

# [ Theories to resolve hierarchy ]

Despite the worry that maybe we have it all wrong, I must still bow to the 25 years of theory that have been predicated on the idea that the standard model must be changed at or around 1 TeV

I will briefly touch on the principles behind

- Extra Dimensions
- Technicolor
- Little Higgs Models
- Supersymmetry

and say a few words about their phenomenology

# Extra Dimensions

$$v / M_{pl} \sim 10^{-16}$$

- Perhaps the problem with the hierarchy is that we are using the wrong  $M_{pl}$
- Perhaps four-dimensional quantum field theory breaks down at the TeV scale.
- *If there are extra dimensions, the  $M_{pl}$  we measure is not the “fundamental” one of the full  $D+1 = (3+1) + (D-3 \text{ extra})$  dimensional theory.*

Einstein Lagrangian in  $D+1$  dimensions

Volume of extra dimensions

$$\begin{aligned}
 M_{pl,D}^{D-2} \int d^{D+1}x \sqrt{g_{D+1}} R &= M_{pl,D}^{D-2} \int d^{3+1}x \sqrt{g_{3+1}} R \int d^{D-3}x \sqrt{g_{D-3}} \\
 &= M_{pl,D}^{D-2} V_{D-3} \int d^{3+1}x \sqrt{g_{3+1}} R
 \end{aligned}$$

Determinant of metric

Scalar Curvature

$$M_{pl,3}^2 = M_{pl,D}^{D-2} V_{D-3}$$

# Possible scenarios

$$M_{pl,3}^2 = M_{pl,D}^{D-2} V_{D-3}$$

Thus the measured  $M_{pl,3}$  may be large because

- The  $M_{pl,D}$  dimensional Planck scale is at  $10^{18}$  GeV and the extra dimensional volume is around the same scale
- The  $M_{pl,D}$  Planck scale is of order 1 TeV, and the extra-dimensional volume is very large, of order  $10^{30}$  GeV<sup>(3-D)</sup>
- Some combination of both; perhaps the Planck scale is at 100 TeV, out of immediate reach

What would happen if the second case were true?

A big volume means low energy effects --- so why haven't we seen evidence for them already?

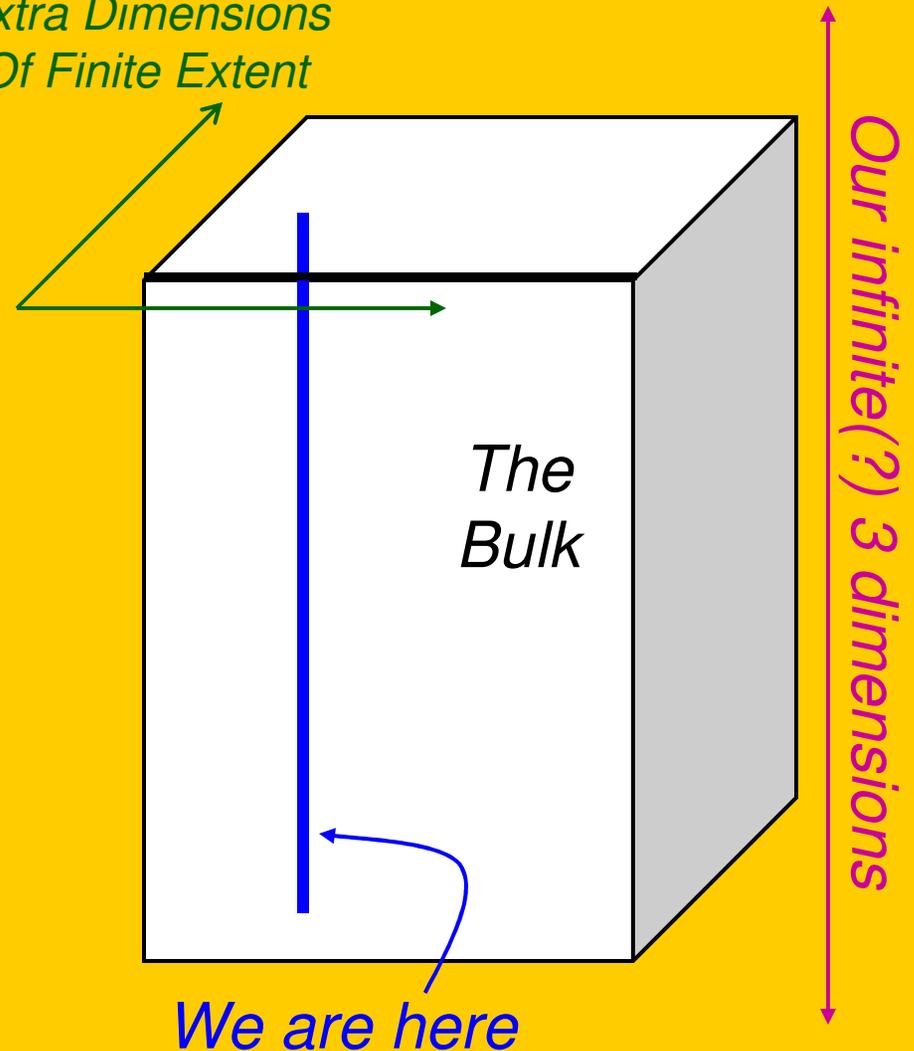
# [ How to allow $M_{pl} = \text{TeV}$ ]

- A simple scenario: we're trapped on a 3+1 dimensional subspace in the extra dimensions. ('Brane')
- The only way to find out about the extra dimensions that are larger than .01 fm is through gravity itself
- If extra bulk dimensions have length  $L$ , gravity will look 3+1 dimensional at distances larger than  $L$
- At shorter distances, Newton's law changes
- But most collider physics does not change until energy reaches  $\sim M_{pl}$ .

## Analogy:

*electrons trapped in a wire inside of a waveguide*

*Extra Dimensions  
Of Finite Extent*



# [ Phenomena: $M_{pl} = \text{TeV}$ ]

- As scattering energy increases, easier to radiate higher-dimensional graviton

$$q \bar{q} \rightarrow \gamma + \text{invisible graviton} \quad , \quad g g \rightarrow g + \text{invisible graviton}$$

- Gravitons in s-channel, either as many weakly-coupled light particles or as isolated strong resonances, can affect

$$q \bar{q} \rightarrow \mu^+ \mu^- \quad , \quad \text{etc.}$$

- At very high energies, could even see non-linear **quantum gravitational** processes, including black holes or even excited strings from string theory

$$q q \rightarrow \text{boom}$$

multiple particles, crudely thermal spectrum with  $T \sim M_{pl}$  , or  $M_{string}$

CAUTION: Statements in literature that gravity is **universal**, and leptons and photons will be produced abundantly along with quarks and gluons by decaying strings or black holes, are *only true at ultra-high energies* ; they need not be the case at LHC!

# [ Second case: $R_{extra} = \text{TeV}$ ]

- Suppose  $M_{pl}$  somewhat higher, so we may not see gravitational effects, but...
- There are new TeV-scale dimensions in which not all SM fields are points
- Those fields in the extra dimensions have a new feature: Kaluza-Klein states ('KK modes') with masses  $\sim 1/R_{extra}$
- The principle behind KK modes is similar to harmonics on violin string

String modes labelled  $n$ , frequency  $\omega_n = n \omega_1$ , wave function  $\sin [\pi n y/L]$

$$\phi(y) = \sum_n \alpha_n \sin [\pi n y/L] \quad n > 0$$

Higher modes have higher energy.

Better analogy:  
KK modes are more like  
the modes of photons  
in a wave guide  
See Jackson E+M book

# Field in extra dimensions

- Massless scalar field in 4+1 dimensions (with **periodic** extra dimension  $y$ ) has similar mode expansion

$$\phi(x, y) = \sum_n \phi_n(x) \cos [2\pi n y / L] \quad n \text{ any integer \quad [sin also]}$$

- There are an infinite number of 3+1 dimensional scalar fields, with masses  $m_n = n/L$  (and  $m_0 = 0$ , the massless 3+1 dimensional scalar)
- Similar tower of states for gluon (or photon or top quark or Higgs or graviton) in 4+1; a massless ordinary gluon plus a tower of KK gluons of increasing mass,  $m_n = n m_1$ .

