Triviality and the Higgs mass lower bound

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Outline

- Experimental $m_{\rm Higgs}$ bounds
- Standard Model $m_{\rm Higgs}$ bounds: Landau pole and vacuum instability — Fakes
- Triviality and true $m_{\rm Higgs}$ bounds
- $m_{\rm Higgs}$ lower bound & lattice
- Theoretical bounds give range of allowed $m_{\rm Higgs}$
- Bounds and $m_{\rm Higgs}$ measurement tells us where Standard Model breaks down
- Proposal: Lattice test of Standard Model — important & timely

Experimental m_{Higgs} bounds

Higgs not seen

$m_{\text{Higgs}} > 114.4$ GeV (95% C.L.)

Observables $(m_W, \Gamma_Z, \sin^2\theta_W, \ldots)$ depend on m_{Higgs}

Fit measurements with m_{Higgs} as free parameter

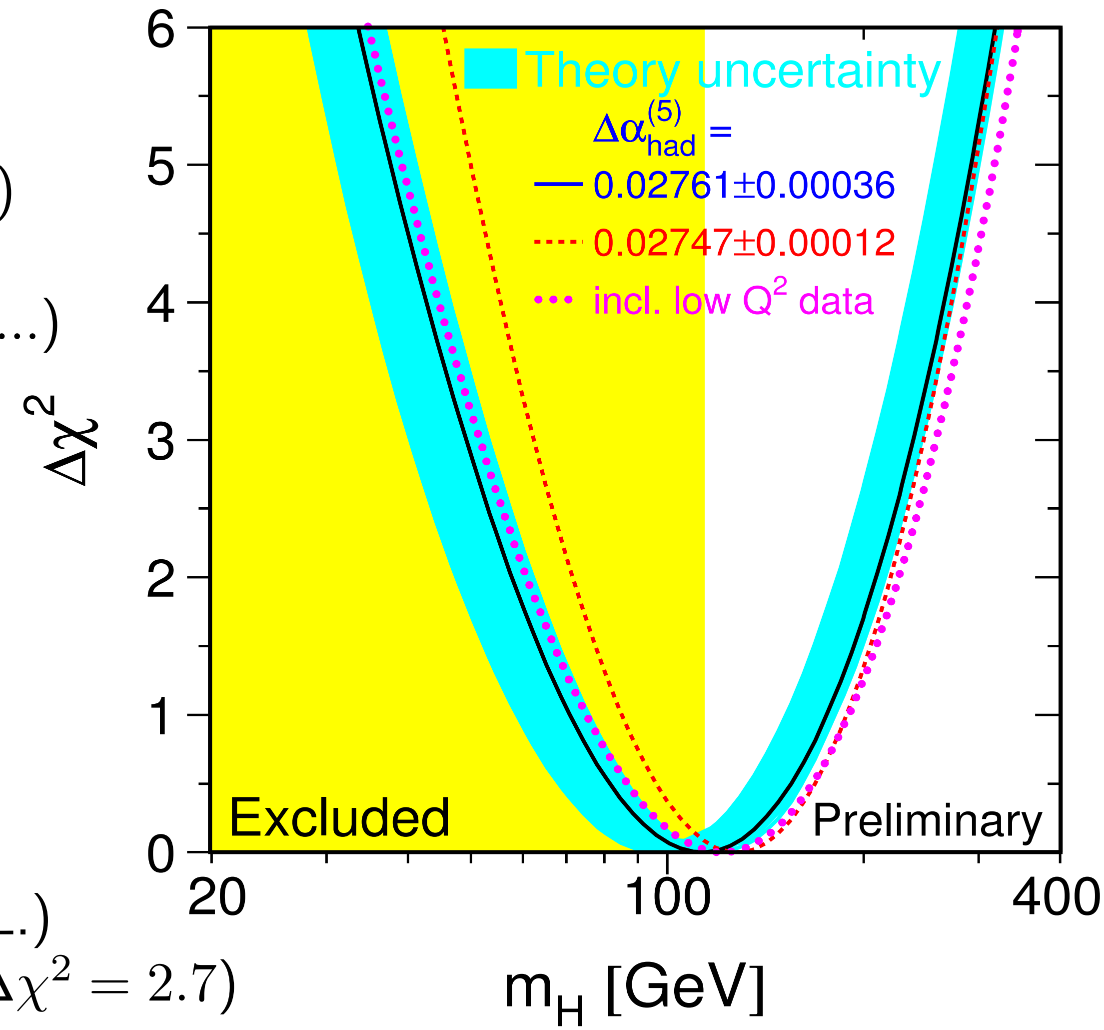
$\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}}$ quality of fit

Fit

$m_{\text{Higgs}} = 113^{+62}_{-42}$ GeV (68% C.L.)

$m_{\text{Higgs}} < 237$ GeV (95% C.L. $\Delta\chi^2 = 2.7$)

Higgs is light if Standard Model is correct!



