



CERN–Fermilab Collider School  
Fermilab  
9 – 18 August 2006

LUND UNIVERSITY

# Theory of Hadronic Collisions

## Part II: Phenomenology

Torbjörn Sjöstrand

Lund University

- 1. (today) Introduction and Overview; Parton Showers**
2. (tomorrow) Matching Issues; Multiple Interactions I
3. (on Monday) Hadronization; MI II/LHC; Generators & Conclusions

# Apologies

These lectures will *not* cover:

- ★ Heavy-ion physics:
  - without quark-gluon plasma formation, or
  - with quark-gluon plasma formation.
- ★ Specific physics studies for topics such as
  - B production,
  - Higgs discovery,
  - SUSY phenomenology,
  - other new physics discovery potential.
- ★ The modelling of elastic and diffractive topologies.

They *will* cover the “normal” physics that is there at the Tevatron, and will be there in (essentially) all LHC pp events, from QCD to exotics:

- ★ the “dressing up” of a hard process by parton showers,
- ★ the addition of an underlying event,
- ★ the transition from partons to observable hadrons.

Event generators often only realistic tool, so end with a few words on

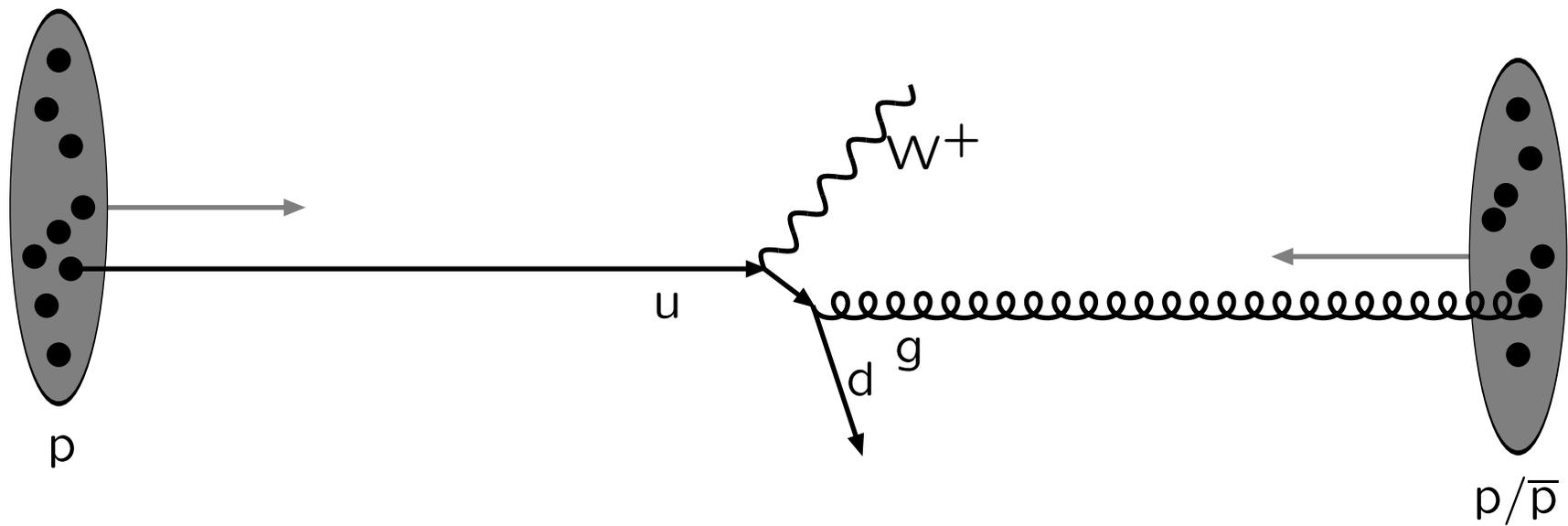
- ★ the status and evolution of general-purpose generators.

# The structure of an event

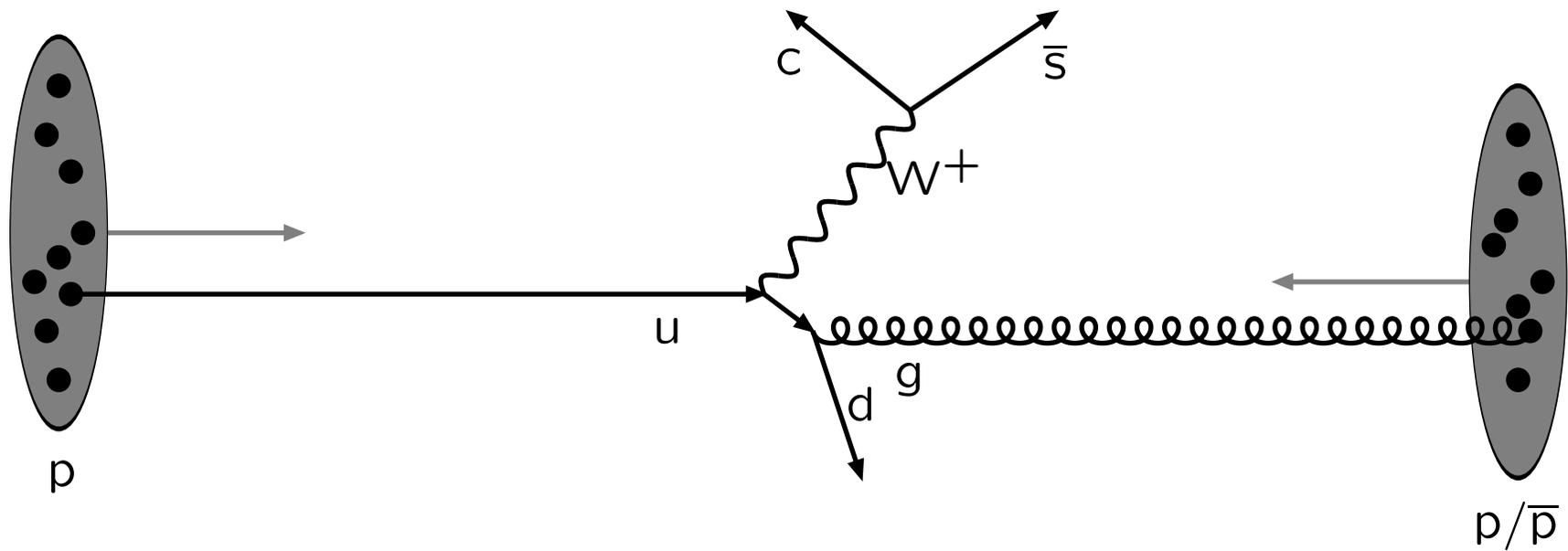
Warning: schematic only, everything simplified, nothing to scale, ...



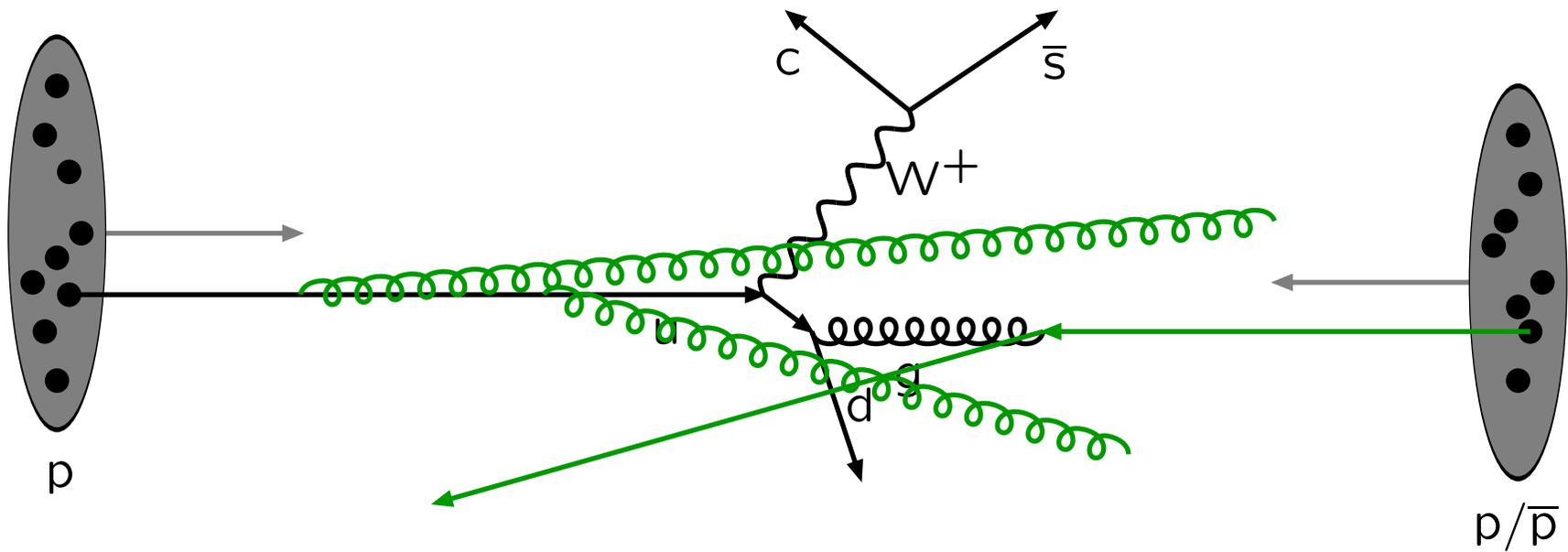
Incoming beams: parton densities



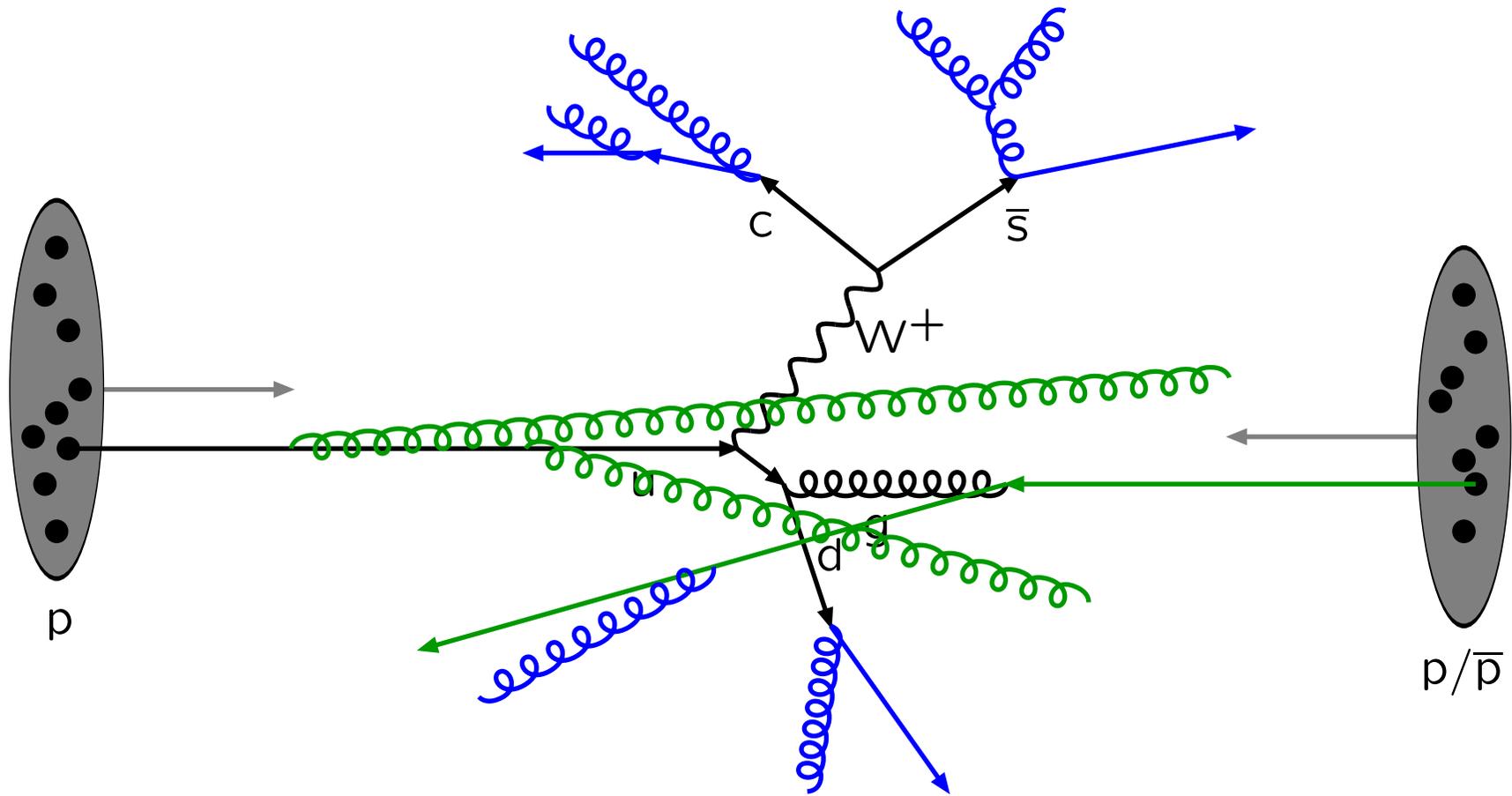
Hard subprocess: described by matrix elements