*But equal spacing of KK mode masses is not a robust prediction, nor is equal spin*

- Not equally spaced:
  - suppose 2 extra dimensions of length  $R_1, R_2$ ; modes specified by integers  $n_1, n_2$ ; masses are

$$\sqrt{n_1^2 / R_1^2 + n_2^2 / R_2^2}$$

- Not necessarily same-spin;
  - gluon in 5+1 dims  $\rightarrow$  tower of vectors  $(A_0, A_1, A_2, A_3)$  and scalars  $(A_4, A_5)$  in 3+1 dims
  - A spin-1/2 fermion in 3+1 dims can descend from a higher-spin fermion in higher dims

# Spectrum not predicted

- Suppose dimensions are not uniform
  - Consider nonuniform violin string
  - Can get any spectrum, depending on density profile
  - Only prediction is a tower of states
  - Nonuniform extra dimensions have similar possibilities

Better analogy:

A wave guide of arbitrary cross  
can have a nearly arbitrary  
spectrum of photon modes

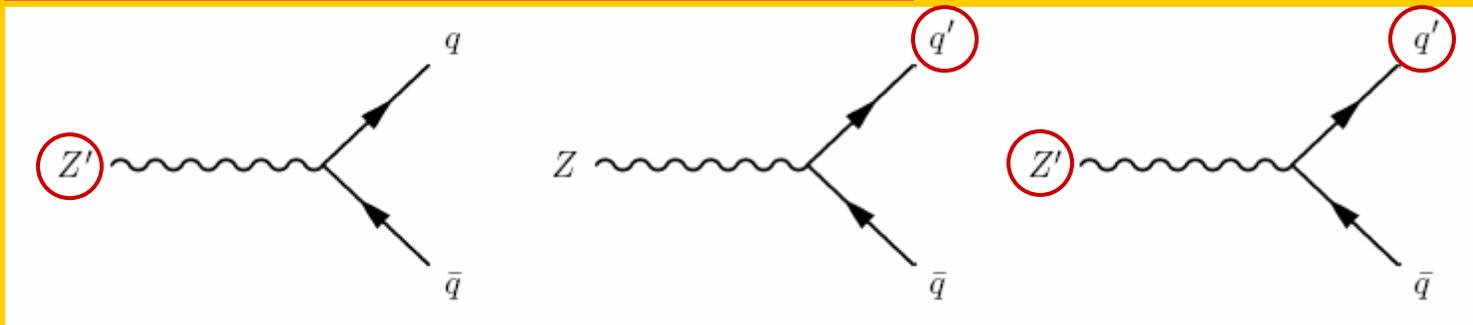
- One fairly reliable prediction: towers of repeaters ---  
including new particles of the same color, electric charge, weak isospin, possibly spin, but higher mass
- In many models, repeaters for  $W$ ,  $Z$ ,  $h$ ,  $t$  ( $b$ ).
- Repeater for the gluon, graviton, other matter etc. possible too.

“Universal Extra Dimensions”: all SM particles have a tower with similar splittings

# Physics of Repeaters

- Production modes:
  - Consider any  $2 \rightarrow 1$  SM process, put a repeater in the final state
  - Consider any SM  $2 \rightarrow 2$  production process, put one or two repeaters in final state
- Decay modes: Consider any  $1 \rightarrow 2$  SM process, put a repeater in the initial state
- Couplings *will not be identical* to those of SM,
  - typically some are of similar order while others are highly suppressed.

## Repeater interactions from the $Zq\bar{q}$ interaction

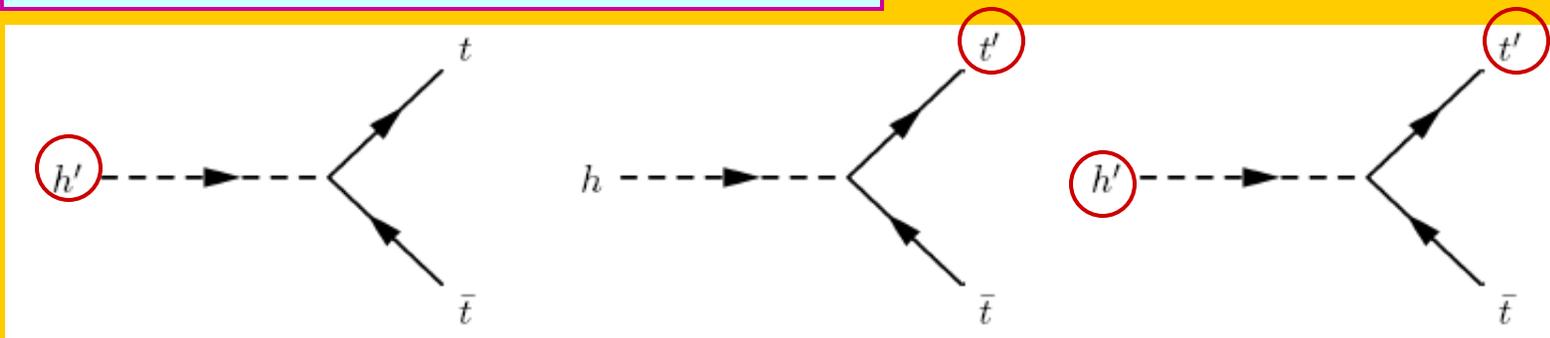


*but these couplings may only be large for some quarks and not others; likely either to be flavor-universal or heavy-flavor-weighted to avoid big flavor-changing neutral currents*

# Physics of Repeaters

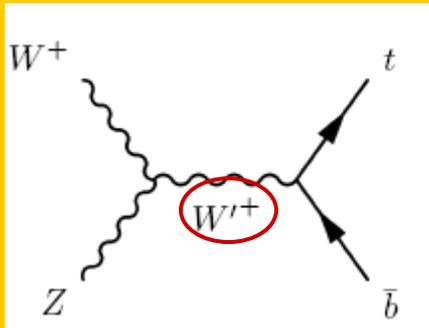
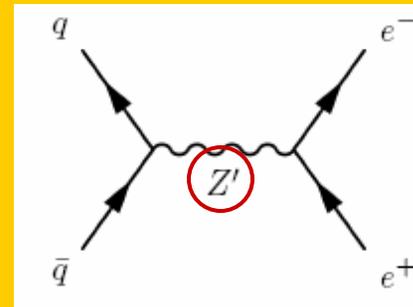
- Production modes:
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- Couplings *will not be identical* to those of SM,
  - typically some are of similar order while others are highly suppressed.

Repeater interactions from the  $h\bar{t}t$  interaction



# [ A few examples ]

If the couplings of a  $Z'$  are flavor universal, we may produce it directly via  $q\bar{q}$ , see it as lepton-pair resonance

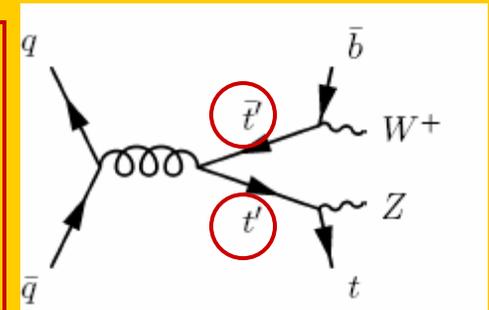


If the couplings of a  $W'$  are heavy-flavor weighted, we may produce it in Vector Boson Fusion, see it in heavy flavor or diboson final states (resonance in 4 jets with  $b$ -tags, 2 leptons + 2 jets)

If a  $t'$  repeater has a small coupling to gluon plus top, it may be mainly pair-produced and decay to  $Wb$ ,  $Zt$ ,  $ht$ . Up to 10 jets!

But large energy release, so  $W, Z, t$  are boosted; look for four fat jets with substructure and two  $b$  tags, or three fat jets and a lepton near a  $b$ , etc.

Two equal-mass resonances to reconstruct.



# [ Back to Hierarchy Problem ]

- **Maybe the hierarchy is due to nonperturbative strongly-interacting dynamics in quantum field theory**
- Not all mass scales have a hierarchy problem.
- QCD has taught us a great deal
- Review
  - Abelian Higgs Model
  - Standard Model Higgs Model
- A third Higgs-like model: Chiral symmetry breaking in QCD.
- Using this last model to break electroweak symmetry
  - ➔ **technicolor – solution to the hierarchy problem – problematic**
- Generalizing this model and playing some games
  - ➔ **little Higgs –mitigation of the hierarchy problem – less problematic**

# [ No Hierarchy Problem in QCD ]

If  $v / M_{pl} \ll 1$  is a “problem”, why isn’t  $\Lambda_{QCD} / M_{pl} \ll 1$  a “problem”?