LEP EW working group 2004

m_{Higgs} bounds: Landau pole & vacuum instability

Upper and lower bounds for SM m_{Higgs}

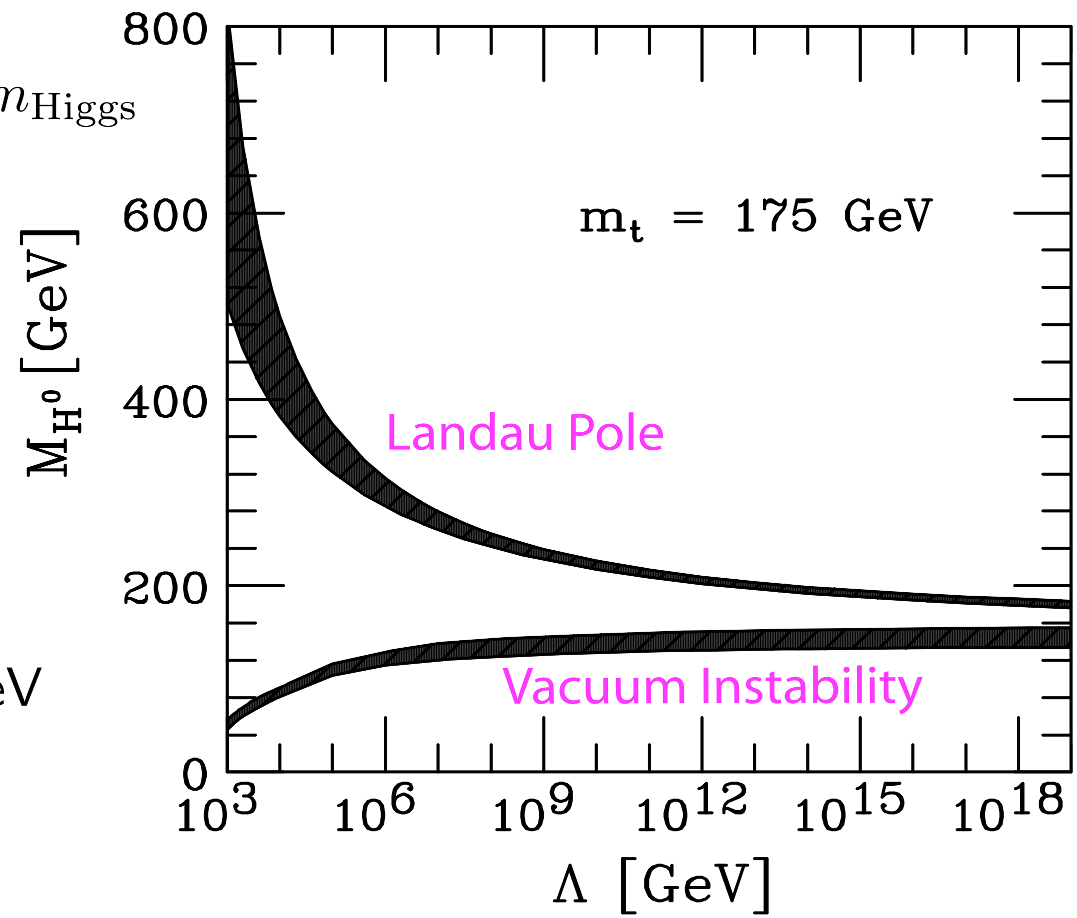
SM valid up to Λ, scale of "new physics", for allowed range of m_{Higgs}

