The Importance of the TeV Scale

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Lecture 3

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The Standard Model Works

- Any discussion of the Standard Model has to start with its success
- This is unlikely to be an accident!

Issues with the Standard Model

- Unitarity
- Landau pole
- Triviality
- Dependence of Higgs mass on high scale physics

Unitarity

- Consider 2 $\rightarrow$ 2 elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

- a_l are the spin l partial waves

Unitarity

- $P_l(\cos\theta)$ are Legendre polynomials:

$$\int_{-1}^{1} dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$$

$$\sigma = \frac{8\pi}{s} \sum_{l=0}^{\infty} (2l+1) \sum_{l'=0}^{\infty} (2l'+1) a_l a_{l'}^* \int_{-1}^{1} d\cos\theta P_l(\cos\theta) P_{l'}(\cos\theta)$$

$$= \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2$$

Sum of positive definite terms

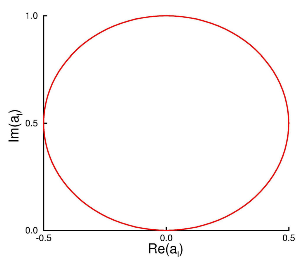
More on Unitarity

- Optical theorem $\sigma = \frac{1}{s}\mathrm{Im}\left[A(\theta=0)\right] = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)\left|a_l\right|^2$

$$\boxed{\mathrm{Im}(a_l) = \left|a_l\right|^2}$$

Optical theorem derived assuming only conservation of probability

- Unitarity requirement:



$$\left|\mathrm{Re}(a_l)\right| \leq \frac{1}{2}$$

More on Unitarity

- Idea: Use unitarity to limit parameters of theory

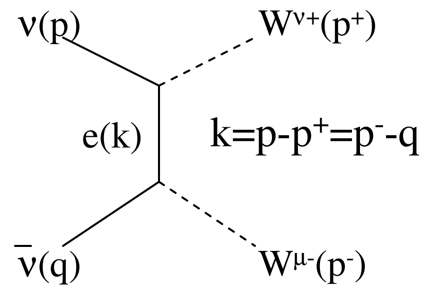
Cross sections which grow with energy always violate unitarity at some energy scale

Consider W^+W^- pair production

Example: $\nu\bar{\nu}\to W^+W^-$

- t-channel amplitude:

$$\boxed{A_t(\nu\bar{\nu}\to W^+W^-)=-i\frac{g^2}{8}\bar{v}(q)\gamma^{\mu}(1-\gamma_5)\frac{k}{k^2}\gamma^{\nu}(1-\gamma_5)u(p)\varepsilon_{\mu}(p^-)\varepsilon_{\nu}(p^+)}$$



- In center-of-mass frame:

$$p=\frac{\sqrt{s}}{2}(1,0,0,1)$$
$$q=\frac{\sqrt{s}}{2}(1,0,0,-1)$$
$$p^+=\frac{\sqrt{s}}{2}(1,0,\beta_W\sin\theta,\beta_W\cos\theta)$$
$$p^-=\frac{\sqrt{s}}{2}(1,0,-\beta_W\sin\theta,-\beta_W\cos\theta)$$

$$s=(p+q)^2$$
$$t=k^2=(p-p')^2$$
$$\beta_W=\sqrt{1-4M_W^2/s}$$

W^+W^- pair production, 2

- Interesting physics is in the longitudinal W sector:

$$\boxed{A_t(\nu\bar{\nu} \to W_L{}^+ W_L{}^-) = i\frac{g^2}{4M_W^2}\bar{v}(q)k(1+\gamma_5)u(p)}$$

$$\boxed{\varepsilon^+ \to \frac{p^+}{M_W} + O\left(\frac{M_W^2}{s}\right)}$$

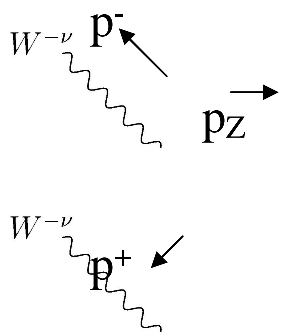
- Use Dirac Equation: $\not{p}u(p)=0$

$$\longrightarrow \quad \boxed{\left|A_t(\nu\bar{\nu} \to W_L{}^+ W_L{}^-)\right|^2 = 2G_F^2 s^2 \sin^2\theta + O\left(\frac{M_W^2}{s}\right)}$$