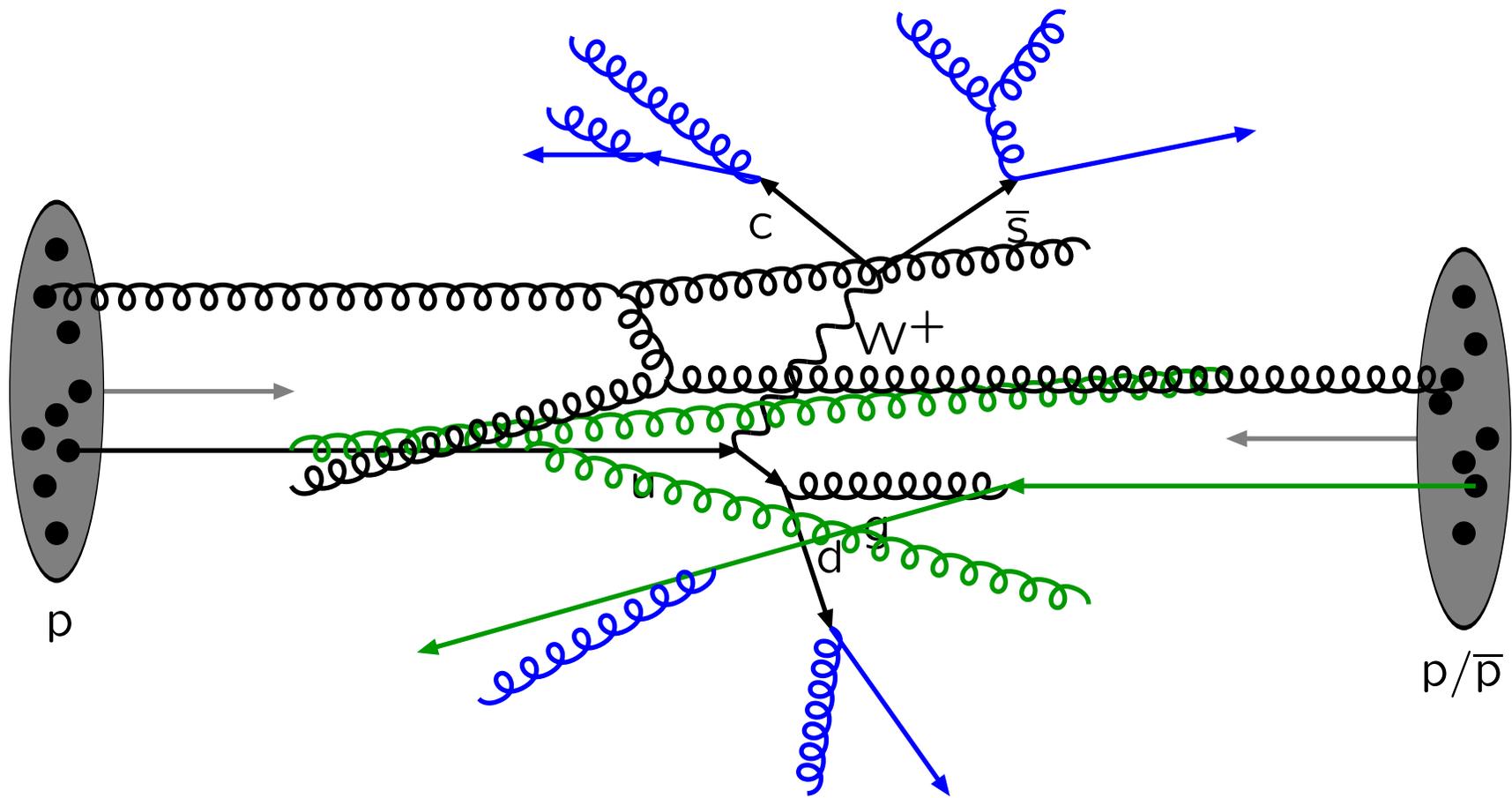
Resonance decays: correlated with hard subprocess



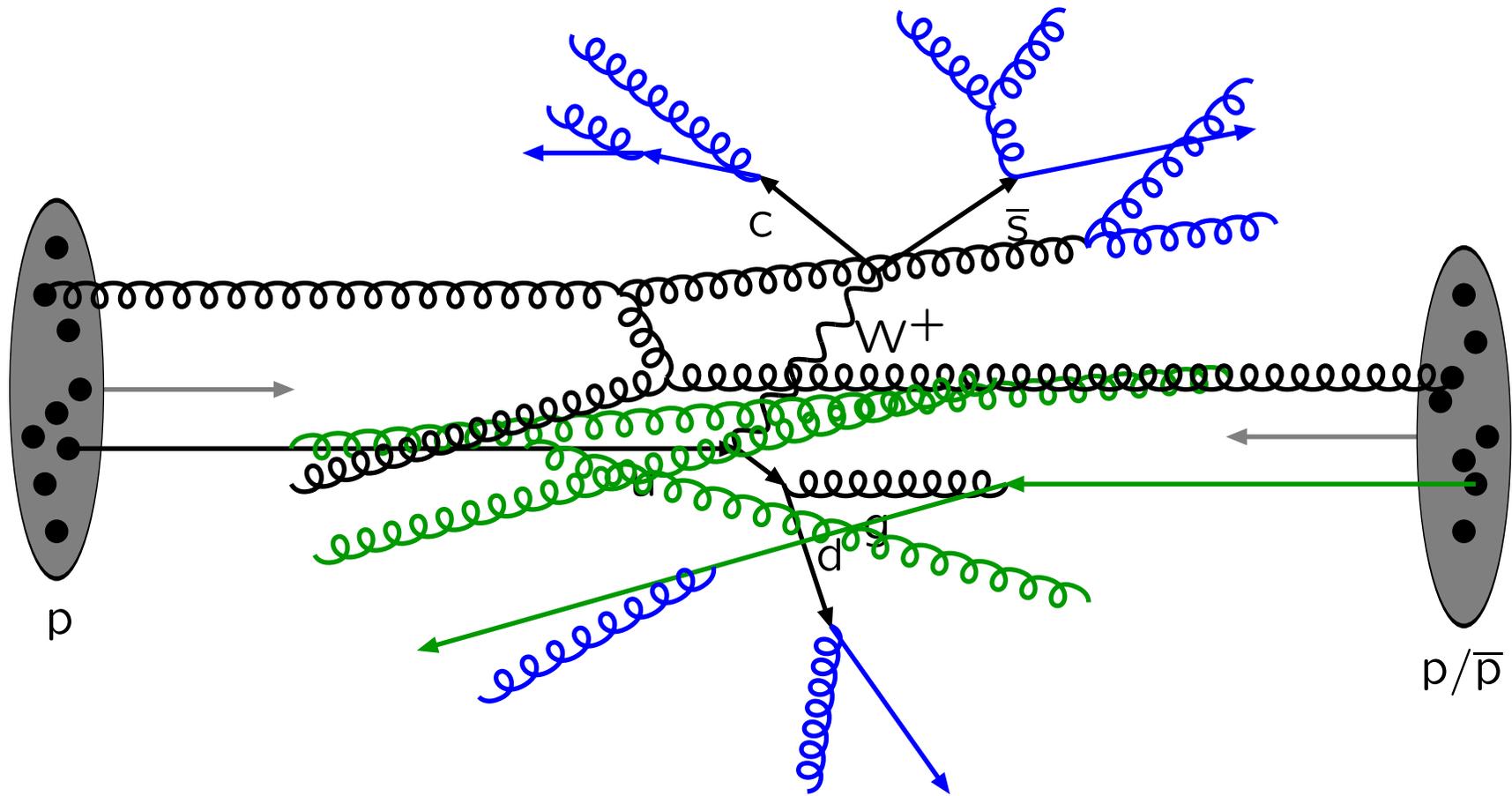
Initial-state radiation: spacelike parton showers



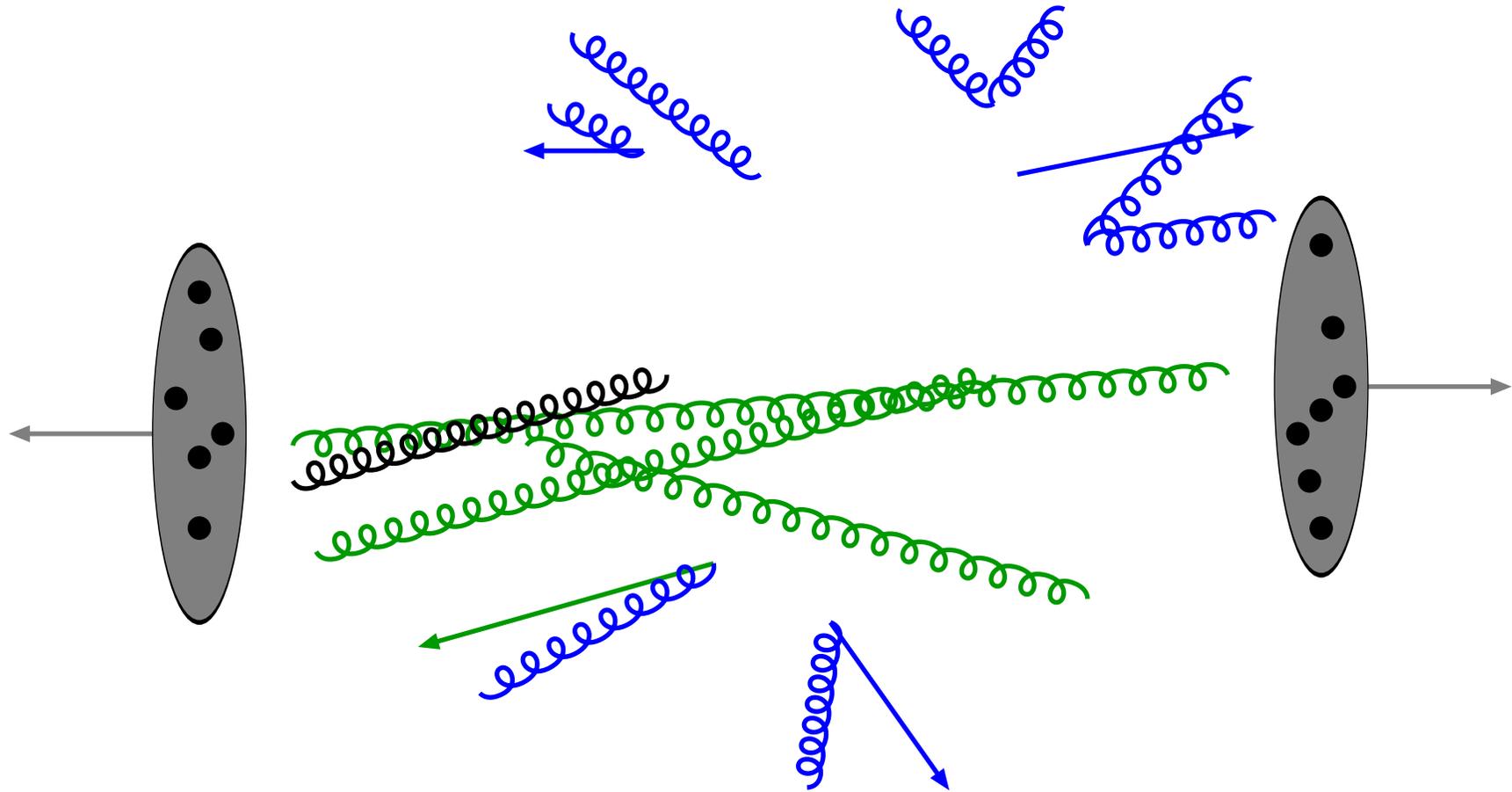
Final-state radiation: timelike parton showers



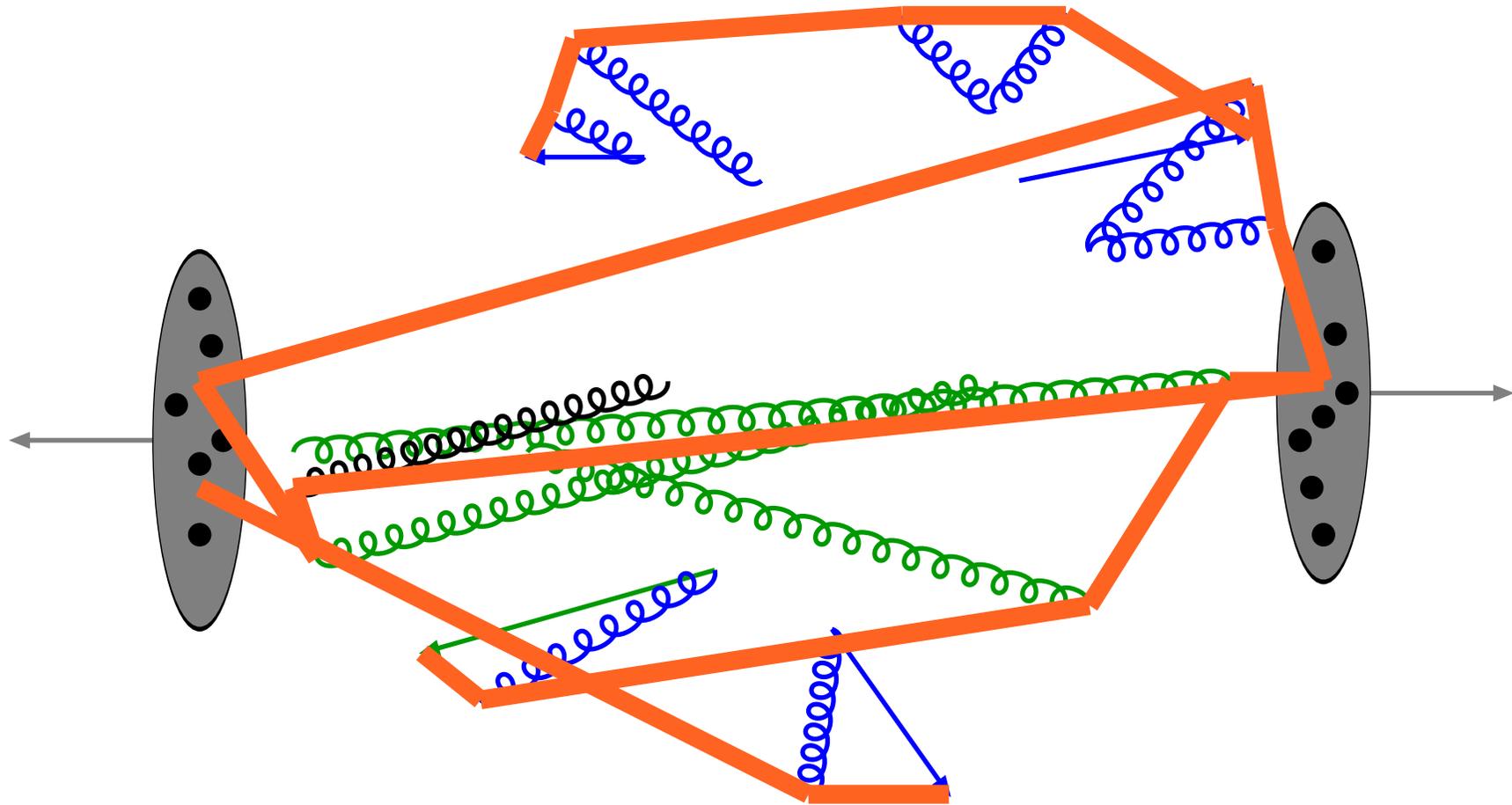
Multiple parton-parton interactions ...



