Symmetry and the Origin of Mass

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In 10 years the written laws of physics will be different than they are now.

The Energy Frontier will reveal new organizing principles of nature

Conventional Wisdom:

Supersymmetry

Extra Dimensions

(a heretic)

All theoretical candidate organizing principles of nature, in our modern perspective, involve hypothetical new *symmetries*.

Symmetry is fundamental to our understanding of the basic laws of physics

Mass is always associated with symmetry breaking

Symmetry was (sort of) a Twentieth Century Revelation

"Noether's Theorem" Connects Symmetry to Laws of Physics

Noether's Theorem is as important to us now as the Pythagorean Theorem



Emmy Noether 1882-1935

Symmetry is:

Invariance of a system or object under a transformation or collection of transformations.

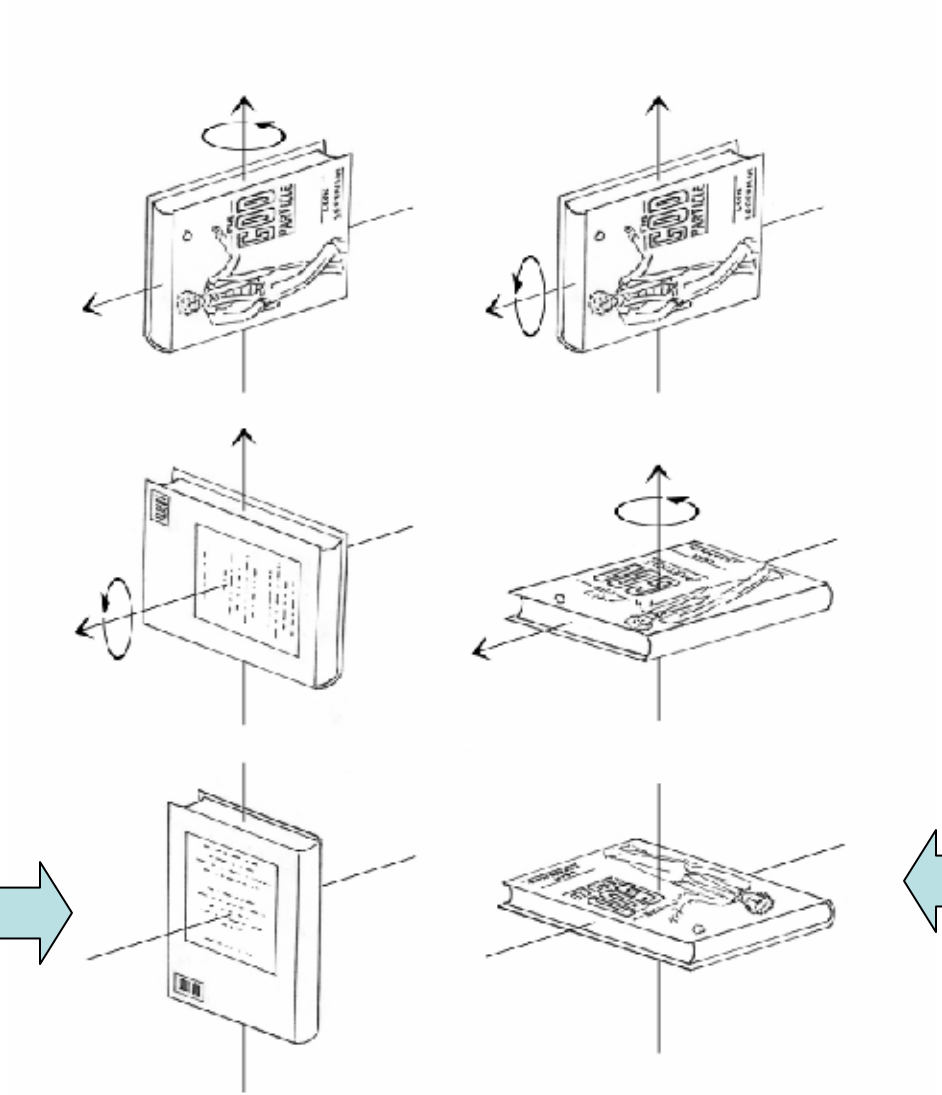
Symmetry Groups

Symmetry Groups

- A group G is a collection of elements $\{ r_j \}$
- G has a "multiplication" operation: $r_j \times r_k = r_k$ where r_k is in G
- There is a unique identity in G, 1, such that $1 \times r_k = r_k \times 1 = r_k$
- Each element r_k has a unique inverse r_k^{-1} such that $r_k^{-1} \times r_k = r_k \times r_k^{-1} = 1$
- Group multiplication is associative

Commutation generally doesn't hold in nature:

$$A \times B \neq B \times A$$



Continuous (simple, compact) Symmetry Groups

Cartan Classification

- Spheres in N dimensions: O(2), O(3), ..., SO(N)
- Complex Spheres in N dimensions: U(1), SU(2), ..., SU(N)
- N dimensional phase space Sp(2N)
- Exceptional Groups: G_2, F_4, E_6, E_7, E_8

Continuous symmetries for the basis of Noether's Theorem, and the fundamental forces in nature described as local gauge symmetries.

Noether's Theorem

For every continuous symmetry there is a conservation law

For every conservation law there is a continuous symmetry

Conservation law ⟺ conserved current

Space Translations (3 d.o.f.) ⟺ Momentum (3 d.o.f.)

Time Translations (1 d.o.f.) ⟺ Energy (3 d.o.f.)

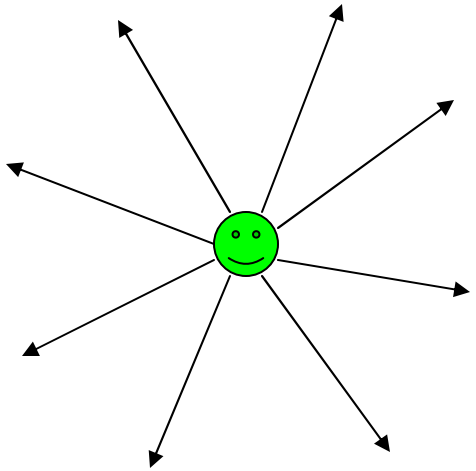
Rotations (3 d.o.f.) ⟺ Angular Momentum (3 d.o.f.)

⇕

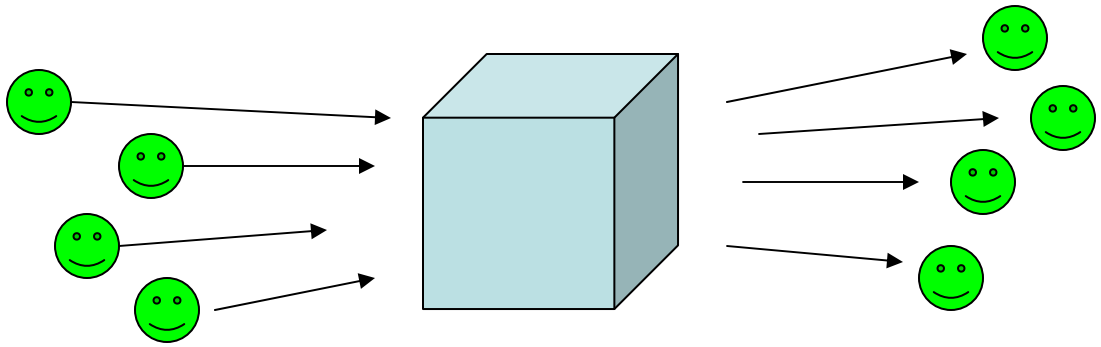
Boosts (3 d.o.f.) ⟺ Relativistic Angular Momentum (3 d.o.f.)

Electricity and Magnetism

Electric charge:



Electric charge is conserved:



What is the continuous symmetry that leads, by Noether's Theorem, to electric charge conservation?

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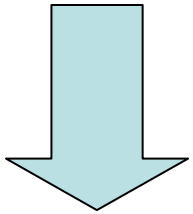
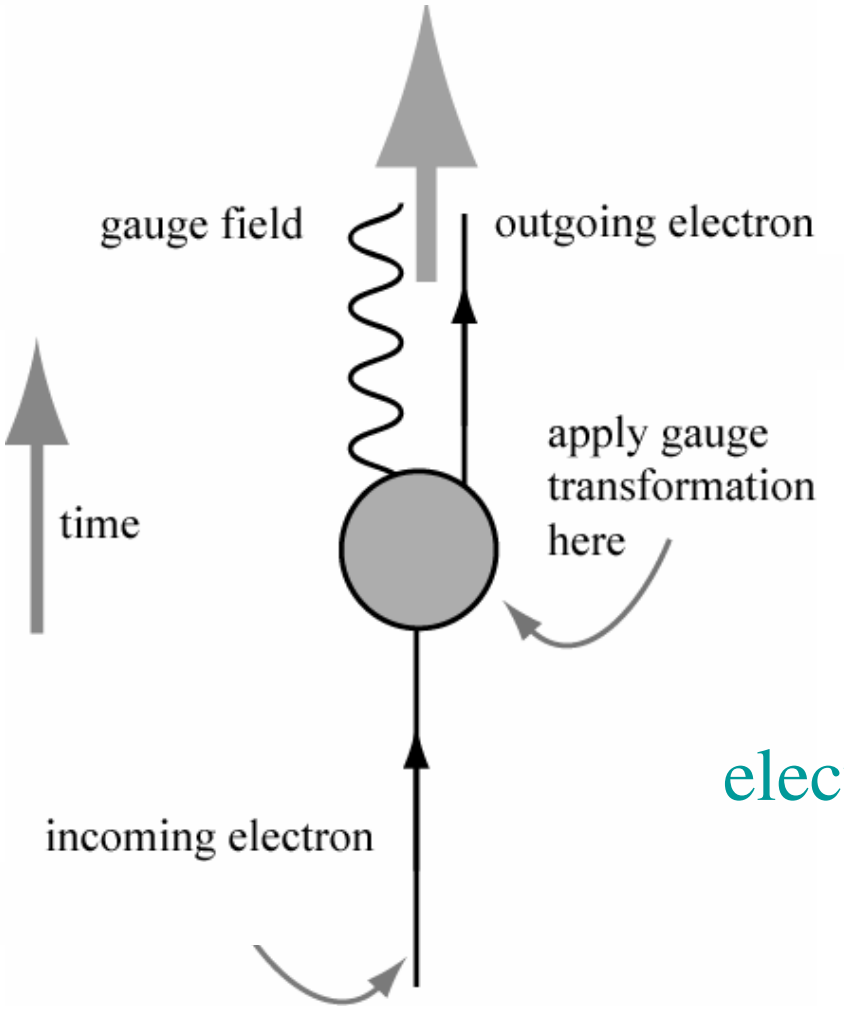


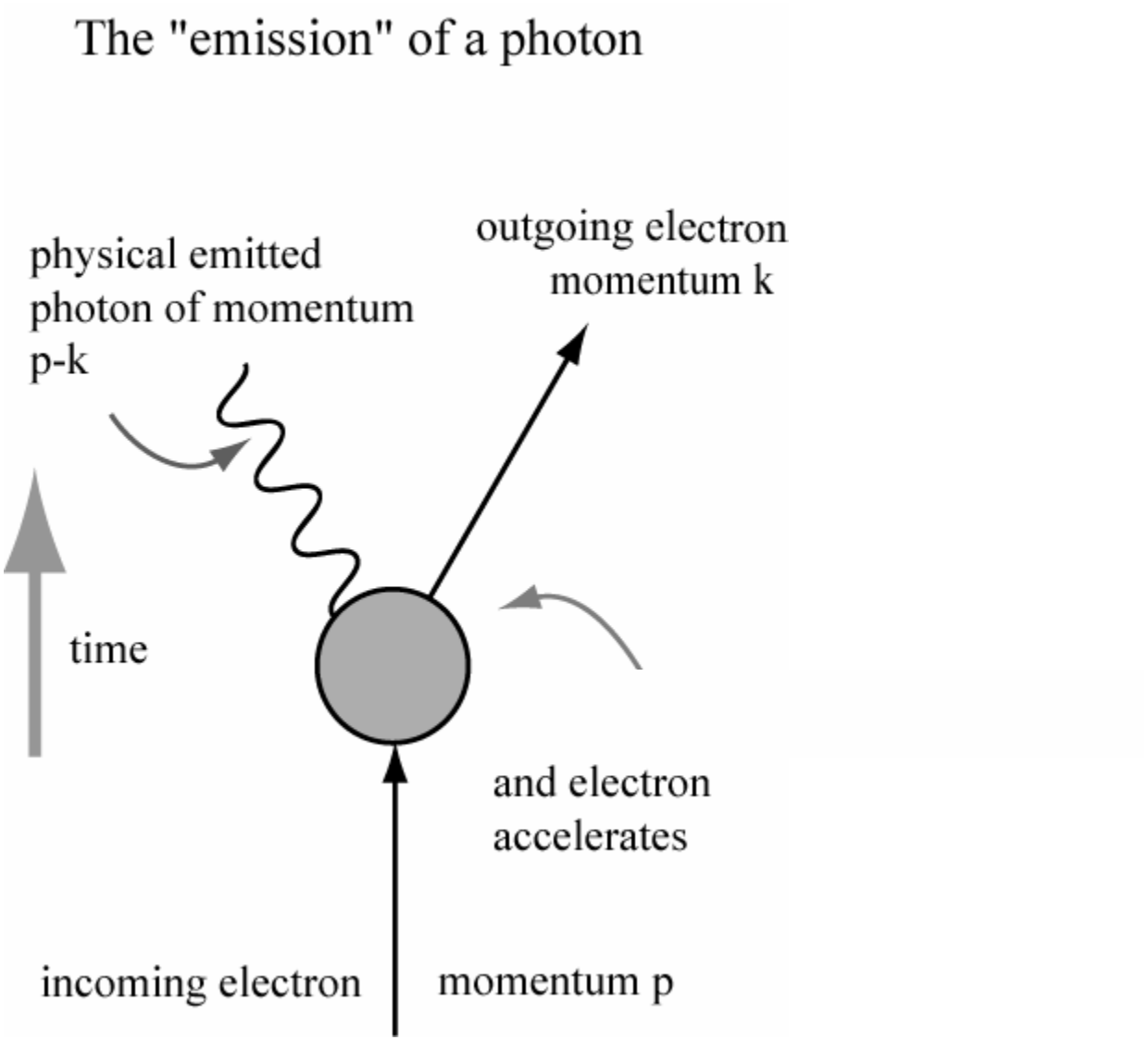
Defines the full structure of electrodynamics

Local Gauge Symmetry U(1):