Grows with energy

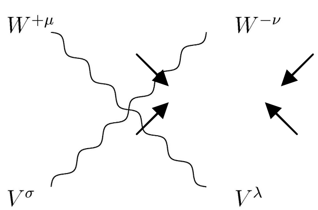
Feynman Rules for Gauge Boson Vertices

$$\boxed{\begin{aligned} g_{WW\gamma} &= e \\ g_{WWZ} &= e\cot\theta_W \end{aligned}}$$



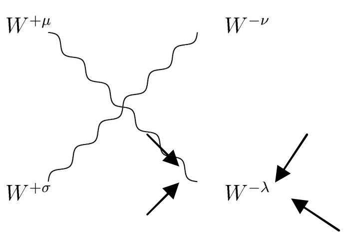
$$ig_{WWV}\, V^{\mu\nu\sigma}(p^+, p^-, p_Z)$$

$$V^{\mu\nu\sigma}(p^+, p^-, p_Z) = (p^+ - p^-)^\sigma g^{\mu\nu} + (p^- - p_Z)^\mu g^{\sigma\nu} + (p_Z - p^+)^\nu g^{\mu\sigma}$$



$$-ig_{WWVV}\left(2g^{\sigma\lambda}g^{\mu\nu} - g^{\sigma\mu}g^{\lambda\nu} - g^{\lambda\mu}g^{\sigma\nu}\right)$$

$$\boxed{\begin{aligned} g_{WW\gamma\gamma} &= e^2 \\ g_{WW\gamma Z} &= e^2\cot\theta_W \\ g_{WWZZ} &= e^2\cot^2\theta_W \end{aligned}}$$



$$ig^2\left(2g^{\sigma\mu}g^{\lambda\nu} - g^{\mu\nu}g^{\lambda\sigma} - g^{\lambda\mu}g^{\sigma\nu}\right)$$

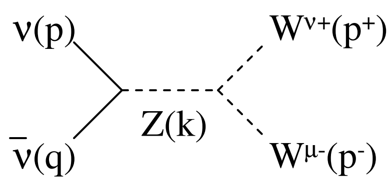
W^+W^- pair production, 3

- SM has additional contribution from s-channel Z exchange

$$A_s(\nu\bar{\nu}\to W^+W^-) = -i\frac{g^2}{4(s-M_Z^2)}\bar{v}(q)\gamma_\mu(1-\gamma_5)u(p)\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{M_Z^2}\right)\left[g_{\lambda\rho}(p^- - p^+)_\nu + g_{\lambda\nu}(p^+ + k)_\rho - g_{\rho\nu}(p^- + k)_\lambda\right]\varepsilon^\lambda(p^+)\varepsilon^\rho(p^-)$$

- For longitudinal W's

$$A_s(\nu\bar{\nu}\to W_L{}^+W_L{}^-) = i\frac{g^2}{4M_W^2}\bar{v}(q)(p^+ - p^-)(1-\gamma_5)u(p)$$



$$A_s(\nu\bar{\nu}\to W_L{}^+W_L{}^-) = -i\frac{g^2}{4M_W^2}\bar{v}(q)k(1+\gamma_5)u(p)$$