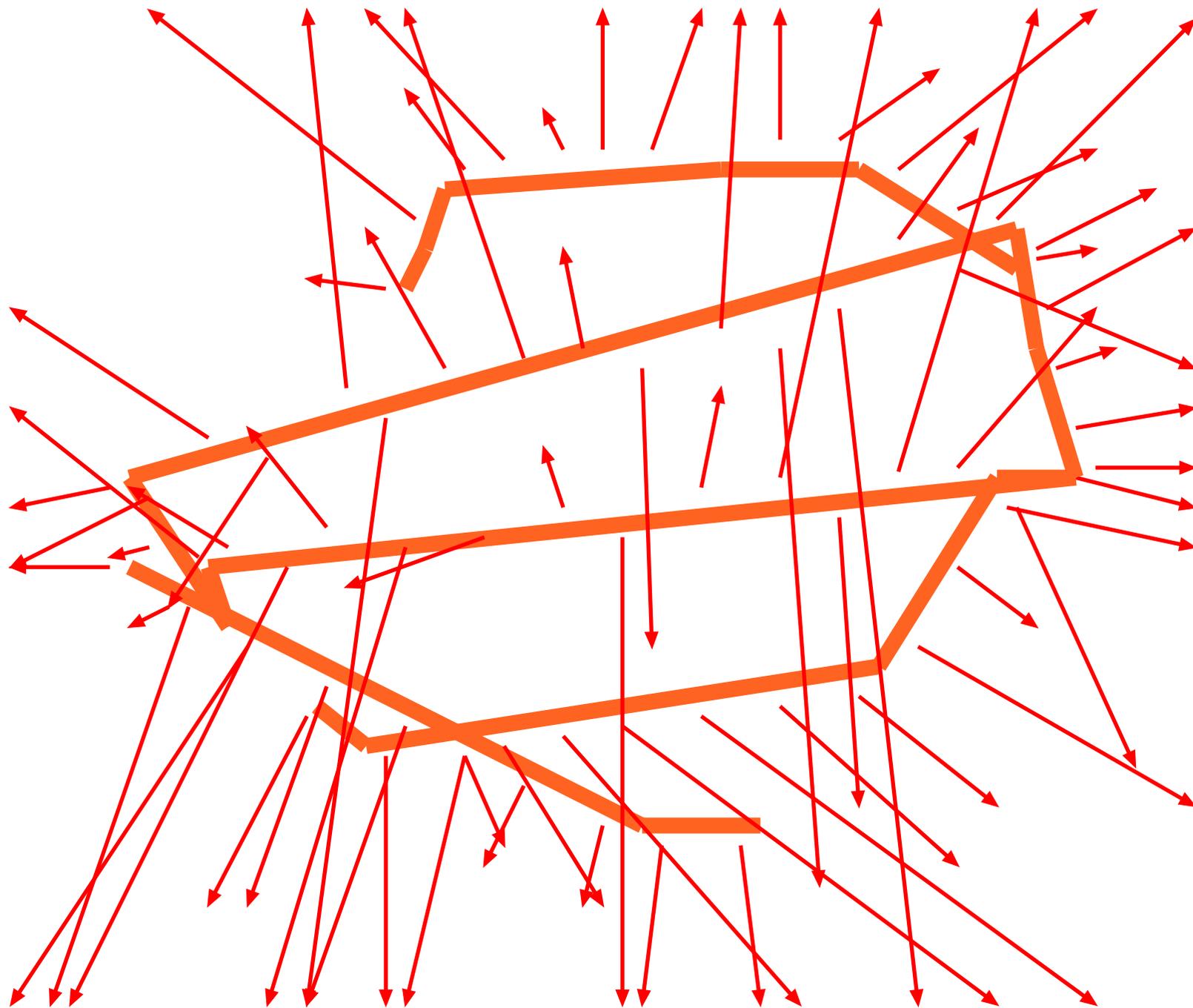
... with its initial- and final-state radiation



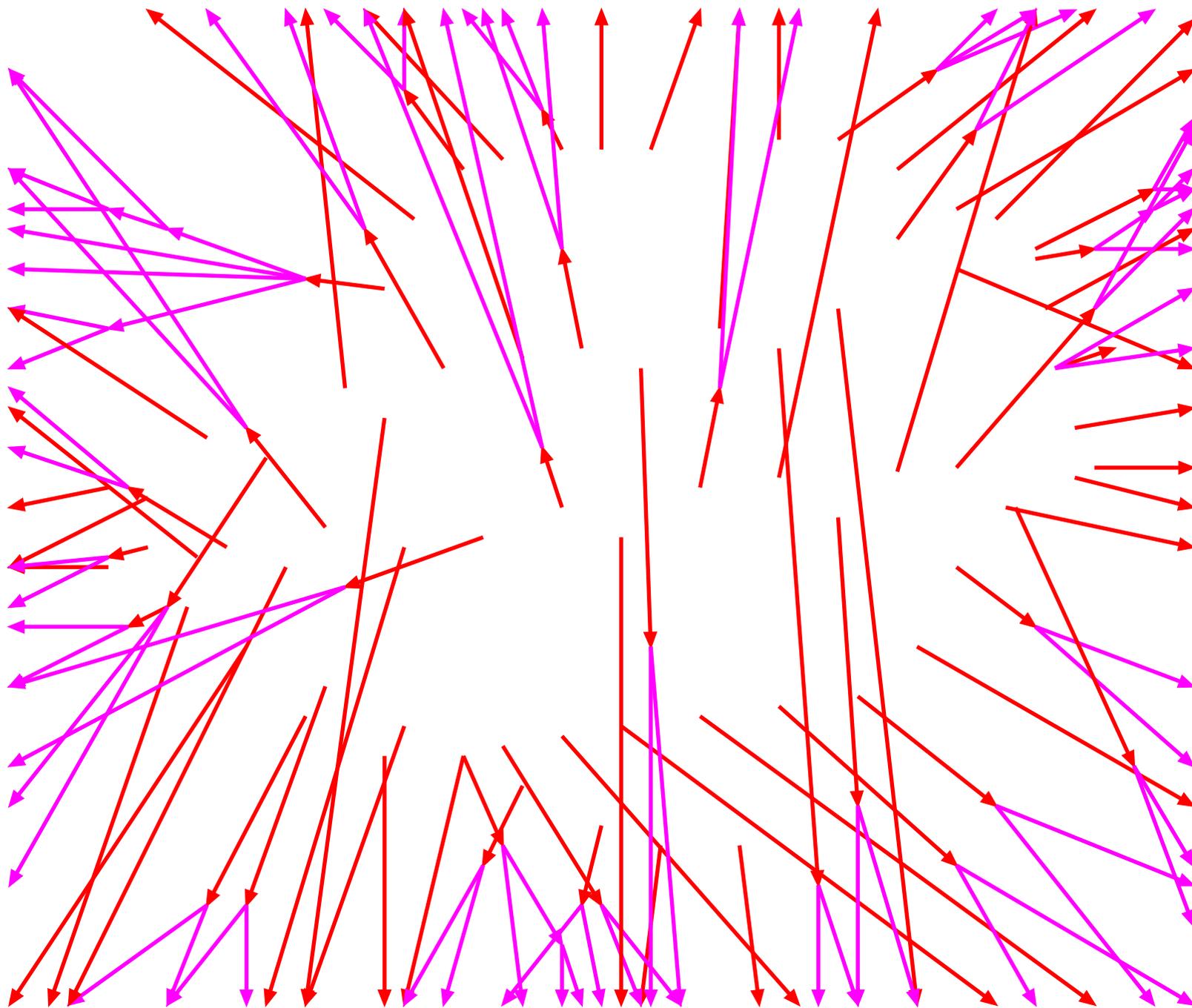
Beam remnants and other outgoing partons



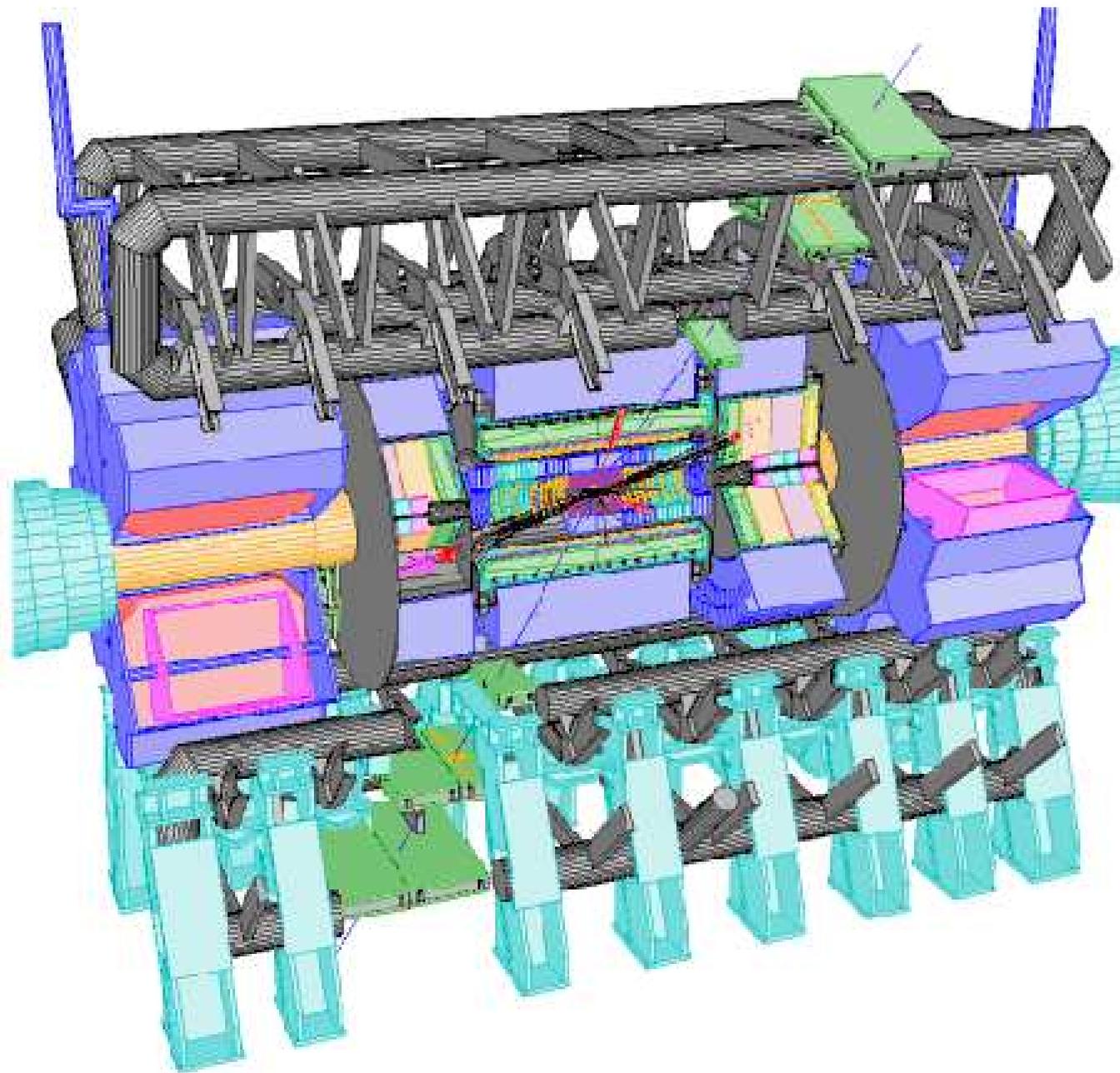
Everything is connected by colour confinement strings  
Recall! Not to scale: strings are of hadronic widths



The strings fragment to produce primary hadrons

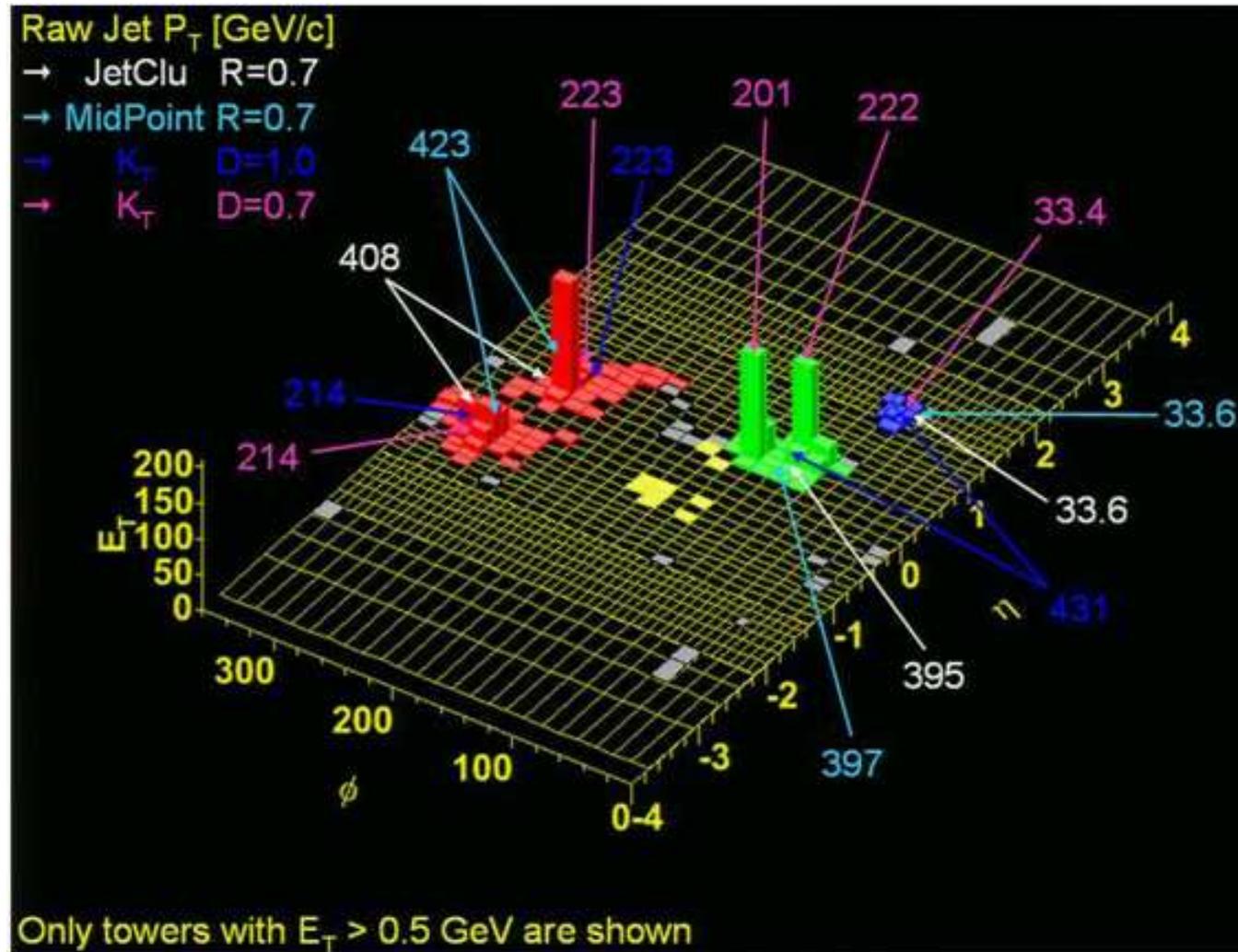


Many hadrons are unstable and decay further



These are the particles that hit the detector

# Parton Showers



- Final-State (Timelike) Showers
- Initial-State (Spacelike) Showers
  - Matching to Matrix Elements