Bounds have uncertainty — can be reduced, in principle

SM cannot sustain $m_{\text{Higgs}} = 1$ TeV

If $m_{\text{Higgs}} = 120$ GeV, SM breaks down at $\Lambda = 10^2 – 10^3$ TeV

If $m_{\text{Higgs}} = 150 – 180$ GeV, SM could be valid up to 10^{19} GeV

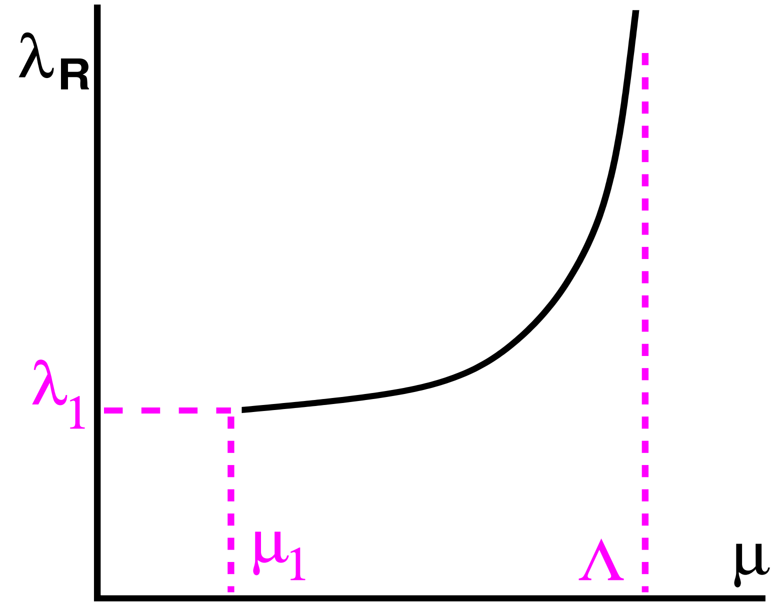


PDG 2002

Lower bound very relevant today — but these bounds are FAKE

Fake: Landau pole and $m_{\rm Higgs}$ upper bound

4-d $\lambda\phi^4$ $\lambda_{\rm R}(\mu)$ $\lambda_{\rm R}(v_{\rm R}) = 3m^2_{\rm Higgs}/v^2_{\rm R}$



$\beta(\lambda_{\rm R}) = 3\lambda^2_{\rm R}/16\pi^2 + \mathcal{O}(\lambda^3_{\rm R}) > 0$

$\lambda_{\rm R}(\mu) = \frac{1}{[1/\lambda_1 - (3/16\pi^2)\ln(\mu/v_{\rm R})]}$ $\mu_1 = v_{\rm R}$

Divergence at $\mu = \Lambda$ Landau pole

If $\lambda_{\rm R}(v_{\rm R}) \leq \lambda_1$, then SM valid up to $\mu = \Lambda$

Λ decreases as λ_1 increases

$m_{\rm Higgs}$ upper bound increases if SM valid only for lower energies

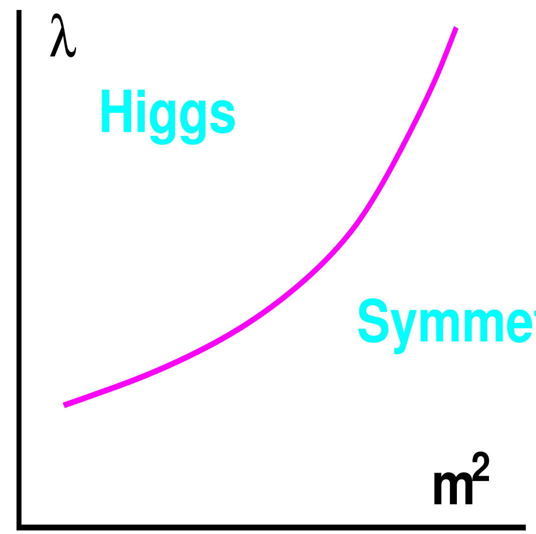
Landau pole supposedly represents "new physics" $\Rightarrow$ used for PDG upper bound

Wrong: breakdown of perturbation theory

Triviality and true $m_{\rm Higgs}$ upper bound

4-d $\lambda\phi^4$ $\quad \mathcal{L} = \frac{1}{2}m^2\phi^2 + \frac{1}{24}\lambda\phi^4 + \frac{1}{2}(\partial_\mu\phi)^2 \quad \lambda, m^2$ bare

Regulate the theory e.g. lattice cut-off $\Lambda = \pi/a$



Measurements in lattice units e.g. $v_{\rm R}a, m_{\rm Higgs}a$

Critical curve for 2nd order phase transition

$v_{\rm R}a \to 0 \qquad \xi/a = 1/(m_{\rm Higgs}a) \to \infty$

Use $v_{\rm R} = 246$ GeV to convert cut-off
into physical units

In Higgs phase, close to boundary $\Lambda/v_{\rm R} = \pi/v_{\rm R}a \to \infty$

Cut-off theory physically acceptable only if $\Lambda/v_{\rm R}$ sufficiently large

What is maximal $m_{\rm Higgs}/v_{\rm R}$ if $\Lambda/v_{\rm R}$ held fixed? (Dashen and Neuberger 1983)