Contributions which grow with energy cancel between t- and s-channel diagrams

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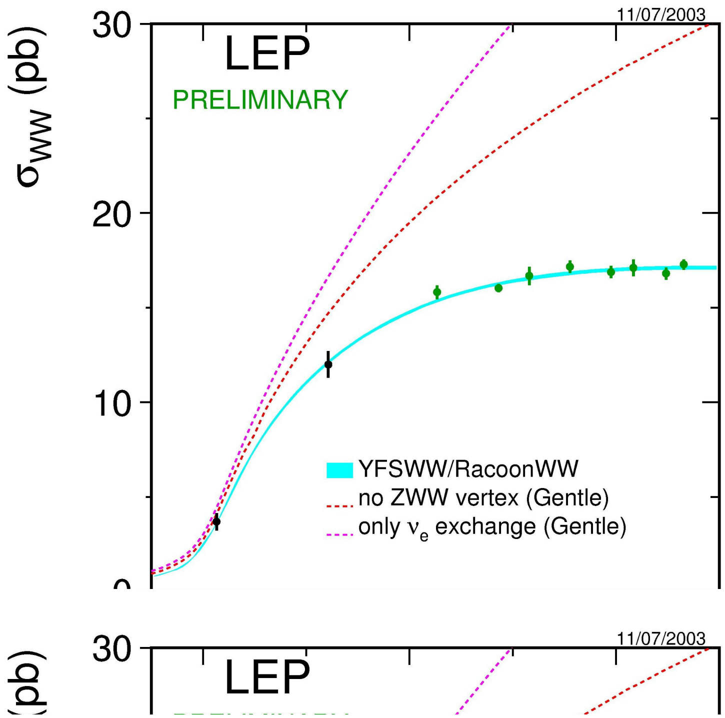
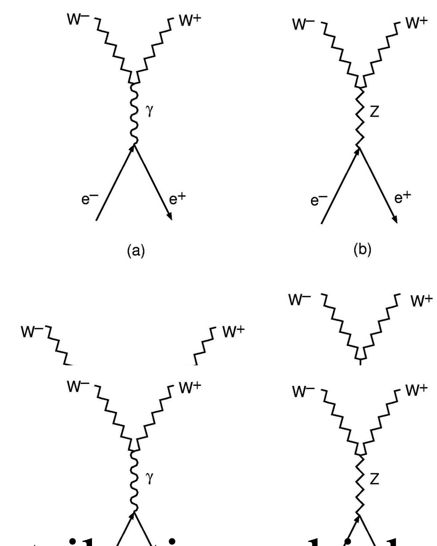
Depends on special form of 3-gauge boson couplings

No deviations from SM at LEP2



LEP EWWG, hep-ex/0312023

No evidence for Non-SM
3 gauge boson vertices



Contribution which grows like $m_e^2 s$ cancels between Higgs diagram and others

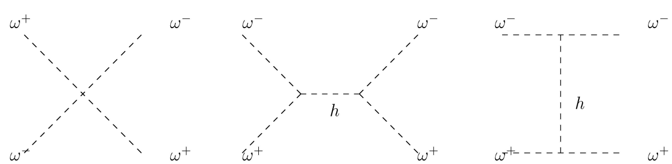
Example: $W^+W^- \rightarrow W^+W^-$

- Recall scalar potential (Include Goldstone Bosons)

$$V = \frac{M_h^2}{2} h^2 + \frac{M_h^2}{2v} h\left(h^2 + z^2 + 2\omega^+\omega^-\right) + \frac{M_h^2}{8v^2}\left(h^2 + z^2 + 2\omega^+\omega^-\right)^2$$

- Consider Goldstone boson scattering:

$\omega^+\omega^- \rightarrow \omega^+\omega$



$$iA(\omega^+\omega \rightarrow \omega^+\omega^-) = -2i\frac{M_h^2}{v^2} + \left(-i\frac{M_h^2}{v}\right)^2 \frac{i}{t - M_h^2} + \left(-i\frac{M_h^2}{v}\right)^2 \frac{i}{s - M_h^2}$$

$\omega^+\omega^-\rightarrow\omega^+\omega^-$

- Two interesting limits:
  - s, t >> M_h^2

$$A(\omega^+\omega^- \rightarrow \omega^+\omega^-) \rightarrow -2\frac{M_h^2}{v^2}$$

$$\boxed{a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}}$$

  - s, t << M_h^2

$$A(\omega^+\omega^- \rightarrow \omega^+\omega^-) \rightarrow -\frac{u}{v^2}$$

$$\boxed{a_0^0 \rightarrow -\frac{s}{32\pi v^2}}$$

Use Unitarity to Bound Higgs

$$\left|\mathrm{Re}(a_l)\right| \leq \frac{1}{2}$$

- High energy limit:

$$\boxed{a_0^0 \to -\frac{M_h^2}{8\pi v^2}}$$

$\boxed{M_h < 800 \text{ GeV}}$

- Heavy Higgs limit

$$\boxed{a_0^0 \to -\frac{s}{32\pi v^2}}$$

$E_c \sim 1.7$ TeV

$\rightarrow$ New physics at the TeV scale

Can get more stringent bound from coupled channel analysis

Electroweak Equivalence Theorem

$$A(V_L^1 \ldots V_L^N \to V_L^1 \ldots V_L^{N'}) = (i)^N (-i)^{N'} A(\omega_1 \ldots \omega_N \to \omega_1 \ldots \omega_{N'})$$
$$+ O\left(\frac{M_W^2}{E^2}\right)$$