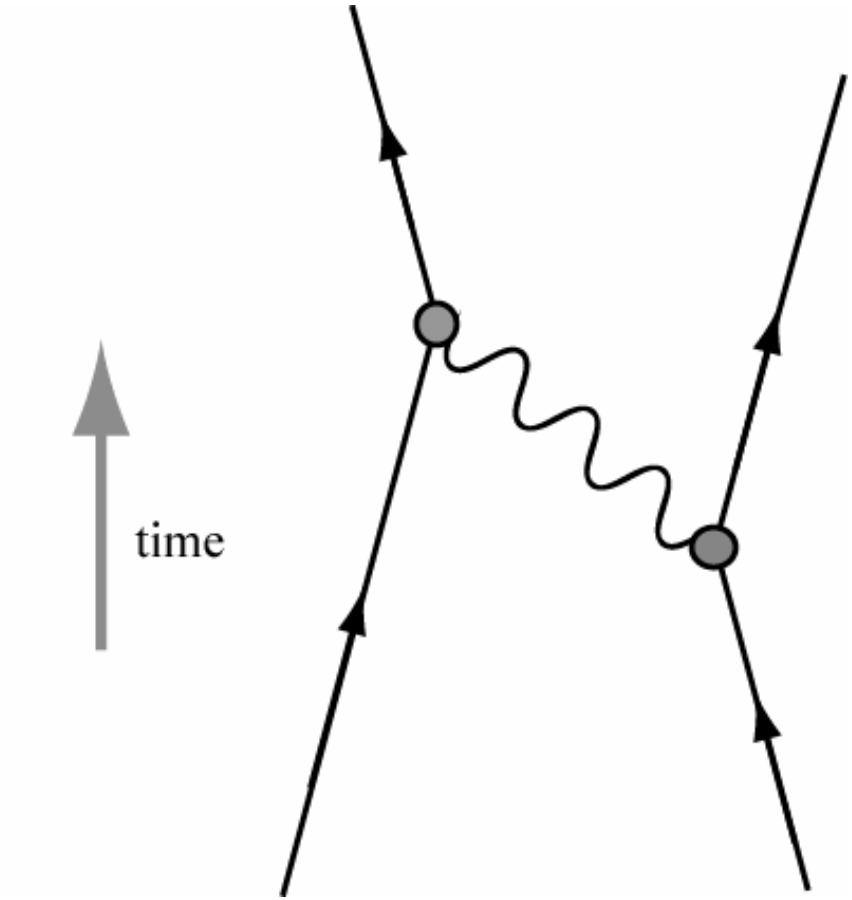


phase of electron's
wave function is
strictly unobservable

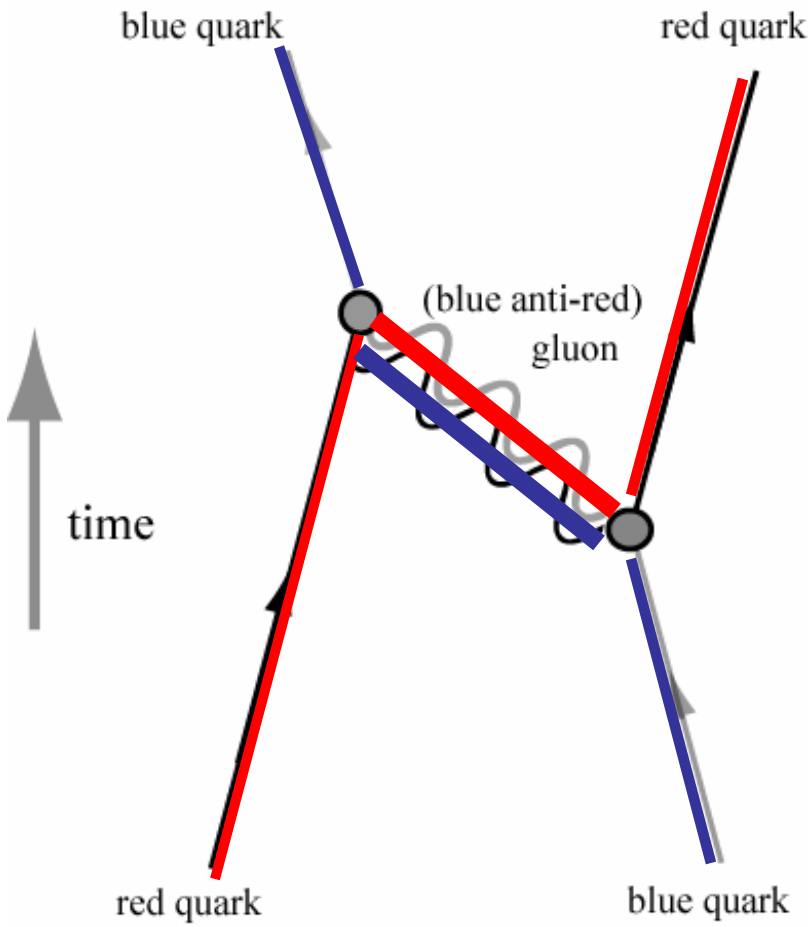
electron = electron + collinear
gauge field


The "emission" of a photon
physical emitted
photon of momentum
p-k
outgoing electron
momentum k
time
and electron
accelerates
incoming electron
momentum p

Electromagnetic force U(1)



Quark color force SU(3)



Local U(1) Gauge Invariance Wallet Card

$$\Psi(x,t) \longrightarrow e^{i\theta(x,t)}\Psi'(x,t) \quad \text{Local}$$

$$D_\mu\psi(x,t) \longrightarrow e^{i\theta(x,t)}D'_\mu\psi'(x,t)$$

$$D_\mu = \partial_\mu - ieA_\mu(x,t)$$

$$A_\mu(x,t) \longrightarrow A_\mu(x,t)' + \tfrac{1}{e}\partial_\mu\theta$$

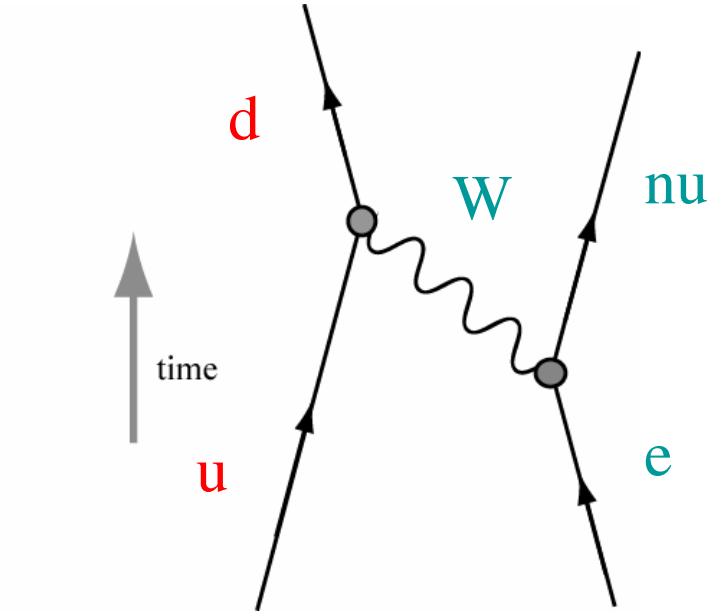
$$F_{\mu\nu} = \tfrac{i}{e}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Psi)^\dagger D^\mu\Psi - M^2\Psi^\dagger\Psi$$

$$\mathcal{L}(\Psi, A_\mu) = \mathcal{L}(\Psi', A_\mu{}') \quad \text{Gauge Invariance}$$

Standard Model

Weak Force (left-handed fields):



$SU(2)_L$ x U(1)

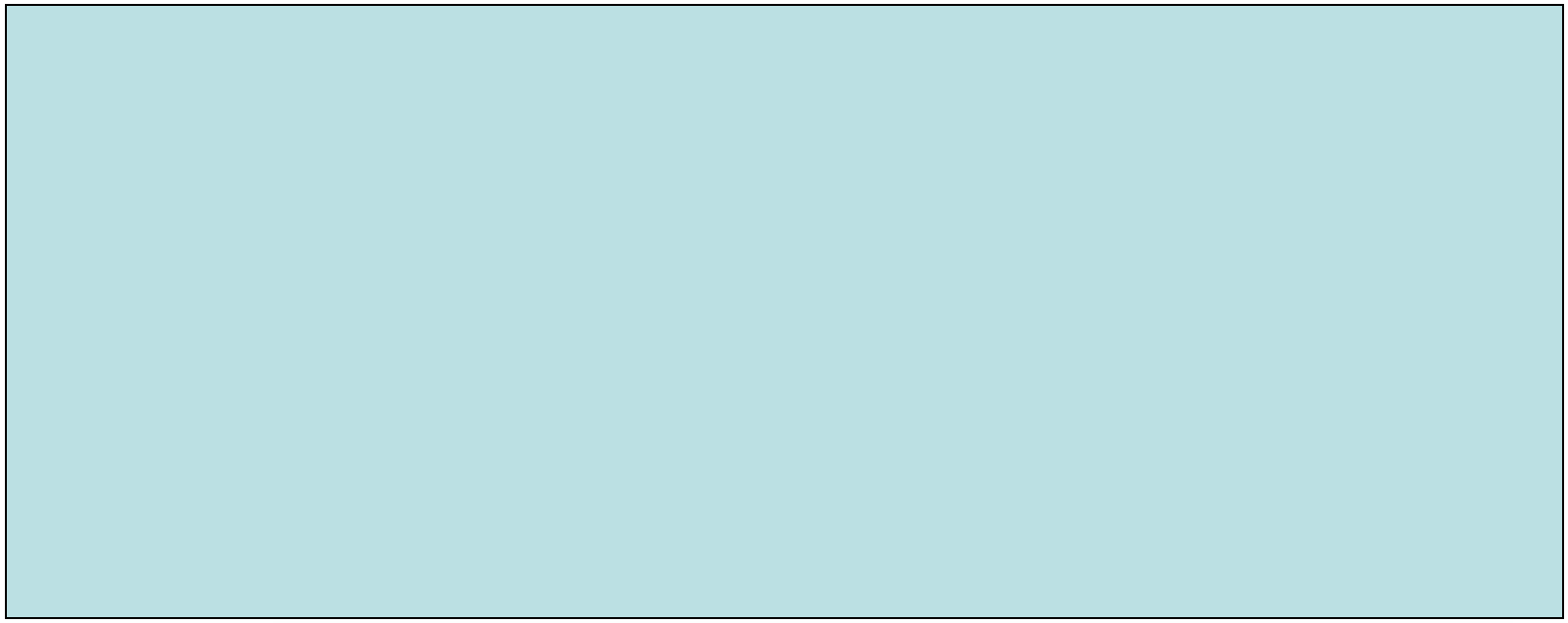
What gives rise to the masses of W and Z boson?

Can a gauge field have a mass and still have gauge symmetry?

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 A_\mu A^\mu$$

$$A_\mu(x,t) \longrightarrow A_\mu(x,t)' + \tfrac{1}{e}\partial_\mu\theta$$

$$\mathcal{L} \rightarrow -\tfrac{1}{4}F'_{\mu\nu}F^{\mu\nu\prime} + \tfrac{1}{2}m^2(A'_\mu + \tfrac{1}{e}\partial_\mu\theta)^2 \quad \text{NO!!!}$$



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$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 A_\mu A^\mu + \tfrac{1}{2}(\partial_\mu\phi)^2 - mA_\mu\partial^\mu\phi$$

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2(A_\mu - \tfrac{1}{m}\partial_\mu\phi)^2$$

massless scalar field

$$\mathcal{L} \rightarrow -\tfrac{1}{4}F'_{\mu\nu}F^{\mu\nu\prime} + \tfrac{1}{2}m^2(A'_\mu - \tfrac{1}{m}\partial_\mu\phi')^2$$

$$\phi \longrightarrow \phi' + \tfrac{m}{e}\theta \qquad \text{OK!!!}$$

London's Theory of Superconductivity

$$\mathcal{L} = -\frac{1}{2m_e}[(-i\vec{\nabla} - \frac{e}{c}\vec{A})\psi]^\dagger(-i\vec{\nabla} - \frac{e}{c}\vec{A})\psi + ...$$

"rigid wavefunction" $\leftrightarrow$ energy "gap" $\leftrightarrow$

$$\psi \sim \psi_0 e^{i\phi(\vec{x},t)} \qquad |\psi_0|^2 = n_e$$

$$\mathcal{L} \to -\frac{1}{2m_e}(\vec{\nabla}\phi - \frac{e}{c}\vec{A})|\psi_0|^2$$

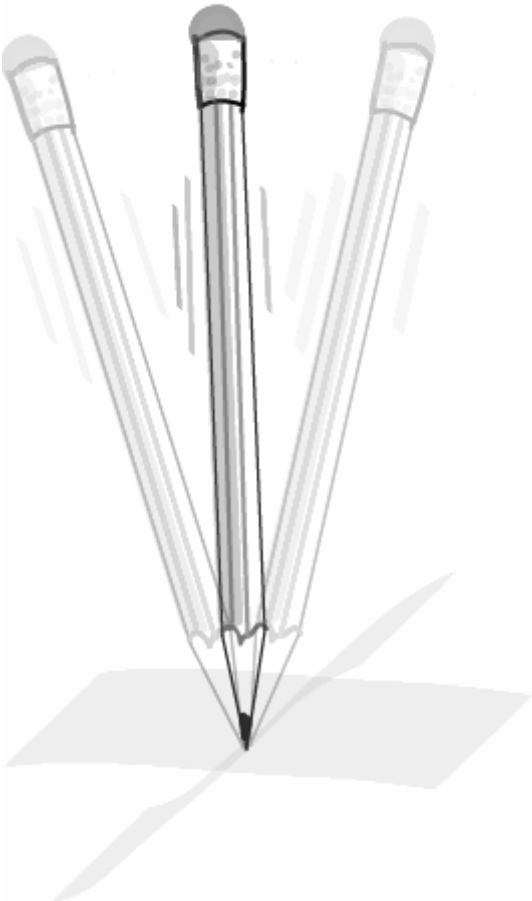
gauge invariant: $\vec{A} \to \vec{A}' + \frac{c}{e}\vec{\nabla}\theta, \quad \phi \to \phi' + \theta$

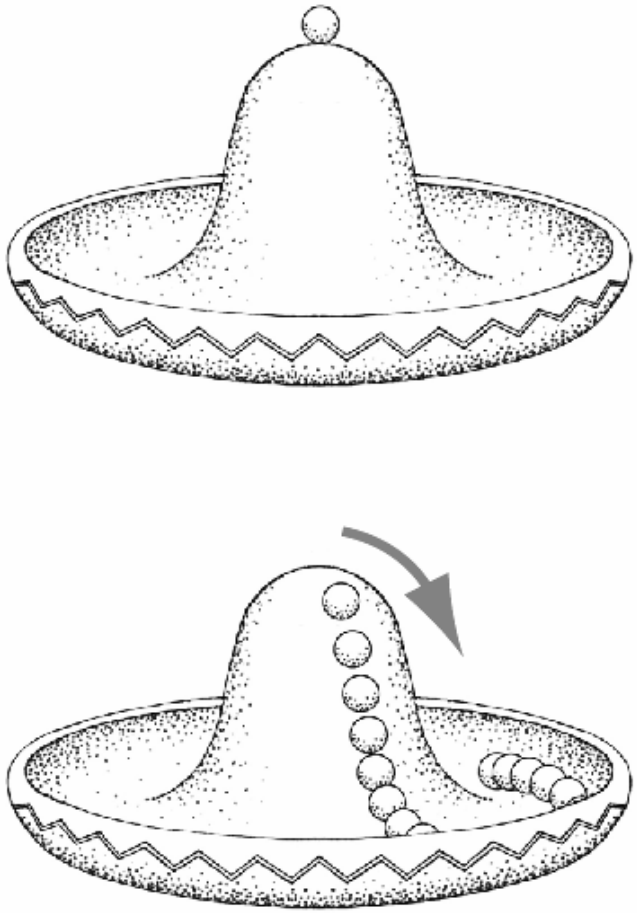
photon mass: $m^2 = \frac{e^2 n_e}{m_e c}$ $2\times$ too small!!!

BCS Theory: $\psi \to \psi(x)\psi(y)$ Cooper Pair

Where can we find a massless scalar?