# Divergences

Emission rate  $q \rightarrow qg$  diverges when

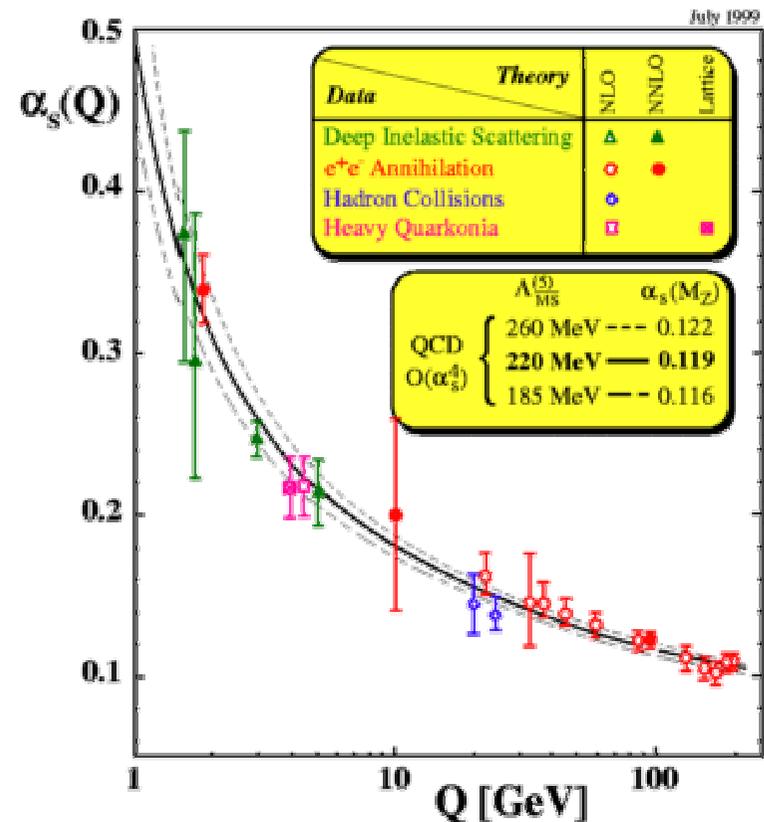
- collinear: opening angle  $\theta_{qg} \rightarrow 0$
- soft: gluon energy  $E_g \rightarrow 0$

Almost identical to  $e \rightarrow e\gamma$

(“bremsstrahlung”),

but QCD is non-Abelian so additionally

- $g \rightarrow gg$  similarly divergent
- $\alpha_s(Q^2)$  diverges for  $Q^2 \rightarrow 0$   
(actually for  $Q^2 \rightarrow \Lambda_{\text{QCD}}^2$ )

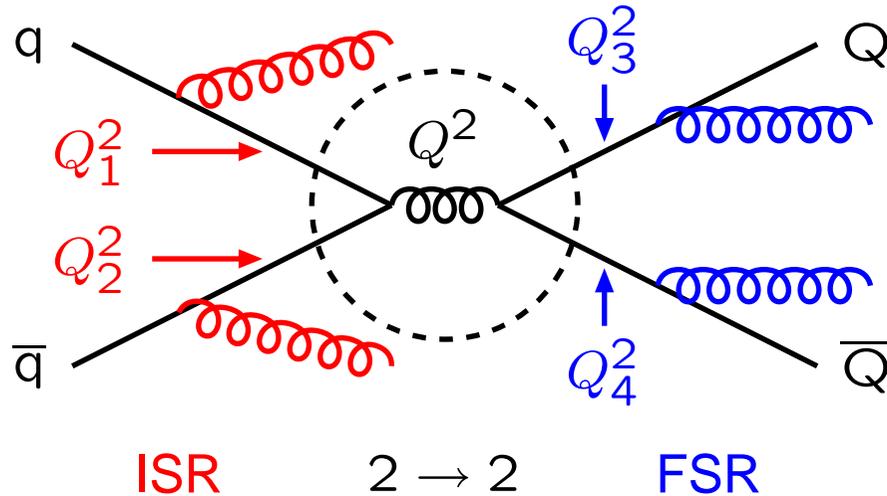


Big probability for one emission  $\implies$  also big for several  
 $\implies$  with ME's need to calculate to high order **and** with many loops  
 $\implies$  extremely demanding technically (not solved!), and  
involving big cancellations between positive and negative contributions.

Alternative approach: **parton showers**

# The Parton-Shower Approach

$$2 \rightarrow n = (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR}$$



FSR = Final-State Rad.;  
timelike shower

$Q_i^2 \sim m^2 > 0$  decreasing

ISR = Initial-State Rad.;  
spacelike shower

$Q_i^2 \sim -m^2 > 0$  increasing

$2 \rightarrow 2 =$  hard scattering (on-shell):

$$\sigma = \iiint dx_1 dx_2 d\hat{t} f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

Shower evolution is viewed as a probabilistic process,

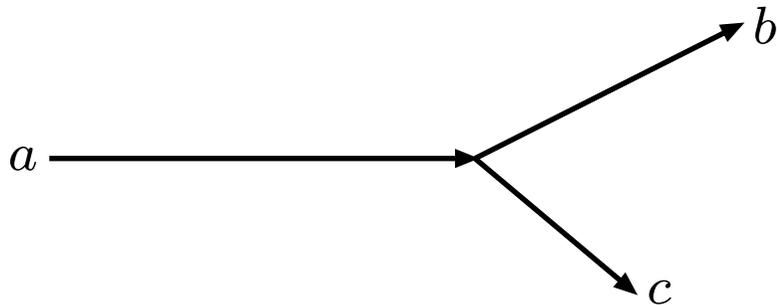
which occurs with unit total probability:

*the cross section is not directly affected,*

*but indirectly it is, via the changed event shape*

# Technical aside: why timelike/spacelike?

Consider four-momentum conservation in a branching  $a \rightarrow b c$



$$\begin{aligned} \mathbf{p}_{\perp a} = 0 &\Rightarrow \mathbf{p}_{\perp c} = -\mathbf{p}_{\perp b} \\ p_+ = E + p_L &\Rightarrow p_{+a} = p_{+b} + p_{+c} \\ p_- = E - p_L &\Rightarrow p_{-a} = p_{-b} + p_{-c} \end{aligned}$$

Define  $p_{+b} = z p_{+a}$ ,  $p_{+c} = (1 - z) p_{+a}$

Use  $p_+ p_- = E^2 - p_L^2 = m^2 + p_{\perp}^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1 - z) p_{+a}}$$

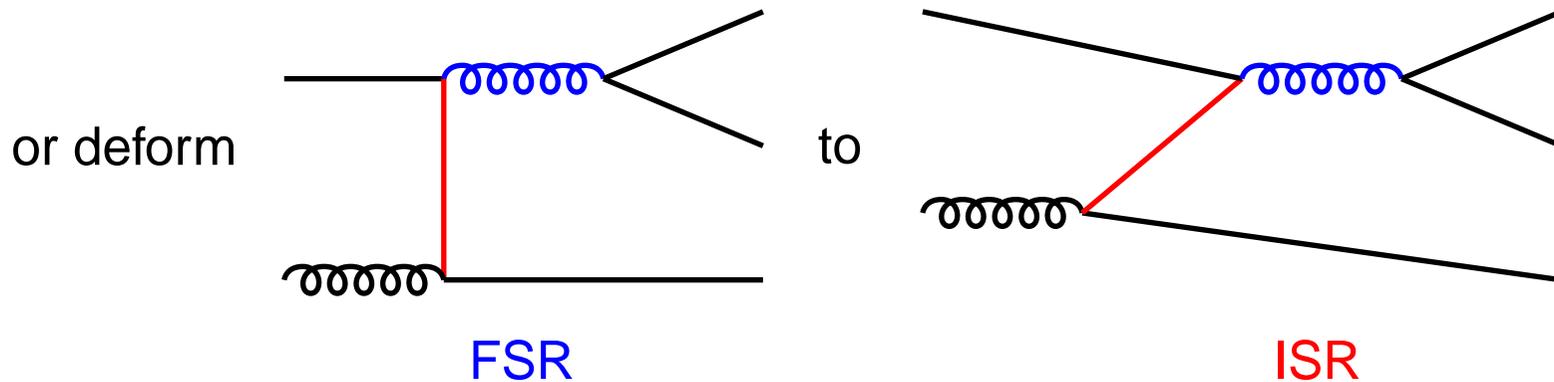
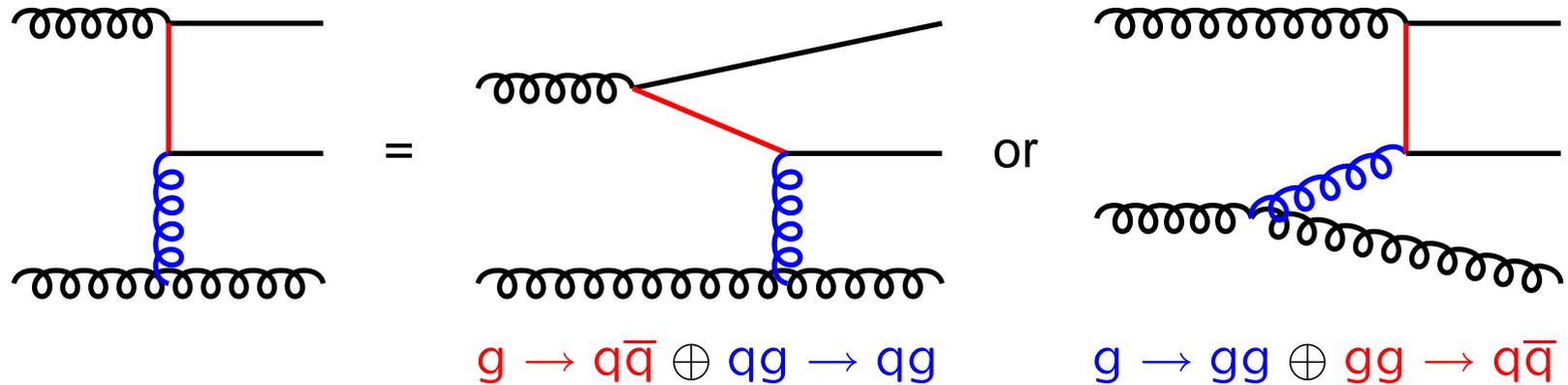
$$\Rightarrow m_a^2 = \frac{m_b^2 + p_{\perp}^2}{z} + \frac{m_c^2 + p_{\perp}^2}{1 - z} = \frac{m_b^2}{z} + \frac{m_c^2}{1 - z} + \frac{p_{\perp}^2}{z(1 - z)}$$

Final-state shower:  $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_{\perp}^2}{z(1 - z)} > 0 \Rightarrow$  timelike

Initial-state shower:  $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_{\perp}^2}{1 - z} < 0 \Rightarrow$  spacelike

# Doublecounting

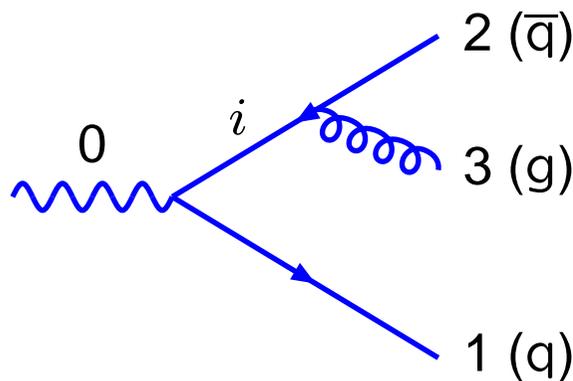
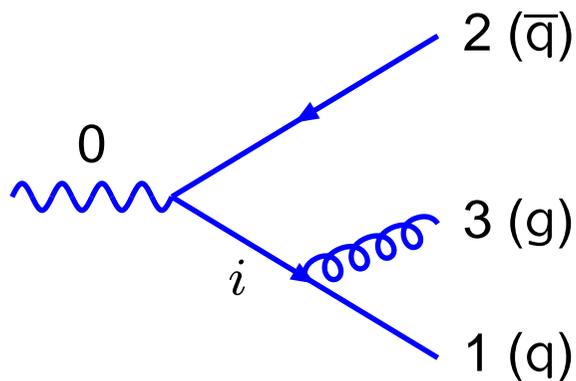
A  $2 \rightarrow n$  graph can be “simplified” to  $2 \rightarrow 2$  in different ways:



*Do not doublecount:  $2 \rightarrow 2 = \text{most virtual} = \text{shortest distance}$*

Conflict: theory derivations often assume virtualities strongly ordered;  
interesting physics often in regions where this is not true!