This is a statement about scattering amplitudes, NOT individual Feynman diagrams

Landau Pole

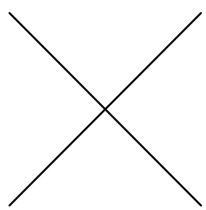
- M_h is a free parameter in the Standard Model
- Can we derive limits on the basis of consistency?
- Consider a scalar potential:

$$V = \frac{M_h^2}{2} h^2 + \frac{\lambda}{4} h^4$$

- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way

Consider $hh \rightarrow hh$

- Real scattering, $s+t+u=4M_h^2$
- Consider momentum space-like and off-shell: $s=t=u=Q^2<0$
- Tree level: $iA_0=-6i\lambda$



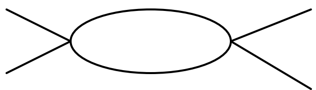
$bb \rightarrow bb$, #2

- One loop:

$$iA_s = (-6i\lambda)^2 \frac{1}{2} \int \frac{d^n k}{(2\pi)^2} \frac{i}{k^2 - M_h^2} \frac{i}{(k+p+q)^2 - M_h^2}$$

$$= \frac{9\lambda^2}{8\pi^2} (4\pi\mu^2) \Gamma(\varepsilon) \left(M_h^2 - Q^2 x(1-x) \right)^{-\varepsilon}$$

- $A = A_0 + A_s + A_t + A_u$



$$A = -6\lambda \left(1 + \frac{9\lambda}{16\pi^2} (4\pi\mu^2) \Gamma(\varepsilon) \left(M_h^2 - Q^2 x(1-x) \right)^{-\varepsilon} + \ldots \right)$$

$hh \rightarrow hh$, #3

- Sum the geometric series to define running coupling

$$A = -6\lambda\left(1 + \frac{9\lambda}{16\pi^2}\log\frac{Q^2}{M_h^2}\right) + \ldots$$

$$A = \frac{6\lambda}{1 - \frac{9\lambda}{8\pi^2}\log\left(\frac{Q}{M_h}\right)} \equiv 6\lambda(Q)$$

- $\lambda(Q)$ blows up as $Q \rightarrow \infty$ (called Landau pole)

$bb \rightarrow bb$, #4

- This is independent of starting point
- BUT…. Without $\lambda\phi^4$ interactions, theory is non-interacting
- Require quartic coupling be finite

$$\frac{1}{\lambda(Q)} > 0$$

$hh \rightarrow hh$, #5

- Use $\lambda = M_h^2/(2v^2)$ and approximate $\log(Q/M_h) \rightarrow \log(Q/v)$
- Requirement for $1/\lambda(Q)>0$ gives upper limit on M_h

$$M_h^2 < \frac{32\pi^2 v^2}{9\log\left(\frac{Q^2}{v^2}\right)}$$

- Assume theory is valid to 10^{16} GeV
  - Gives upper limit on $M_h < 180$ GeV
- Can add fermions, gauge bosons, etc.

High Energy Behavior of λ

- Renormalization group scaling $\frac{1}{\lambda(Q)} = \frac{1}{\lambda(\mu)} + (\ldots)\log\left(\frac{Q}{\mu}\right)$

$$\boxed{16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12 g_t^4 + (gauge)}$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right) \qquad g_t = \frac{M_t}{v}$$

- *Large* λ *(Heavy Higgs):* self coupling causes λ to grow with scale
- *Small* λ *(Light Higgs):* coupling to top quark causes λ to become negative

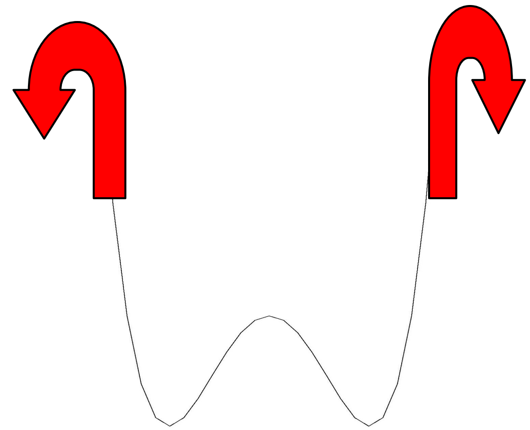
Does Spontaneous Symmetry Breaking Happen?