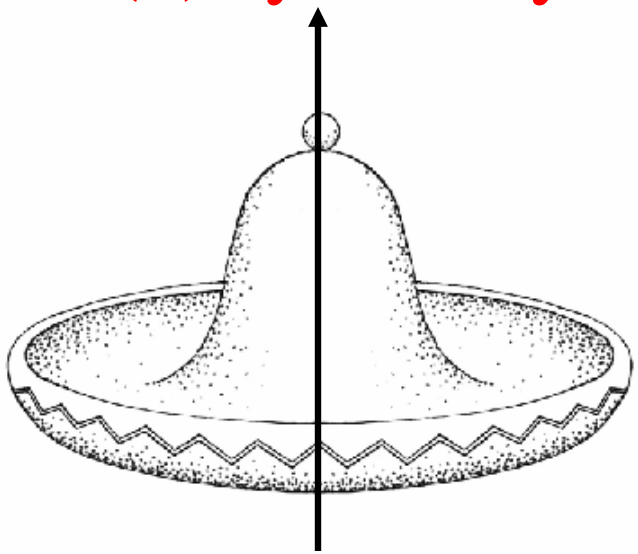
Spontaneous Continuous Symmetry Breaking

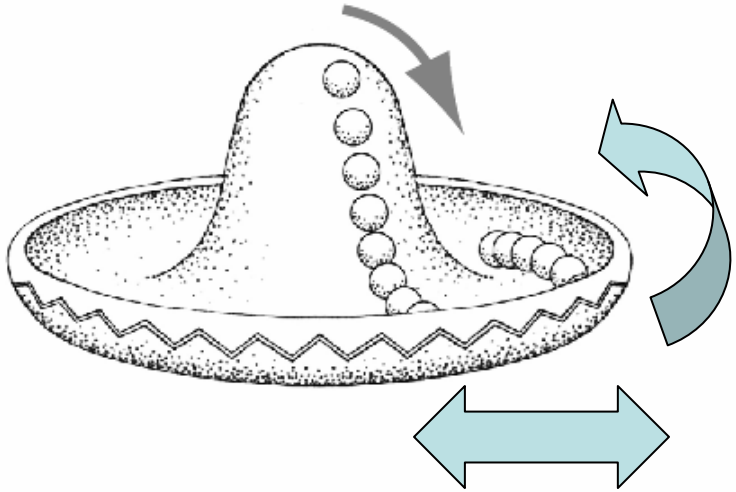




U(1) symmetry

Goldstone Theorem



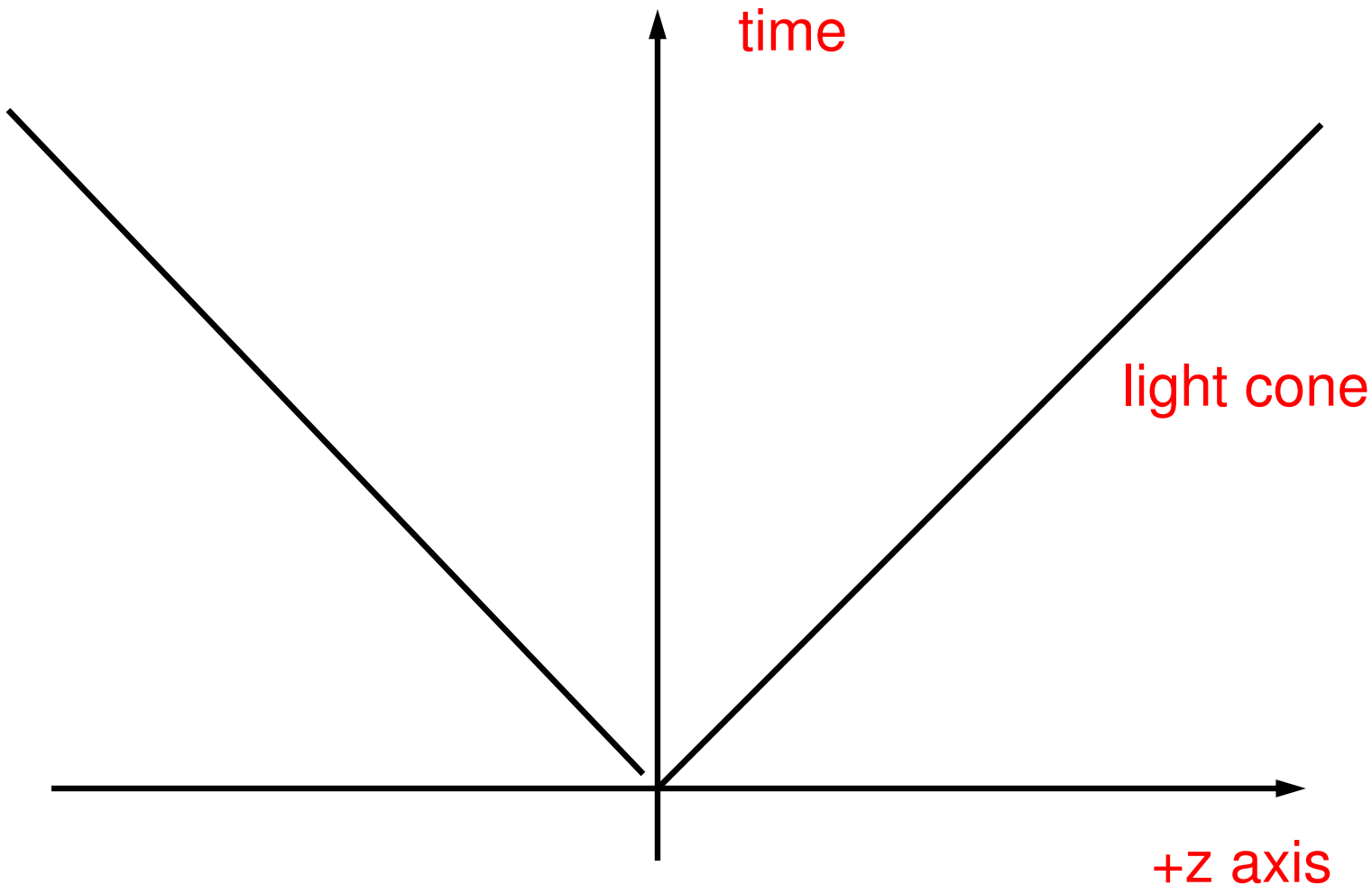


Nambu-Goldstone Boson:
angular motion with no cost
in energy ⇔ massless mode

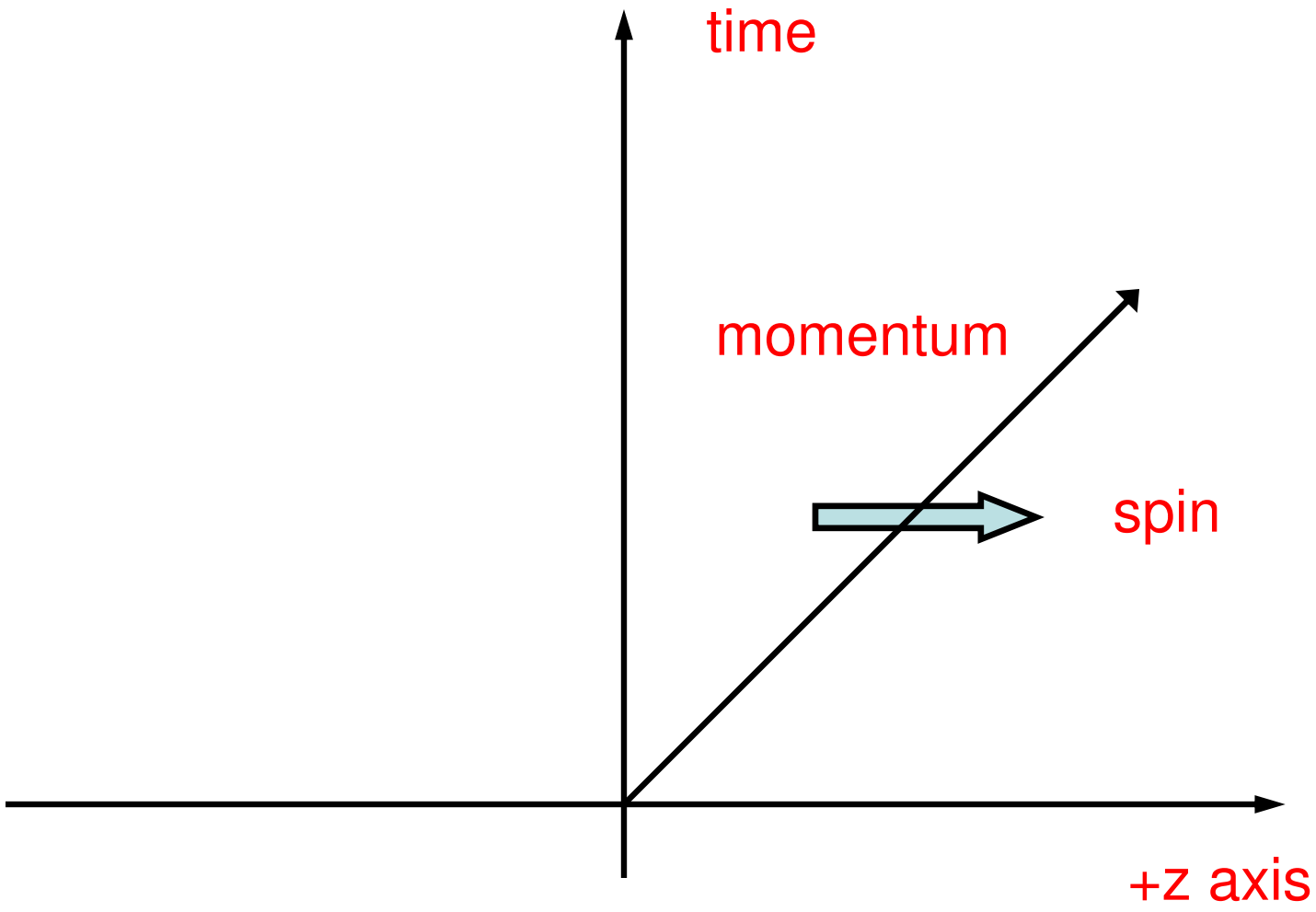
Higgs Boson: small radial oscillations ⇔ massive mode

Fermion Mass and Chirality

Fermion Mass and Chirality



A massless right-handed fermion
$s_z = +1/2$



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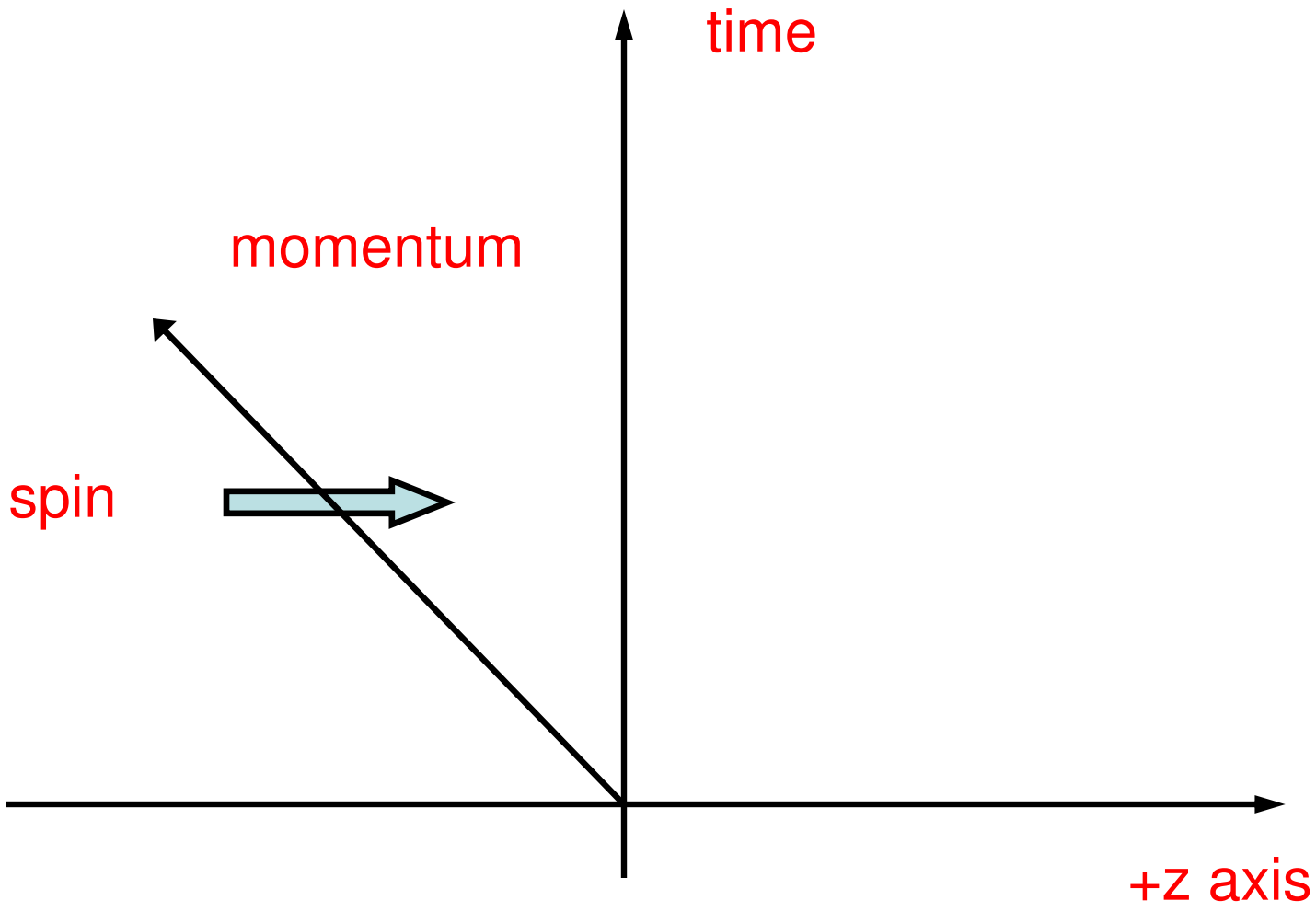
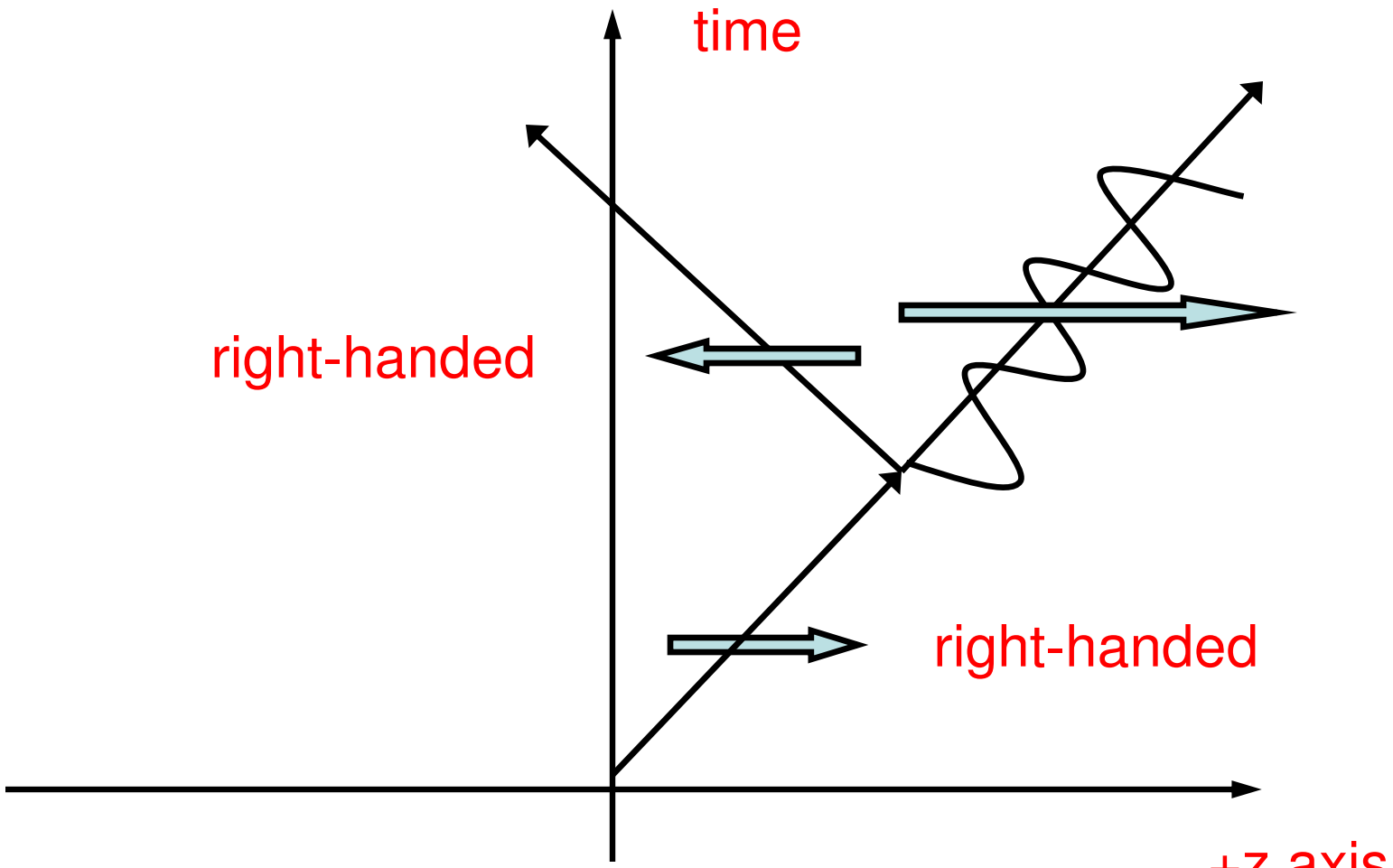
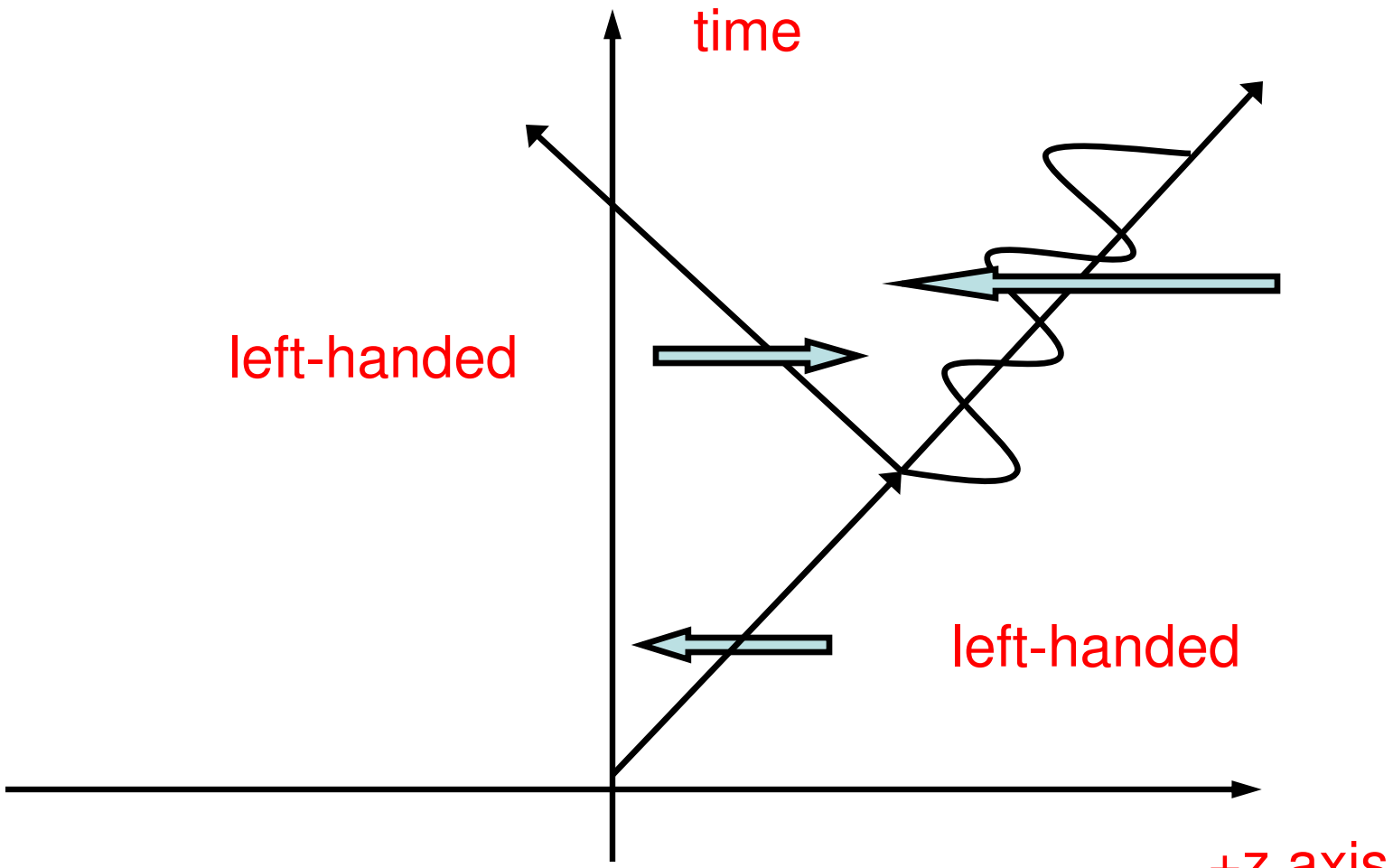


