x)$$

The splitting functions $P_{ab}(x)$ are now known through order α_s^3.

Determining PDFs

PDFs are determined by comparing perturbative QCD calculations to experimental results.

Experiments are sensitive to different combinations of the PDFs, over differing ranges of parton momentum fraction x and are performed at a variety of values of Q^2.

The fitting procedure must take the evolution in Q^2 between experiments into account.

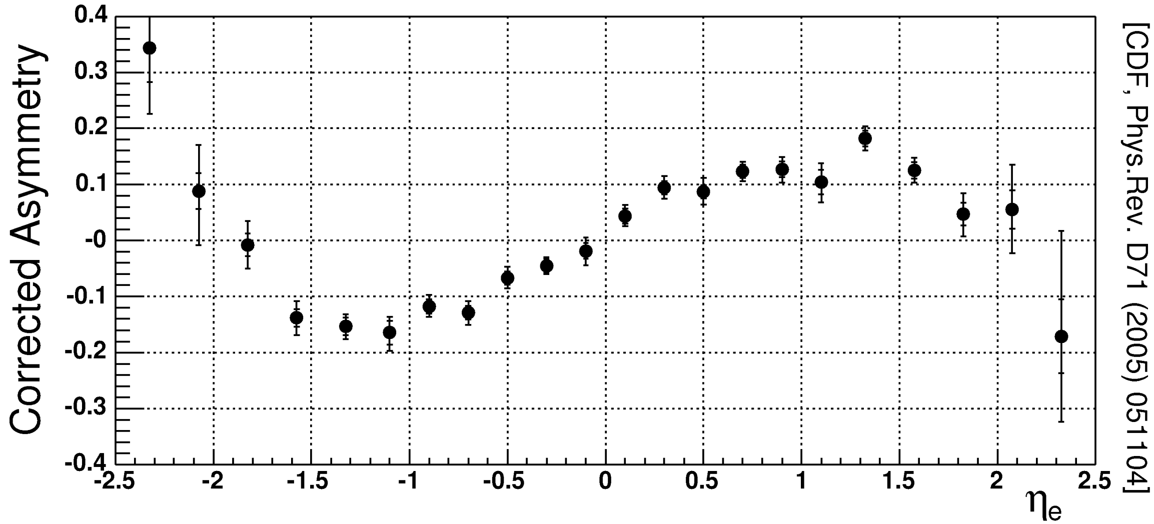
Fitting PDFs

A wide variety of data are used to fit PDFs.

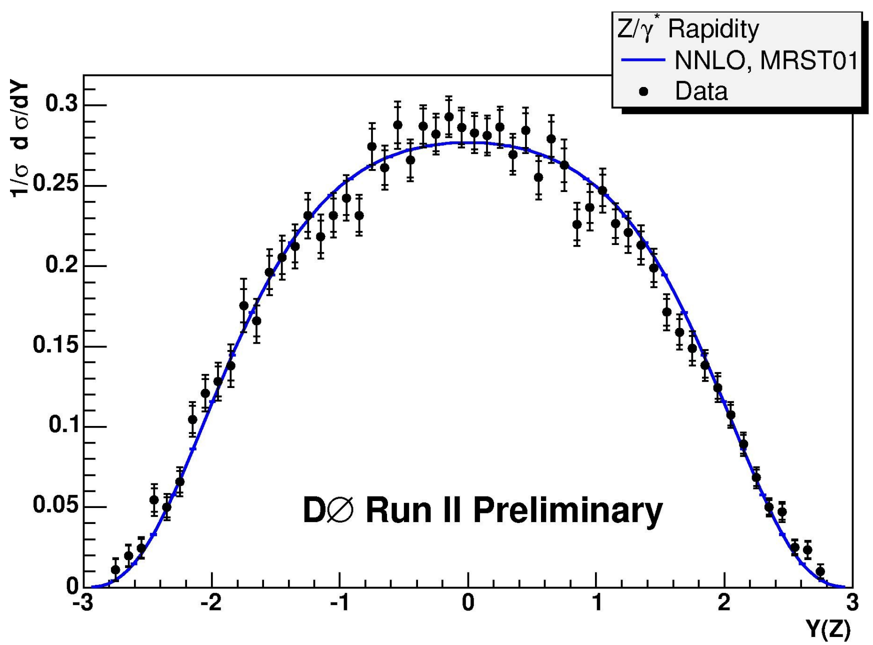
- DIS Structure Functions at H1 and ZEUS
- W (lepton) asymmetry at CDF
- Inclusive Jet Production at Tevatron
- Fixed target DIS (proton and deuteron)
- Fixed target Drell-Yan (proton and deuteron)
- Neutrino DIS (nuclear target)

The low energy data often require corrections to deal with "higher twist" effects. Deuterium and nuclear data require still more corrections.