- $\Lambda_{QCD}$  doesn’t appear in Lagrangian;
- What appears is  $\alpha_s$ , but unless  $\alpha_s(M) \sim 1$ ,  $\Lambda_{QCD} \ll M$
- Thus it is easy to obtain  $\Lambda_{QCD} \ll M$ ! the *opposite* of the situation with  $m_h$
- **There is no mystery why the rho meson (770 MeV) is light.**
- *Can we get a composite Higgs boson created through new strong forces? With a mass that is light for the same reason that the rho meson is light?*

$$\Lambda_{QCD} \sim M e^{-2\pi/b\alpha_s(M)}$$

One-loop beta  
function coefficient  
 $\sim 5$  in QCD

# [ Abelian Higgs ]

Abelian Higgs model: U(1) global symmetry or gauge ‘symmetry’

- Complex scalar  $\phi$ , potential  $V(|\phi|)$
- Minimum at  $\phi = v e^{i\alpha}$  ( any angle  $\alpha$  )
- This breaks the U(1), since under U(1) rotation  $\alpha$  would shift.
- If U(1) symmetry is *global*, fluctuations of  $\alpha$  are a massless Goldstone boson
- If U(1) symmetry is *gauged*, fluctuations of  $\alpha$  are absorbed by photon, which becomes massive
- In either case, fluctuations of  $|\phi|$  around  $v$ , the “Higgs boson”, are a physical massive particle

# Standard Higgs

Standard Higgs model SU(2) x U(1) global symmetry or gauge 'symmetry'

- Complex doublet scalar  $\phi$ , potential  $V(|\phi^\dagger\phi|)$
- Minimum at  $\phi = (v \sin \theta e^{i\alpha}, v \cos \theta e^{i\beta})$  ( any angle  $\alpha, \beta, \theta$  )
- This breaks the U(1), since under U(1) rotation  $\alpha, \beta$  would shift.
- This breaks the SU(2), since under SU(2) rotation  $\alpha, \beta, \theta$  would shift.
- But a diagonal U(1) is unbroken:

- to see this, first rotate vev to

$$\phi = (0, v)$$

Then clearly the following rotation leaves the vev  $\langle\phi\rangle$  unchanged.

first rotate by SU(2):  $(\phi^+, \phi^0) \rightarrow (e^{i\eta} \phi^+, e^{-i\eta} \phi^0)$

then rotate by U(1):  $(\phi^+, \phi^0) \rightarrow (e^{i\eta} \phi^+, e^{i\eta} \phi^0)$

which altogether is  $(\phi^+, \phi^0) \rightarrow (e^{2i\eta} \phi^+, \phi^0)$

- If SU(2) x U(1) symmetry is *global*, fluctuations of  $\alpha, \beta, \theta$  are massless Goldstone bosons: three pions.
- If SU(2) x U(1) symmetry is *gauged*, fluctuations of  $\alpha, \beta, \theta$  are absorbed by three of the four gauge bosons, which become massive W,Z, and a massless photon remains.
- In either case, fluctuations of  $|\phi|$  around  $v$ , the "Higgs boson", are a physical massive particle

# [ Chiral Symmetry Breaking ]

In QCD, and technicolor, these ideas are generalized; the field  $\phi$  is composite, and its vev is generated by complicated quantum effects.

- In QCD with  $N_f$  massless quarks, the theory has a  $SU(N_f)_L \times SU(N_f)_R$  chiral symmetry
- This chiral symmetry is spontaneously broken at low energy because of strong nonperturbative dynamics
- A *composite* operator, a quark-antiquark bilinear  $\phi = \bar{q}_i q_j$ , an  $N_f \times N_f$  matrix scalar

develops a vev proportional to the unit matrix

$$\text{for two flavors } \phi = \begin{bmatrix} \bar{u}u & \bar{u}d \\ \bar{d}u & \bar{d}d \end{bmatrix}$$

- It thereby breaks the  $SU(N_f)_L \times SU(N_f)_R$  chiral symmetry to the diagonal  $SU(N_f)_{L+R}$ , generating  $N_f^2 - 1$  massless Goldstone bosons.
- Real QCD has quarks with small masses,  $m_u, m_d \ll m_s < \Lambda_{QCD}$ , but the fact that
  - $2^2 - 1 = 3$  particles must be massless when  $m_u, m_d \rightarrow 0$ , and
  - $3^2 - 1 = 8$  particles must be massless when  $m_u, m_d, m_s \rightarrow 0$

ensures that the  $\pi$ s,  $K$ s,  $\eta$  are all light, with the  $\pi$ s the lightest of all.

# QCD-based Higgs-like Model

$N_f = 2$  QCD model:  $SU(2)_L \times SU(2)_R$  global symmetry or gauge 'symmetry'

- **2 x 2 matrix scalar**  $\phi = \begin{bmatrix} \bar{u}u & \bar{u}d \\ \bar{d}u & \bar{d}d \end{bmatrix}$
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- This vev preserves the *diagonal* subgroup  $SU(2)_{L+R}$ , with rotations  $\mathbf{U}_R = \mathbf{U}_L^{-1}$
- If  $SU(2) \times SU(2)$  symmetry is *global*, fluctuations of  $\mathbf{U}_R = \mathbf{U}_L$ , broken combination of the  $SU(2)$ 's, are massless Goldstone bosons – three pions
- If  $SU(2) \times U(1)$  symmetry is *gauged*, fluctuations  $\mathbf{U}_R = \mathbf{U}_L$  are absorbed by three of the four gauge bosons, which become massive  $W, Z$ , and a massless photon remains. *Nothing remains of the rest of  $SU(2)_R$ ; it is not a symmetry.*
  
- In all cases, fluctuations of  $|\phi|$  around  $v$ , the “Higgs boson”, are a physical massive particle, known in QCD as the  $\sigma$  or the  $f_0$ ; (very wide in QCD because  $\sigma \rightarrow \pi\pi$  very rapidly.)

# [ QCD-based Higgs-like Mo

If the quarks aren't quite massless, the  $SU(2)_L \times SU(2)_R$  symmetry isn't exact, and the pions aren't quite massless either

$N_f = 2$  QCD model:  $SU(2)_L \times SU(2)_R$  global symmetry or gauge 'symmetry'

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## What if there were no Higgs vev in the real world?

- the up and down quarks would have been massless;
- $SU(2) \times SU(2)$  would have been exact, with  $SU(2) \times U(1)$  gauged;
- the QCD vev would have broken electroweak  $SU(2) \times U(1) \rightarrow U(1)$  in the standard way;
- the  $W$  and  $Z$  bosons would have eaten the pions and would have had mass of tens of **MeV**;
- and the  $\sigma$  **would** have been the Higgs boson!

- In all cases, fluctuations of  $|\phi|$  around  $v$ , the "Higgs boson", are a physical massive particle, known in QCD as the  $\sigma$  or the  $f_0$ ; (very wide in QCD because  $\sigma \rightarrow \pi\pi$  very rapidly.)

# Technicolor – Scaled-up QCD

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- the  $W$  and  $Z$  bosons would have eaten the pions and would have had mass of tens of **MeV**;
- and the  $\sigma$  **would** have been the Higgs boson!

- Add a new QCD-like group – **technicolor** -- to SM
  - with two massless flavors of techniquarks,
  - symmetry group  $SU(2)_L \times SU(2)_R \times U(1)_B$ , and
  - strong coupling scale  $\Lambda_{TC} \sim 1$  TeV
- Gauge part of symmetry group:  $SU(2)_W \times U(1)_Y$
- Then the strong technicolor interactions cause a composite operator, a techniquark-antitechniquark bilinear, to break
$$SU(2)_W \times U(1)_Y \rightarrow U(1)_{em}$$
- Three TC-pions are absorbed into  $W^+, W^-, Z$ ; photon remains massless
- $\sigma_{TC}$  = Higgs boson: heavy, wide resonance; decays very rapidly to  $WW, ZZ$
- This solves the hierarchy problem, because  $\Lambda_{TC} / M_{pl}$  is naturally small

# [ Why doesn't this work easily? ]

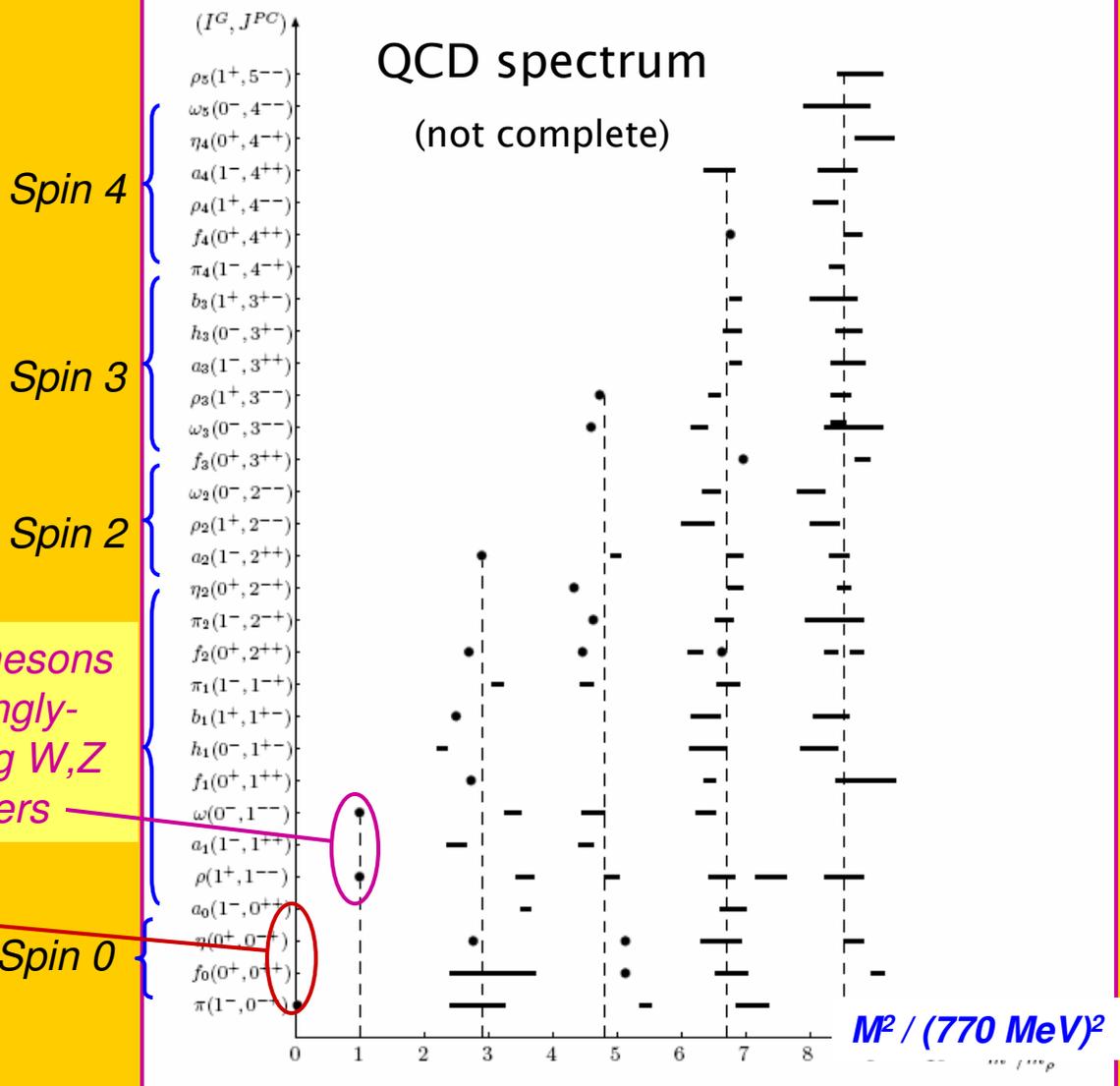
- If  $\Lambda_{\text{TC}} = 1 \text{ TeV}$ , expect quantum corrections to classical SM of order  $(m_W/\Lambda_{\text{TC}})^2 \sim 1 \%$ 
  - *but SM precision tests work to 0.1 % – 1 %*
- Composite Higgs must couple to SM fermions and strongly distinguish different flavors
  - *but get large flavor changing neutral currents, disallowed by experiment*
- Composite Higgs boson must couple strongly to top quark
  - *Possibly make top quark [and bottom quark?] composite too? Or at least strongly coupled to technicolor sector.*
  