Couple electron to the photon



Chirality is conserved!

Couple electron to the photon

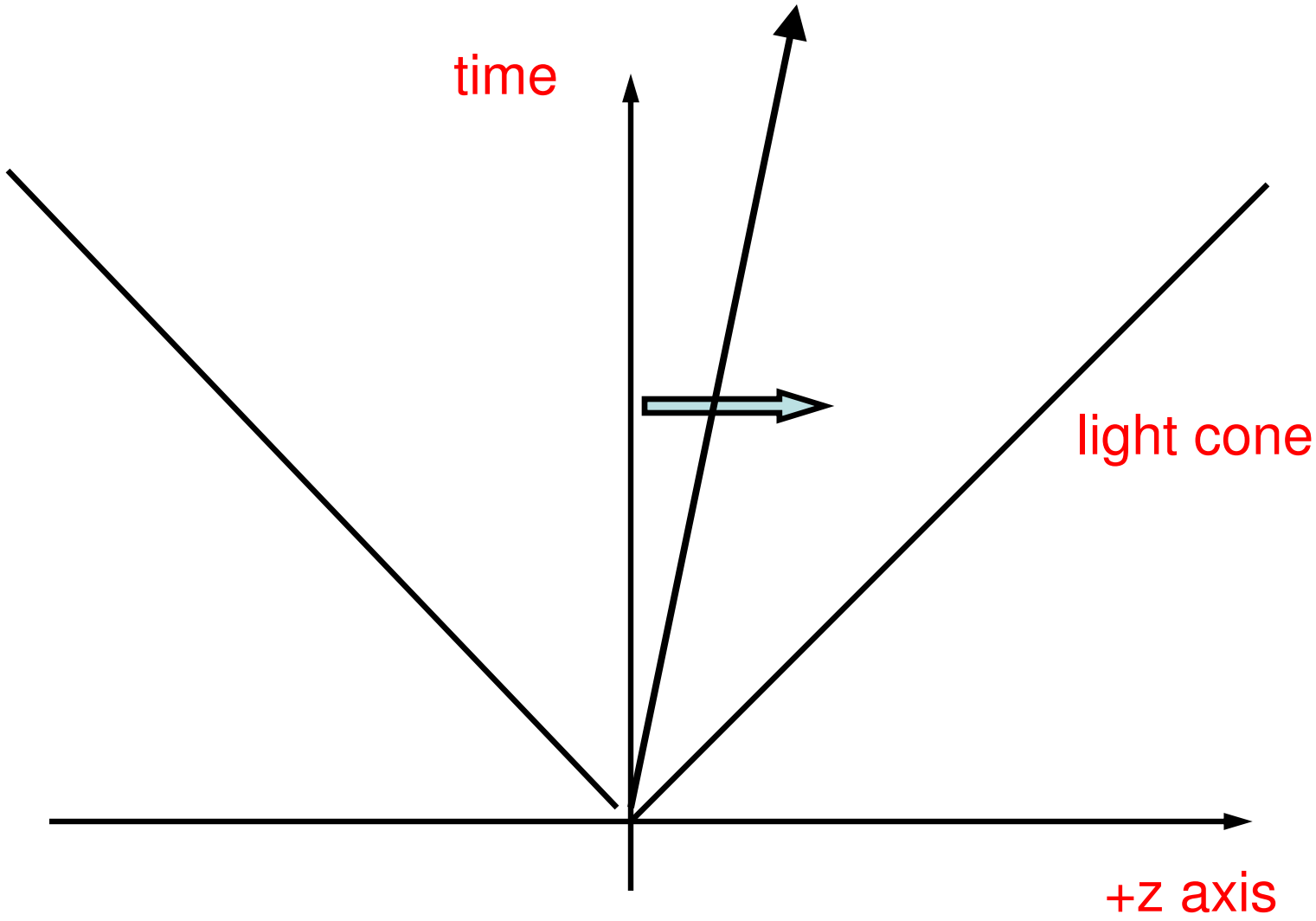


Chirality is conserved

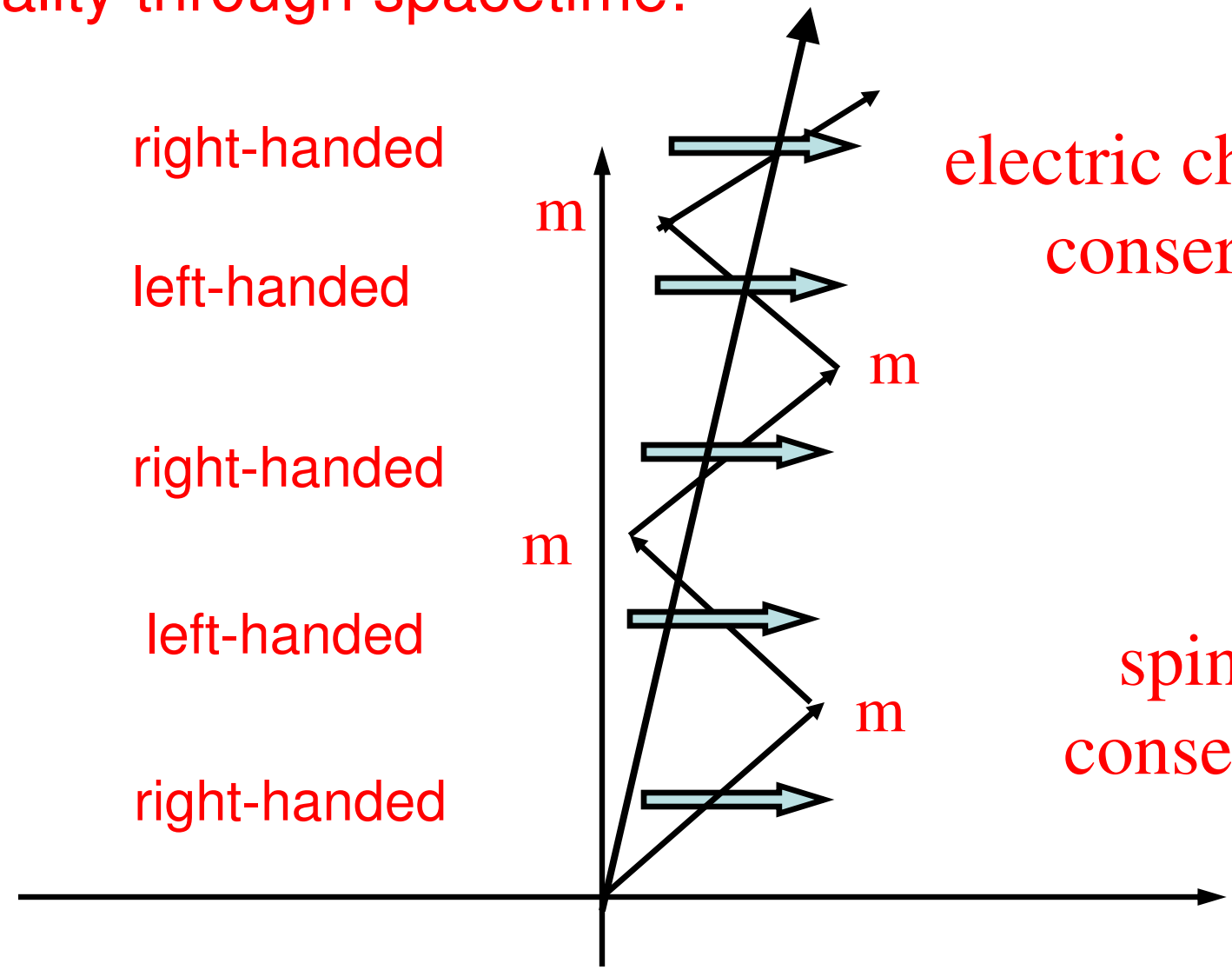
The left-handed and right-handed electrons have the same electric charge

QED is “vectorlike”

How do we make a massive electron?



A massive fermion oscillates in chirality through spacetime:



electric charge is conserved

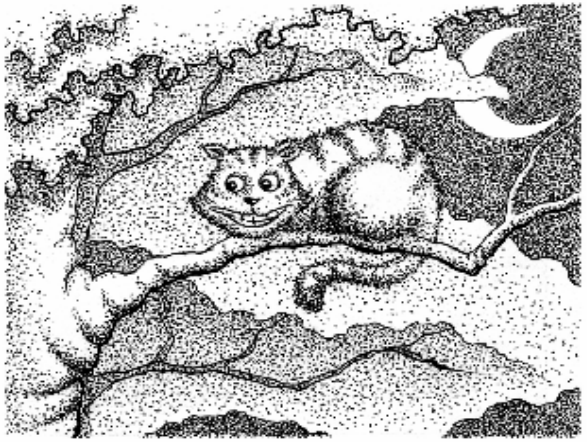
spin is conserved

Chirality is not conserved by mass!

Only left-handed fermions have electroweak charge and form doublets under SU(2)

Right handed's are "sterile" under SU(2)