Triviality and true $m_{\rm Higgs}$ upper bound

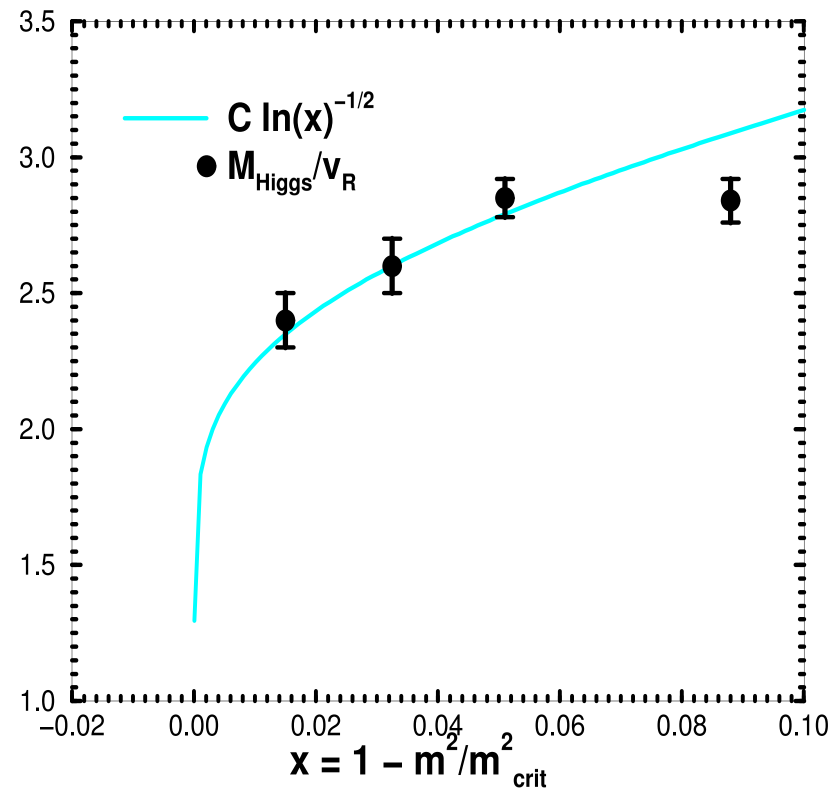
$\lambda_{\rm R} = 3m^2_{\rm Higgs}/v^2_{\rm R} \propto 1/\ln(\Lambda/v_{\rm R})$

$\lambda_{\rm R} \to 0$ as $\Lambda/v_{\rm R} \to \infty$

Non-interacting TRIVIAL theory

Interaction requires finite $\Lambda/v_{\rm R}$

$x = 0.1 \Rightarrow \Lambda/v_{\rm R} \approx 8 \quad m_{\rm Higgs}/v_{\rm R} \approx 2.8$



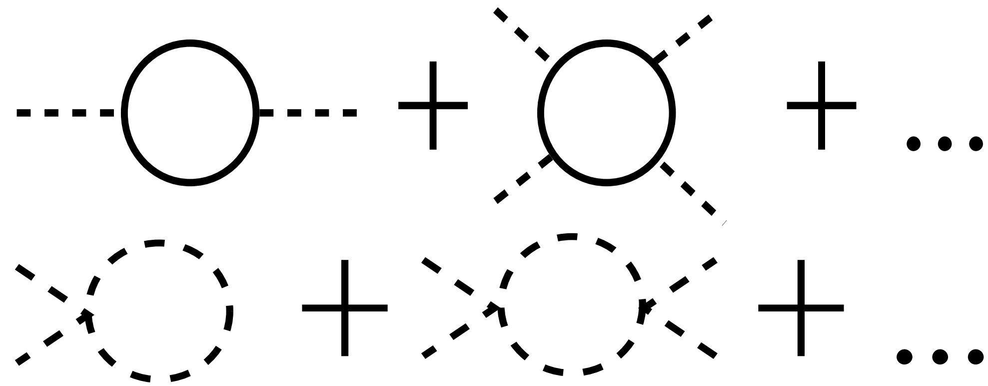
Kuti et al. (1988) 4-d $O(4)$ $\lambda\phi^4$

1. For fixed $\Lambda/v_{\rm R}$, $m_{\rm Higgs}/v_{\rm R}$ largest
when $\lambda \to \infty$ non-perturbative

2. For $\xi/a = 1/(m_{\rm Higgs}a) \geq 2$, violation
of Euclidean invariance less than a few % (Lüscher and Weisz 1989)

$\Lambda \sim 4$ TeV, $m_{\rm Higgs} < 600 - 700$ GeV $\quad \Lambda \sim 10^{19}$ GeV, $m_{\rm Higgs} < 140 - 150$ GeV
Regulator-Dependent (Kuti,Neuberger,Lüscher,Bhanot,Jansen,Hasenfratz,Weisz,...)

Vacuum instability



Higgs–Yukawa system

N_{F} degenerate fermions

Higgs effective potential $V_{\mathrm{eff}}(\phi)$

$$V_{\mathrm{eff}}(\phi) = \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{24}\lambda\phi^4 + \tfrac{1}{2}\int_k \ln[1+\lambda\phi^2/2k^2] - 2N_{\mathrm{F}}\int_k \ln[1+y^2\phi^2/k^2]$$

After renormalization (regulate, counter-terms, send cut-off $\longrightarrow \infty$)

$$V^{\mathrm{R}}_{\mathrm{eff}}(\phi_{\mathrm{R}}) = -\tfrac{1}{4}m^2_{\mathrm{R}}\phi^2_{\mathrm{R}} + \tfrac{1}{24}\lambda_{\mathrm{R}}\phi^4_{\mathrm{R}} + \{\phi^4_{\mathrm{R}}\ln[\phi^2_{\mathrm{R}}/\mu^2]/16\pi^2\}\{\lambda^2_{\mathrm{R}}/16 - N_{\mathrm{F}}y^4_{\mathrm{R}}\}$$

If $\lambda_{\mathrm{R}} \ll y^2_{\mathrm{R}}$, $V^{\mathrm{R}}_{\mathrm{eff}}(\phi_{\mathrm{R}})$ unstable at large ϕ_{R} due to negative fermion term

Does not require λ_{R} or y^2_{R} to be large — perturbative (unlike Landau pole)

Linde, Politzer, Wolfram, Cabibbo, Maiani, Parisi, Altarelli, Sher, Casas, Espinosa, Quiros,...