PDF Fits



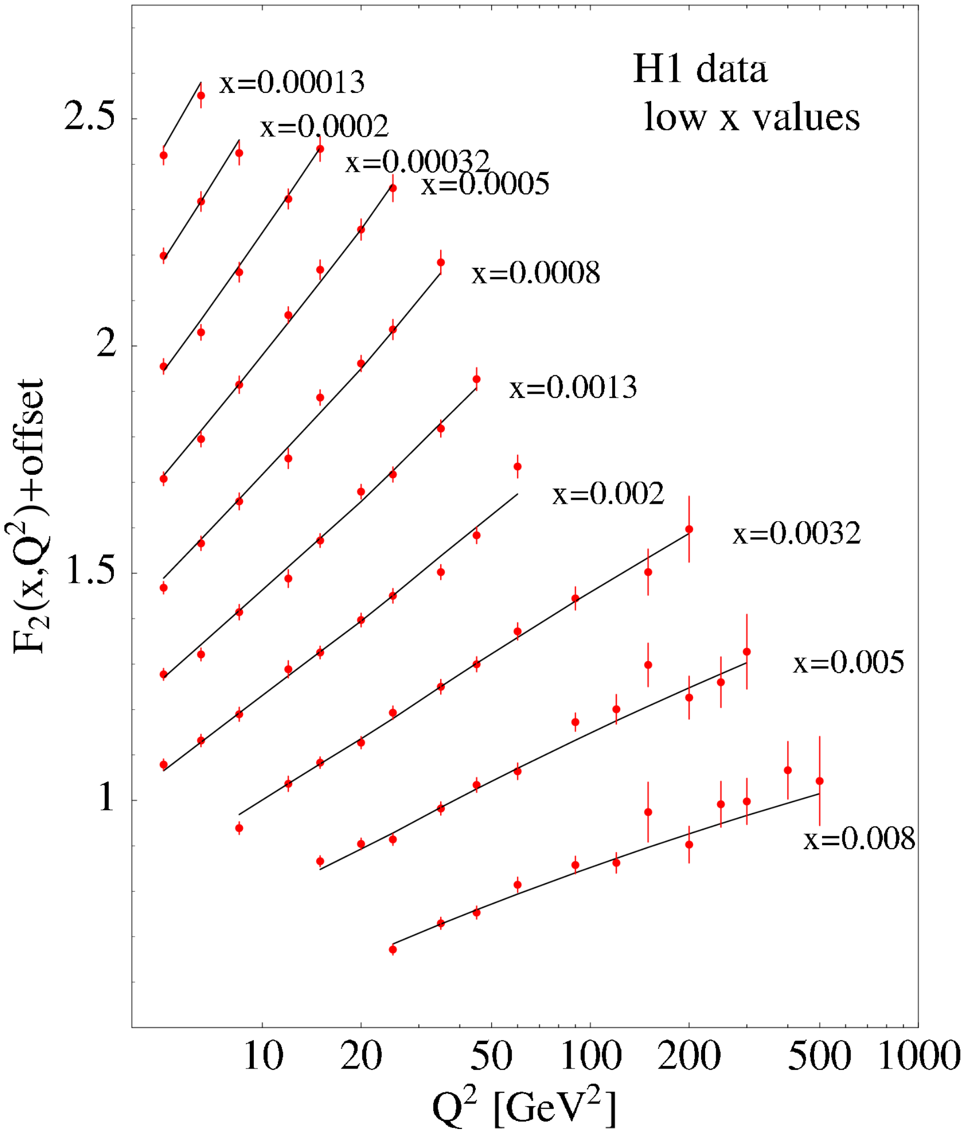
W boson charge asymmetry

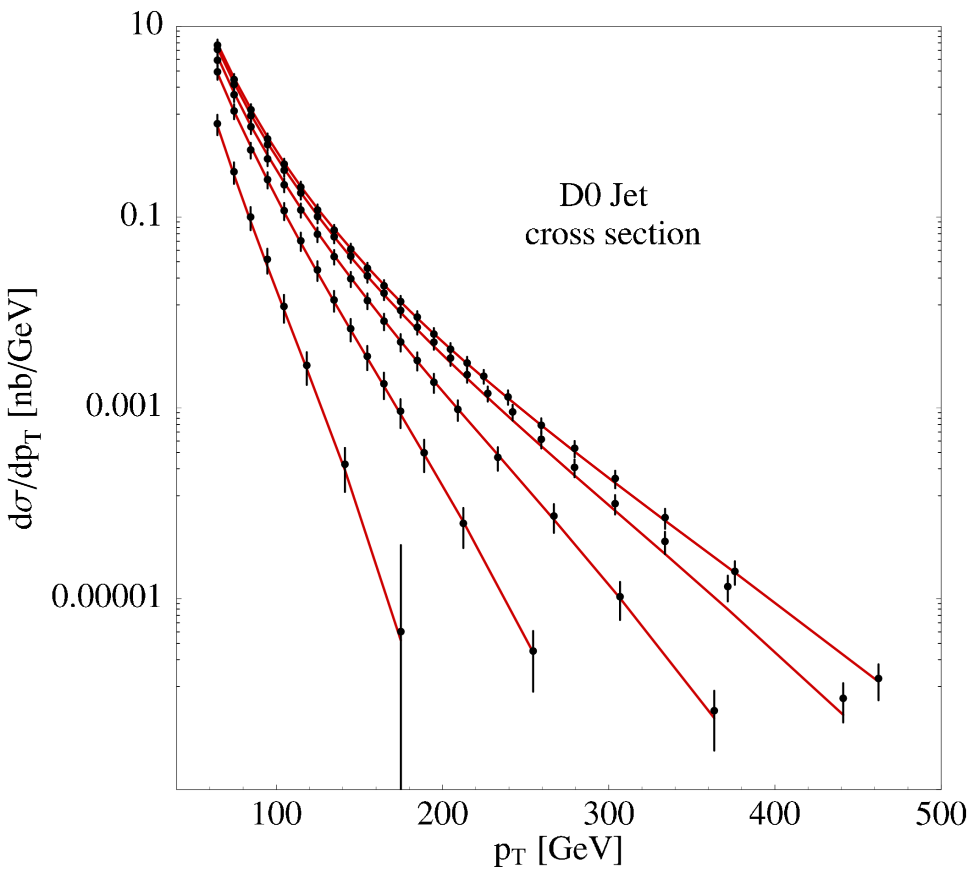


Z/γ^* rapidity distribution

PDF Fits

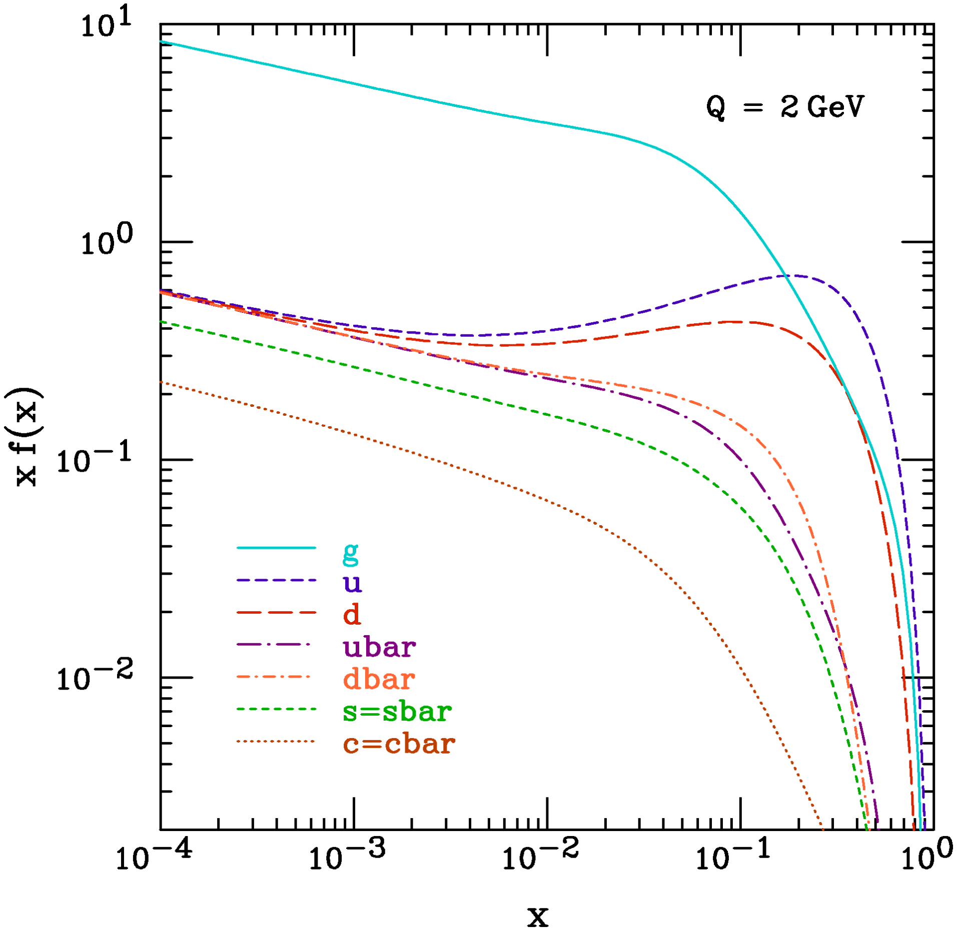
Modern PDFs fit the available data very well.