# From Matrix Elements to Parton Showers



$$e^+e^- \rightarrow q\bar{q}g$$

$$x_j = 2E_j/E_{\text{cm}} \Rightarrow x_1 + x_2 + x_3 = 2$$

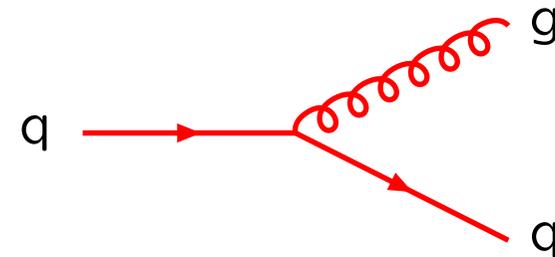
$$m_q = 0 : \frac{d\sigma_{\text{ME}}}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} dx_1 dx_2$$

Rewrite for  $x_2 \rightarrow 1$ , i.e. q-g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\text{cm}}^2} = \frac{Q^2}{E_{\text{cm}}^2} \Rightarrow dx_2 = \frac{dQ^2}{E_{\text{cm}}^2}$$

$$x_1 \approx z \Rightarrow dx_1 \approx dz$$

$$x_3 \approx 1 - z$$



$$\Rightarrow d\mathcal{P} = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{dx_2}{(1-x_2)} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1-x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1+z^2}{1-z} dz$$

## Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

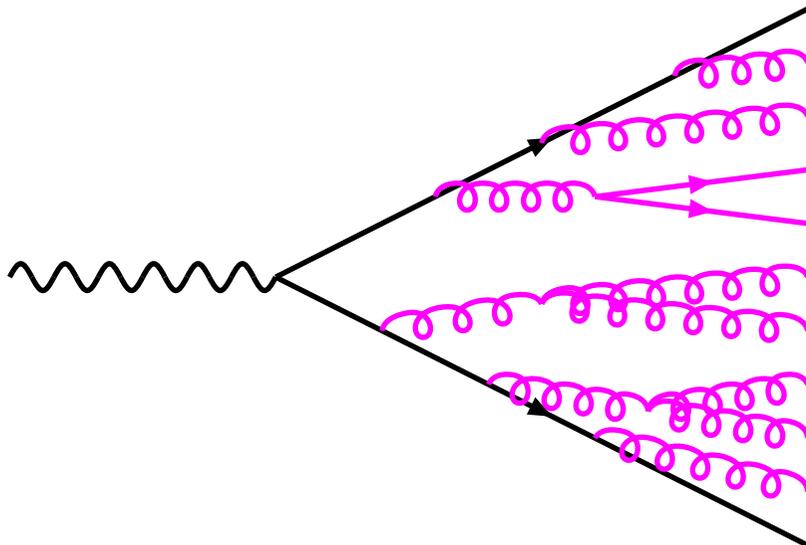
$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz$$

$$P_{q \rightarrow qg} = \frac{4}{3} \frac{1+z^2}{1-z}$$

$$P_{g \rightarrow gg} = 3 \frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{g \rightarrow q\bar{q}} = \frac{n_f}{2} (z^2 + (1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

Iteration gives final-state parton showers



Need soft/collinear cut-offs  
to stay away from  
nonperturbative physics.

Details model-dependent, e.g.

$Q > m_0 = \min(m_{ij}) \approx 1 \text{ GeV}$ ,

$z_{\min}(E, Q) < z < z_{\max}(E, Q)$

or  $p_{\perp} > p_{\perp \min} \approx 0.5 \text{ GeV}$

# The Sudakov Form Factor

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

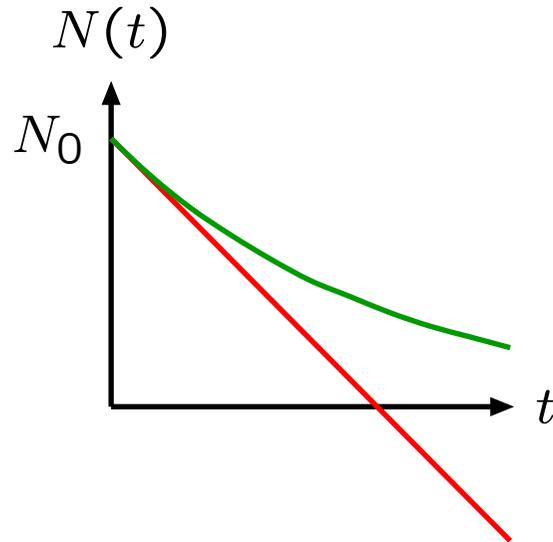
“multiplicativeness” in “time” evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$$

Subdivide further, with  $T_i = (i/n)T$ ,  $0 \leq i \leq n$ :

$$\begin{aligned} \mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\ &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \left( 1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left( - \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \\ \implies d\mathcal{P}_{\text{first}}(T) &= d\mathcal{P}_{\text{something}}(T) \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \end{aligned}$$

Example: radioactive decay of nucleus



naively:  $\frac{dN}{dt} = -cN_0 \Rightarrow N(t) = N_0 (1 - ct)$

depletion: a given nucleus can only decay once

correctly:  $\frac{dN}{dt} = -cN(t) \Rightarrow N(t) = N_0 \exp(-ct)$

generalizes to:  $N(t) = N_0 \exp\left(-\int_0^t c(t') dt'\right)$

or:  $\frac{dN(t)}{dt} = -c(t) N_0 \exp\left(-\int_0^t c(t') dt'\right)$

sequence allowed: nucleus<sub>1</sub> → nucleus<sub>2</sub> → nucleus<sub>3</sub> → ...

Correspondingly, with  $Q \sim 1/t$  (Heisenberg)

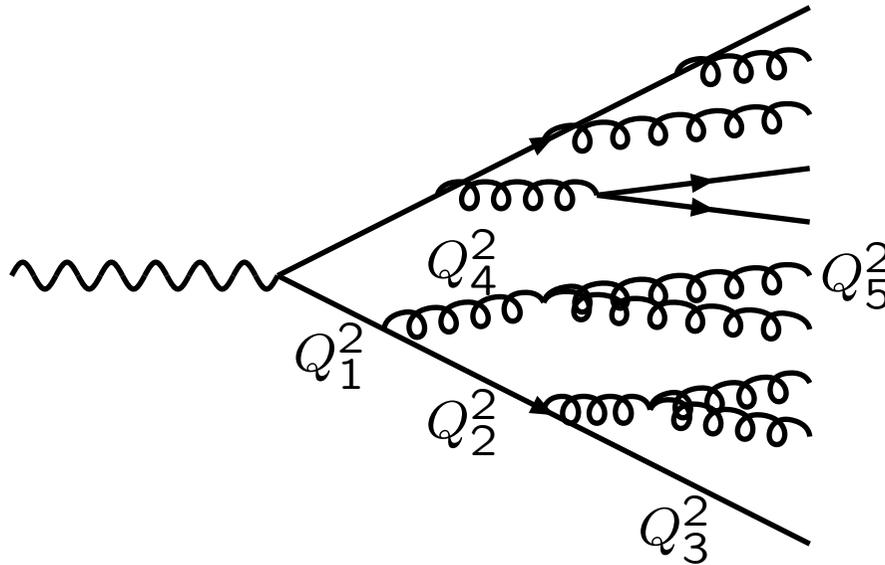
$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz \exp\left(-\sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz'\right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that  $\sum_{b,c} \int \int d\mathcal{P}_{a \rightarrow bc} \equiv 1 \Rightarrow$  convenient for Monte Carlo

( $\equiv 1$  if extended over whole phase space, else possibly nothing happens)



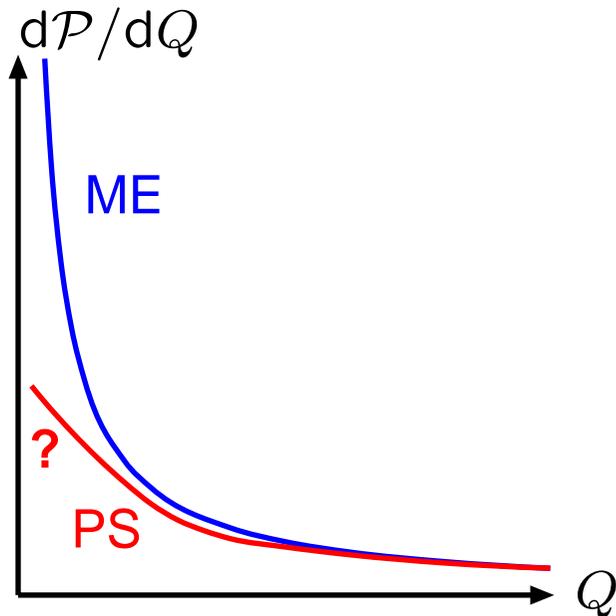
Sudakov form factor provides  
 “time” ordering of shower:  
 lower  $Q^2 \iff$  longer times

$$Q_1^2 > Q_2^2 > Q_3^2$$

$$Q_1^2 > Q_4^2 > Q_5^2$$

etc.

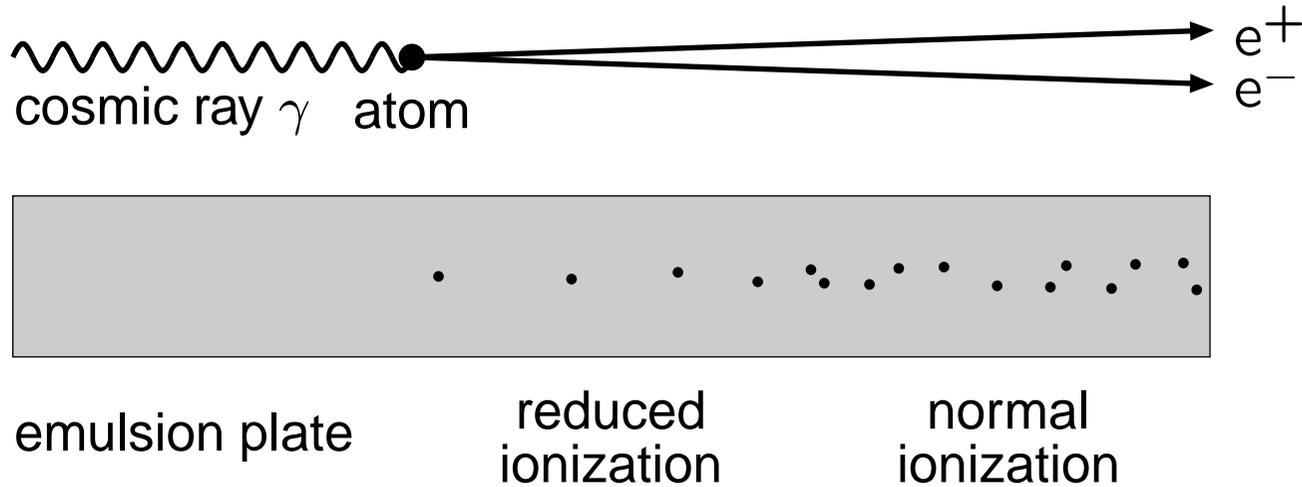
Sudakov regulates singularity for *first* emission ...



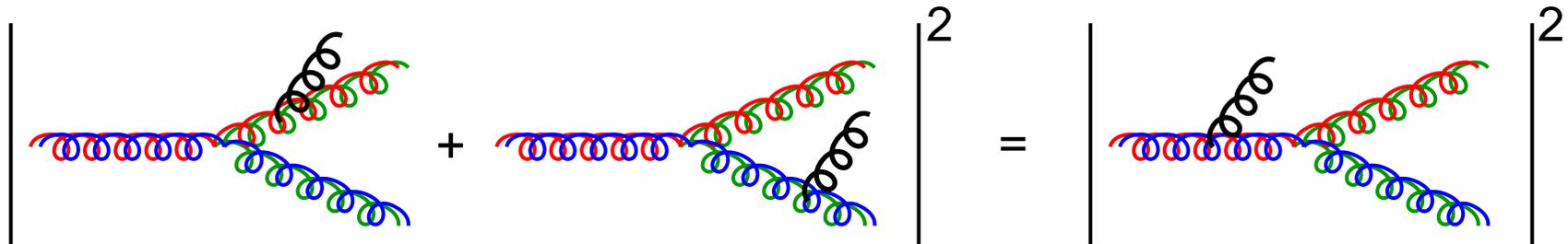
... but in limit of *repeated soft*  
 emissions  $q \rightarrow qg$   
 ( $g \rightarrow gg, g \rightarrow q\bar{q}$  not considered)  
 one obtains the same inclusive  
 $Q$  emission spectrum as for ME,  
 i.e. divergent ME spectrum  
 $\iff$  infinite number of PS emissions

# Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission

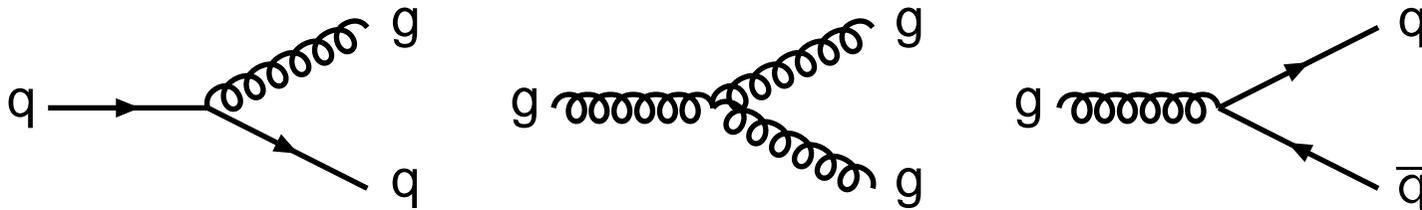


- solved by
- requiring emission angles to be decreasing
  - or
  - requiring transverse momenta to be decreasing

# The Common Showering Algorithms

Three main approaches to showering in common use:

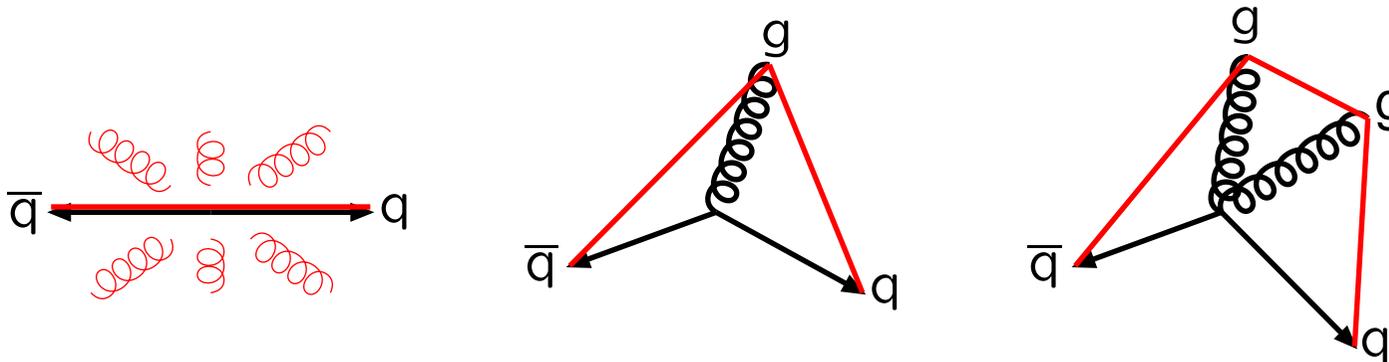
Two are based on the standard shower language  
of  $a \rightarrow bc$  successive branchings:



HERWIG:  $Q^2 \approx E^2(1 - \cos \theta) \approx E^2\theta^2/2$

PYTHIA:  $Q^2 = m^2$  (timelike) or  $= -m^2$  (spacelike)

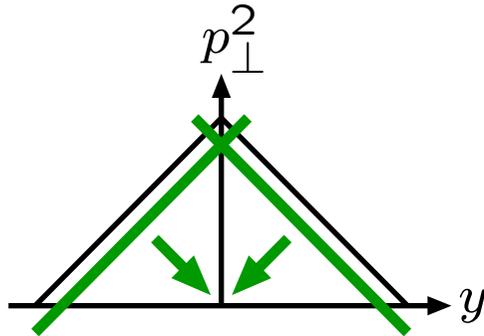
One is based on a picture of dipole emission  $ab \rightarrow cde$ :



ARIADNE:  $Q^2 = p_{\perp}^2$ ; FSR mainly, ISR is primitive;  
there instead LDCMC: sophisticated but complicated

# Ordering variables in final-state radiation

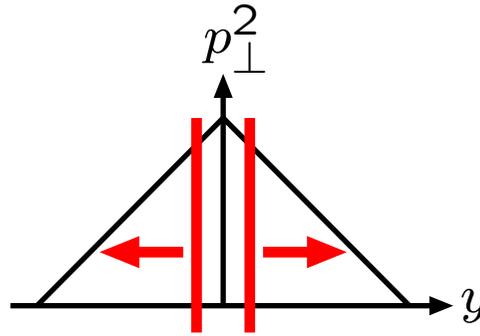
PYTHIA:  $Q^2 = m^2$



large mass first  
 $\Rightarrow$  “hardness” ordered  
**coherence brute force**

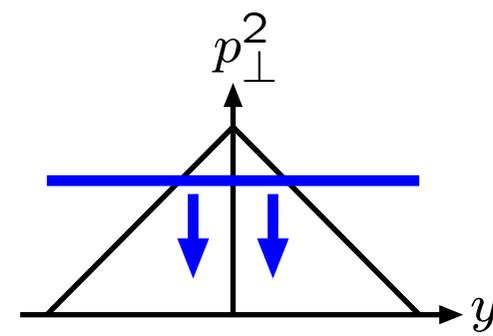
covers phase space  
 ME merging simple  
 $g \rightarrow q\bar{q}$  simple  
**not Lorentz invariant**  
 no stop/restart  
 ISR:  $m^2 \rightarrow -m^2$

HERWIG:  $Q^2 \sim E^2\theta^2$



large angle first  
 $\Rightarrow$  **hardness not ordered**  
 coherence inherent  
**gaps in coverage**  
**ME merging messy**  
 $g \rightarrow q\bar{q}$  simple  
**not Lorentz invariant**  
 no stop/restart  
 ISR:  $\theta \rightarrow \theta$

ARIADNE:  $Q^2 = p_{\perp}^2$

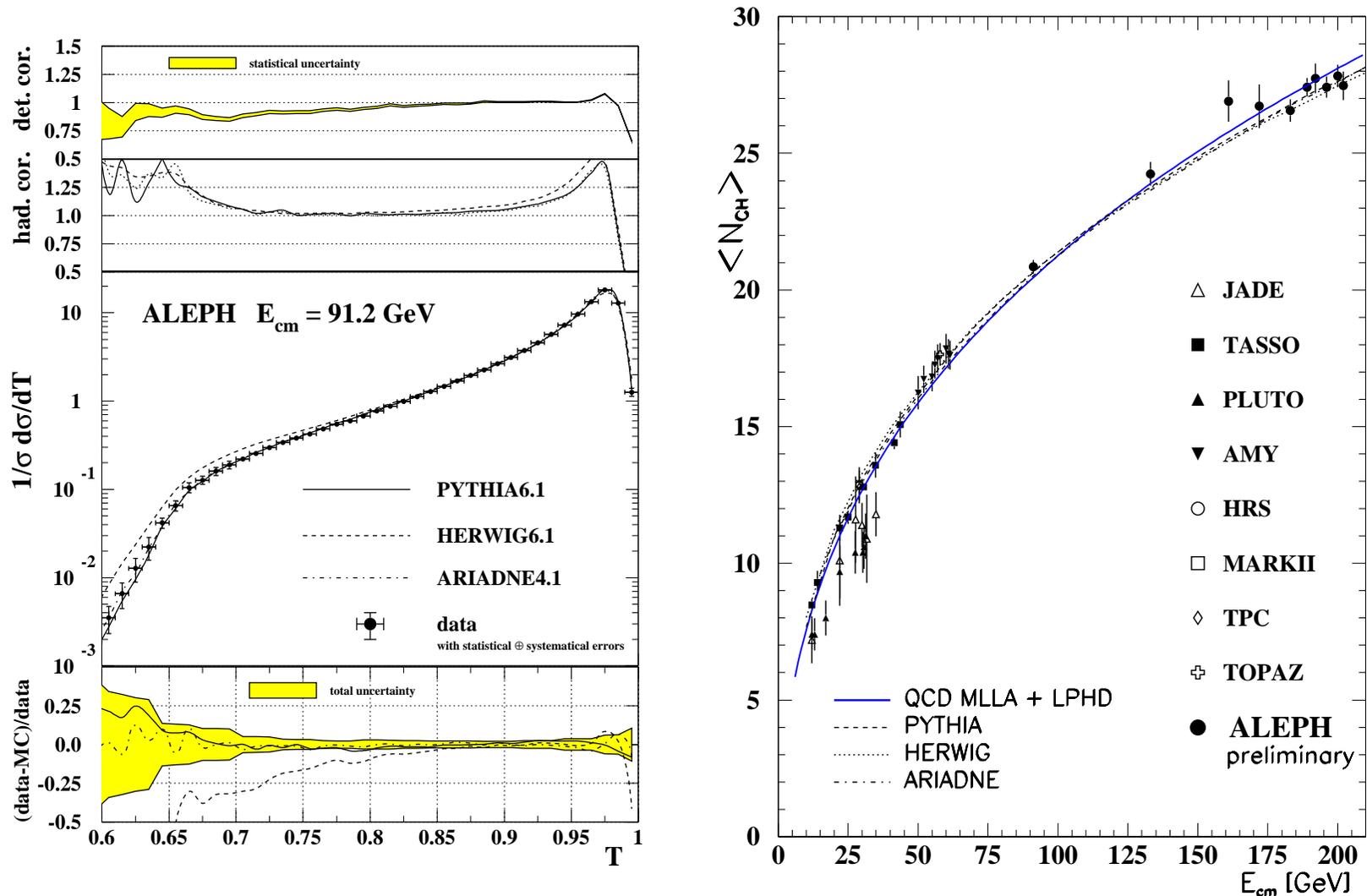


large  $p_{\perp}$  first  
 $\Rightarrow$  “hardness” ordered  
 coherence inherent

covers phase space  
 ME merging simple  
 $g \rightarrow q\bar{q}$  **messy**  
 Lorentz invariant  
 can stop/restart  
**ISR: more messy**

# Data comparisons

All three algorithms do a reasonable job of describing LEP data, but typically  $\text{ARIADNE } (p_{\perp}^2) > \text{PYTHIA } (m^2) > \text{HERWIG } (\theta)$



... and programs evolve to do even better ...

# Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\begin{aligned}\mathcal{P}_{q \rightarrow qg} &\approx \int \frac{dQ^2}{Q^2} \int dz \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z} \\ &\approx \alpha_s \ln \left( \frac{Q_{\max}^2}{Q_{\min}^2} \right) \frac{8}{3} \ln \left( \frac{1-z_{\min}}{1-z_{\max}} \right) \sim \alpha_s \ln^2\end{aligned}$$

Rate for  $n$  emissions is of form:

$$\mathcal{P}_{q \rightarrow qng} \sim (\mathcal{P}_{q \rightarrow qg})^n \sim \alpha_s^n \ln^{2n}$$

Next-to-leading log (NLL): inclusion of *all* corrections of type  $\alpha_s^n \ln^{2n-1}$

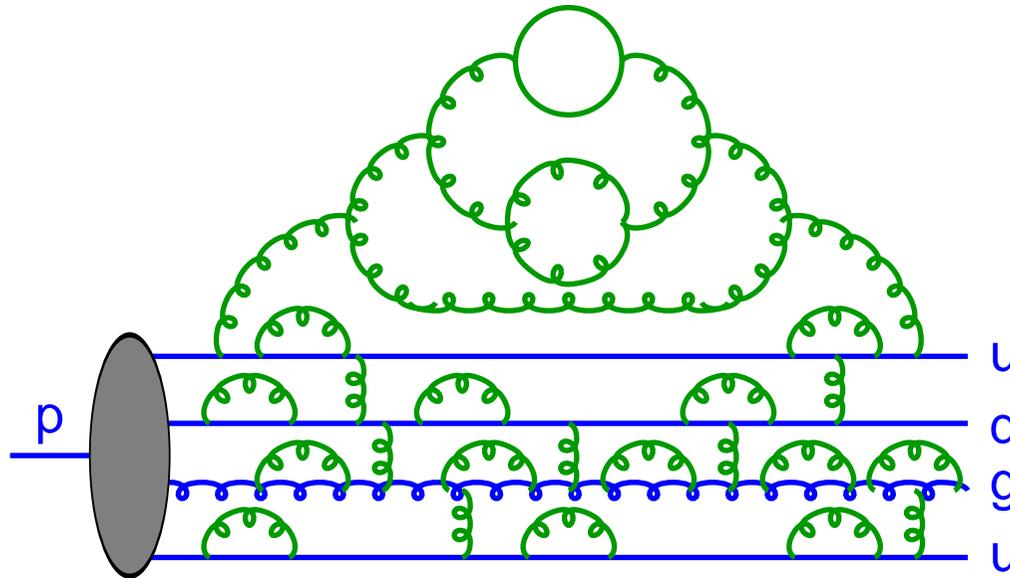
No existing pp/p $\bar{p}$  generator completely NLL, but

- energy-momentum conservation (and “recoil” effects)
- coherence
- $2/(1-z) \rightarrow (1+z^2)/(1-z)$
- scale choice  $\alpha_s(p_{\perp}^2)$  absorbs singular terms  $\propto \ln z, \ln(1-z)$  in  $\mathcal{O}(\alpha_s^2)$  splitting kernels  $P_{q \rightarrow qg}$  and  $P_{g \rightarrow gg}$
- ...

$\Rightarrow$  far better than naive, analytical LL

# Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2)$  = number density of partons  $i$   
at momentum fraction  $x$  and probing scale  $Q^2$ .

Linguistics (example):

$$F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2)$$

structure function

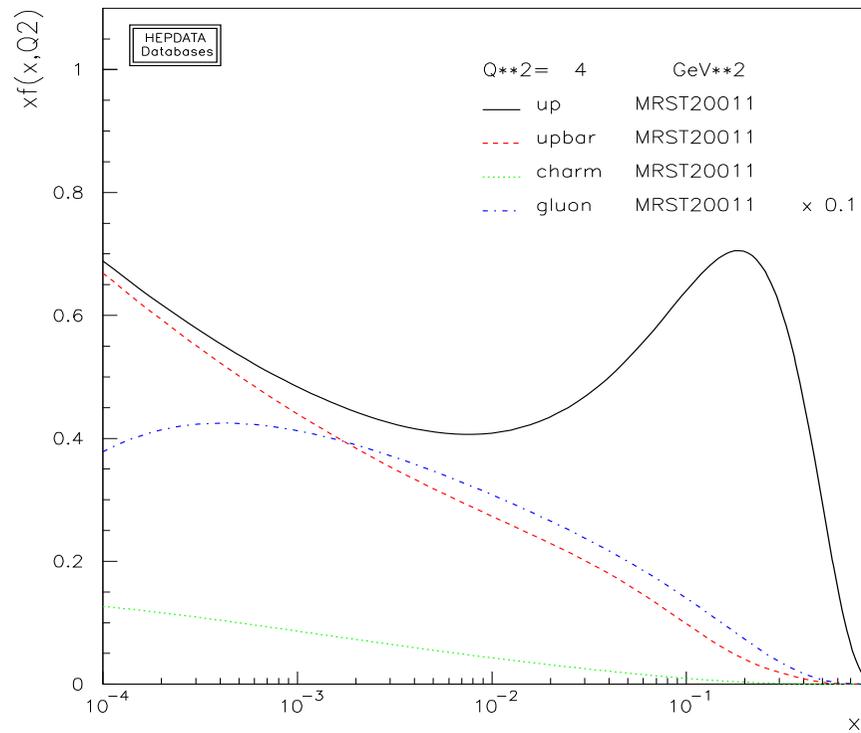
parton distributions

Absolute normalization at small  $Q_0^2$  unknown.