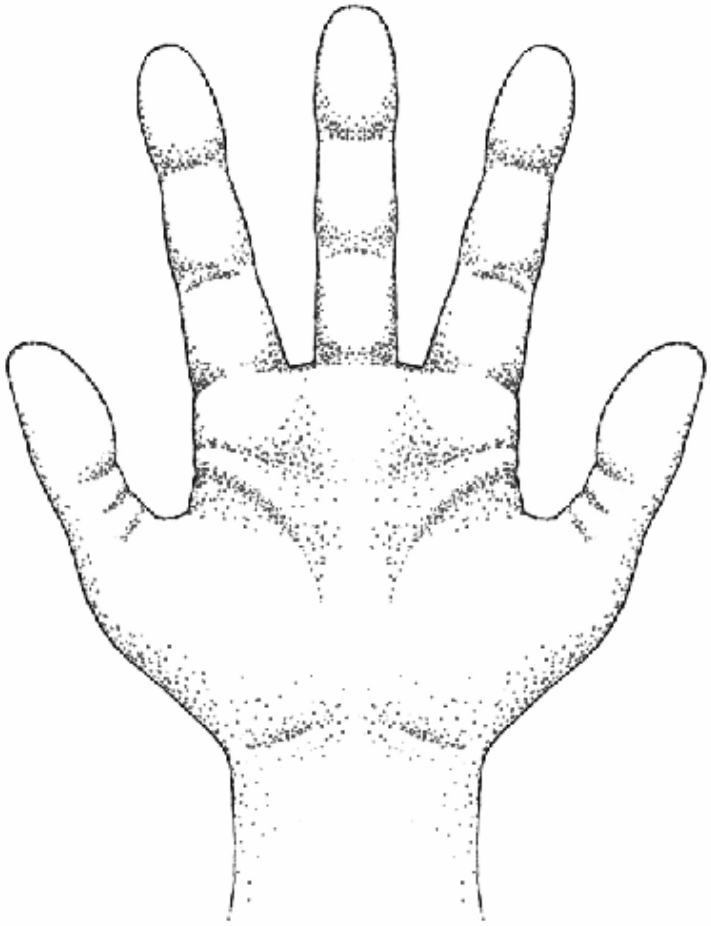
Reflection Symmetry



Tum

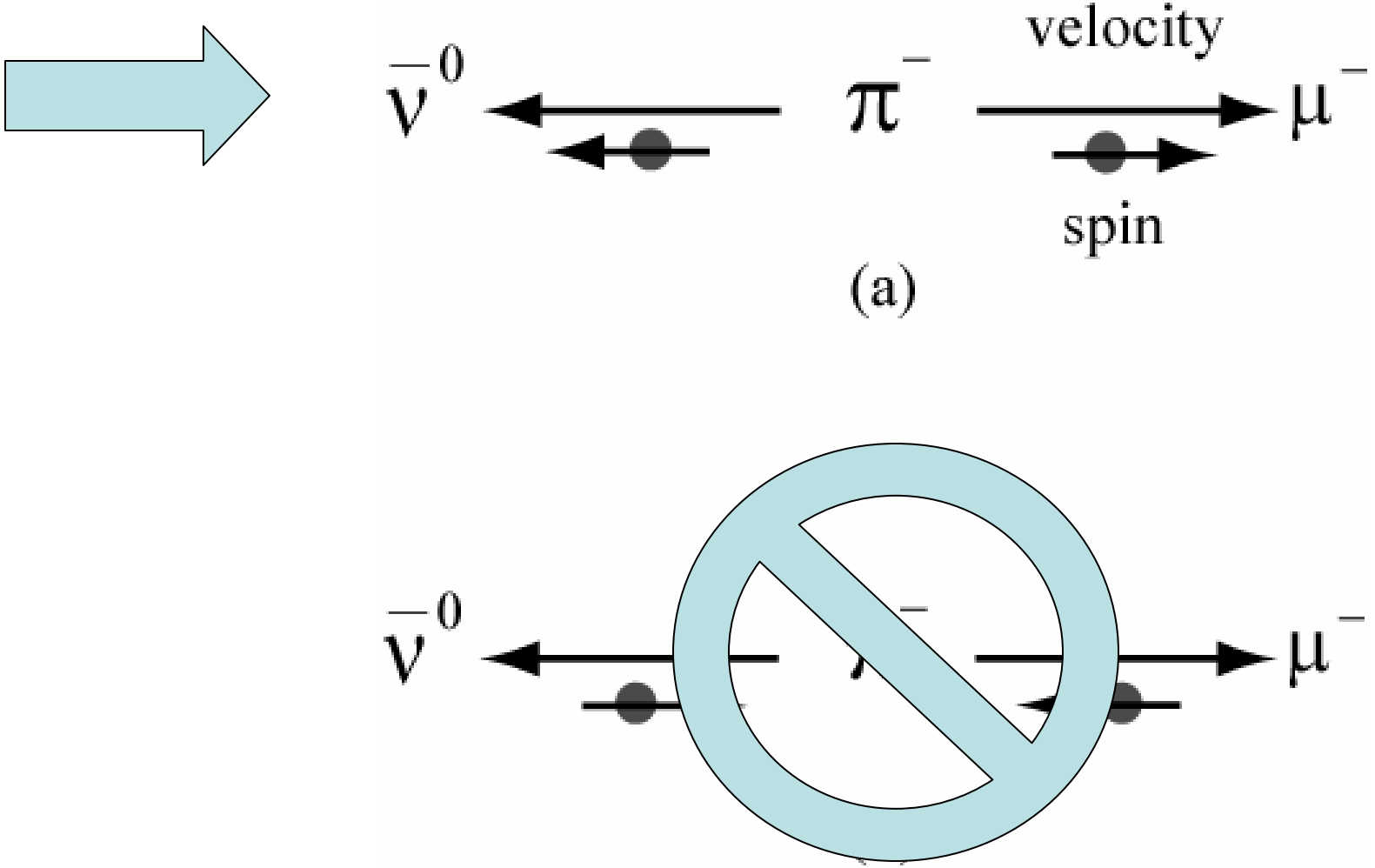
Mut

doublets

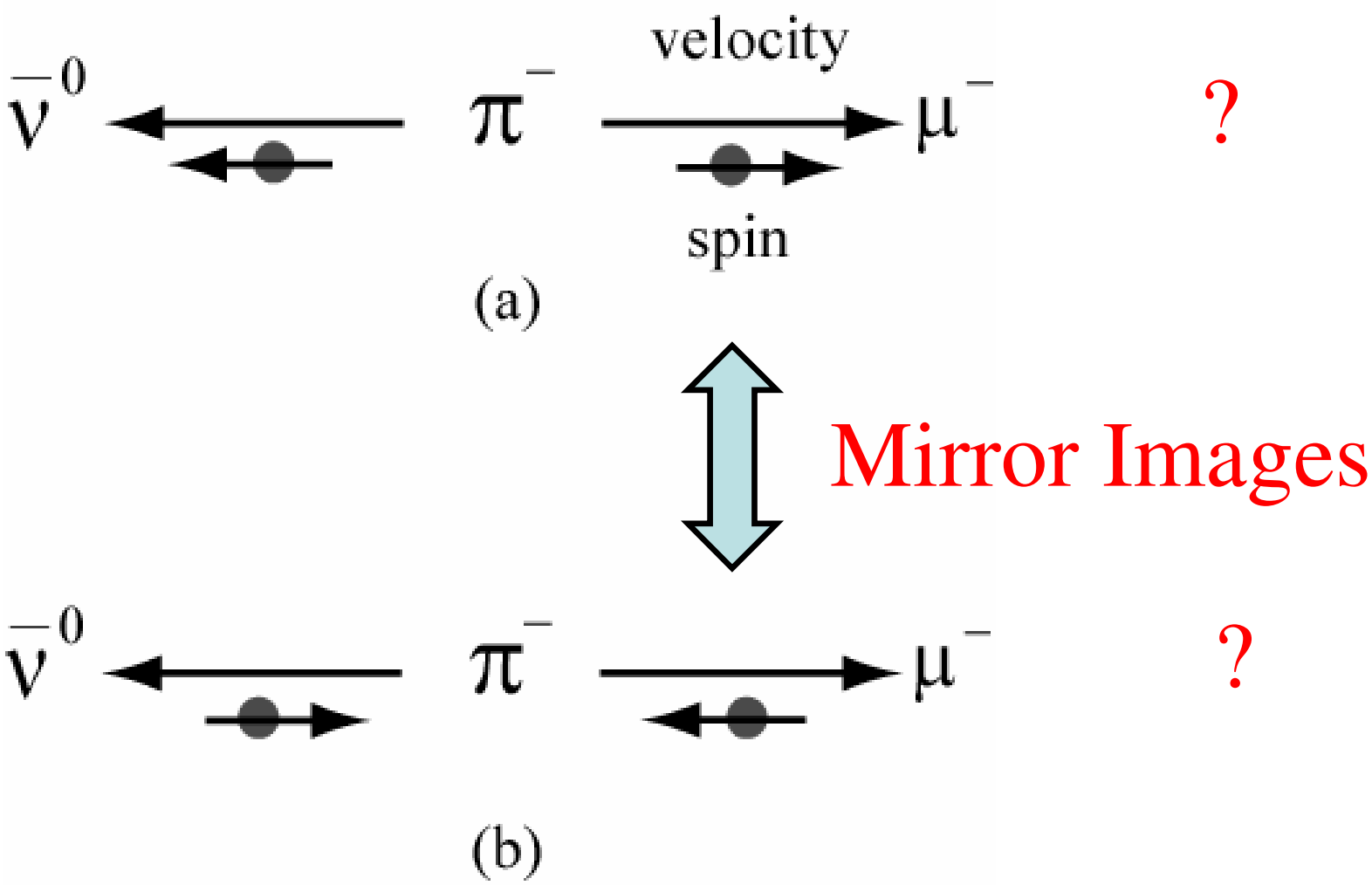


singlets

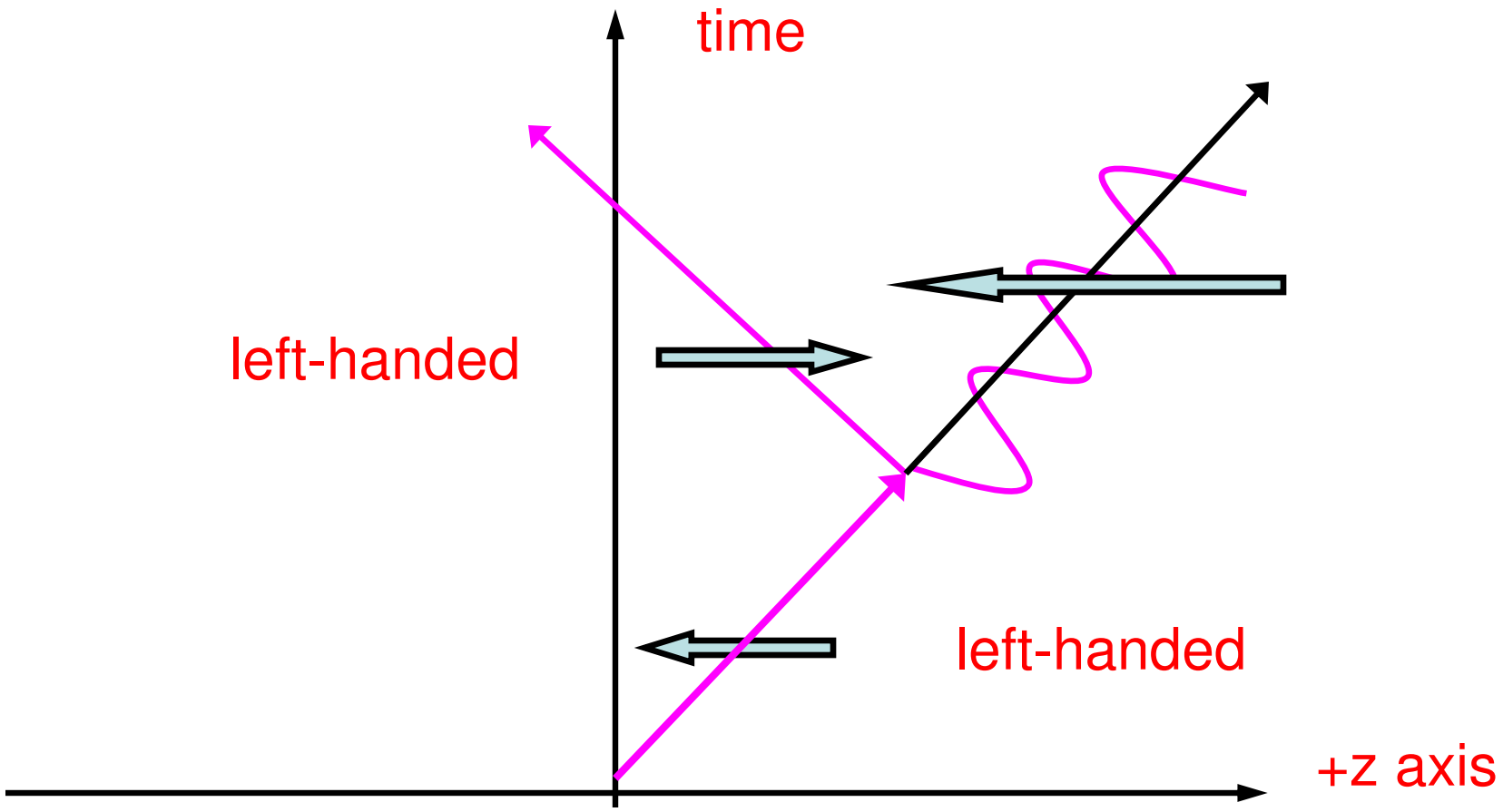
Parity is violated in pion decay: (Lederman)



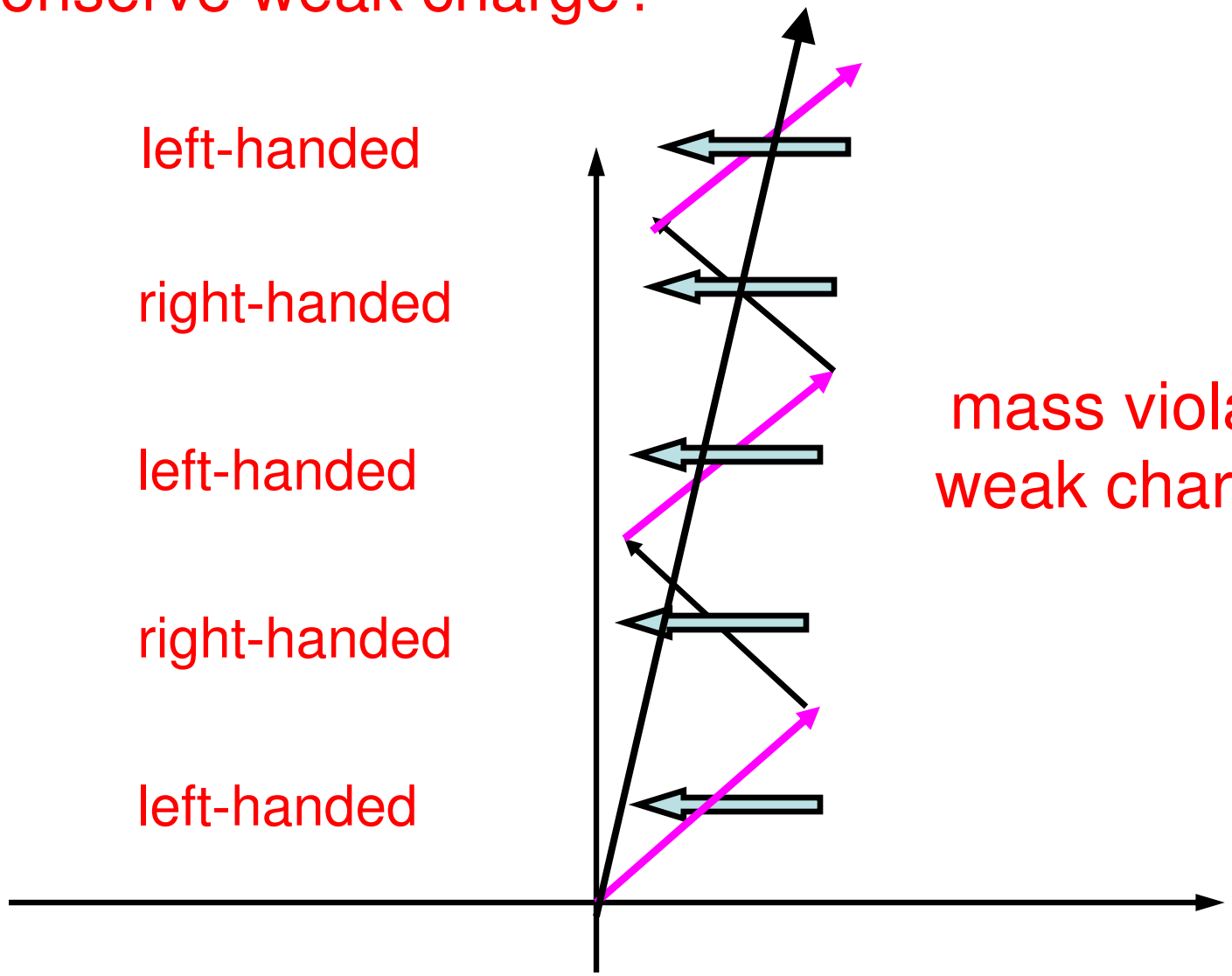
Helicity of decay products in pion decay:



Couple LH fermions to the W-boson



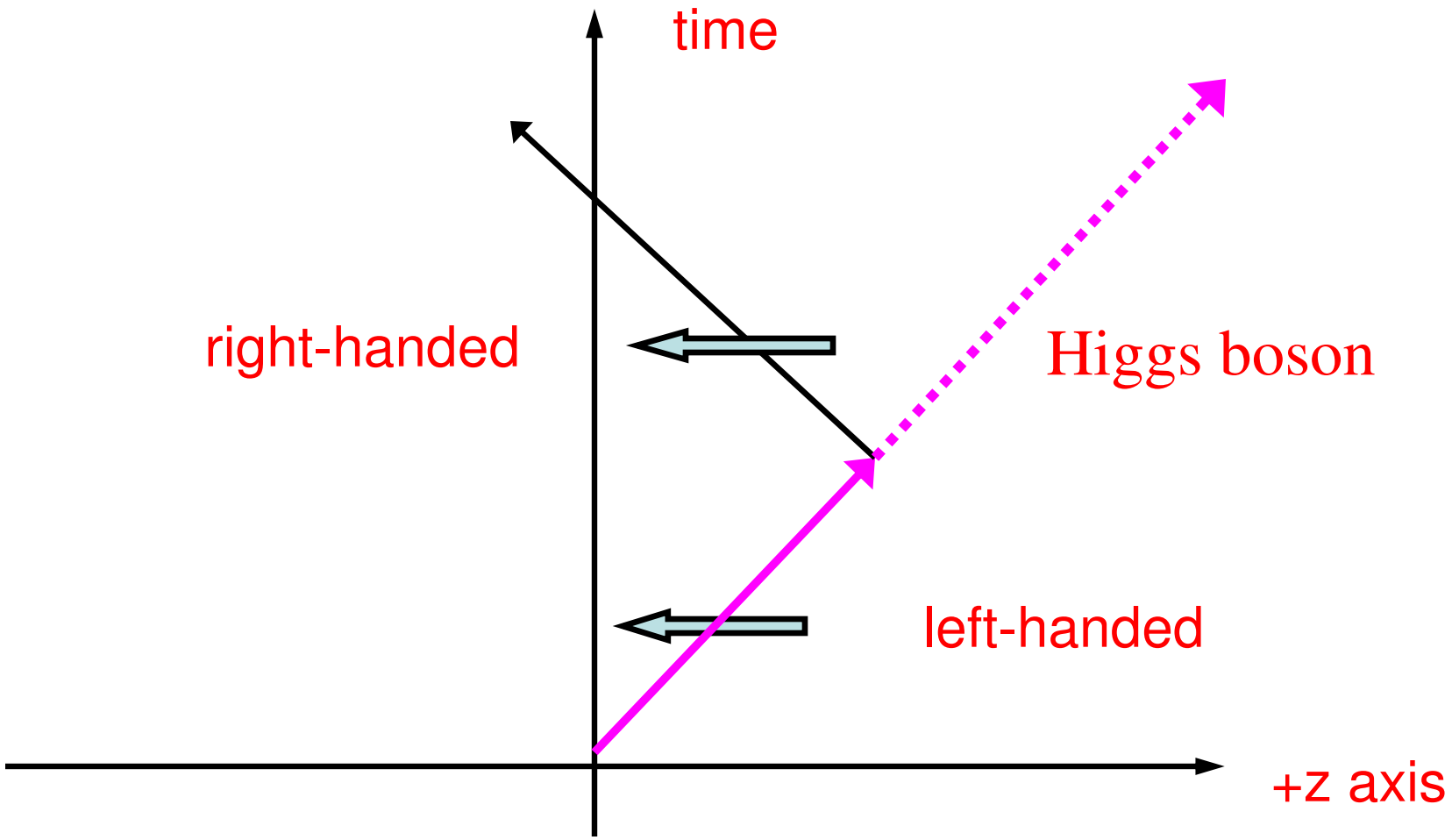
How do we make a massive fermion
but conserve weak charge?



mass violates
weak charge!!!