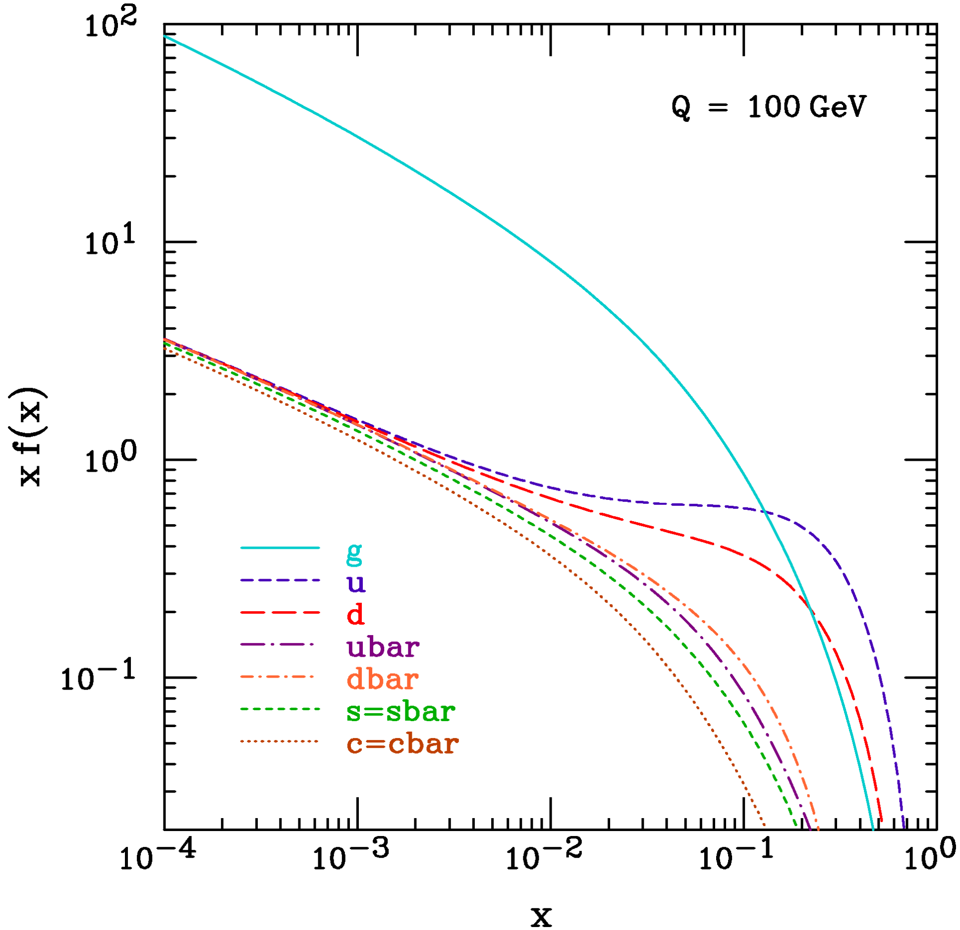




PDF Fits

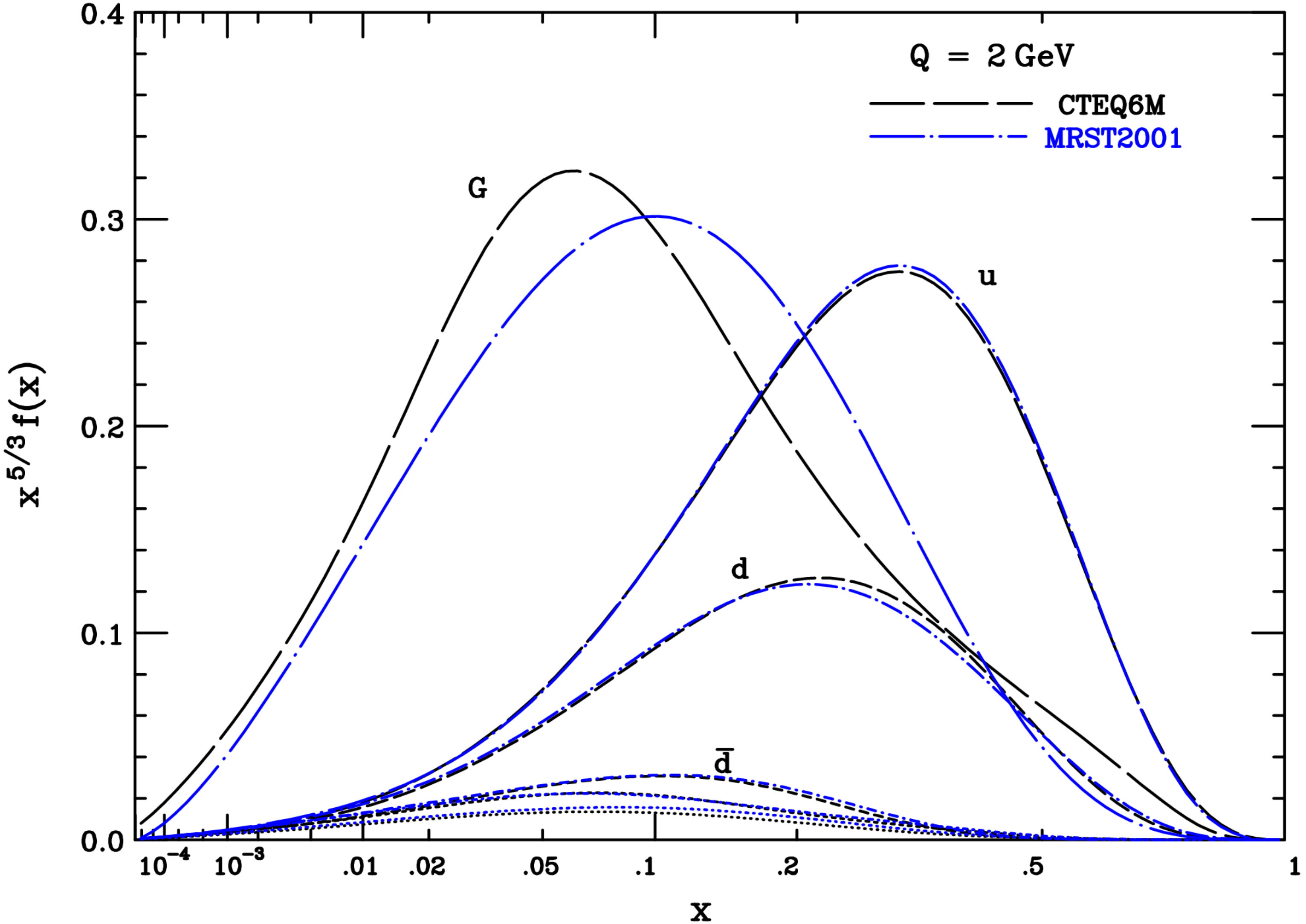
CTEQ6M at two different values of Q:





PDF Fits

Still the gluon is hard to constrain.

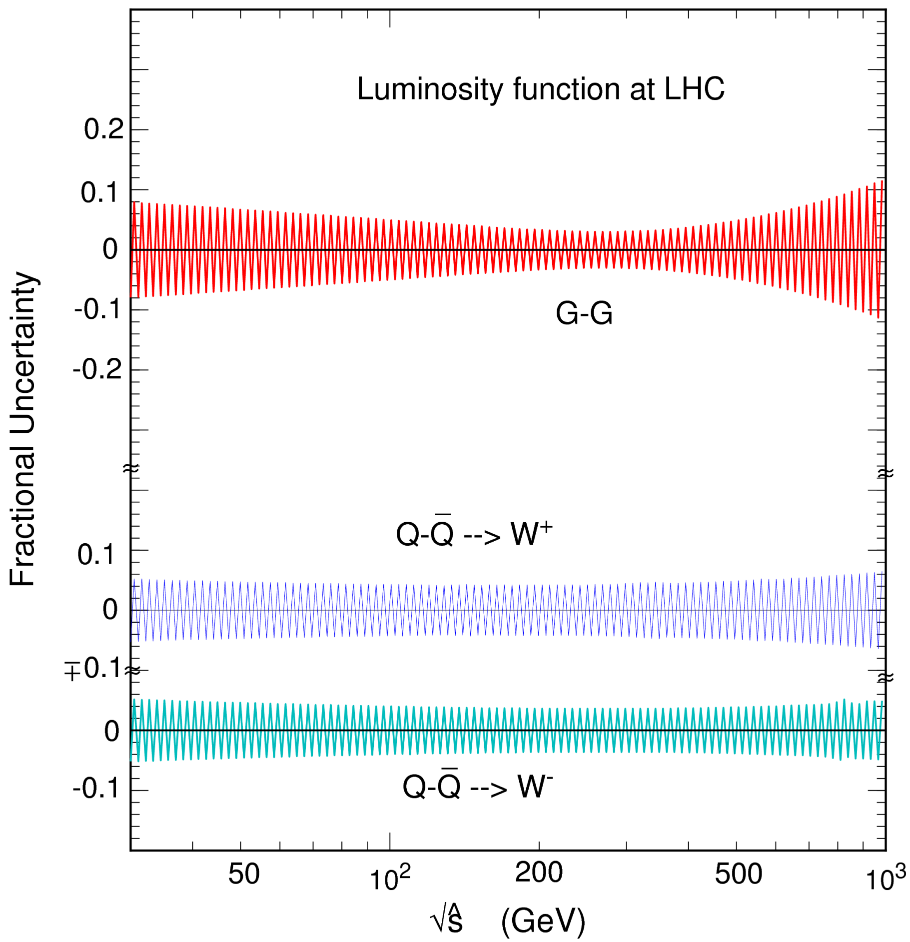


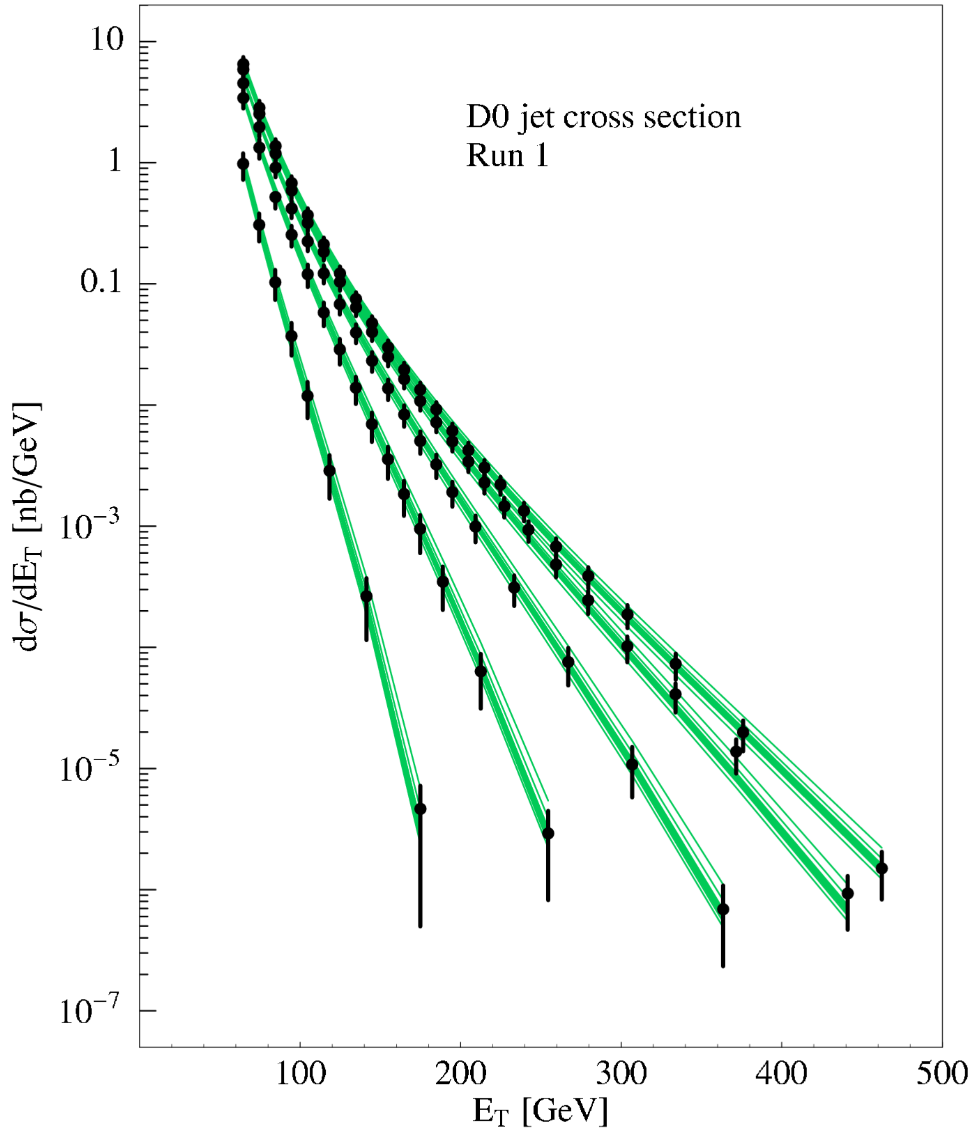
PDF Uncertainties

For many years, PDF "best fits" were distributed without any serious attempt to quantify how good the best fits were.

It is now common for PDF fitters to produce sets of PDFs that map out a range of "good" fits. Averaging over the sets introduces uncertainty to Monte Carlo calculations that reflect the uncertainty in the input PDFs.

PDF Uncertainties (CTEQ6)





The Hard Scattering

The PDFs contain all of the initial state long-distance physics. The short-distance physics is contained in the hard-scattering cross section, often called the partonic cross section.

The partonic cross section is computed by using the Feynman Rules to calculate on-shell matrix elements of (usually) massless quarks and gluons, which are then integrated over the phase space of the final state partons.