- SM requires spontaneous symmetry
- This requires $V(v) < V(0)$
- For small λ

$$\boxed{16\pi^2 \frac{d\lambda}{dt} \approx -16 g_t^4}$$

- Solve

$$\boxed{\lambda(\Lambda) \approx \lambda(v) - \frac{3g_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)}$$



Does Spontaneous Symmetry Breaking Happen? (#2)

- $\lambda(\Lambda) > 0$ gives lower bound on M_h

$$\boxed{M_h^2 > \frac{3v^2}{2\pi^2}\log\left(\frac{\Lambda^2}{v^2}\right)}$$

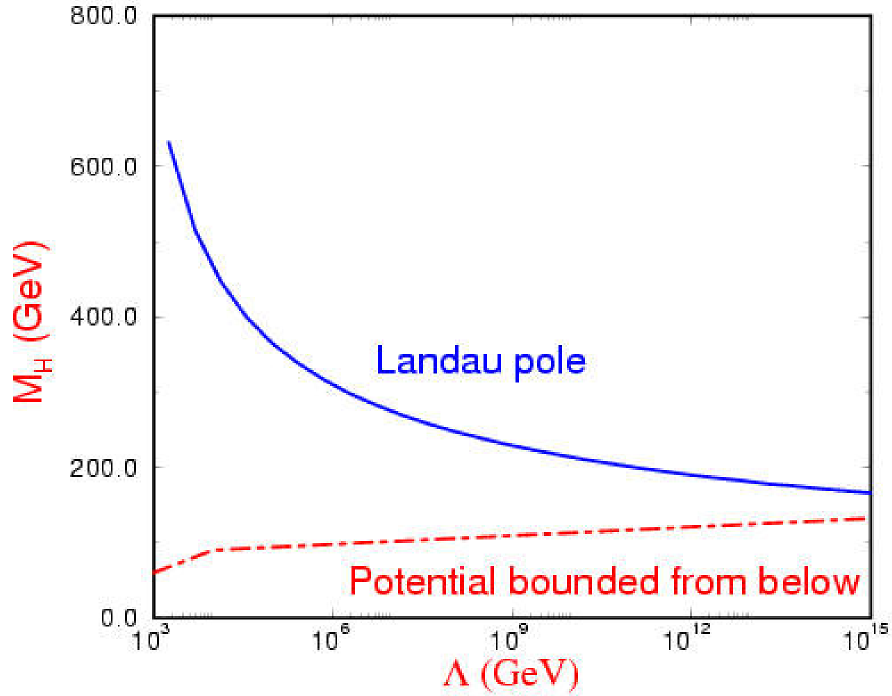
- If Standard Model valid to 10^{16} GeV

$$\boxed{M_h > 130\, GeV}$$

- For any given scale, Λ, there is a theoretically consistent range for M_h

Bounds on SM Higgs Boson

- If SM valid up to Planck scale, only a small range of allowed Higgs Masses



More Problems

- We often say that the SM cannot be the entire story because of the quadratic divergences of the Higgs Boson mass

Masses at one-loop

- First consider a fermion coupled to a massive complex Higgs scalar

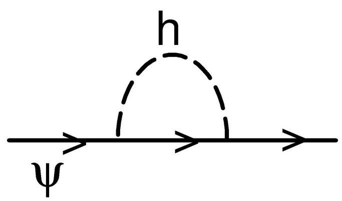
$$L = \overline{\Psi}(i\partial)\Psi + \left|\partial_\mu \phi\right|^2 - m_s\left|\phi\right|^2 - \left(\lambda_F \overline{\Psi}_L \Psi_R \phi + h.c.\right)$$

- Assume symmetry breaking as in SM:

$$\phi = \frac{(h+v)}{\sqrt{2}} \qquad\qquad m_F = \frac{\lambda_F v}{\sqrt{2}}$$