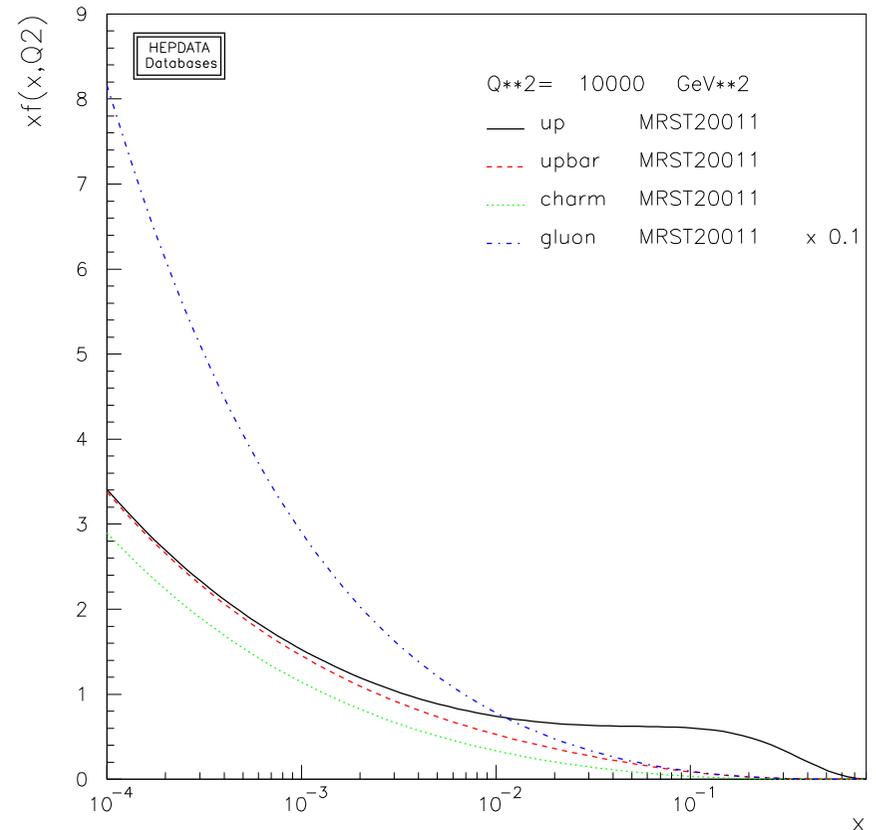
Resolution dependence by DGLAP:

$$\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc} \left( z = \frac{x}{x'} \right)$$

$Q^2 = 4 \text{ GeV}^2$

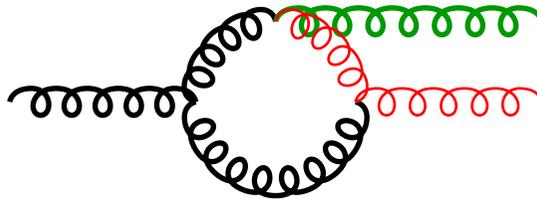


$Q^2 = 10000 \text{ GeV}^2$

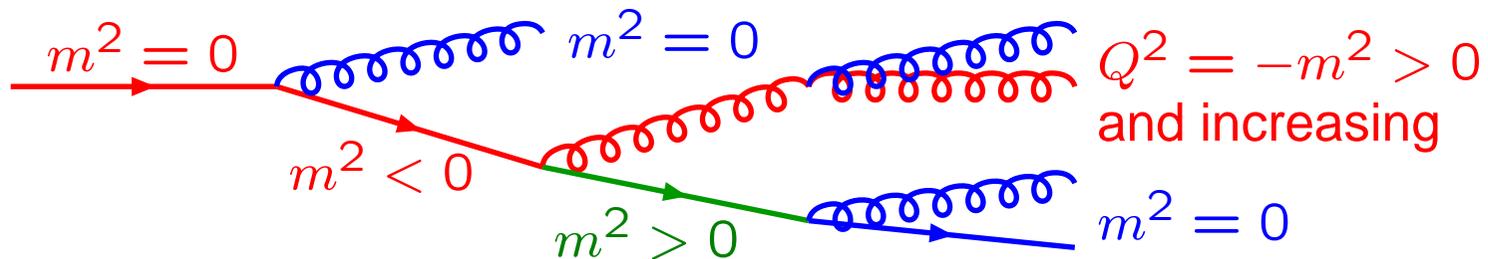


# Initial-State Shower Basics

- Parton cascades in  $p$  are continuously born and recombined.
- Structure at  $Q$  is resolved at a time  $t \sim 1/Q$  *before* collision.
- A hard scattering at  $Q^2$  probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



- Convenient reinterpretation:



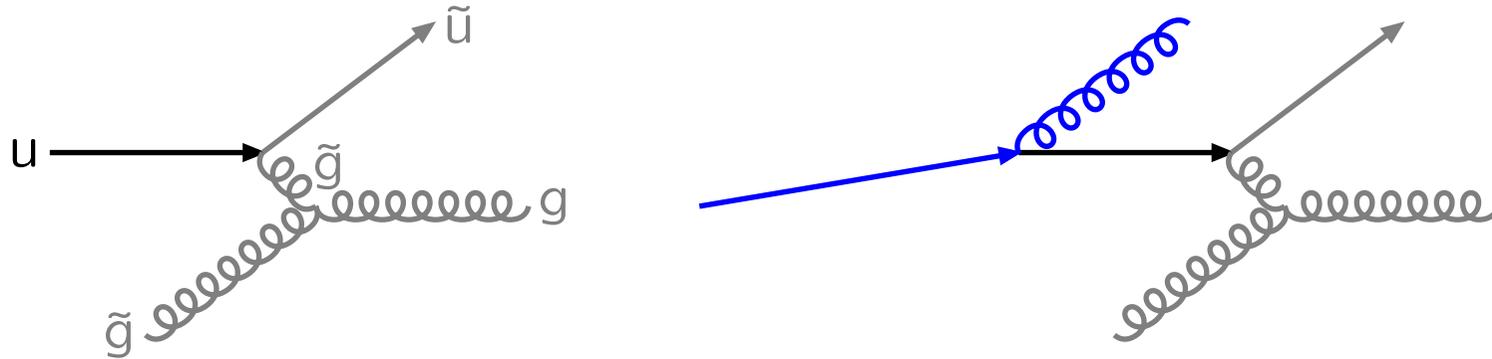
Event generation could be addressed by **forwards evolution**:  
pick a complete partonic set at low  $Q_0$  and evolve, see what happens.

**Inefficient:**

- 1) have to evolve and check for *all* potential collisions, but 99.9...% inert
- 2) impossible to steer the production e.g. of a narrow resonance (Higgs)

# Backwards evolution

**Backwards evolution** is viable and  $\sim$ equivalent alternative:  
start at hard interaction and trace what happened “before”



Monte Carlo approach, based on *conditional probability*: recast

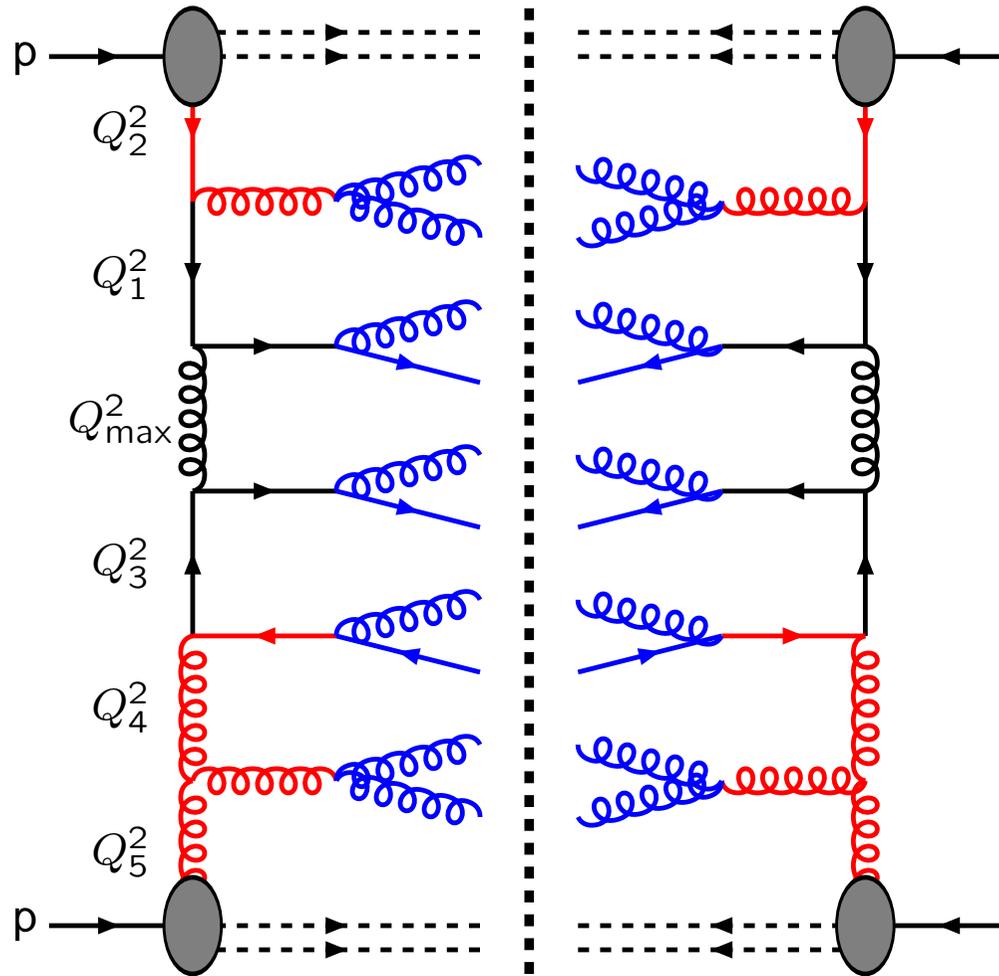
$$\frac{df_b(x, Q^2)}{dt} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z)$$

with  $t = \ln(Q^2/\Lambda^2)$  and  $z = x/x'$  to

$$d\mathcal{P}_b = \frac{df_b}{f_b} = |dt| \sum_a \int dz \frac{x' f_a(x', t)}{x f_b(x, t)} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z)$$

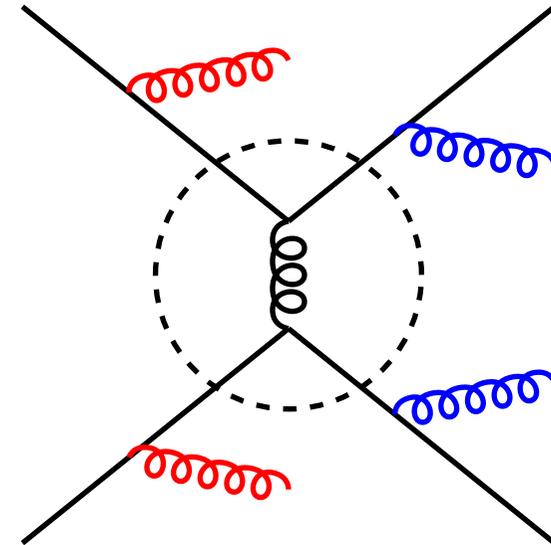
then solve for *decreasing*  $t$ , i.e. backwards in time,  
starting at high  $Q^2$  and moving towards lower,  
with Sudakov form factor  $\exp(-\int d\mathcal{P}_b)$

Ladder representation combines whole event:



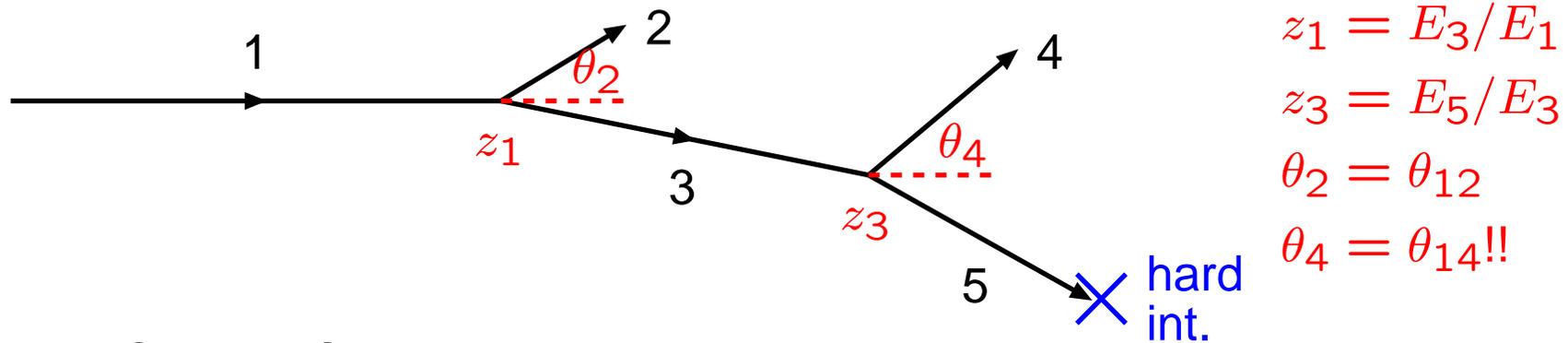
DGLAP:  $Q_{\max}^2 > Q_1^2 > Q_2^2 \sim Q_0^2$   
 $Q_{\max}^2 > Q_3^2 > Q_4^2 > Q_5^2 \sim Q_0^2$

cf. previously:



- One possible Monte Carlo order:
- 1) Hard scattering
  - 2) Initial-state shower  
from center outwards
  - 3) Final-state showers

# Coherence in spacelike showers



with  $Q^2 = -m^2 = \text{spacelike virtuality}$

- kinematics only:

$$Q_3^2 > z_1 Q_1^2, Q_5^2 > z_3 Q_3^2, \dots$$

i.e.  $Q_i^2$  need not even be ordered

- coherence of leading collinear singularities:

$$Q_5^2 > Q_3^2 > Q_1^2, \text{ i.e. } Q^2 \text{ ordered}$$

- coherence of leading soft singularities (more messy):

$$E_3 \theta_4 > E_1 \theta_2, \text{ i.e. } z_1 \theta_4 > \theta_2$$

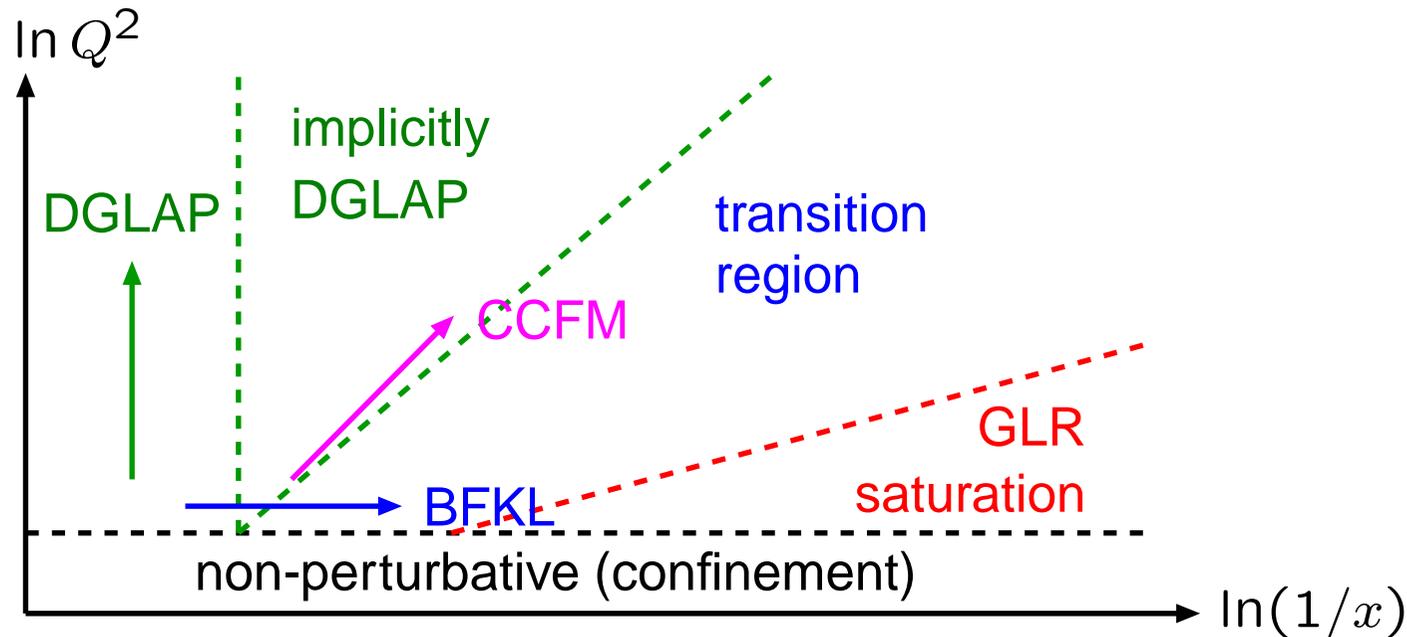
$$z \ll 1: E_1 \theta_2 \approx p_{\perp 2}^2 \approx Q_3^2, E_3 \theta_4 \approx p_{\perp 4}^2 \approx Q_5^2$$

i.e. reduces to  $Q^2$  ordering as above

$$z \approx 1: \theta_4 > \theta_2, \text{ i.e. angular ordering of soft gluons}$$