Feynman Rules for pQCD

$$\mathscr{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} + \bar{q}_i \, i \not{D}_{ij} q_j - m_q \bar{q}_i q_i - \bar{\eta}^a \partial^\mu D^{ac}_\mu \eta^c$$

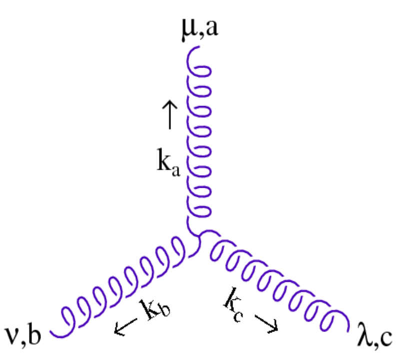
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D^\mu_{ij} = \partial^\mu \delta_{ij} - i g t^a_{ij} A^{a\mu}, \qquad D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu$$

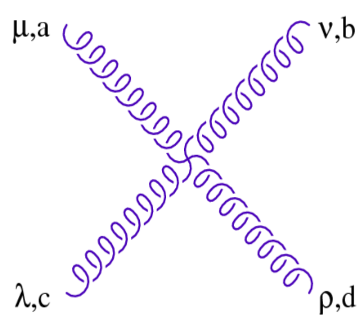
$$\frac{-i\eta^{\mu\nu}\delta^{ab}}{k^2+i\varepsilon}$$

$$\frac{i\delta^{ij}}{\not{k}+i\varepsilon}$$

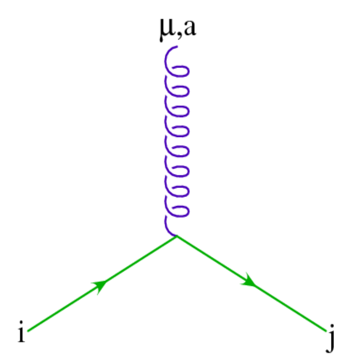
$$\frac{i\delta^{ab}}{k^2+i\varepsilon}$$



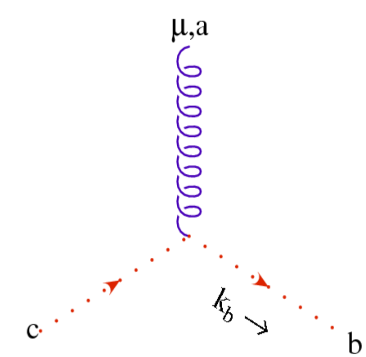
$$gf^{abc}[\eta_{\mu\nu}(k_b-k_a)_\lambda + \eta_{\mu\lambda}(k_a-k_c)_\nu + \eta_{\lambda\nu}(k_c-k_b)_\mu]$$



$$-ig^2[f^{xab}f^{xcd}(\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\lambda}) + f^{xac}f^{xbd}(\eta_{\mu\nu}\eta_{\lambda\rho}-\eta_{\mu\rho}\eta_{\nu\lambda}) + f^{xad}f^{xbc}(\eta_{\mu\nu}\eta_{\lambda\rho}-\eta_{\mu\lambda}\eta_{\nu\rho})]$$



$$ig\, T^a_{ji}\gamma_\mu$$



$$gf^{abc}k_{b\mu}$$

Applications of Perturbation Theory in QCD

There are several techniques for applying perturbation theory to QCD:

Fixed Order: All contributions are computed up to a specified order of α_s.

Resummation: For some observables, perturbation theory breaks down due to log enhancements ($\alpha_s \ln \xi \sim 1$). but, one can resum to all orders.

Parton Showers: (See Sjostrand's lectures) Provide more realistic events than fixed order, but are usually based on lowest order matrix elements.

Fixed Order Calculations in QCD

This is the simplest technique and is also the easiest to carry forward to higher orders.

The idea is to compute all quantities up to a certain order of α_s. However, different processes start at different orders of α_s.

Drell-Yan starts at order α_s^0, while n-jet production starts at order α_s^n.

Higher Order corrections.

For any process, the lowest non-trivial order of α_s is called Leading order, or LO.

Leading Order (LO) calculations are performed at the Born level.

Next-to-Leading Order (NLO) calculations include one-loop corrections to the Born process and Single Real Radiation corrections

Next-to-Next-to-Leading Order (NNLO) calculations include two-loop corrections to Born, one-loop corrections to Single Real Radiation terms and Double Real Radiation correction.

Limitations of Fixed Order Calculations

Experience has shown that LO calculations are of only qualitative value, often getting the normalization and shapes of distributions to within 10-20 percent. Often they do worse.

NLO is the first serious approximation. Unfortunately, the state of the art currently allows for loop calculations with 5 (6 is coming) external partons.

NNLO is only available in a few special cases.

Limitations of Fixed Order Calculations

Many important backgrounds will be computed at NLO in the near future. The improved accuracy will be a boon, but fixed order still leaves a lot to be desired in terms of event simulation.

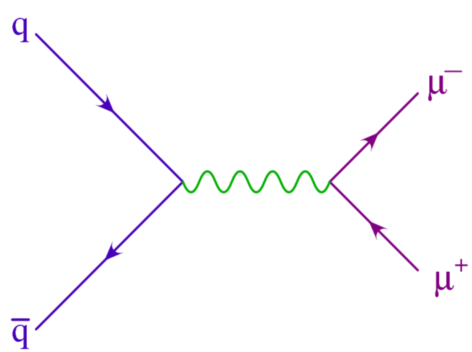
At LO, each parton is identified with a jet. A LOT of structure is being left out. Even at NNLO, a jet can contain at most 3 partons!

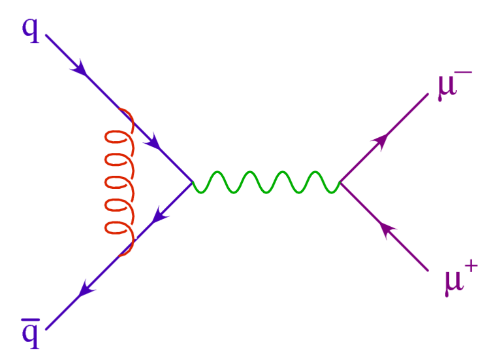
There is great demand for combining the accuracy of NLO with the event simulation of parton showers.

Example: Inclusive Drell-Yan at NLO

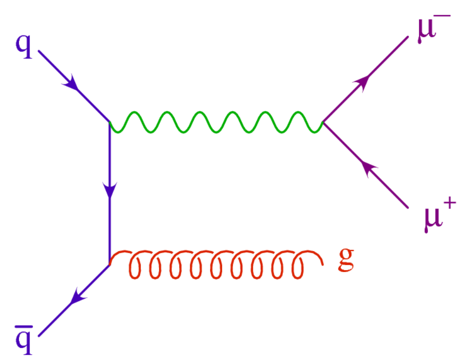
We compute all terms at orders $\alpha_s^{\,0}$ and $\alpha_s^{\,1}$:

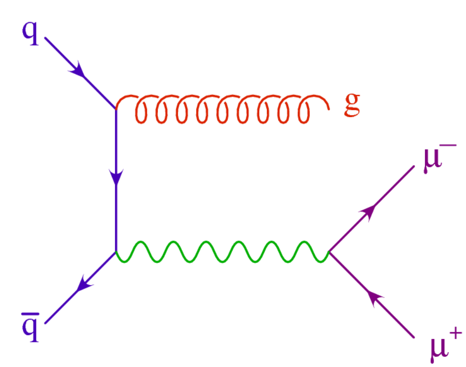
$q\bar{q}$ Born and virtual terms.



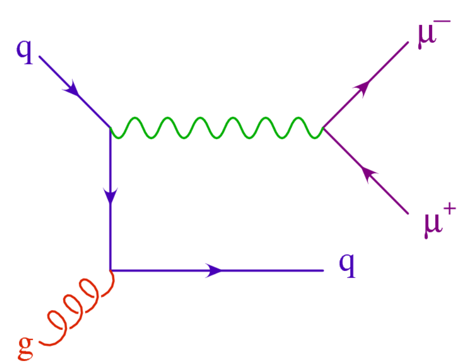


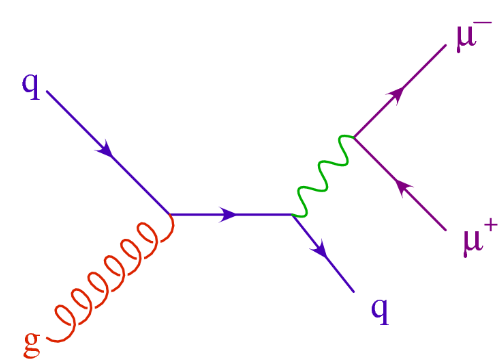
$q\bar{q}$ Real emission terms.





qg Real emission terms.