- *Resolve most problems with non-QCD-like “Walking Technicolor” (some dynamical assumptions necessary)*
  - *See for example*  
Walking technicolor and electroweak radiative corrections.  
[Raman Sundrum, Stephen D.H. Hsu](#)  
Nucl.Phys.B391:127-146,1993 , hep-ph/9206225

# Technicolor spectrum

- The spectrum of QCD has far more than just the  $\pi, \sigma$ .
- Similarly technicolor has much more than just the  $W, Z, h$
- **Repeaters** for the  $W, Z, h$
- **Repeaters** for top? bottom?
- Other spins, other group representations
- Repeaters are expected for any composite system (think of atomic physics)

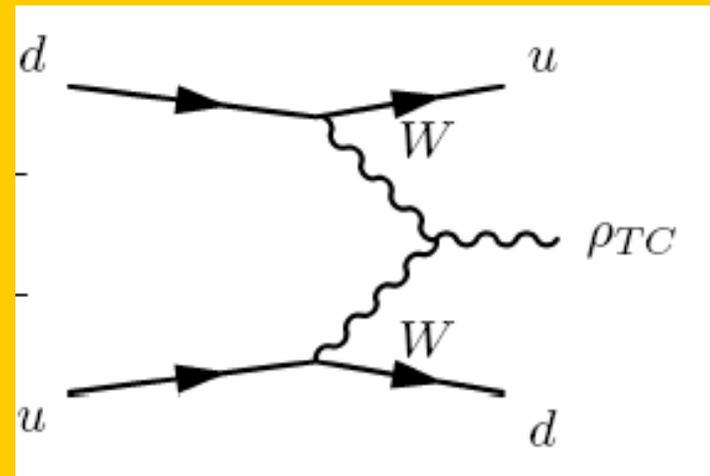
*TC-Rho mesons are strongly-interacting W,Z repeaters*

*3 TC-Pions absorbed into W,Z*



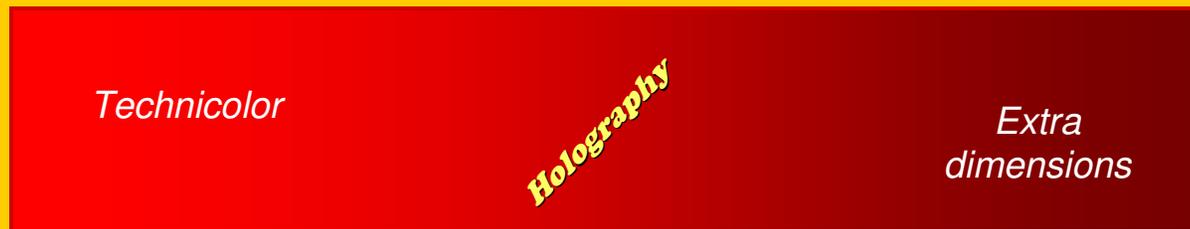
# [ Detecting Technicolor? ]

- Unfortunately precision electroweak constraints imply large  $\Lambda_{TC}$
- Only lowest states above  $W, Z$  probably in reach:  $\rho_{TC}$  mesons, 2-3 TeV
- Produce  $\rho_{TC}$  in  $WW, WZ$  Vector Boson Fusion, though very challenging to detect and study at LHC
- More work needed to understand best techniques, reach



# [ TC vs. Extra Dims ]

- Both technicolor and fields in TeV extra dimensions predict towers of repeaters at  $\sim$  TeV
- How do we tell the difference between these classes of models?
- ***WE CAN'T TELL THE DIFFERENCE! Even in principle!***



- ***More precisely, we can't **always** tell the difference, even in principle.***
  - Some technicolor models really don't look extra-dimensional
  - Some extra-dimensional models really don't look QCD-like
  - But there is an overlap region where the models are quantum mechanically identical, with buzzwords: "holographic theories", string-gauge theory duality, Randall-Sundrum TeV-brane models.
- *Caution: the statement that RS and technicolor models are identical is glib; the precise relation is known! but beware naïve statements in the literature!*
- It is true that those RS TeV-brane models that solve the hierarchy problem do so in effectively the same way technicolor does

# [ Technicolor vs. TeV xtra dims ]

- **And we may not be able to tell the difference in practice**
  - To properly identify a technicolor or TeV-scale extra-dimensional model, need to study its towers of states
  - *But LHC will give us only the first one or two repeaters --- probably insufficient, at least in early stages, for distinguishing models*
- Still, production mechanisms and branching fractions tell us much about repeaters –
  - Which SM fermions do they mainly couple?
    - *(Quarks only? L-handed fermions only? Heavy-flavor only? All fermions equally?)*
  - Which SM vector bosons do they mainly couple to?
    - *(Couple to Ws and Zs? To Zs only? To gluons?)*
- This information will be very helpful in separating models; but –
- We need to measure rates and branching ratios with moderate precision. *Tough at LHC!*

# [ Evade constraints? ]

- All of the models we have just considered have drastically new physics at the weak scale
- All are highly constrained by electroweak precision measurements, which typically demand new physics be above a few TeV  
[but see 3<sup>rd</sup> lecture for caveats...]
- *This is bad for LHC, but it is also bad for keeping the Higgs boson relatively light without large corrections from loops or from breakdown of four-dimensional quantum field theory*
- *That suggests that perhaps nature has a light Higgs along with new weakly-coupled four-dimensional field-theory physics at a TeV*
- Could we make the Higgs a Goldstone boson? This would be naturally light and, at low enough energy, weakly interacting (since all Goldstone-boson interactions must vanish at zero momentum)
- The Little Higgs --- pushes the strongly interacting physics up to 10 TeV at the expense of weakly-interacting physics at 1 TeV.