$\implies$  reduced phase space

# Evolution procedures



**DGLAP:** Dokshitzer–Gribov–Lipatov–Altarelli–Parisi  
evolution towards larger  $Q^2$  and (implicitly) towards smaller  $x$

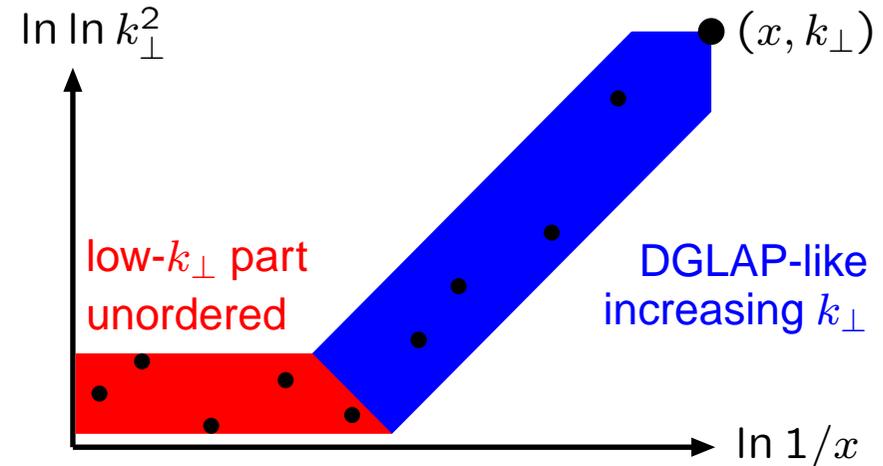
**BFKL:** Balitsky–Fadin–Kuraev–Lipatov  
evolution towards smaller  $x$  (with small, unordered  $Q^2$ )

**CCFM:** Ciafaloni–Catani–Fiorani–Marchesini  
interpolation of DGLAP and BFKL

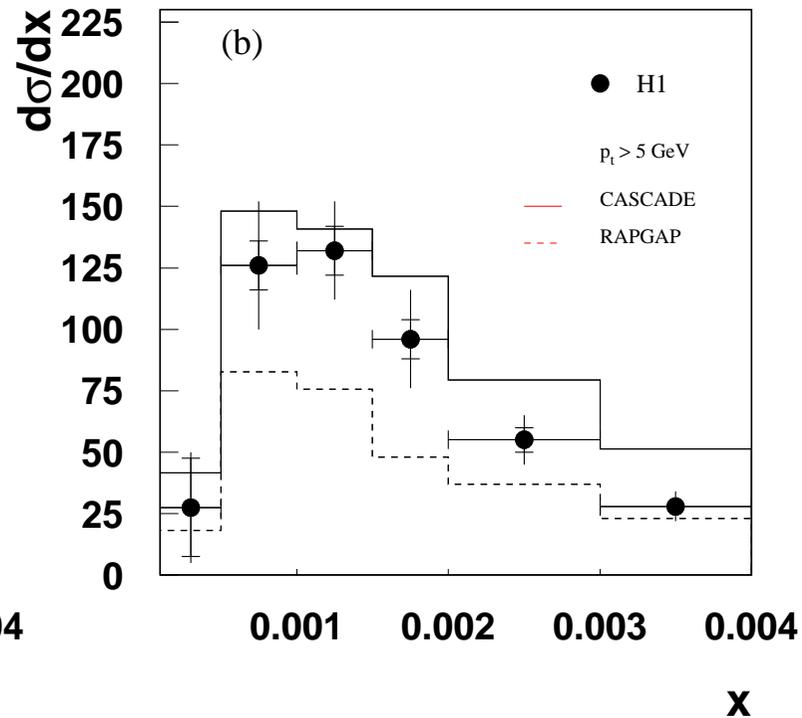
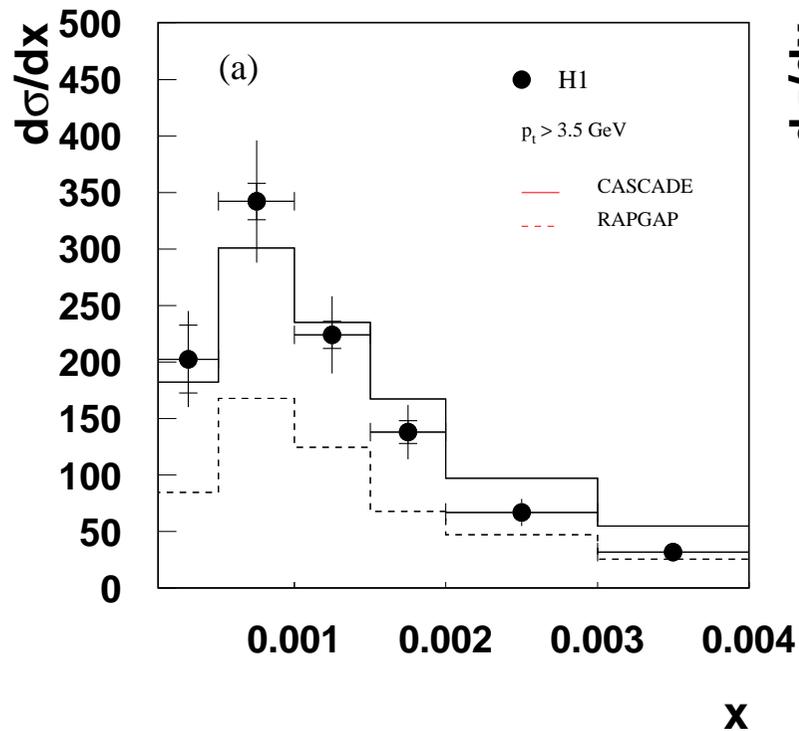
**GLR:** Gribov–Levin–Ryskin  
nonlinear equation in dense-packing (saturation) region,  
where partons recombine, not only branch

# Initial-State Shower Comparison

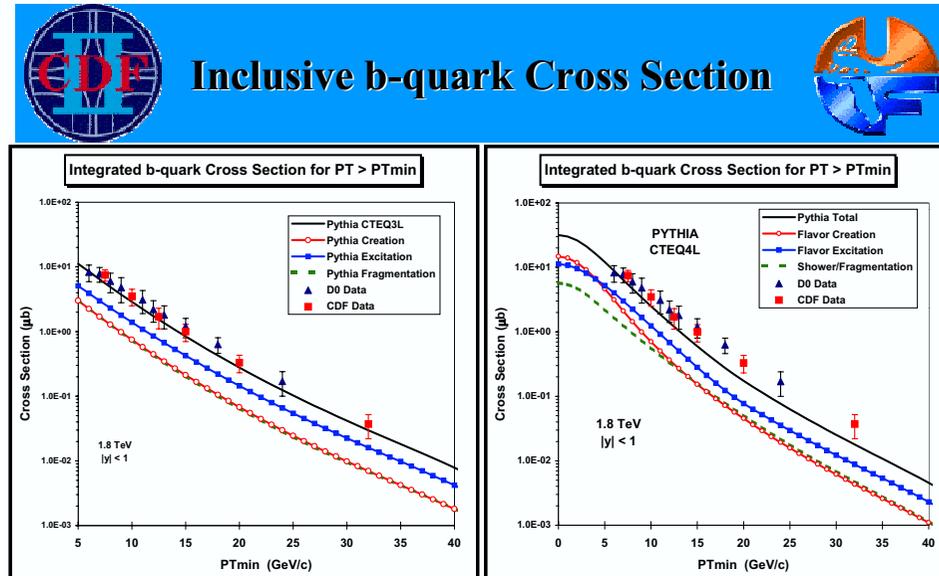
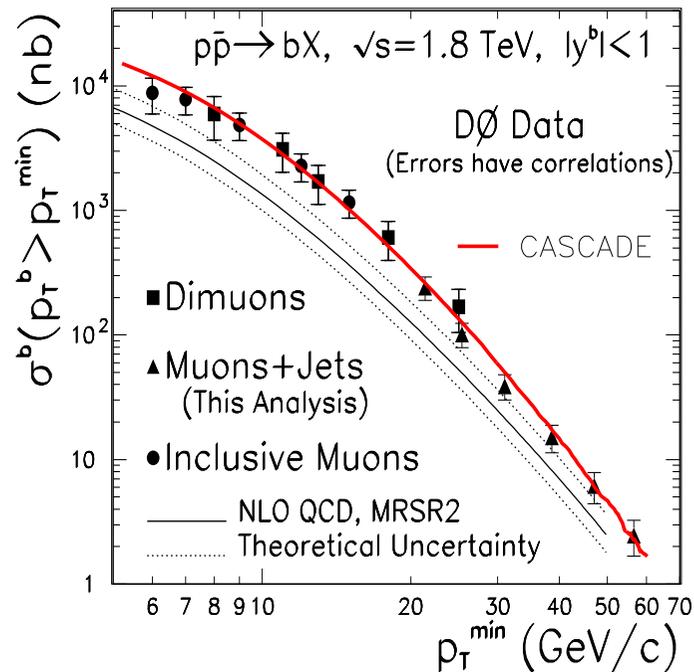
Two(?) CCFM Generators:  
 (SMALLX (Marchesini, Webber))  
 CASCADE (Jung, Salam)  
 LDC (Gustafson, Lönnblad):  
 reformulated initial/final rad.  
 $\implies$  eliminate non-Sudakov



Test 1) forward (= p direction) jet activity at HERA



## 2) Heavy flavour production



➔ Data on the integrated b-quark total cross section ( $P_T > P_{Tmin}$ ,  $|y| < 1$ ) for proton-antiproton collisions at 1.8 TeV compared with the QCD Monte-Carlo model predictions of PYTHIA 6.115 (CTEQ3L) and PYTHIA 6.158 (CTEQ4L). The four curves correspond to the contribution from **flavor creation**, **flavor excitation**, **shower/fragmentation**, and the resulting **total**.

DPF2002  
May 25, 2002

Rick Field - Florida/CDF

Page 5

but also explained by DGLAP with leading order pair creation  
+ flavour excitation ( $\approx$  unordered chains)  
+ gluon splitting (final-state radiation)

CCFM requires off-shell ME's + unintegrated parton densities

$$F(x, Q^2) = \int^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \mathcal{F}(x, k_{\perp}^2) + (\text{suppressed with } k_{\perp}^2 > Q^2)$$

so not ready for prime time in pp

# Initial- vs. final-state showers

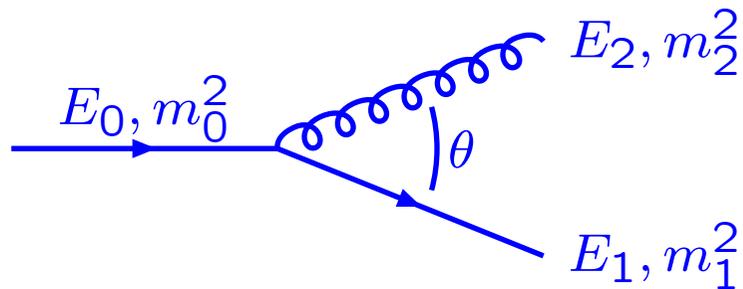
Both controlled by same evolution equations

$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz \cdot \text{(Sudakov)}$$

but

Final-state showers:

$Q^2$  timelike ( $\sim m^2$ )



decreasing  $E, m^2, \theta$

both daughters  $m^2 \geq 0$

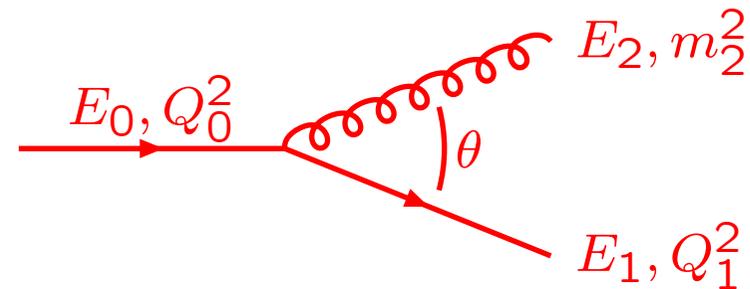
physics relatively simple

$\Rightarrow$  “minor” variations:

$Q^2$ , shower vs. dipole, ...

Initial-state showers:

$Q^2$  spacelike ( $\approx -m^2$ )



decreasing  $E$ , increasing  $Q^2, \theta$

one daughter  $m^2 \geq 0$ , one  $m^2 < 0$

physics more complicated

$\Rightarrow$  more formalisms:

DGLAP, BFKL, CCFM, GLR, ...

# Future of showers

Showers still evolving:

HERWIG has new evolution variable better suited for heavy particles

$$\tilde{q}^2 = \frac{q^2}{z^2(1-z)^2} + \frac{m^2}{z^2} \quad \text{for } q \rightarrow qg$$

Gives smooth coverage of soft-gluon region, no overlapping regions in FSR phase space, but larger dead region.

PYTHIA is moving to transverse-momentum ordered showers (borrowing some of ARIADNE dipole approach, but still showers)

$$p_{\perp \text{evol}}^2 = z(1-z)Q^2 = z(1-z)M^2 \quad \text{for FSR}$$

$$p_{\perp \text{evol}}^2 = (1-z)Q^2 = (1-z)(-M^2) \quad \text{for ISR}$$

Guarantees better coherence for FSR, hopefully also better for ISR.

However, main evolution is *matching to matrix elements*  $\Rightarrow$  tomorrow