Drell-Yan at NLO

If we are fully inclusive, we treat the μ pair as a massive vector boson and thus have a 2→1 virtual process. We integrate the squared amplitudes over phase space,

$$\sigma_V = \frac{1}{2\hat{s}} \int \frac{d^{3-2\epsilon} q}{(2\pi)^{3-2\epsilon} 2q^0} (2\pi)^{4-2\epsilon} \delta^{4-2\epsilon}(p_1 + p_2 - q) |M_V|^2$$

$$\sigma_R = \frac{1}{2\hat{s}} \frac{d^{3-2\epsilon} q \, d^{3-2\epsilon} k}{(2\pi)^{3-2\epsilon} 4q^0 k^0} (2\pi)^{4-2\epsilon} \delta^{4-2\epsilon}(p_1 + p_2 - q - k) |M_R|^2$$

and combine real and virtual terms

$$\sigma_{Tot} = \sigma_V + \sigma_{R.}$$

Drell-Yan at NLO

$$\sigma^{(n)} = \sigma_0 \left(\frac{\alpha_s}{\pi}\right)^n \Delta^{(n)}(x), \qquad \sigma_0 = \frac{4\pi\alpha^2}{9Q^4}, \qquad \Delta^{(0)} = \delta(1-x), \qquad x \equiv \frac{Q^2}{\hat{s}}, \qquad \mathcal{D}_n = \left[\frac{\ln^n(1-x)}{1-x}\right]_+$$

$$\Delta^{(1)}_{q\bar{q},V} = C_F\,\delta(1-x)[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}(\frac{1}{2}+\ln\frac{\mu^2}{Q^2}) - \frac{5}{2} + \frac{7}{2}\zeta_2 - \frac{1}{2}\ln\frac{\mu^2}{Q^2} - \frac{1}{2}\ln^2\frac{\mu^2}{Q^2}]$$

$$\Delta^{(1)}_{q\bar{q},R} = C_F\,\delta(1-x)[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}(1-\ln\frac{\mu^2}{Q^2}) - \frac{3}{2}\zeta_2 - \ln\frac{\mu^2}{Q^2} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2}] + 2C_F\,\mathcal{D}_0(1-x)[-\frac{1}{\epsilon} + 1 - \ln\frac{\mu^2}{Q^2}]$$
$$+\, 4C_F\,\mathcal{D}_1(1-x) + C_F(1+x)[\frac{1}{\epsilon} - 1 + \ln\frac{\mu^2}{Q^2} - 2\ln(1-x)] - C_F\frac{1+x^2}{1-x}\ln(x)$$

$$\Delta^{(1)}_{q\bar{q},V+R} = C_F\,\delta(1-x)[-\frac{3}{2}\frac{1}{\epsilon} - \frac{5}{2} + 2\zeta_2 - \frac{3}{2}\ln\frac{\mu^2}{Q^2}] + 2C_F\,\mathcal{D}_0(1-x)[-\frac{1}{\epsilon} + 1 - \ln\frac{\mu^2}{Q^2}]$$
$$+\, 4C_F\,\mathcal{D}_1(1-x) + C_F(1+x)[\frac{1}{\epsilon} - 1 + \ln\frac{\mu^2}{Q^2} - 2\ln(1-x)] - C_F\frac{1+x^2}{1-x}\ln(x)$$

$$\Delta^{(1)}_{qg,R} = \frac{T_R}{2}(1-2x+2x^2)[-\frac{1}{\epsilon} + 2\ln(1-x) - \ln(x) - \ln\frac{\mu^2}{Q^2}] + \frac{T_R}{4}(3+2x-3x^2)$$

The result still has poles in ϵ! Something is missing

Mass Factorization

The parts of the real-emission terms where the final state parton is collinear with the beam have already been included in the parton distributions. Those pieces must therefore be removed from the real emission terms.

This is done by adding in the Mass Factorization Counterterms, which are convolutions of lower-order terms with the DGLAP splitting functions.

$$\sigma_{ij} = \sum_{ij=q,\bar{q},g} \hat{\sigma}_{ab} \otimes \Gamma_{ai} \otimes \Gamma_{bj}, \qquad \Gamma_{ij}(x) = \delta(1-x)\delta_{ij} - \frac{\alpha_s}{\pi}\frac{P^{(0)}_{ij}(x)}{\epsilon} + \ldots,$$

$$(f \otimes g)(x) = \int_0^1 dy \int_0^1 dz\, f(y)\, g(z)\, \delta(x - yz)$$

Drell-Yan at NLO

Adding in the Mass Factorization Counterterms

$$\Delta^{(1)}_{q\bar{q},MF} = C_F\left(\frac{1}{\epsilon}-1\right)\left[\frac{3}{2}\delta(1-x)+2\mathcal{D}_0(1-x)-1-x\right]$$

$$\Delta^{(1)}_{qg,MF} = \frac{T_R}{2}\left(\frac{1}{\epsilon}-1\right)\left[1-2\mathrm{x}+2\mathrm{x}^2\right]$$

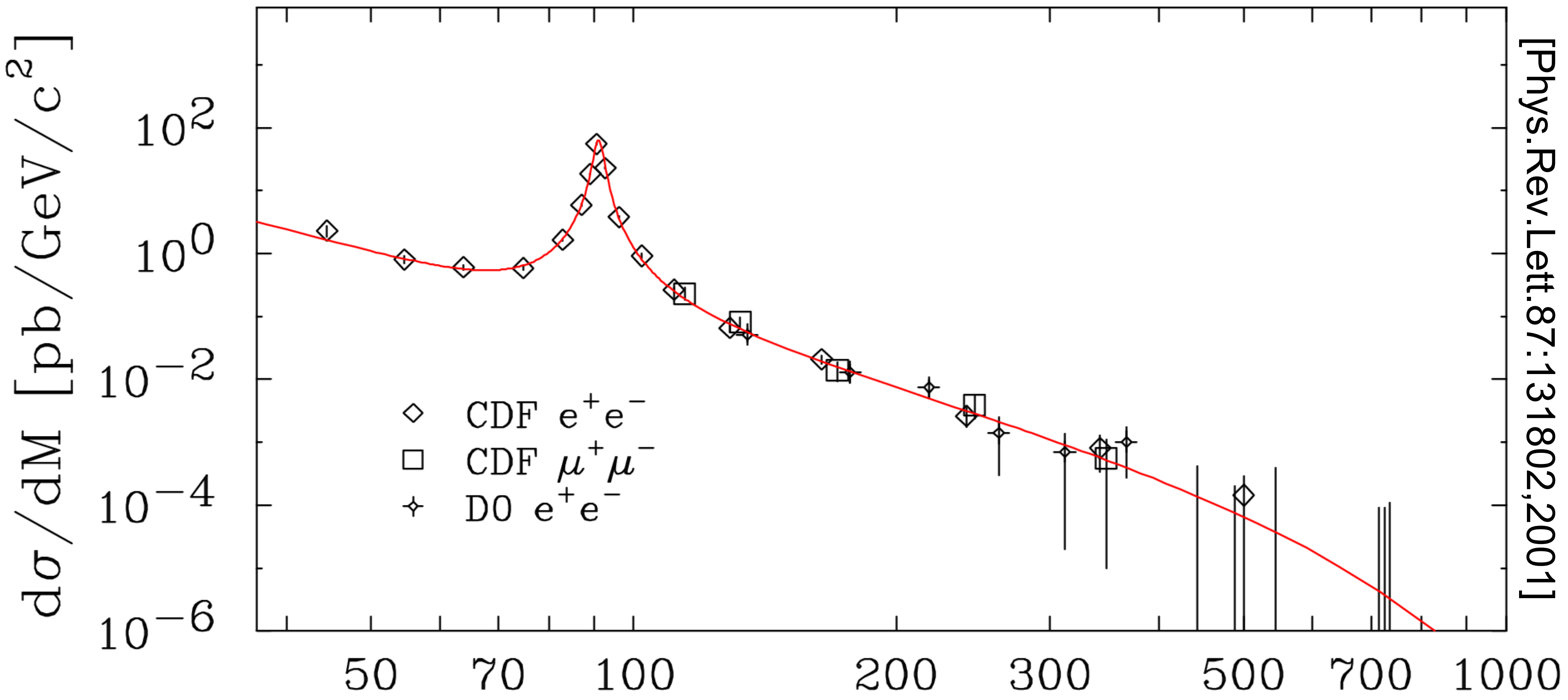
we get the correct (finite!) result:

$$\Delta^{(1)}_{q\bar{q},Tot} = C_F\left[\left(-4-\frac{3}{2}\ln\frac{\mu^2}{Q^2}+2\zeta_2\right)\delta(1-x)-2\mathcal{D}_0(1-x)\ln\frac{\mu^2}{Q^2}+4\mathcal{D}_1(1-x)\right]$$
$$+\,C_F(1+x)\left[\ln\frac{\mu^2}{Q^2}-2\ln(1-x)\right]-C_F\frac{1+x^2}{1-x}\ln(x)$$

$$\Delta^{(1)}_{qg,Tot} = \frac{T_R}{2}(1-2\mathrm{x}+2\mathrm{x}^2)\left[2\ln(1-x)-\ln(x)-\ln\frac{\mu^2}{Q^2}\right]+\frac{T_R}{4}(1+6\mathrm{x}-7\mathrm{x}^2)$$

Drell-Yan at the Tevatron

Experimental Measurements compared to NNLO QCD calculation of Drell-Yan production.