Mass Violates Electroweak Gauge Symmetry!!!

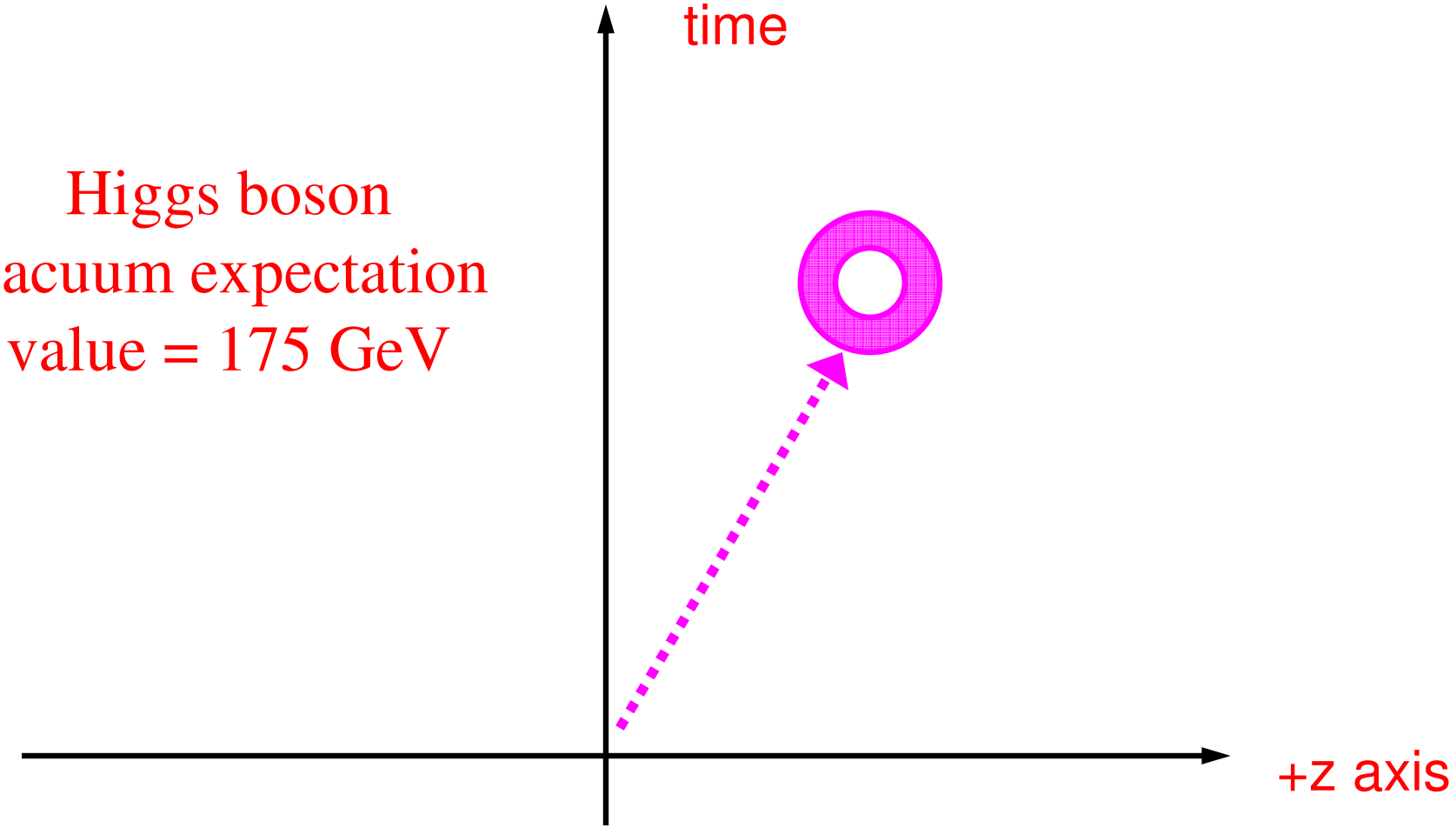
Couple to the Higgs



Weak charge is conserved !

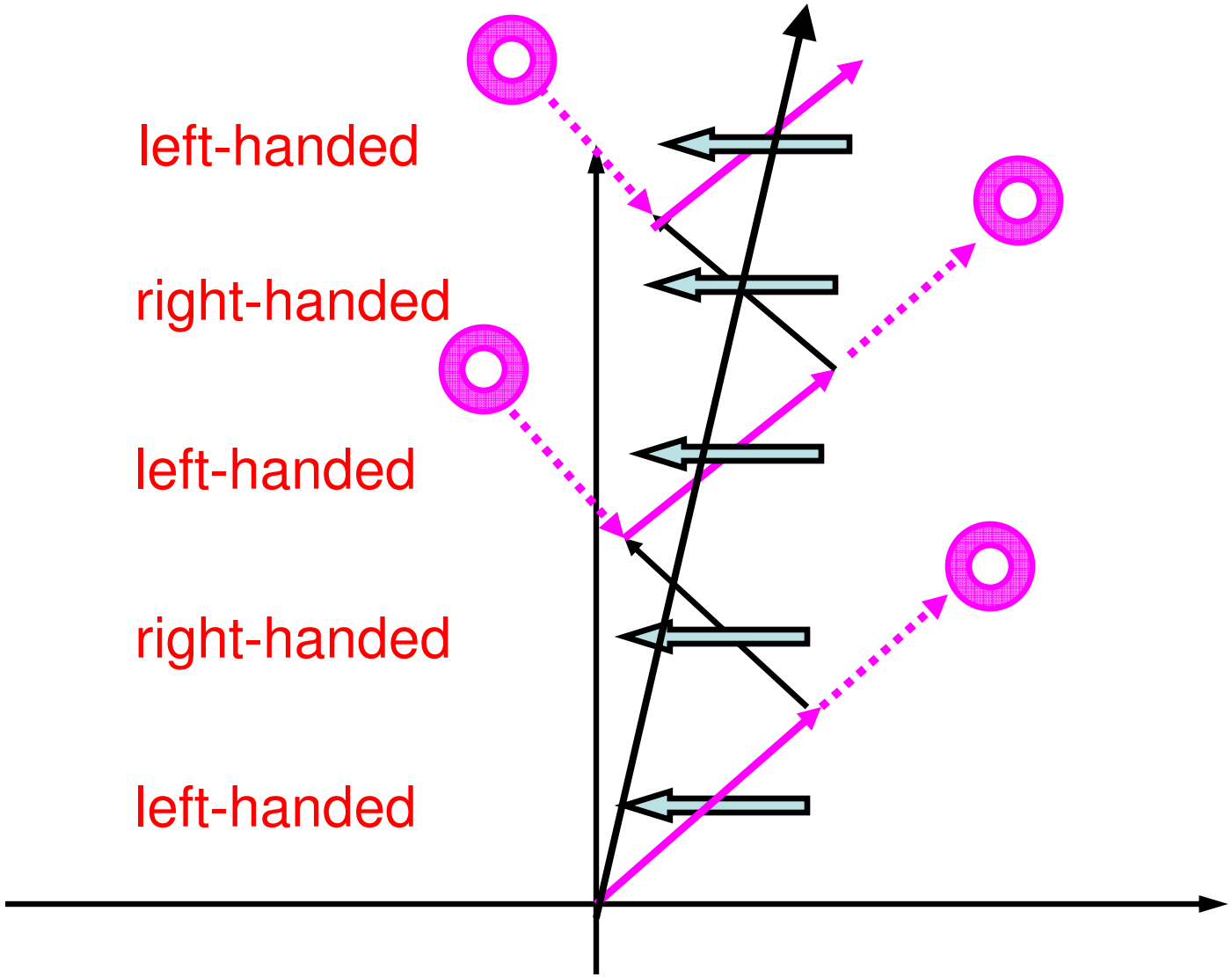
Higgs Boson Condenses in vacuum

Higgs boson
vacuum expectation
value = 175 GeV



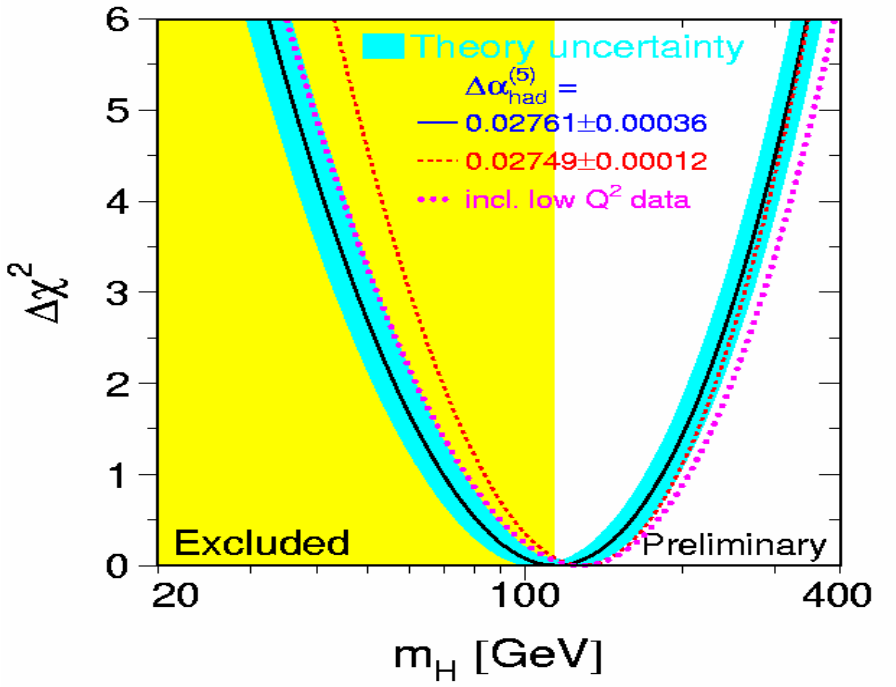
Weak charge is hidden in vacuum !!

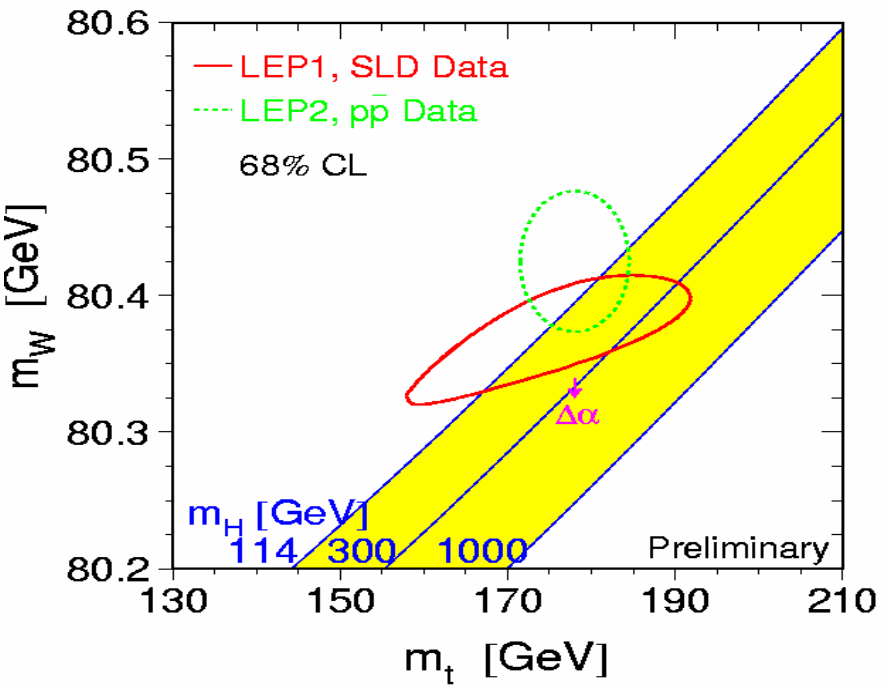
Fermion Masses in Electroweak Theory



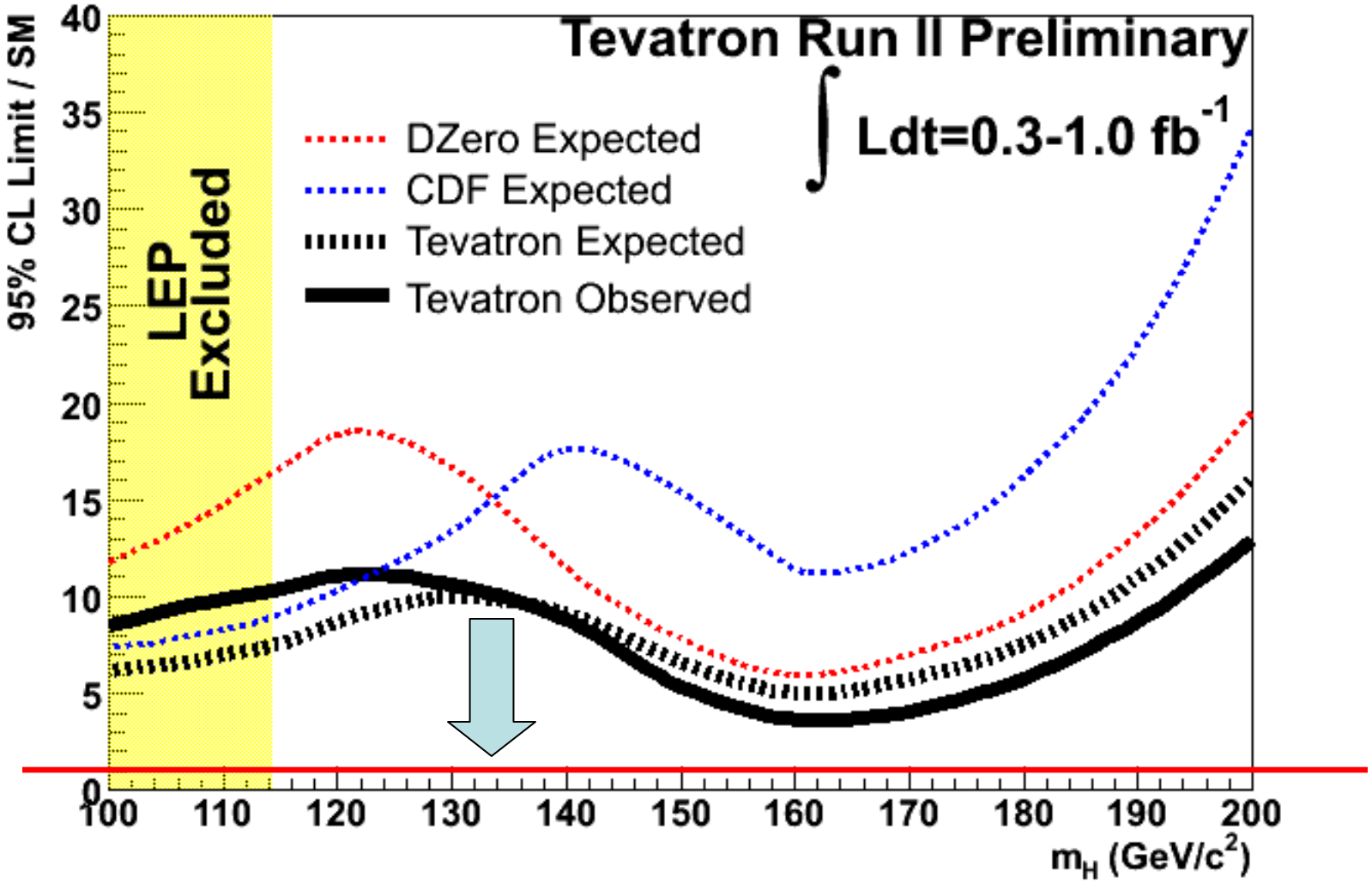
Fermion Mass requires Higgs to maintain Electroweak Gauge Symmetry!!!

Searching for the Higgs
(Vacuum Electroweak Superconductivity)





$114\ \text{GeV} < m_H < 260\ \text{GeV}$


Tevatron Run II Preliminary
∫ Ldt=0.3-1.0 fb-1
DZero Expected
CDF Expected
Tevatron Expected
Tevatron Observed
LEP Excluded
95% CL Limit / SM
0
5
10
15
20
25
30
35
40
100
110
120
130
140
150
160
170
180
190
200
mH (GeV/c2)

Extra Dimensions
Provide Organizing Principles:
A Context for the Higgs boson

Supersymmetry $\leftrightarrow$ Extra Fermionic Dimensions:

- (Graded) extension of Lorentz Group
- New "rotations" and "translations:" fermion $\leftrightarrow$ boson.
- Broken SUSY lifts partners; (*SUSY partners are analogues of Kaluza-Klein modes*).
- High energy Theory: Superstring Theory.
- Low energy Effective Theory: Perturbative Minimal Supersymmetric Standard Model.