Vacuum instability

2–loop SM calculation $V_{\rm eff}^{\rm R}(\phi_{\rm R})$ (solid line)

If $m_{\rm Higgs} = 52$ GeV, $V_{\rm eff}^{\rm R}$ valid for $\phi_{\rm R} \leq 1$ TeV

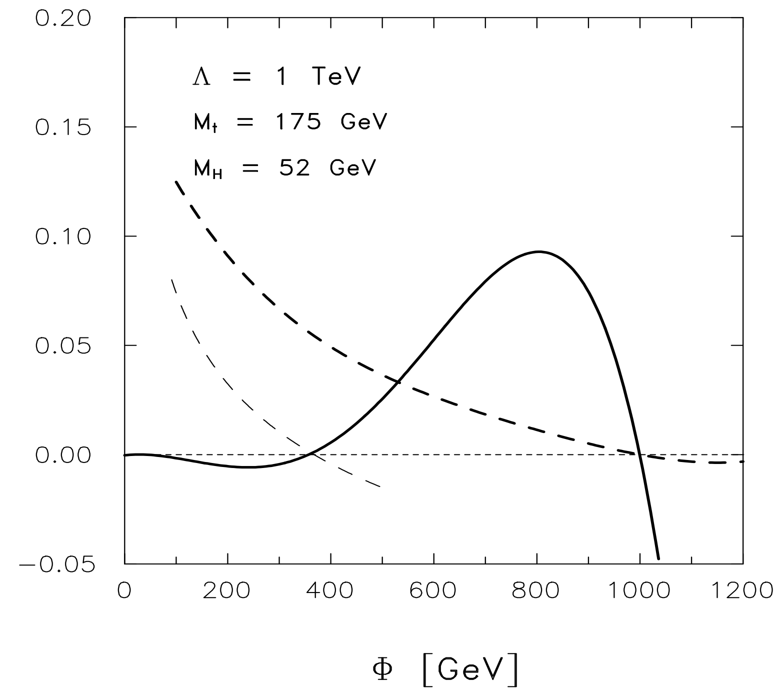
Position of $V_{\rm eff}^{\rm R}$ instability varies with $m_{\rm Higgs}$

$m_{\rm Higgs}$ lower bound if SM valid up to Λ

Equivalent: RG flow for Higgs-Yukawa

$\mu(d\lambda_{\rm R}/d\mu) = \frac{1}{16\pi^2}(3\lambda_{\rm R}^2 + 8N_{\rm F}\lambda_{\rm R}y_{\rm R}^2 - 48N_{\rm F}y_{\rm R}^4)$