[Phys.Rev.Lett.87:131802,2001]

Example II: Jet Production

The previous example of an NLO calculation was special for a number of reasons:

There was no need to renormalize

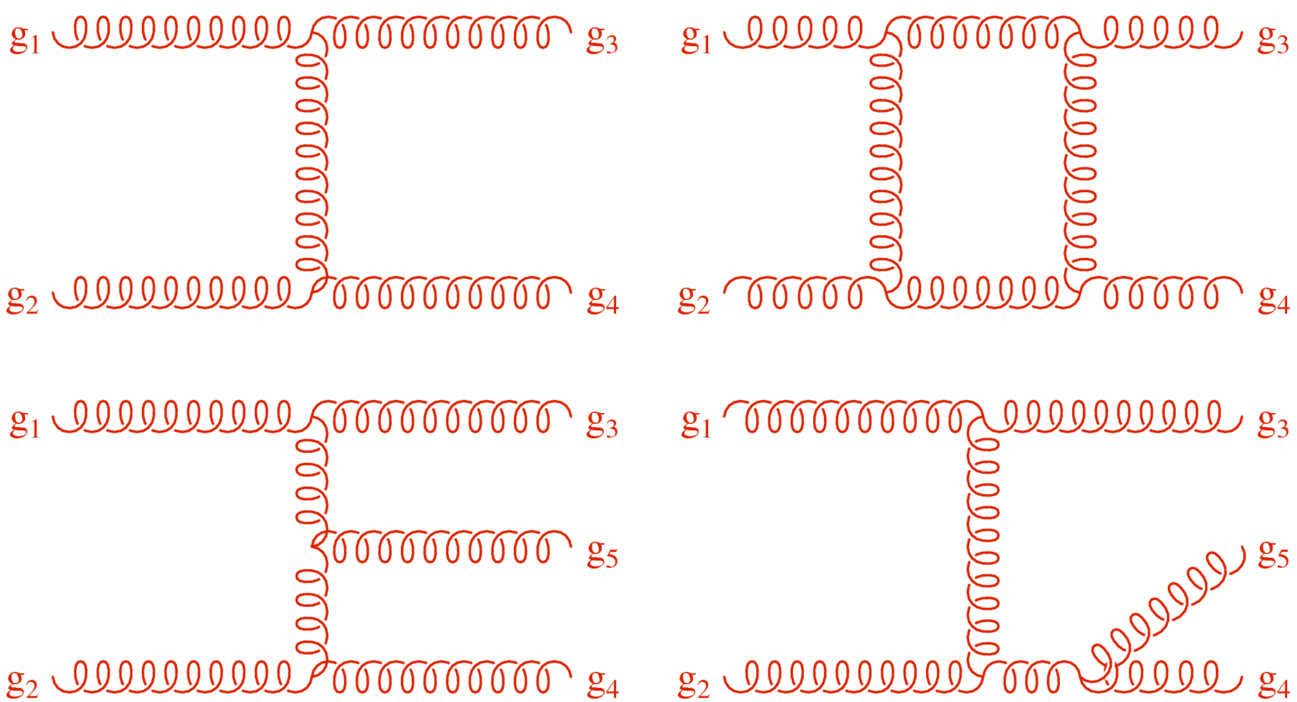
One could perform the total integrals

We were not interested in the hadrons in the final state, so we needed no jet algorithms.

Let us now look at jet production.

Jet Production

Again, we have Born, virtual and real emission terms. For simplicity I have drawn only all-gluon diagrams.



Jet Production

When computing jet production, we can't do the total integrals as we did for Drell-Yan.

Even if we could, there is far more information to be had from differential distributions.

To compute differential distributions, we need to impose acceptance cuts, etc., in order to approximate the experimental environment.

This is virtually impossible in an analytic calculation, so we adopt numerical techniques and perform Monte Carlo integrations over phase space.

Numerical Integration at NLO

The problem with numerical integration at NLO is that there are infrared divergences all over the place. The one-loop amplitudes have explicit infrared poles, while the real radiation terms diverge in soft and collinear configurations.

We need some method of regulating the divergences so that we can compute the (finite!) NLO cross section with good numerical accuracy.

Most of all, we would like a flexible algorithm that can be applied to a variety of processes.

Universality of Infrared Structure

It is possible to develop a multipurpose algorithm for NLO calculations because the infrared structure of QCD amplitudes is universal and the amplitudes factorize.

One loop amplitudes take the form,

$$M^{(1)}(p_{1,}\ldots,p_n) = V^{(1)}(p_{1,}\ldots,p_n)\; M^{(0)} + M^{(1),f}(p_{1,}\ldots,p_n)$$

where V contains all infrared poles and multiplies the Born amplitude. $M^{(1),f}$ is infrared finite.

Universal Infrared Structure

Real radiation amplitudes factorize in the soft and collinear limits.

$$\lim_{p_n \| p_{n+1}} M_{n+1}^{(0)}(p_1,\ldots,p_n,p_{n+1}) = \mathcal{C}(p_n,p_{n+1};K)\times M_n^{(0)}(p_1,\ldots,p_{n-1},K)$$

$$\lim_{p_{n+1}\to 0} M_{n+1}^{(0)}(p_1,\ldots,p_n,p_{n+1}) = \mathcal{S}(p_n,p_{n+1},p_1)\times M_n^{(0)}(p_1,\ldots,p_n)$$

The soft and collinear functions, $\mathcal{S}$ and $\mathcal{C}$, integrated over phase space, generate the infrared poles to cancel those in loop amplitudes.

Integrating over $\mathcal{S}$ and $\mathcal{C}$ to cancel the virtual poles is another way of saying the measurement is "sufficiently inclusive" to be infrared safe.

NLO Jet Production

To summarize: Next-to-Leading Order calculations consist of two contributions:

Virtual Corrections to one loop.

Single Real Emission Corrections at tree-level.

$$\sigma^{NLO} = \int_{n+1} d\sigma_{n+1}^{(0)} + \int_{n} d\sigma_{n}^{(1)}$$

Both terms are infrared singular.

A Multipurpose Approach to NLO

One scheme is to add a local counter-term to the Virtual terms and subtract it from the Real Correction terms, canceling the infrared singularities. This is the Subtraction Method.

$$\sigma^{NLO} = \int_{n+1} \left(d\sigma^{(0)}_{n+1} - \quad d\alpha^{(0)}_{n+1} \right)$$
$$+ \int_{n} d\sigma^{(1)}_{n} + \int_{n+1} d\alpha^{(0)}_{n+1}$$

Both terms are now infrared finite.

The Subtraction Method

Q:How can we construct this local counterterm?

A:By making use of the universality of the infrared structure of QCD amplitudes.

We define the local counterterm in (n+1) body phase space as

$$A_{n+1}^{(0)}(p_{1,}\ldots,p_{n+1}) = \mathcal{D}(p_n, p_{n+1}, p_1; k_n, k_1)\, M_n^{(0)}(k_{1,}p_{2,}\ldots,p_{n-1},k_n) + \ldots$$

where $\mathcal{D}$ is a function of p_1,p_n,p_{n+1}, (which define the momenta k_1,k_n) which has the same infrared structure as the real emission amplitude. M_n is the on-shell n-point Born amplitude.

The Subtraction Method

Phase space can be factorized in such a way that the three particle phase sub-space $d\Phi(p_1,p_n,p_{n+1})$ can be integrated down to the two particle subspace $d\Phi(k_1,k_n)$.

Only $|\mathcal{D}|^2$ varies under this integration, which exposes the infrared poles that cancel those of the loop amplitude as in the KLN theorem.

$$\int_{\Phi_3(p_1,p_n,p_{n+1})/\Phi_2(k_1,k_n)} d\,\Phi_{n+1}(p_1,\ldots,p_{n+1})|A|^2 = V^{(1)}(k_1,\ldots,k_n)|M_n^{(0)}(k_1,\ldots,k_n)|^2$$

Subtraction at NLO

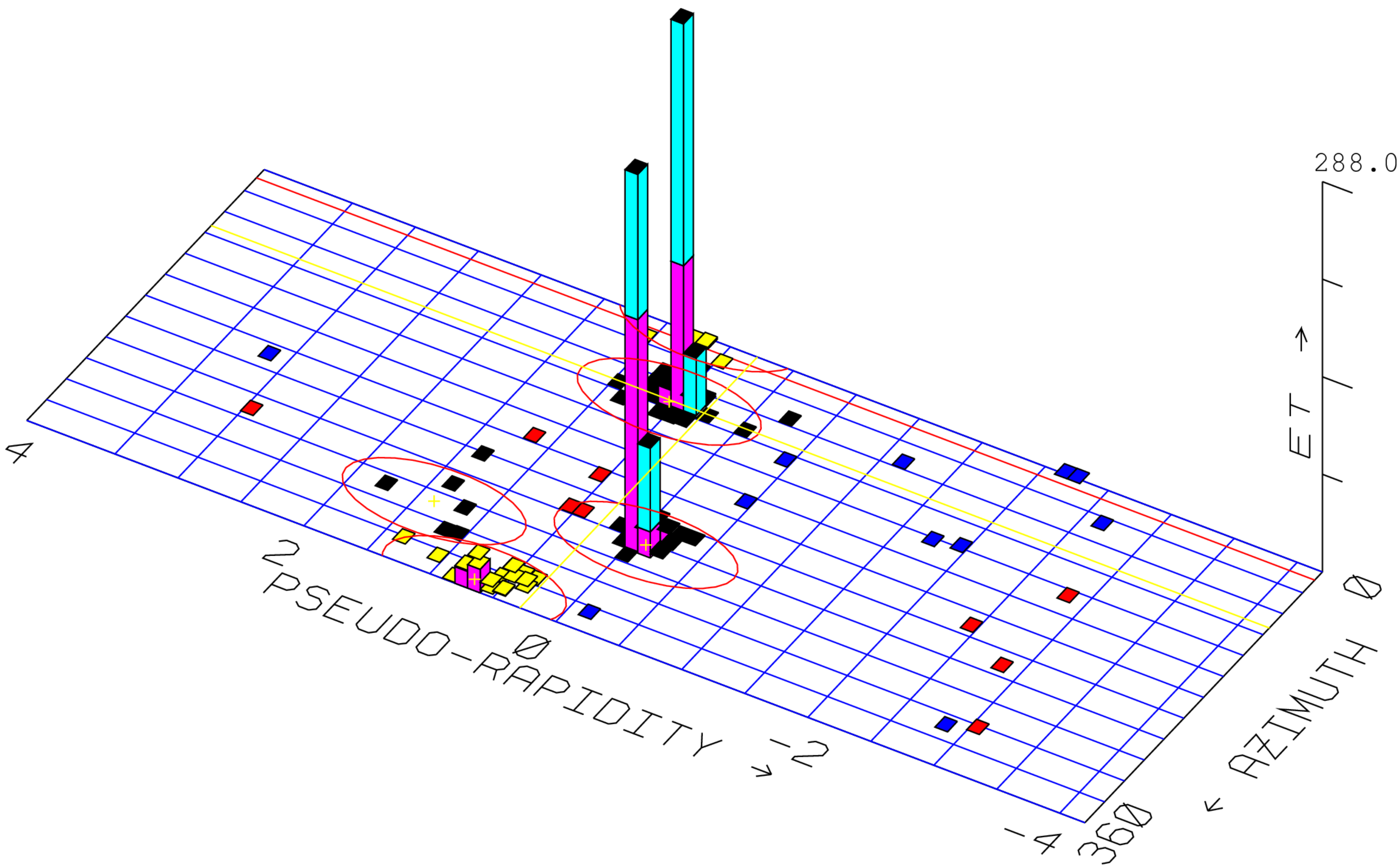
To Summarize:

A subtraction scheme adds (and subtracts back out) a local counter-term to both Virtual and Real Correction terms, canceling the infrared singularities.

$$\sigma^{NLO} = \int_{n+1} \left(d\sigma_{n+1}^{(0)} - d\alpha_{n+1}^{(0)} \right) + \int_{n} d\sigma_{n}^{(1)} + \int_{n+1} d\alpha_{n+1}^{(0)}$$

Both terms are now infrared finite.

Jet Clustering



Jet Clustering

The hadronic clusters clearly reflect some underlying structure and are best treated as individual "jets" than as groups of hadrons.

Two questions:

- What is the best way of grouping the hadrons into jets?
- How do we make contact with the hard scattering processes of perturbative QCD?

Jet Clustering Algorithms

There are two primary algorithms for clustering jets: Cone Algorithms and k_T Algorithms.

Cone algorithms are based on geometry. They group all particles that lie in a cone extending from the beam spot to a "circle" in η-φ space.

k_T Algorithms cluster hadrons (or calorimeter towers) according to their transverse momentum relative to their neighbors.

Partons and Jets

The hard scattering produces quarks and gluons and does not address the hadronization process.

The appearance of jets can be explained by identifying the hard partons as color charge antennæ and the jets as the radiation patterns that develop around the antennæ.

Dynamical considerations would prefer a k_T algorithm, but radiation does tend to cluster around antennæ so cones make sense.

Both are used. Both work fairly well.

Properties of Jet Algorithms

Both cones and k_T can be used to construct good (or bad) jet algorithms. An ideal algorithm has the following properties:

1) It is fully specified. There is a definite procedure for handling any configuration.

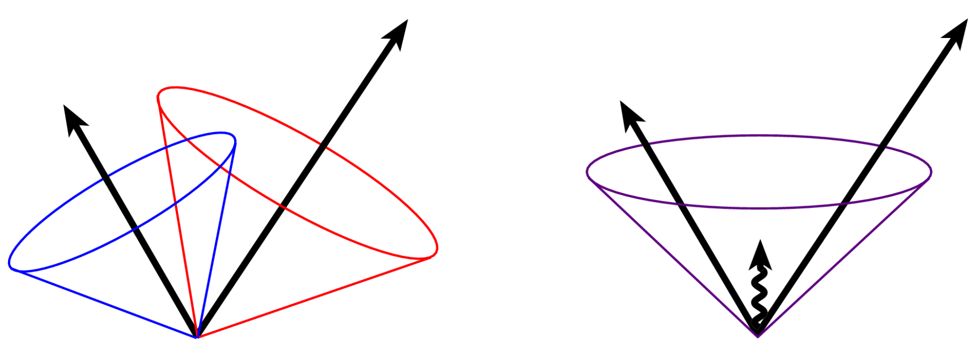
2) It must be theoretically well behaved. In particular, it must be Infrared Safe!

3) It should be detector independent.

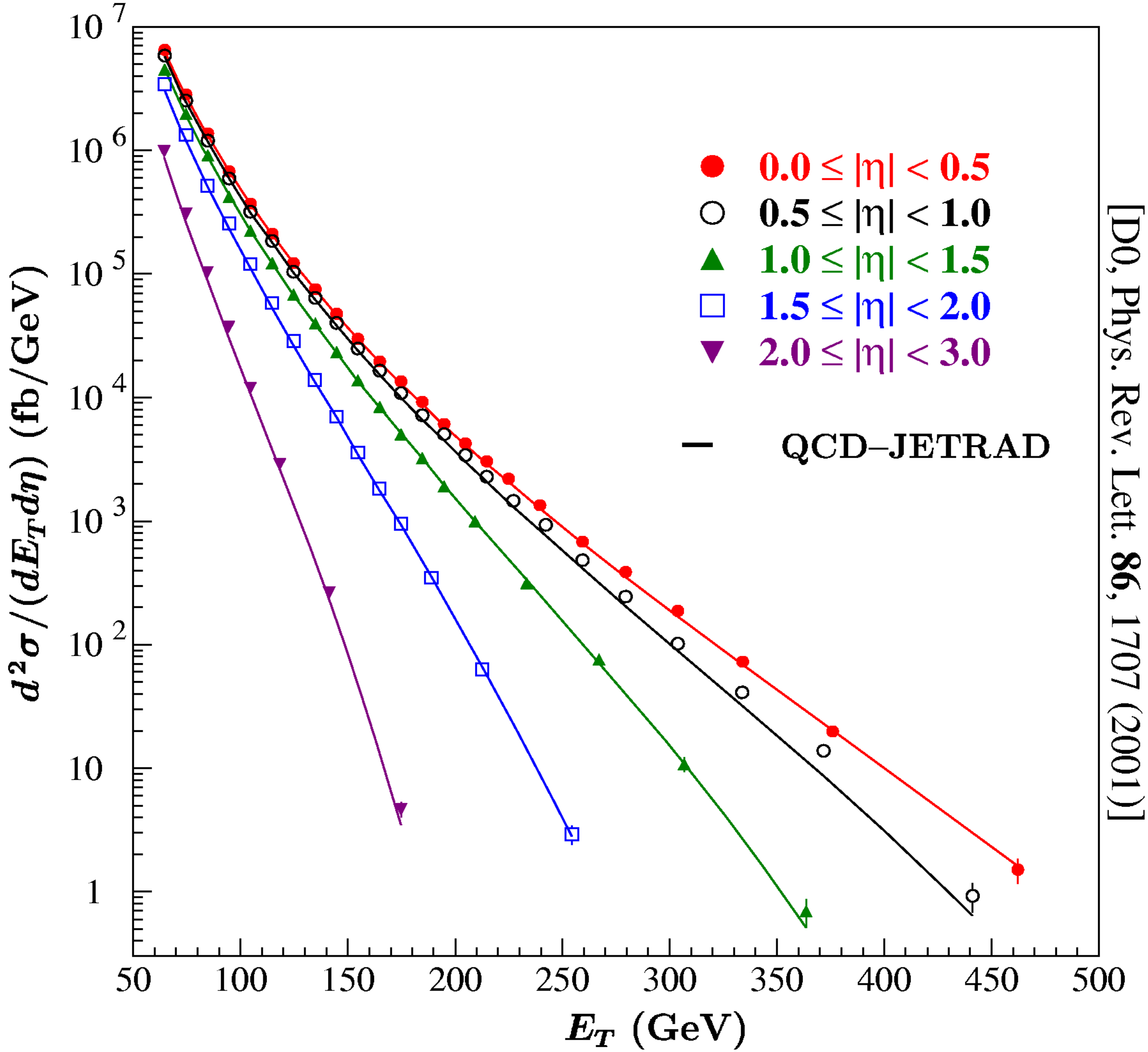
4) It should behave the same whether operating on parton, particles or calorimeter cells.

Infrared Safety in Jet Clustering

When Sterman and Weinberg first discussed jets in e^+e^- collisions, they emphasized that jet clustering must implement infrared safety by summing over soft and collinear configurations. Still, unforeseen problems can arise. The CDF Run-I cone algorithm turned out to have an infrared sensitivity that would affect calculations at NNLO and beyond.