Supersymmetry: Boson  Fermion

Name	Spin	Superpartner	Spin
Graviton	2	**Gravitino**	3/2
Photon	1	**Photino**	1/2
Gluon	1	**Gluino**	1/2
$W^{+,-}$	1	**Wino$^{+,-}$**	1/2
Z^0	1	**Zino**	1/2
Higgs	0	**Higgsino**	1/2

Name	Spin	Superpartner	Spin
Electron	1/2	**Selectron**	0
Muon	1/2	**Smuon**	0
Tau	1/2	**Stau**	0
Neutrino	1/2	**Sneutrino**	0
Quark	1/2	**Squark**	0

SUSY Higgs

In SUSY, two Higgs doublets are needed.
8 DoF, 3 eaten up by the $W^{\pm}$ and Z
=> 5 Higgs fields: h^0, A^0, H^0, $H^{\pm}$

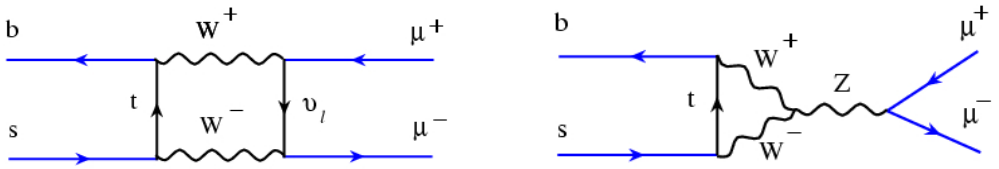
Tree level connection between Higgs masses and gauge boson masses:

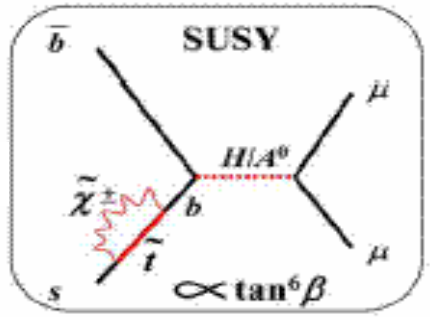
$$M^2_{H\pm} = M^2_A + M^2_W$$

$$\Rightarrow \quad M_{H\pm} > M_W \qquad M_h < M_Z$$

Probe in Flavor Physics:

Indirect constraint: $B_S \rightarrow \mu\mu$

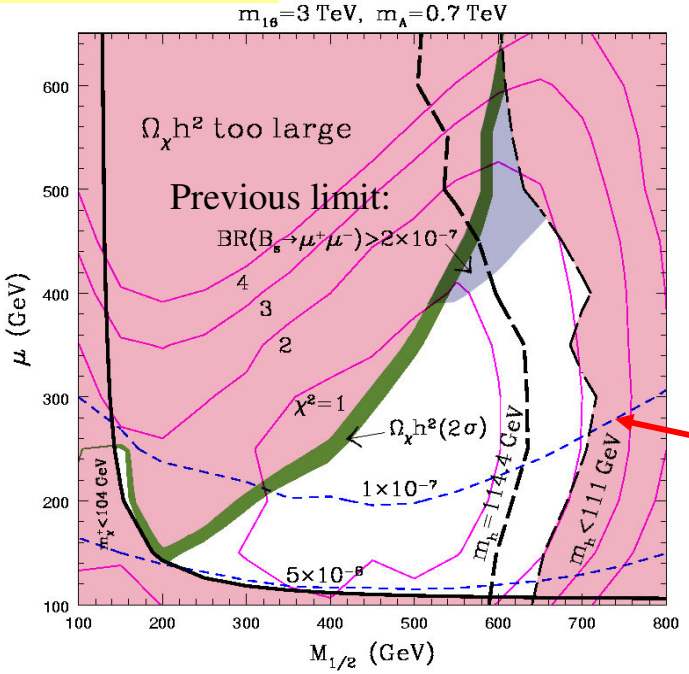




Rare decay, SM branching frac ~10^{-9}
→Loop diagrams with sparticles (or direct decay if RPV) enhance orders of magnitude

Important at high tanβ

hep-ph/0507233

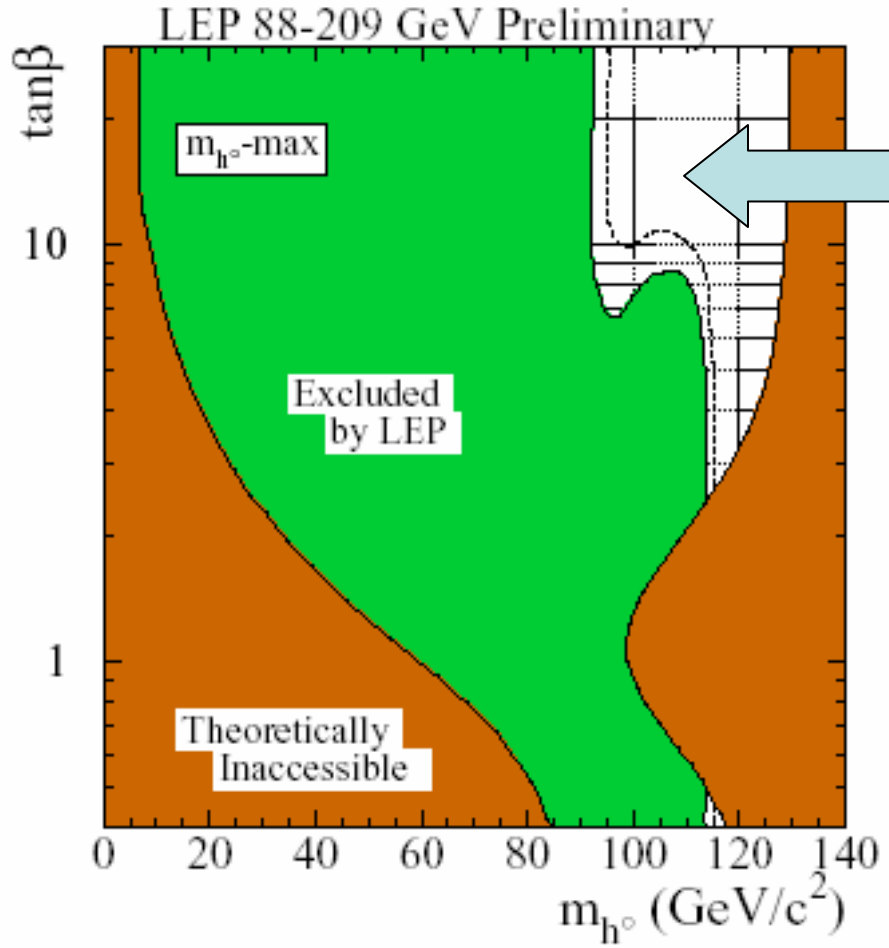


- Look for excess of μμ events in B_s and B_d mass windows
- Background estimation: linear extrapolation from sidebands
- Results compatible with SM backgrounds

$Br(B_s \rightarrow \mu\mu) < 1.0 \times 10^{-7}$ @ 95%CL

--- Closing in on SUSY! ---

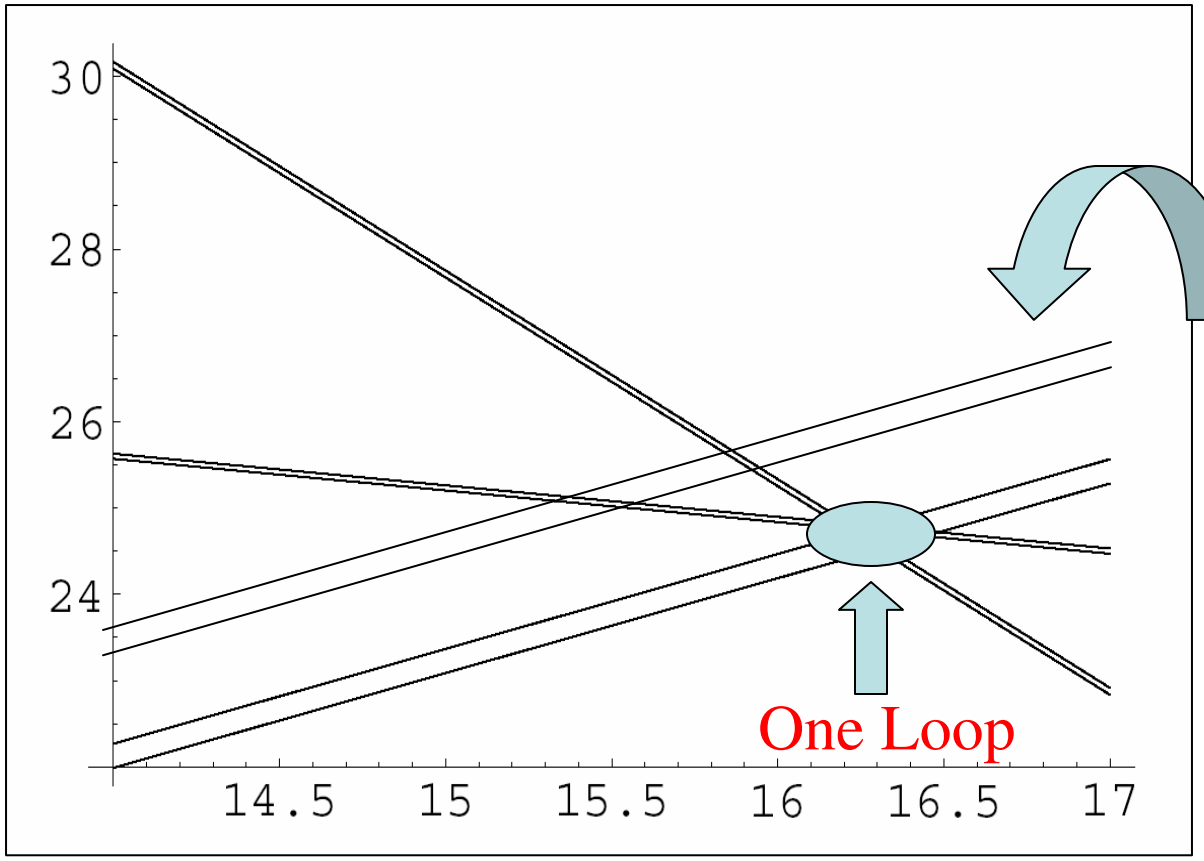
MSSM is squeezed



Fine Tuning
< few % precision

Does MSSM Unify?

S. Raby PDG, 2002



4% discrepancy = 3 sigma discrepancy predicting $\alpha(M_Z)$

Supersymmetry is still our best operational hypothesis !!!

- Perturbative -> Calculable
- Unification works pretty well
- rich spectroscopy to probe, LHC, ILC and flavor physics
- fits into string theory envelope
- Offers a hierarchy custodial symmetry

But....

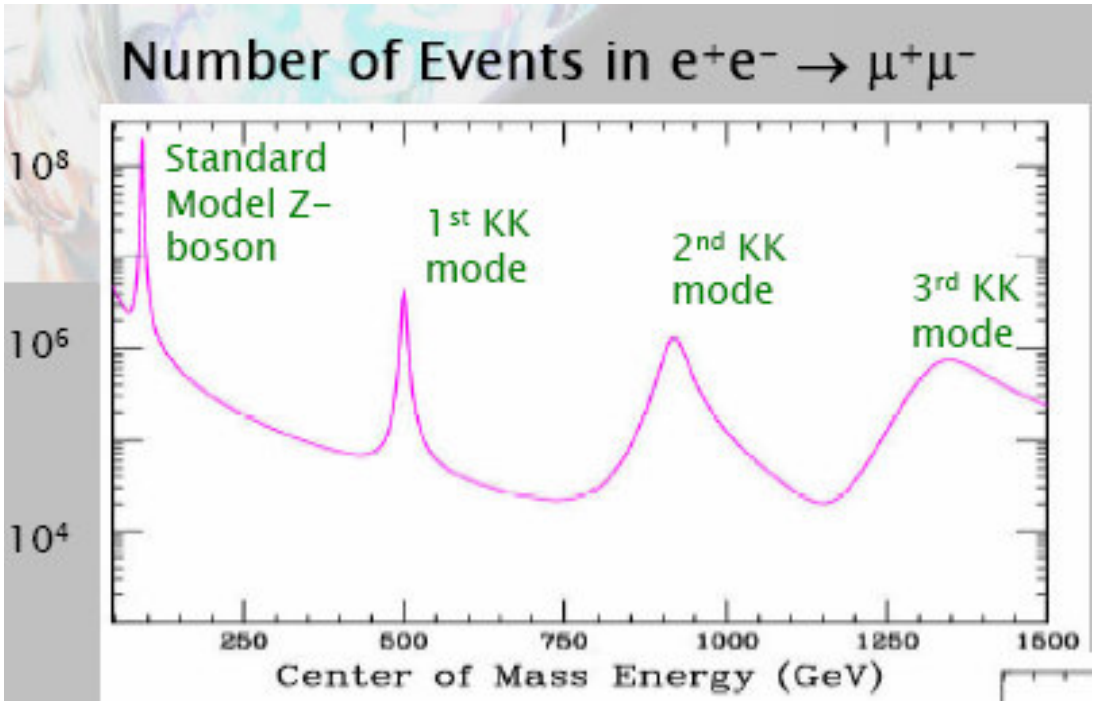
It (probably) won't be MSSM !!!

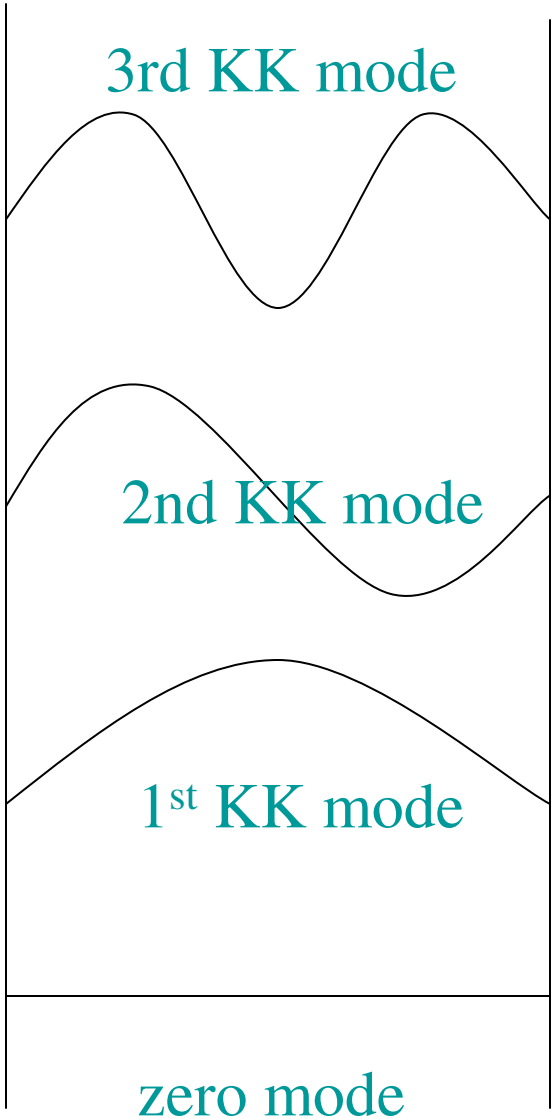
If Not SUSY then What?

Conventional Extra Dimensions?

- Conventional extension of Lorentz Group.
- New "rotations" and "translations."
- Broken Lorentz Invariance by compactification: *Kaluza-Klein modes.*
- High energy Theory: Superstring Theory?
- Extra dimensions can be associated with the electroweak scale, e.g., ~ 1 TeV, rather than M_{Planck} ?