$\mu(dy_{\rm R}^2/d\mu) = \frac{1}{8\pi^2}(3 + 2N_{\rm F})y_{\rm R}^4$

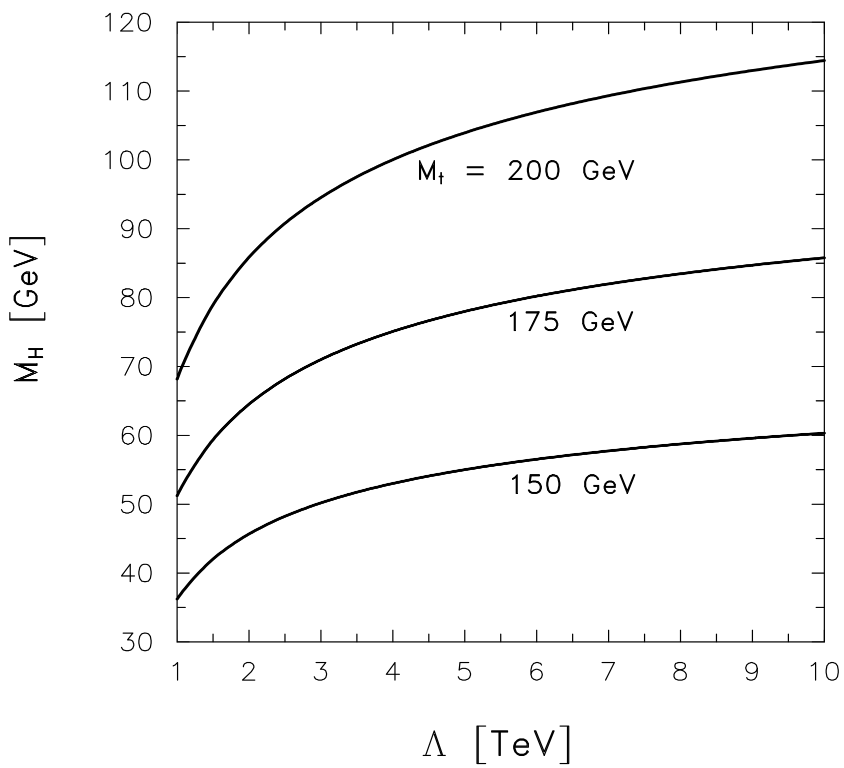


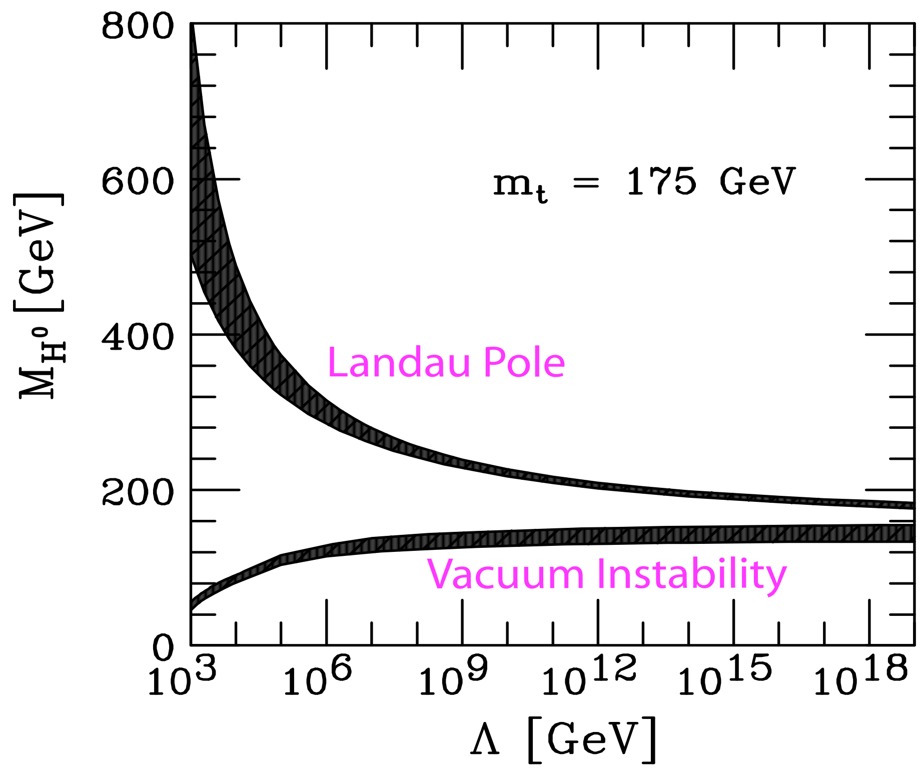
Casas, Espinosa & Quiros (1996)

If $\lambda_{\rm R} \ll y_{\rm R}^2$, $\beta_\lambda < 0$ $\lambda_{\rm R} = 0$ at $\mu = \Lambda$ $\lambda_{\rm R}(\mu = c\phi_{\rm R})$ (thick dashed line)

Vacuum instability appears when $\lambda_{\rm R}$ vanishes

m_{Higgs} lower bound from vacuum instability





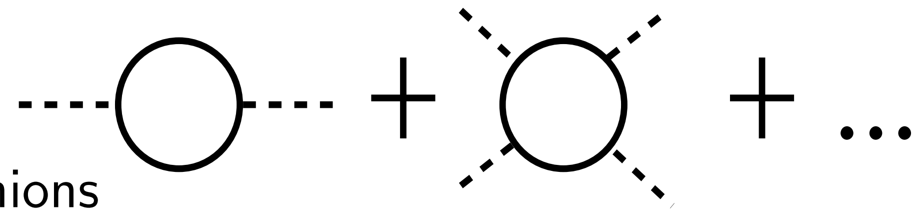
Casas et al. (1996) Theoretical error < 5 GeV similar error from m_{Top} and α_S

A light Higgs would mean SM valid only up to 10^2 – 10^3 TeV

Strong and precise statement — but is it correct? What about Triviality?