# QCD-based Higgs-like Model

$N_f = 2$  QCD model:  $SU(2)_L \times SU(2)_R$  global symmetry or gauge 'symmetry'

- **2 x 2 matrix scalar**  $\phi = \begin{bmatrix} \bar{u}u & \bar{u}d \\ \bar{d}u & \bar{d}d \end{bmatrix}$
- Potential  $V(|\phi^\dagger\phi|)$ , with minimum at  $\langle\phi\rangle = v \mathbf{M}$  which we can always rotate to  $\langle\phi\rangle = v \mathbf{1}$  (unit matrix)
- This vev preserves the *diagonal* subgroup  $SU(2)_{L+R}$ , with rotations  $\mathbf{U}_R = \mathbf{U}_L^{-1}$
- If  $SU(2) \times SU(2)$  symmetry is *global*, fluctuations of  $\mathbf{U}_R = \mathbf{U}_L$ , broken combination of the  $SU(2)$ 's, are massless particles – three massless GBs
- If  $SU(2) \times U(1)$  symmetry is *gauged*, fluctuations  $\mathbf{U}_R = \mathbf{U}_L$  are absorbed by three of the four gauge bosons, which become massive  $W, Z$ , and a massless photon remains. *Nothing remains of the rest of  $SU(2)_R$ ; it is not a symmetry.*
  
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# QCD-based Higgs-like Model

$N_f = 3$  QCD model:  $SU(3)_L \times SU(3)_R$  global symmetry or gauge 'symmetry'

- **3 x 3 matrix scalar**  $\phi = \begin{bmatrix} \bar{u}u & \bar{u}d & \bar{u}s \\ \bar{d}u & \bar{d}d & \bar{d}s \\ \bar{s}u & \bar{s}d & \bar{s}s \end{bmatrix}$
- Potential  $V(|\phi^\dagger\phi|)$ , with minimum at  $\langle\phi\rangle = v \mathbf{M}$  which we can always rotate to  $\langle\phi\rangle = v \mathbf{1}$  (unit matrix)
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- If  $SU(3) \times SU(3)$  symmetry is *global*, fluctuations of  $\mathbf{U}_R = \mathbf{U}_L$ , broken combination of the  $SU(3)$ 's, are massless particles – eight massless GBs:  
what would be pions, Kaons, and the eta in QCD.
- If  $SU(2)_{L+R} \times U(1)_{L+R}$  (subgroup of  $SU(3)_{L+R}$ ) symmetry is *gauged*,
  - The vev does **not** break it, so the four gauge bosons remain massless.
  - However, *nothing remains of the rest of  $SU(3)_L \times SU(3)_R$* ; **there is no broken global symmetry**.
  - The fluctuations  $\mathbf{U}_R = \mathbf{U}_L$  are massive pseudo-GBs, with masses generated at one loop by the gauge bosons.
- Let's consider this case more carefully.

# [ Higgs as PGB? ]

## The Eight Pseudo-Goldstone Bosons

$\#\pi^0 - \#\eta$	$\pi^+$	$K^+$
$\pi^-$	$-\#\pi^0 - \#\eta$	$K^0$
$K^-$	$K^0$	$\#\eta$

$SU(2)_L$   $U(1)_L$   $[-1/2, -1/2, +1]$

$SU(2)_R$   $U(1)_R$   $[+1/2, +1/2, -1]$

$$H = \begin{bmatrix} H^+ \\ H^0 \end{bmatrix}$$

Could Higgs doublets be the “Kaons” of a new strong dynamics? These doublets would naturally be very light. Then we could have  $v / \Lambda \ll 1 \rightarrow$  smaller loop corrections than TC

particle	$SU(2)_{L+R}$	$U(1)_{L+R}$
$\pi^+ \pi^- \pi^0$	3	0
$K^+ K^0$	2	1/2
$K^- K^0$	2	-1/2
$\eta$	1	0

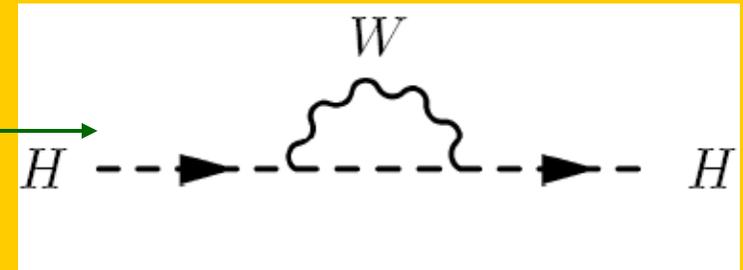
# Little Higgs vs TC

- Technicolor
  - $SU(2) \times SU(2) \rightarrow SU(2)$  global breaking by quark bilinear at scale  $\Lambda$   
breaks electroweak  $SU(2) \times U(1) \rightarrow U(1)$
  - Higgs is  $\sigma$ , fluctuations of vev
  - $\pi^+, \pi^-, \pi^0$  absorbed by  $W^+, W^-, Z^0$
  - Problem to get small loop corrections
- Proto-Little Higgs
  - $SU(3) \times SU(3) \rightarrow SU(3)$  global breaking by quark bilinear at scale  $\Lambda$   
leaves electroweak  $SU(2) \times U(1)$  unbroken
  - Higgs is  $K^0 + \bar{K}^0$ , develops vev  $v \ll \Lambda$
  - $K^+, K^-, K^0 - \bar{K}^0$  absorbed by  $W^+, W^-, Z^0$
  - Problem to keep Kaons light enough, arrange  $v \ll \Lambda$
- *In both cases, getting flavor physics, especially large top quark mass, is a challenge*

# [ Not quite enough yet ]

- What mass do the SM gauge bosons give to the Kaon-like states?

$$m_H^2 = \frac{g_2^2 \Lambda^2}{16\pi^2} \Rightarrow m_H = g_2 \frac{\Lambda}{4\pi} \sim \frac{\Lambda}{10}$$



- This isn't light enough.
- $\Lambda$  still needs to be of order a couple of TeV; we still get pretty big corrections to precision electroweak tests of the SM. We want  $\Lambda$  bigger while keeping the Higgs light.
- But there is a trick to make the PGBs lighter!

# Collective Symmetry Breaking

- Simple case: **enlarge** SM electroweak group to  $SU(2) \times U(1) \times G'$ , with couplings  $g_2, g_1, g'$
- At scale  $\Lambda \sim 10$  TeV, a **new strong interaction** causes  $SU(2) \times U(1) \times G'$  to break to SM  $SU(2) \times U(1)$ . The **resulting massive gauge bosons** get masses  $\sim g \Lambda / 4 \pi \sim 1$  TeV SM gauge bosons are still massless

if all gauge couplings are zero, a large global symmetry group is present and is spontaneously broken by the new strong interaction;  
the Higgs is a massless GB

■ if  $g_1 = g_2 = 0 < g'$ ,

- part of the global symmetry group is broken explicitly by the  $G'$  gauge group;
- a smaller global symmetry group remains and is broken spontaneously
- the Higgs is still a massless GB

■ if  $g_1, g_2 > 0 = g'$ ,

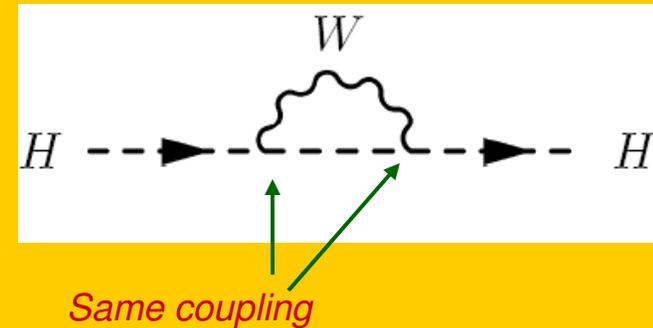
- a different part of the global symmetry group is broken explicitly by the  $SU(2) \times U(1)$  gauge interaction;
- a smaller global symmetry group remains and is broken spontaneously ;
- the Higgs is again still a massless GB

■ If all three gauge couplings are non-zero

- all of the global symmetries broken by the gauge interactions;
- the strong interaction does not spontaneously break any global symmetries, and
- the Higgs boson acquires a mass.

# [ From one loop to two loops ]

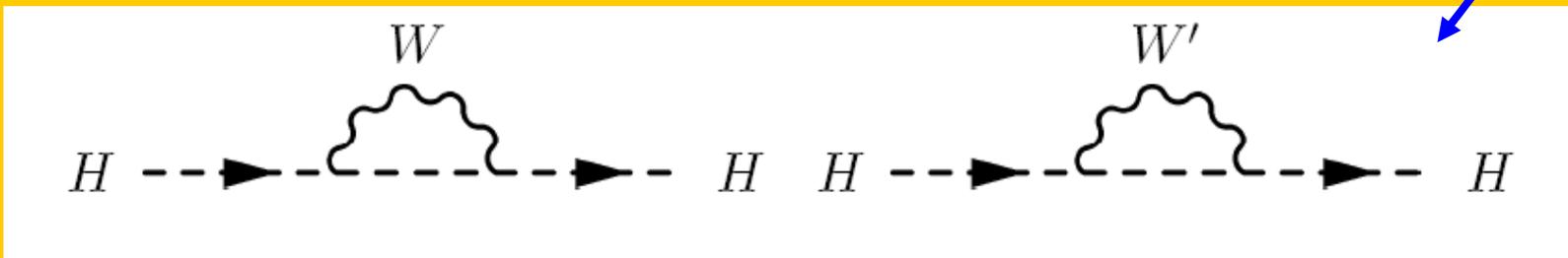
- **ONLY** if both  $g'$  and the SM gauge couplings are non-zero does the Higgs boson acquire a mass.
- Since no one coupling is enough to generate this mass, no one-loop diagram (which always involves squares of one coupling) can be the cause.
- ***The formula for the Higgs mass must involve a product of two couplings at once (so that it vanishes if either coupling is zero) which can only happen in a two-loop graph.***
- Two-loop corrections are small enough to avoid large electroweak corrections.



$$m_H^2 = \frac{g_2^2 g'^2 \Lambda^2}{(16\pi^2)^2} \Rightarrow m_H = \frac{g_2 g' \Lambda}{(4\pi)^2} \sim \frac{\Lambda}{100}$$