Kaluza-Klein Modes



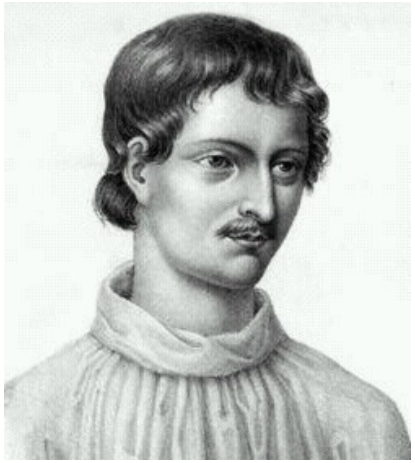


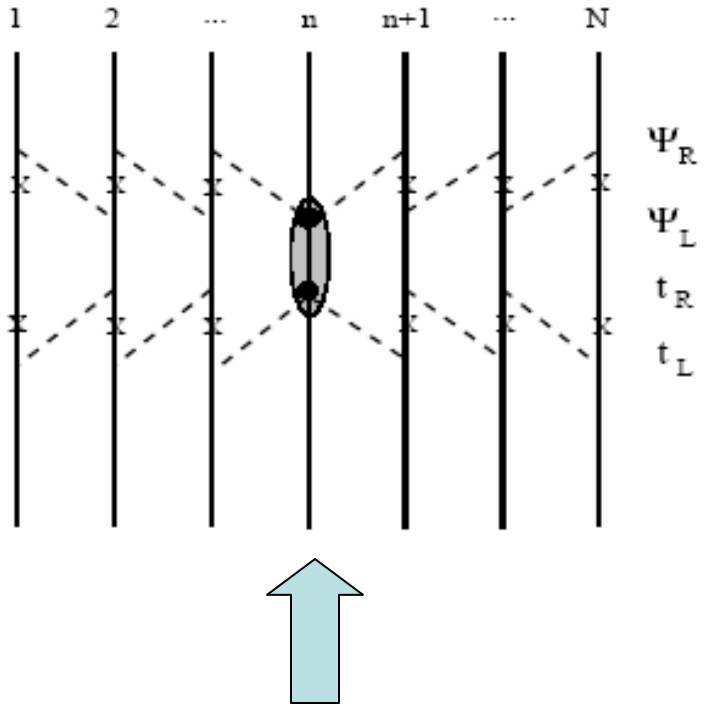
Composite Higgs
"Cooper Pair" of L-R Chiral Fermions

Introduce two flavors of fermions:

$$\Psi = \begin{pmatrix} t \\ b \end{pmatrix}_L , \quad t \sim t_R$$

All chiral zero-modes on brane n with strong couplin



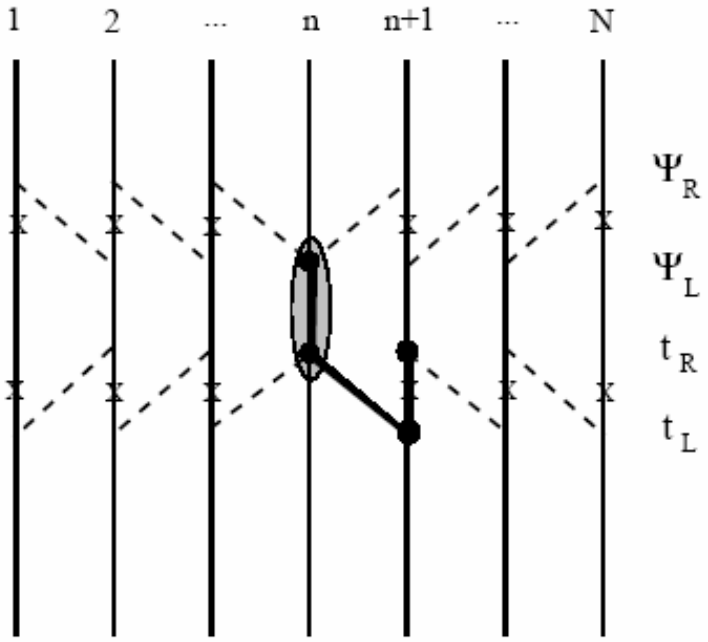


Higgs is composite: $H \sim \begin{pmatrix} \bar{t}_L t_R \\ \bar{b}_L t_R \end{pmatrix}$

Top Quark Seesaw Model

Some links to nearest neighbors may not be completely decoupled.
Background field renormalization.

Mixing with heavy KK fermions occurs in addition to the chiral dynamics on brane n.



Top Seesaw model is prime example of new organizing principle leading to completely different paradigm for the Higgs mechansm!

Mass matrix for $t-\chi$ system is,

$$-\left(\overline{t_L}\ \ \overline{\chi_L}\right)\begin{pmatrix} 0 & \mu \approx 600\ GeV \\ m \approx 1\ TeV & M \approx 4\ TeV \end{pmatrix}\begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{h.c.}$$

Diagonalized:

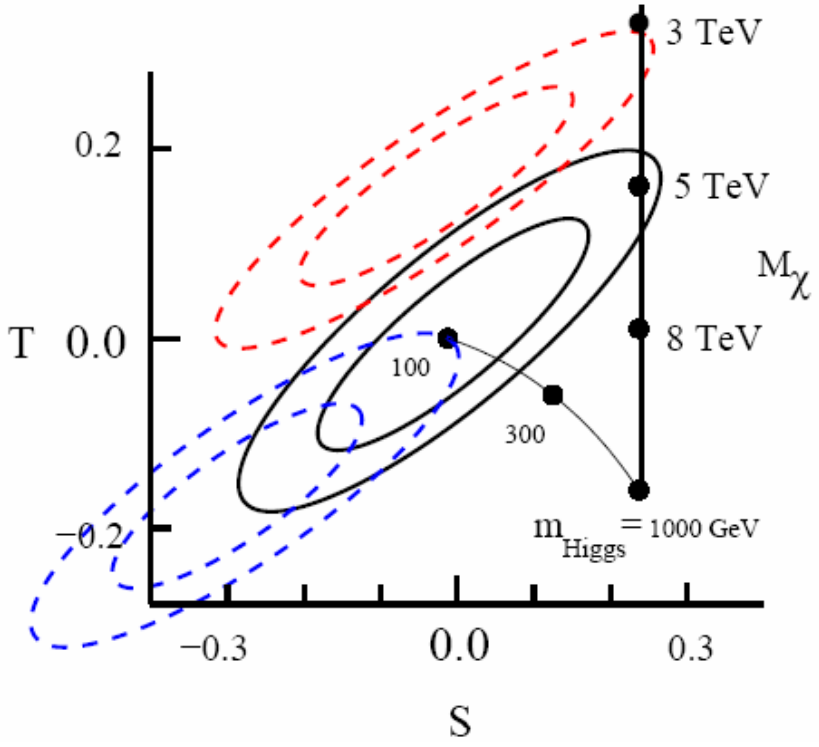
$$\begin{aligned} m_t &\approx \frac{\mu m}{M} \\ m_\chi &\approx M \end{aligned}$$

1998: Top Seesaw DOA (outside of the S-T ellipse $\sim 4\sigma$. (Chivukula, Dobrescu, Georgi, Hill)

1999: S-T error ellipse shifts along major axis towa upper right (predicted by the theory!).

2001: Inconsistencies in data; keep only leptons → Top Seesaw consistent and SM ruled out at $\sim 2\sigma$!!!

Theory consistent for natural values of its paramete at the 2σ level (He, CTH, Tait)



Mass is always associated with conformal symmetry breaking

Broken Scale Symmetry

The Origin of the Nucleon Mass

(aka, the visible mass in the universe)

Murraypalooza July 2005

Gell-Mann and Low:

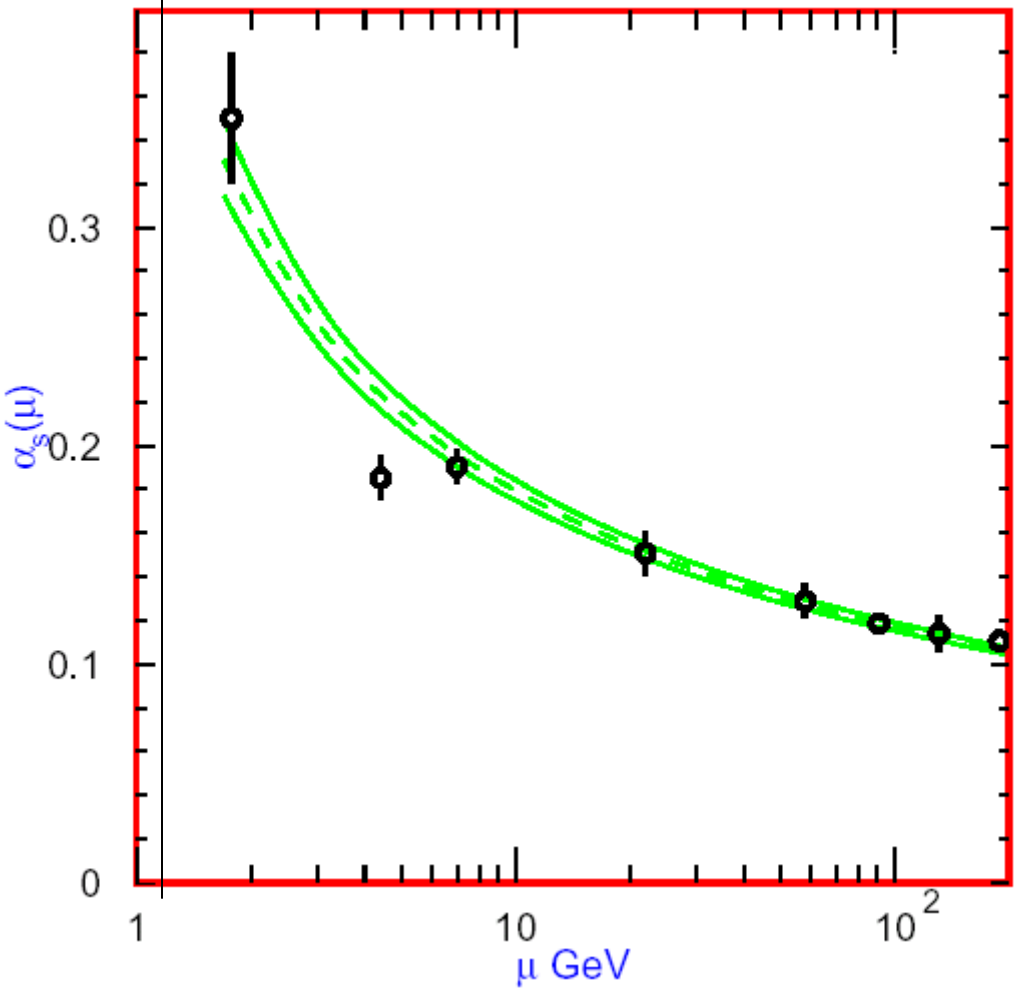
$$\frac{dg}{d\ln\mu} = \beta(g)$$

Gross, Politzer and Wilczek:

$$\beta(g) = \hbar\beta_0 g^3 \qquad \text{where} \qquad \beta_0 = -\frac{1}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)$$

$$\frac{\Lambda_{QCD}}{M_0} = \exp\left(\frac{1}{2\hbar\beta_0 g_0^2}\right)$$

$\Lambda_{MS} = 217^{+25}_{-23}$ MeV



$$\beta(g) = \mu \frac{\partial g}{\partial \mu}.$$

$$\alpha_s(k^2) \equiv \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{\beta_0 \ln(k^2/\Lambda^2)}$$

A Puzzle: Murray Gell-Mann lecture ca 1975

$$S_\mu = x^\nu T_{\mu\nu}$$

$$\partial_\mu S^\mu = T^\mu_\mu$$

$$T_{\mu\nu} = \mathrm{Tr}(G_{\mu\rho}G^\rho_\nu) - \frac{1}{4} g_{\mu\nu}\,\mathrm{Tr}(G_{\rho\sigma}G^{\rho\sigma})$$

$$\partial_\mu S^\mu = T^\mu_\mu = \mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{4}{4}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) = 0 \quad !???$$

QCD is scale invariant!!!???

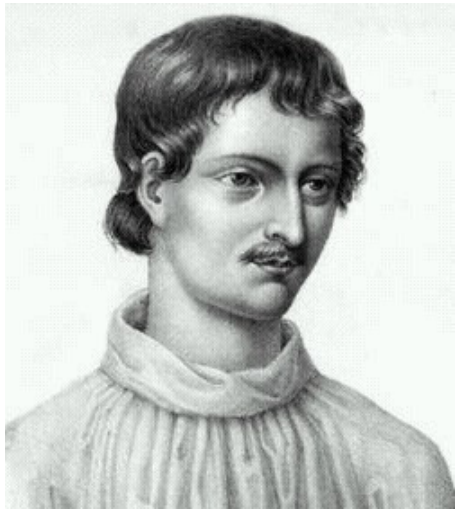
Resolution: The Scale Anomaly

$$\partial_\mu S^\mu = \frac{\beta(g)}{g} \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} = \mathcal{O}(\hbar)$$

Origin of Mass in QCD = Quantum Mechanics

A very heretical conjecture:

All mass scales in physics are intrinsically quantum mechanical and associated with scale anomalies. The $\hbar \to 0$ limit of nature is exactly scale invariant.

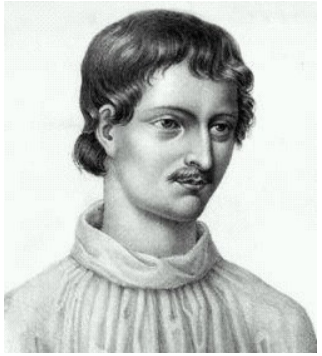


"Predictions" of the Conjecture:

We live in D=4! $$T^{\mu}_{\mu} = \mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \frac{D}{4}\,\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}$$

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated



Testable in the Weak Interactions?

Weyl Gravity in D=4 is QCD-like: $$\frac{1}{h^2}\sqrt{-g}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)$$

Is the Higgs technically natural?

On naturalness in the standard model.
William A. Bardeen (Fermilab) . FERMILAB-CONF-95-391-T, Aug 1995. 5pp.

Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill (Fermilab) . **hep-th/0510177**

Symmetry Principles Define Modern Physics

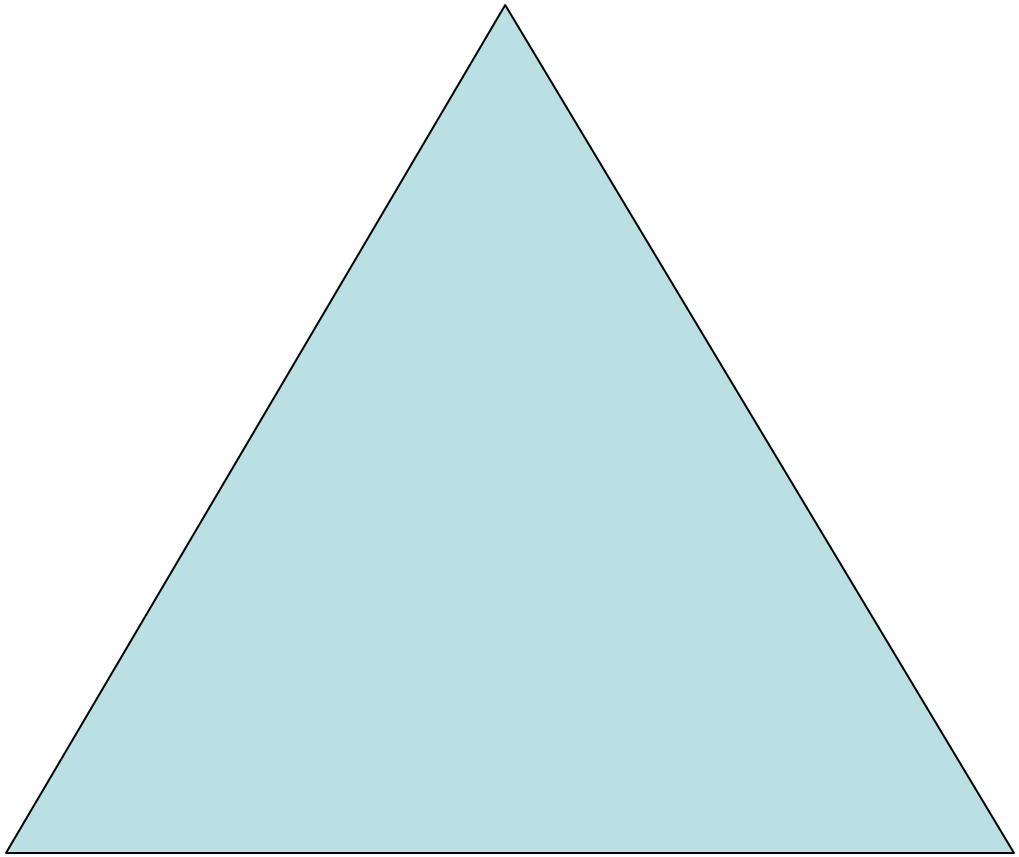
The mission Statement of particle physics:

The mission Statement of particle physics:

Find the symmetries

Discover what breaks them

Symmetry



Beauty

Physics