Higgs-Yukawa and Large N_{F}

Large N_{F} limit of Higgs–Yukawa system



Lore: Vacuum still unstable, due to fermions

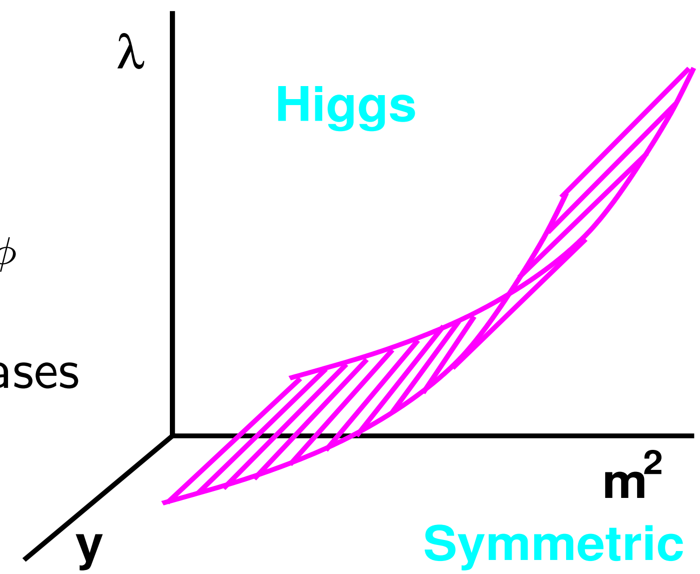
Explore phase diagram of regulated theory   bare parameters λ, y, m^2

Define renormalized couplings

$y_{\mathrm{R}} \equiv m_{\mathrm{Top}}/v_{\mathrm{R}}$ $\lambda_{\mathrm{R}} \equiv 3m^2_{\mathrm{Higgs}}/v^2_{\mathrm{R}}$

$G^{-1}_{\bar{\Psi}\Psi}(0) = m_{\mathrm{Top}}$ $G^{-1}_{\phi}(p^2) = (m^2_{\mathrm{Higgs}} + p^2)/Z_\phi$

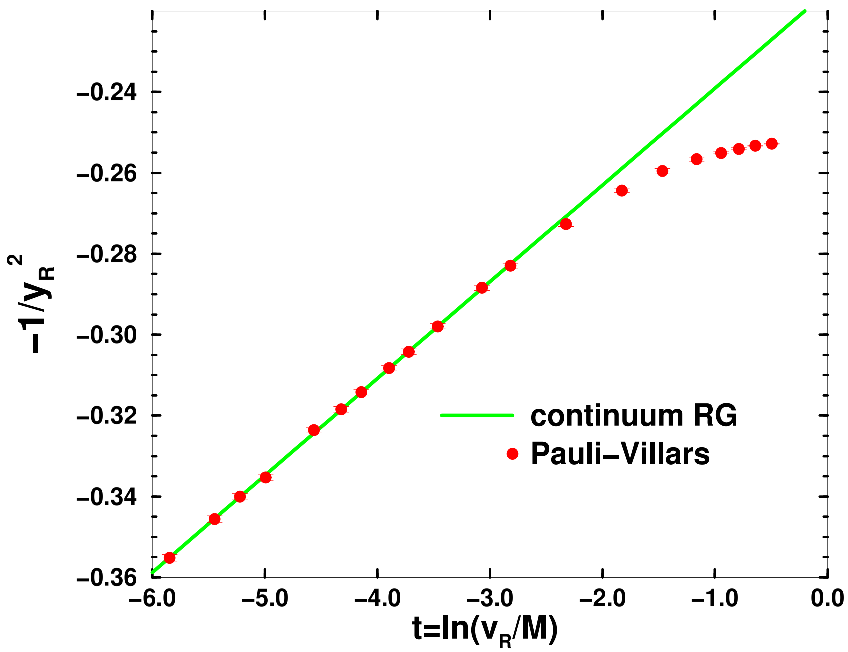
Critical surface between Higgs and Symmetric phases