# How does this happen?

- Magically, the one-loop graphs must cancel. What's doing the canceling?
- At one level, the magic is in the group theory: symmetry.
- On the other hand, there is an explicit cancellation going on.
- The miracle is in the masses and couplings of the **new heavy gauge bosons**, mass about 1 TeV, from the breaking of  $SU(2) \times U(1) \times G'$ ,
- They are precisely what is needed to cancel the one-loop contributions from the W and Z bosons.
- **The cancellation typically requires repeaters for W, Z, h, t, b(?)!**  
(however there are loopholes here)



# [ More Model-Building to do ]

- We're not even close to done: need
  - to ensure Kaons get vev  $v = 246 \text{ GeV} \ll \Lambda$  – more structure, other variants – more states in Higgs sector
  - to ensure large top quark Yukawa coupling generated without giving large Higgs mass, need additional physics in top quark sector  $\rightarrow$  top quark repeaters *too*
- Repeaters for  $W, Z, h, t, [b]$  are common feature in successful models but
  - Other non-repeater states are often generated
  - Not all repeaters are always generated
  - Some models with no repeaters exist; at least one exists [original twin Higgs] with no visible states! (scary!)
- LHC Phenomenology – as we have discussed for earlier repeaters (perhaps plus exotic higgs and quark-like states) but **with weaker couplings at the TeV scale** than in TC and some extra dim models
- In the end,
  - $\Lambda = 10 \text{ TeV}$ , repeaters at 1 TeV,  $v = 246 \text{ GeV}$  appears possible
  - But this is controversial; no fully convincing model exists
- See Schmaltz and Tucker-Smith review for additional reading on models.
  - Ann.Rev.Nucl.Part.Sci.55:229-270,2005 ; e-Print Archive: hep-ph/0502182

# [ Extra Fermionic Dimensions ]

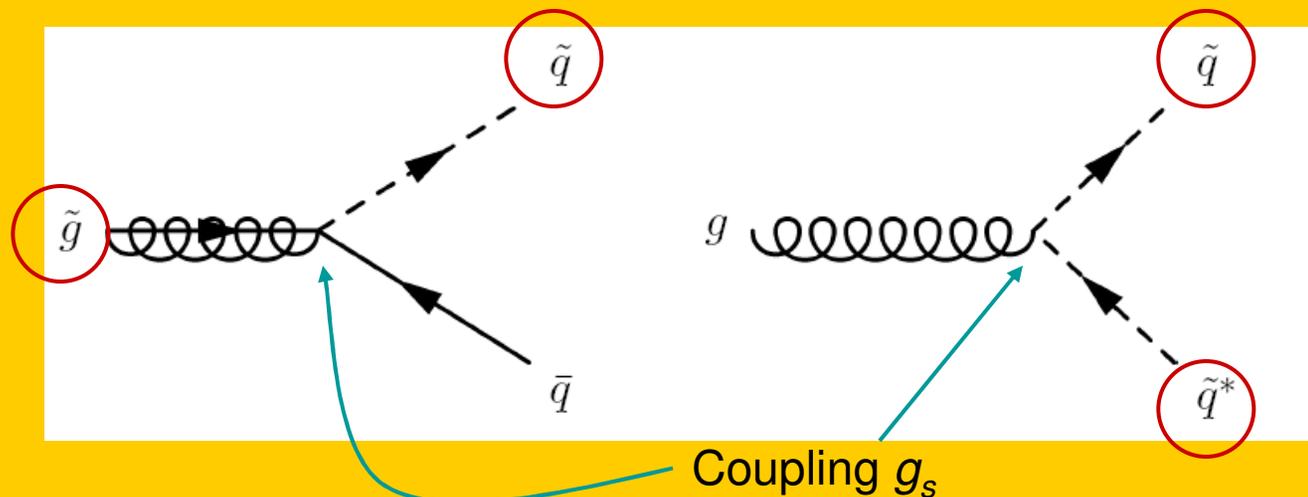
- Can we do better? Can we move  $\Lambda$ , where the Little Higgs is strongly coupled, much higher than 10 TeV?
- Is there a weakly coupled model valid up to the Planck scale with a light Higgs and no hierarchy problem?
- *Yes! Extra fermionic dimensions do the trick.*
- This is also called “supersymmetry”. *I will not study this model in detail – see Polesello 2006 SLAC summer school lectures, Luty TASI 2004 lectures, and many other reviews*
- Just as with ordinary bosonic dimensions, there is a tower of states for each standard model particle
- But there are some key differences
  - There need be only one repeater for each particle
  - The spin of the repeater differs from the original’s spin by 1/2
  - Every particle in the SM *must* have a repeater at scales that cannot differ too much from the weak scale

# [ Or “Supersymmetry” (SUSY) ]

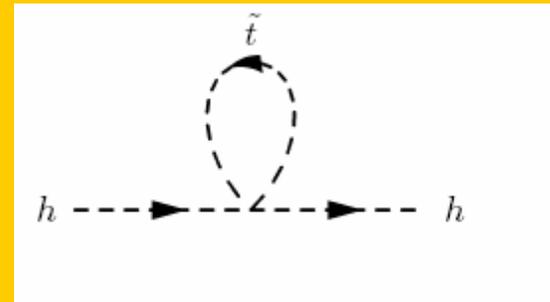
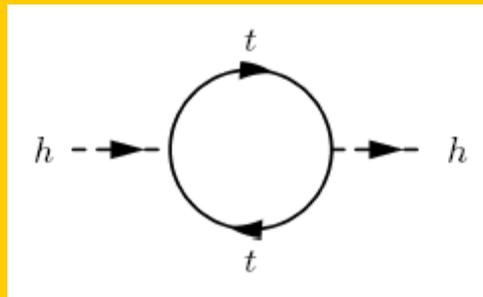
- This means we have to add about thirty new particles:
  - Gluons  $\rightarrow$  gluinos (spin  $\frac{1}{2}$ )
  - 2 Higgs doublets  $\rightarrow$  charged, 2 neutral Higgsinos (spin  $\frac{1}{2}$ )
  - Photon, W, Z  $\rightarrow$  photino(?), Wino, Zino (spin  $\frac{1}{2}$ )
  - Quarks  $\rightarrow$  squarks (spin 0)
  - Leptons, neutrinos  $\rightarrow$  sleptons, sneutrinos (spin 0)
  - Gravitons  $\rightarrow$  gravitinos (spin  $\frac{3}{2}$ )
- But it's a bit more complicated than this, unfortunately...
- Were SUSY exact, all particles and superpartners would have equal masses
- But SUSY is spontaneously broken, so the *masses* are unknowns
- And many of the particles have same quantum numbers, so they *mix*
  - charged Higgsino, Wino  $\rightarrow$  2 charginos
  - neutral Higgsinos, photino(?),  $\rightarrow$  4 neutralinos
- Also we have two Higgs doublets  $\rightarrow h^0 H^0 A^0 H^+ H^-$
- *Altogether something like 100 new parameters!*

# SUSY interactions

- As with models like extra dimensions, take any SM  $2 \rightarrow 1$  process and replace SM particles with repeaters –
- but *unlike previous cases*,
  - coupling constant must be **exactly** the same
  - must replace **two** SM particles with repeaters, never one.
- This last condition is simply Lorentz invariance:
  - Replace two fermions with two bosons **ok**
  - Replace a fermion with a boson and a boson with a fermion **ok**
  - Replace one fermion with one boson **not ok**
- *Some other 4-scalar interactions too, see SUSY review for details.*



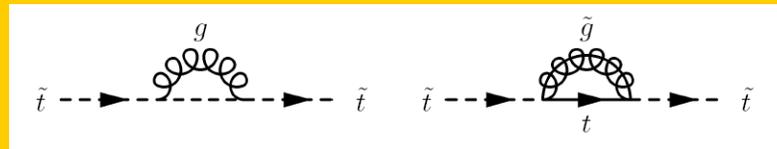
# What's so good about SUSY?



- Just as in little Higgs, the one-loop corrections to the Higgs boson mass cancel between the original and the repeater
- In this case there is a conspiracy of couplings and Bose vs. Fermi statistics
- But --- amazing --- cancellation of  $M^2$  corrections to  $m_h^2$  works to all loops!!! ... as long as all particles have repeaters...
- More precisely, if all repeaters are at 1 TeV, then corrections to  $m_h^2$  are small.