Jet Production at the Tevatron



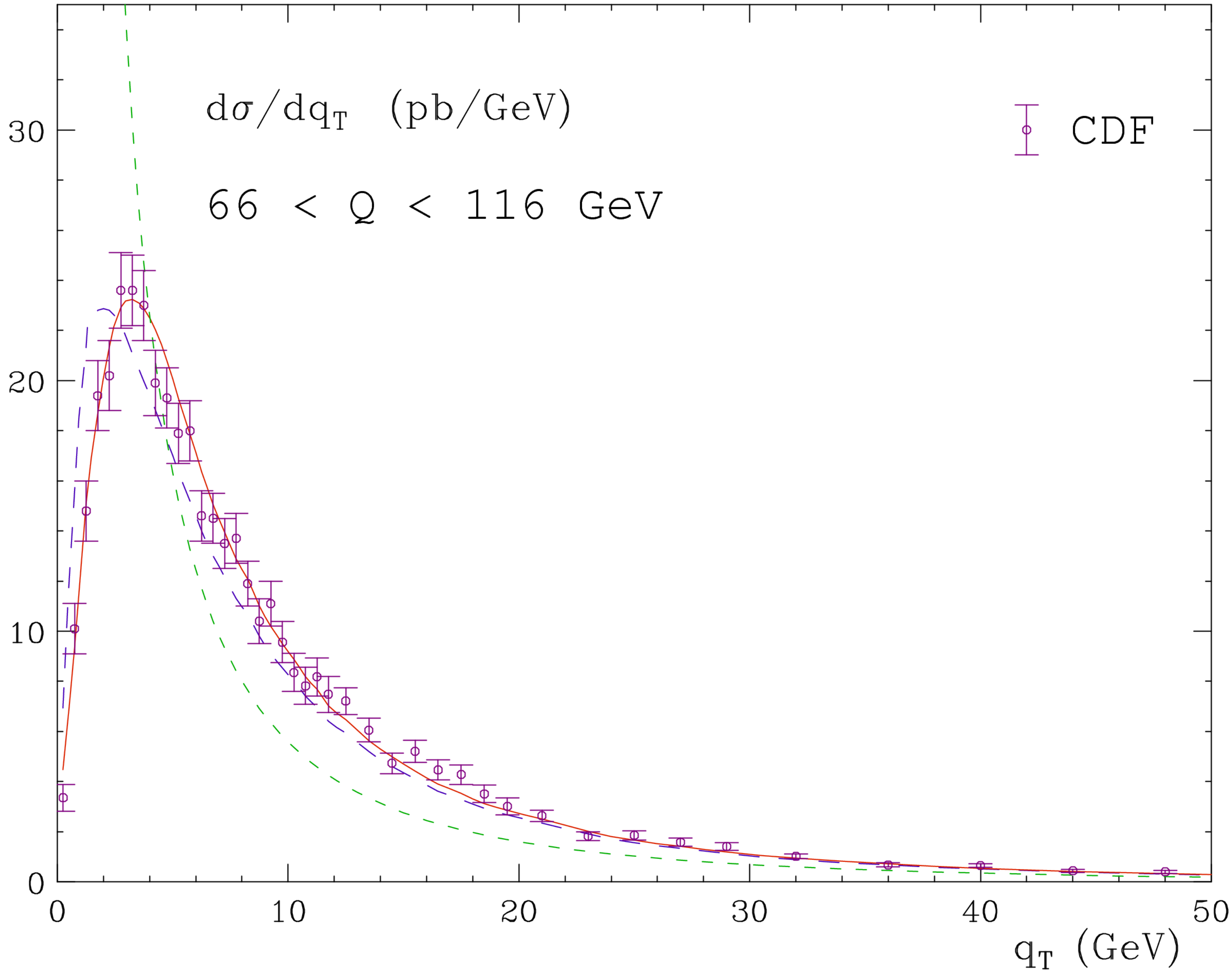
Other Important Issues

There are many important issues that I have barely mentioned or ignored completely.

- Resummation: This is very important for threshold production processes and p_T spectra.
- Fragmentation: Identified hadrons are very important to B physics and to Higgs searches.
- Diffraction: Inelastic scattering where the protons remain intact.

A very important topic to be taken up by T. Sjostrand is that of parton showering Monte Carlo calculations.

Resummation can be Essential



[Phys.Rev.D66:014011,2002]

Identified Hadrons

If there are identified hadrons in the final state, (say ,J/ψ, Υ mesons or photons) these are included through "Fragmentation Functions", which are to some degree the inverse of the parton distributions:

$$D_{h/q_j}(x,\mu)=\frac{1}{12\pi}\sum_X\int dy^- e^{-ip^+y^-/z}\,Tr\,\gamma^+\langle 0|\psi_j(0,y^-,\boldsymbol{y}_T)\boldsymbol{W}(y^-,0)|h(p)X\rangle$$

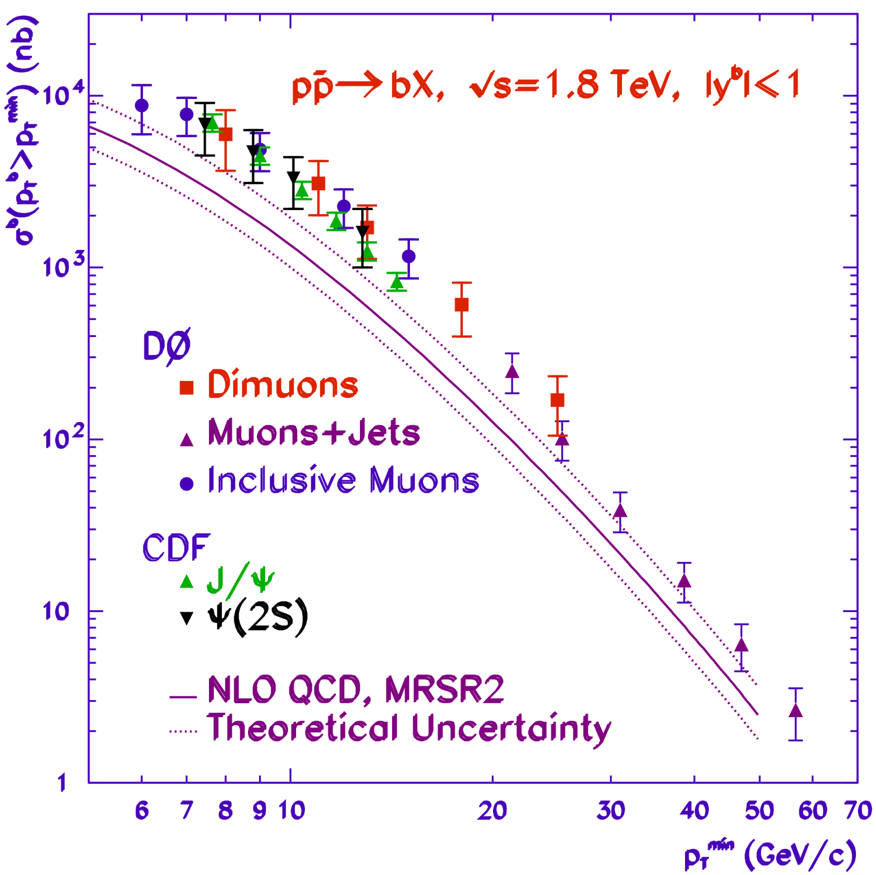
$$\times\langle h(p)X|\boldsymbol{W}(y^-,0)\overline{\psi}_j(0)|0\rangle_R$$

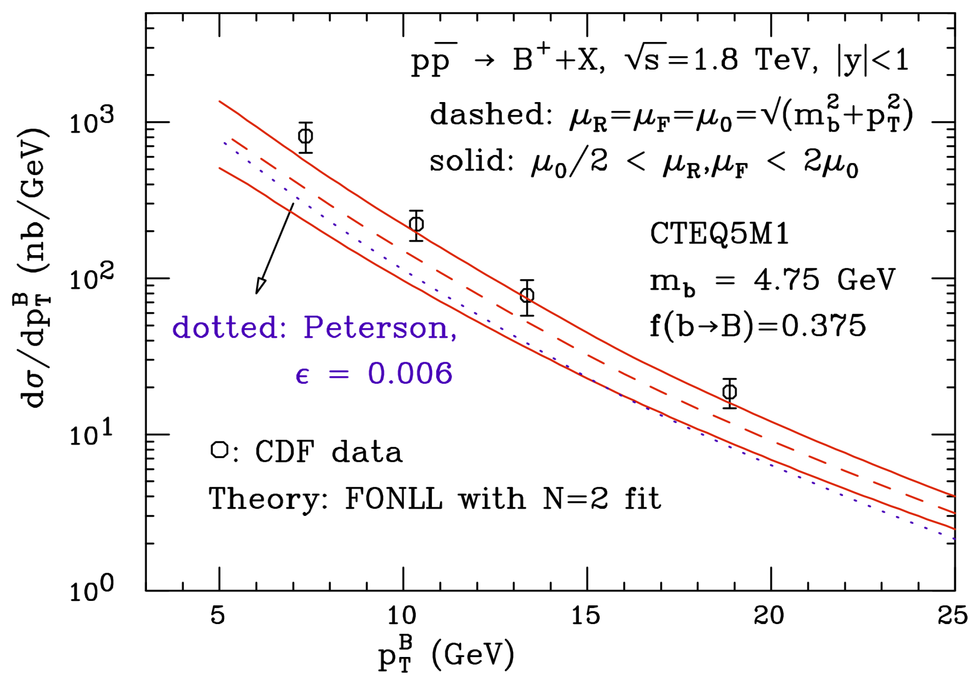
The Fragmentation Function $D_{h/c}(z)$ represents the probability of finding hadron h in the decay products (jet) of a parton of type c, carrying fraction z of the parton's momentum.

Identified Hadrons

The fragmentation functions are non-perturbative objects and must be fit to data!

Bad fragmentation functions give wrong results!





Conclusions

The basic Parton Model picture of hadronic collisions has been given a rigorous theoretical definition within Quantum ChromoDynamics.

The application of perturbation theory is limited by the demand of Infrared Safety which follows from QCD dynamics and its infrared structure.

Realistic applications of perturbative QCD require a vast machinery involving PDFs, loop-level scattering processes, resummations, fragmentations, parton showers and more.