Masses at one-loop, #2

- Calculate mass renormalization for Ψ

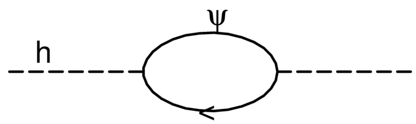


$$\delta m_F = -\frac{3\lambda_F^2 m_F}{32\pi^2}\log\left(\frac{\Lambda^2}{m_F^2}\right) + \ldots..$$

Symmetry and the fermion mass

- $\delta m_F \approx m_F$
  - m_F=0, then quantum corrections vanish
  - When m_F=0, Lagrangian is invariant under
    - $\Psi_L \rightarrow e^{i\theta L} \Psi_L$
    - $\Psi_R \rightarrow e^{i\theta R} \Psi_R$
  - $m_F \rightarrow 0$ increases the symmetry of the threoy
  - Yukawa coupling (proportional to mass) breaks symmetry and so corrections $\approx m_F$

Scalars are very different



$$\delta M_h^2 = \Sigma_S(m_s^2) = -\frac{\lambda_F^2\Lambda^2}{8\pi^2} + \left(m_s^2 - m_F^2\right)\log\left(\frac{\Lambda}{m_F}\right)$$

$$+(2m_F^2 - \frac{m_s^2}{2})\left(1 + I_1\left(\frac{m_s^2}{m_F^2}\right)\right) + O\left(\frac{1}{\Lambda^2}\right)$$

$$I_1(a) = \int_0^1 dx \;\; \log\left(1 - ax(1-x)\right)$$

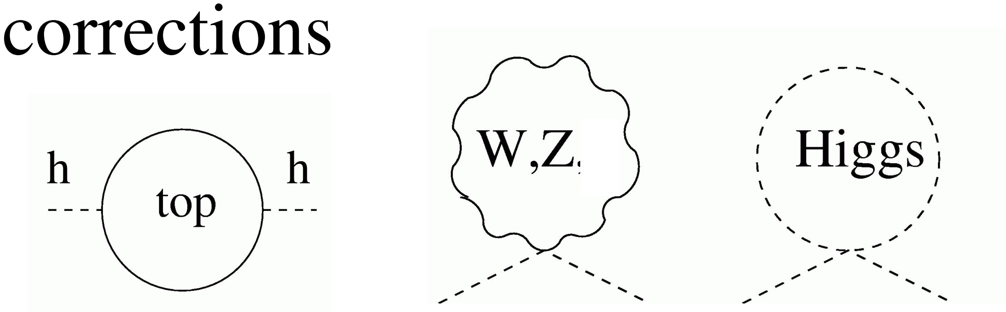
- M_h diverges quadratically!
- This implies quadratic sensitivity to high mass scales

Scalars (#2)

- M_h diverges quadratically!
- Requires large cancellations (hierarchy problem)
- Can do this in Quantum Field Theory
- h does not obey decoupling theorem
  - Says that effects of heavy particles decouple as $M \rightarrow \infty$
- $M_h \rightarrow 0$ doesn't increase symmetry of theory
  - Nothing protects Higgs mass from large corrections

Light Scalars are Unnatural

- Higgs mass grows with scale of new physics, Λ
- No additional symmetry for M_h=0, no protection from large corrections



$$\delta M_h^2 = \frac{G_F}{4\sqrt{2}\pi^2}\Lambda^2\left(6M_W^2 + 3M_Z^2 + M_h^2 - 12M_t^2\right)$$

$$= -\left(\frac{\Lambda}{0.7\,\mathrm{TeV}}\,200\,\mathrm{GeV}\right)^2$$

$M_h \leq 200$ GeV requires large cancellations

What's the problem?

- Compute M_h in dimensional regularization and absorb infinities into definition of M_h

$$M_h^2 = M_{h0}^2 + \frac{1}{\varepsilon}(\ldots)$$

- Perfectly valid approach
- Except we know there is a high scale

Try to cancel quadratic divergences by adding new particles

- SUSY models add scalars with same quantum numbers as fermions, but different spin
- Little Higgs models cancel quadratic divergences with new particles with same spin

We expect something at the TeV scale

- If it's a SM Higgs then we have to think hard about what the quadratic divergences are telling us
- SM Higgs mass is highly restricted by requirement of theoretical consistency