# Why light superpartners?

- We need the top squark to be light to cancel the top correction to the Higgs mass
- But the top squark is a ***scalar*** – this is what makes SUSY different from the other models with repeaters...
- And its mass also gets ***additive corrections*** of order  $M^2$  from a gluon loop...
- ...unless there is a gluino whose mass is of order a TeV



- The presence of light scalars requires *all repeaters be light*  
(to avoid extreme fine-tuning)
- **But not *that* light...** The loops come with coupling constants and loop factors!
- To be precise: if repeaters for  $W, Z, h, t, [b]$  are below 1 TeV, and others are **not *too* high**, corrections to  $m_h^2$  are small.
- It is possible that most of the repeaters *could* be too heavy to see at the LHC –

The More Minimal Supersymmetric Standard Model  
hep-ph/9607394 Cohen, Kaplan, Nelson

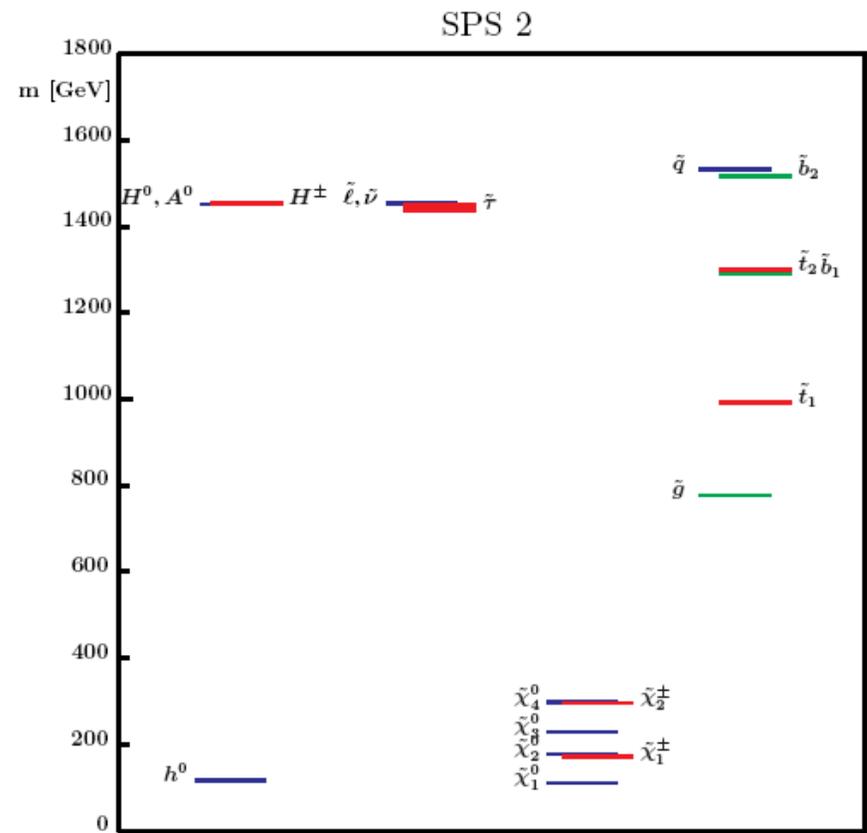
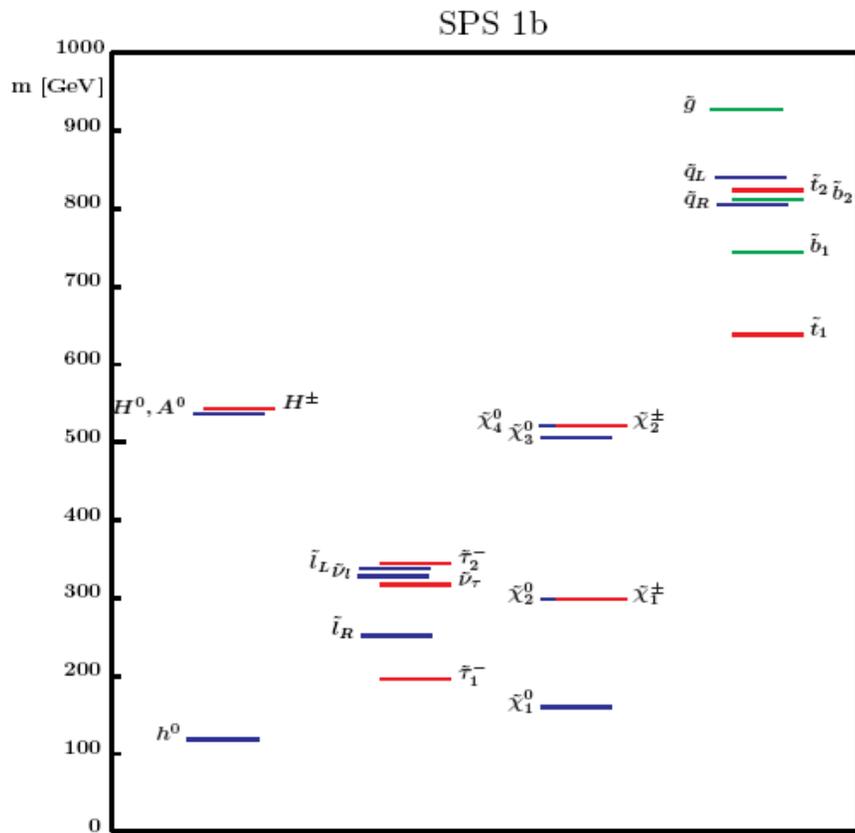
# [ Other good things about SUSY(?) ]

- Unification of couplings – yes, a feature – it really is true the couplings unify better with SUSY partners [*This does not imply Grand Unification of Three Forces! GUTs not required.*]
- “SUSY provides a candidate for dark matter” ?  
*Not necessarily – actually the logic is different, see below.*
- “SUSY required by string theory” ?  
*Not necessarily – certainly not TeV-scale SUSY-breaking.*
- In any case, a *minimal* SUSY model is not required! And so the phenomenology may not look at all like standard SUSY models, even if SUSY is at the weak scale.

# [ But SUSY is not so predictive ]

- As we saw, SUSY, as a solution to the hierarchy problem, predicts at least some superpartners will be found at the LHC
- But with 100 new parameters, we are not able to make any many predictions for LHC physics unless we try to guess how these parameters are determined
- The “mechanism” of SUSY breaking is not known; many possibilities:
  - Gauge mediation
  - Hidden sector mediation
  - Gaugino mediation
  - Anomaly mediation
  - Minimal Supergravity??? See below.
- Different mechanisms predict different **relations** among these many parameters, but rarely predict any one parameter; we theorists have no idea which of these many mechanisms to prefer.

# [ Some SUSY spectra ]



Supersymmetric Benchmarks with  
Non-Universal Scalar Masses

A. De Roeck, J. Ellis, F. Gianotti,  
F. Moortgat, K. A. Olive and L. Pape

All these models have degenerate heavy Higgs bosons, well-split neutralinos, gluino-squark near-degeneracy

This is not a representative sample – but it is typical of studies.

Supersymmetric spectra in NUHM and GDM benchmarks  
calculated with ISASUGRA 7.69

Result depends on program

Model	$\alpha$	$\beta$	$\gamma$	$\delta$	$\epsilon$	$\zeta$	$\eta$
$m_{1/2}$	293	370	247	750	440	1000	1000
$m_0$	206	225	328	500	20	100	20
$\tan\beta$	10	10	20	10	15	21.5	23.7
$\text{sign}(\mu)$	+	+	+	+	+	+	+
$A_0$	0	0	0	0	-25	-127	-25
$m_t$	178	178	178	178	178	178	178
Masses							
$ \mu $	375	500	325	920	569	1186	1171
$h^0$	115	117	115	122	119	124	124
$H^0$	267	328	241	1159	626	1293	1261
$A^0$	265	325	240	1152	622	1285	1253
$H^\pm$	278	337	255	1162	632	1296	1264
$\chi_1^0$	113	146	95	310	175	417	417
$\chi_2^0$	215	282	180	600	339	805	804
$\chi_3^0$	380	503	332	925	574	1192	1176
$\chi_4^0$	400	518	352	935	587	1200	1184
$\chi_{1\pm}$	215	283	180	601	340	807	806
$\chi_{2\pm}$	399	518	352	935	587	1200	1184
$\tilde{g}$	711	880	619	1691	1026	2191	2191
$e_L, \mu_L$	299	351	378	713	306	684	677
$e_R, \mu_R$	216	241	328	572	171	387	374
$\nu_e, \nu_\mu$	287	340	368	703	290	669	662
$\tau_1$	213	239	315	565	153	338	319
$\tau_2$	300	352	378	712	309	677	670
$\nu_\tau$	287	340	365	700	288	660	653
$u_L, c_L$	674	826	636	1604	935	1991	1998
$u_R, c_R$	661	808	629	1550	902	1911	1908
$d_L, s_L$	679	831	642	1606	938	1993	1990
$d_R, s_R$	652	797	621	1544	899	1903	1900
$t_1$	492	622	453	1219	710	1545	1553
$t_2$	662	800	611	1486	900	1842	1840
$b_1$	609	752	558	1456	852	1807	1804
$b_2$	641	785	603	1516	883	1851	1846

Notice these input parameters do not correspond closely to any physical masses.

Remember rates fall like  $\sim 1/m^5$  or faster, so 1 TeV is quick, but 2 TeV is not

Table 4: Proposed NUHM and GDM benchmark points and mass spectra (in GeV), as calculated using ISASUGRA 7.69 [21] and adapting the input parameters to give the best match to the SSARD [20] spectra shown in Table 2, as described in the text.

# mSUGRA – a digression

- **Minimal Supergravity ???**
- **Good news: only 5 parameters**
- **Bad news:**
  - Not supergravity (was once thought to be)
  - In fact, not a theory
  - At best a scenario; mainly a fairly arbitrary parameter reduction scheme
  - **Few realistic models naturally reduce to these parameters**
  - **Potentially deeply misleading for LHC phenomenology**
- Unfortunately, for historical reasons, the vast majority of studies for supersymmetry are based on it.
- Beware studies that limit themselves to this scheme; their conclusions are potentially deeply misleading about what SUSY will actually look like, or how constrained it will be.
- *But that's not what's really bad about this scheme.*
- **Worse news: the 5 parameters are parameters in the effective Lagrangian at  $M = M_{GUT} \sim 10^{16}$  GeV**

# [ How not to use data ]

- Data → GUT-scale Theory is a bad idea
  - Many theoretical assumptions must be made.
  - Very difficult to use or interpret results; constantly have to run couplings down to TeV in one theory, then back up in another theory.
  - Limits on masses and couplings in Lagrangian correspond poorly to constraints on physical masses and couplings.
  - Tiny variations in Lagrangian parameters can make large variations in physical quantities, and vice versa.
  - Even one additional nonminimal particle can *completely* change the results.
- I strongly encourage you to push your colleagues to avoid publishing data exclusively in this format!

# [ How to use data ]

- Data  $\rightarrow$  TeV-Scale Theory  $\leftarrow$  High-Scale Theory is a much better idea
  - Experimentalists can perform the first step without adopting nearly as many theoretical assumptions about unmeasurable quantities
  - Relatively easy for different theorists, or experimentalists, to convert from their favorite High-scale theory to TeV scale theory
  - Limits on masses and couplings in Lagrangian correspond well to constraints on physical masses and couplings.
  - Tiny variations in Lagrangian parameters can only make small variations in physical quantities.
  - One additional nonminimal particle is much less likely to change the results substantially
- Data published in this form is far easier to interpret and far more usable for the community; it properly captures what we actually know from data with far less theoretical bias

# [ Another way to use data? ]

Something to think about in the early days of LHC:

- Data  $\rightarrow$  Fragment of TeV-Scale Theory  $\rightarrow$   
Complete TeV-Scale Theory  $\leftarrow$  High-Scale Theory
  - Experimentalists can perform the first step with even fewer assumptions about unmeasured quantities
  - A fragment of a theory need not be fully consistent, need not be renormalizable, etc, but still makes predictions, can be run through Monte Carlo, and will be useful in trying to figure out the complete theory
  - A good place for theorists and experimentalists to work together (since a useful fragment of a theory can only be designed once the data comes in and a signal is detected)

An example of a partial theory is given in our Summer 2005 LHC Olympics analysis

# [ Testing SUSY ]

Need to see ALL the superpartners

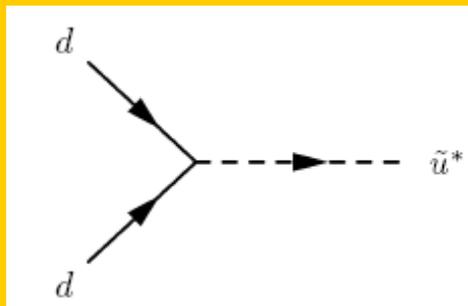
- *Unfortunately, we're not likely to see them all at LHC*

Need to measure coupling constant relations that SUSY predicts

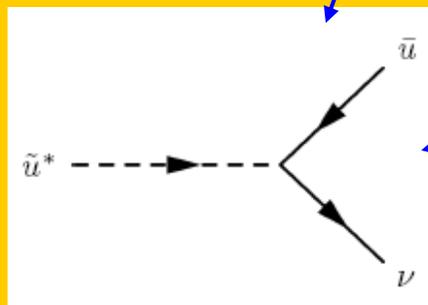
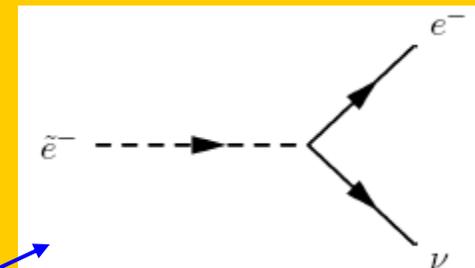
- *Unfortunately, because of mixing among states, coupling constants get mixed up with mixing angles; so this too is a really tough thing to do at the LHC*
- Only in very special circumstances, and usually with some experimentally-unjustifiable assumptions, can SUSY be verified at the LHC
- ***But discovery of any light scalar fields carrying the same quantum numbers as quarks and leptons will be very, very suggestive. Models can fake this, but not easily.***

# Some Additional Interactions

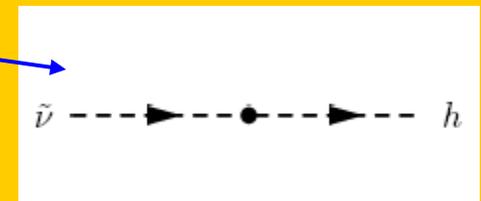
- Earlier, we did not exhaust the list of possible important interactions in SUS, because of new *scalar* states – can't arise with same-spin repeaters.
- New baryon-number- and lepton-number-violating interactions are possible in SUSY, not in other models. These lead to lethally-rapid proton decay. This is bad.



*Violates B*

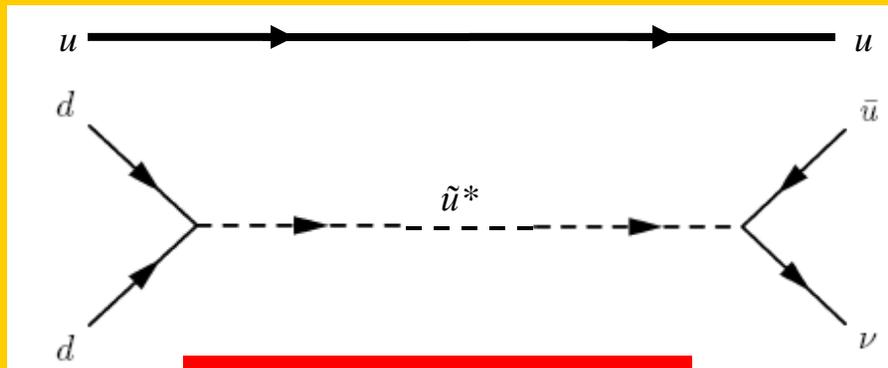


*Violates L*

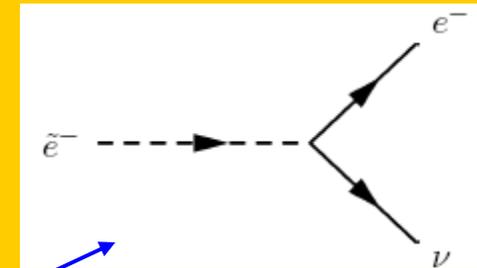


# Some Additional Interactions

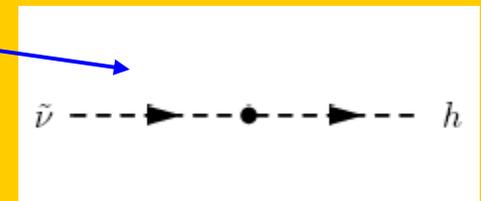
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$$p \rightarrow \pi^0 \nu$$



Violates  $L$



# Some Additional Interactions

- This leads people to often (*but not always*) introduce a new global symmetry on top of supersymmetry: this is called “**R-parity**” for historical reasons, or “matter parity”, which is a less fancy word for the same thing.
- Effectively, it requires that *all interactions must have two or zero superpartners*, never one. More on this soon.
- This new global symmetry has very big consequences:
  - No superpartners can be resonantly produced;  $2 \rightarrow 1$  can't happen. **Superpartners must be produced in pairs.**
  - Once produced, **a superpartner cannot decay to standard model particles alone**. If a superpartner decays, another superpartner comes out in its decay products, often leading to **cascade decays**.
  - Also, the **lightest superpartner must be stable**
    - (but it might not be a standard model superpartner – it might be the gravitino, or a right-handed sneutrino, or some new object)
- ***The above are not directly consequences of SUSY!***

# New Global Symmetries

## R-parity implies:

- No superpartners can be resonantly produced;  $2 \rightarrow 1$  can't happen. **Superpartners must be produced in pairs.**
- Once produced, **a superpartner cannot decay to standard model particles alone.** If a superpartner decays, another superpartner comes out in its decay products, often leading to **cascade decays.**
- Also, the **lightest superpartner must be stable** (but it might not be a standard model superpartner)

- This motivates us to consider more generally the phenomenological consequences of global symmetries that are not carried by SM particles
- We'll consider
  - Effects of exact new global symmetries: exact conservation laws
    - New weakly-coupled sectors that also carry the global symmetry
  - Effect of tiny violation of new global syms: conservation laws barely violated
  - Effect of small violation of new global syms: conservation laws somewhat violated
- Any new exact conserved global symmetry has very big consequences:
  - **Globally-charged particles must be produced in pairs.**
  - **Globally-charged particles cannot decay to standard model particles alone.**
  - **The lightest globally-charged particle must be stable**