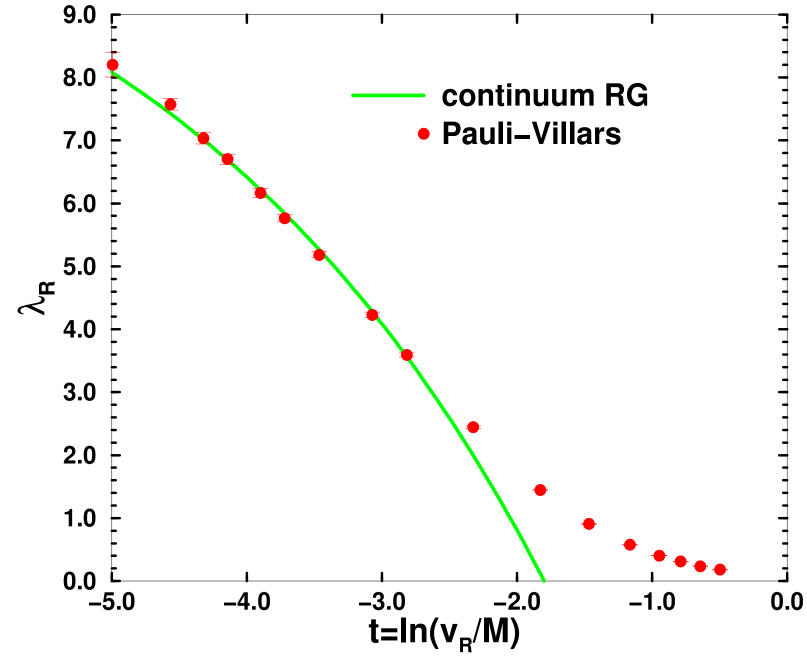


Moving around phase diagram, y_{R} and λ_{R} vary according to RG flow

Should see vacuum instability from $\lambda_{\mathrm{R}} = 0$

Vacuum instability is fake





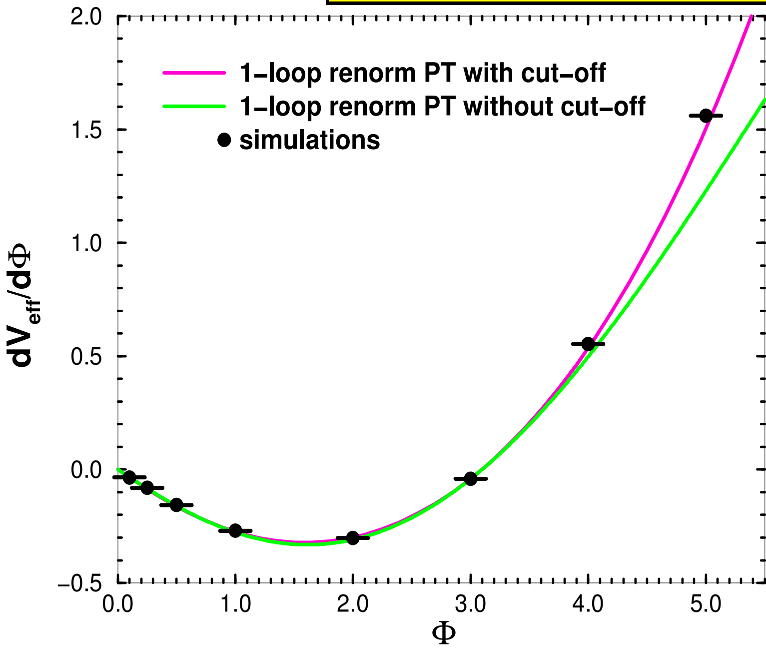
Large $N_{\rm F}$ limit   perturbation theory   Pauli-Villars regulator with mass M

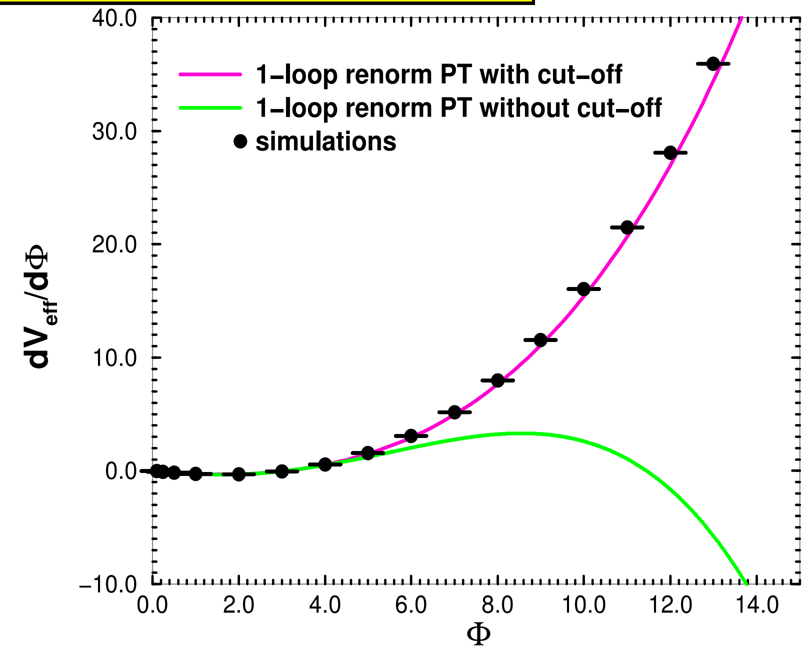
continuum RG $-1/y_{\rm R}^2 = (N_{\rm F}/4\pi^2)t - 1/y_1^2$ $\lambda_{\rm R} = 12y_{\rm R}^2 + c_1 y_{\rm R}^4$ $t = \ln(v_{\rm R}/M)$

Remove cut-off $t = \ln(v_{\rm R}/M) \to -\infty$ $y_{\rm R}, \lambda_{\rm R} \to 0$ TRIVIAL

For $v_{\rm R}/M \sim 1$, RG flow is NOT continuum-like — cut-off effects important
Vacuum instability ($\lambda_{\rm R} < 0$) appears only when cut-off effects ignored

Constraint effective potential $V_{\rm eff}^{\rm c}$





Finite volume Ω $\exp(-\Omega V_{\rm eff}^{\rm c}(\Phi)) = \int [D\phi]\delta(\Phi - 1/\Omega \sum_x \phi(x)) \exp(-S[\phi])$

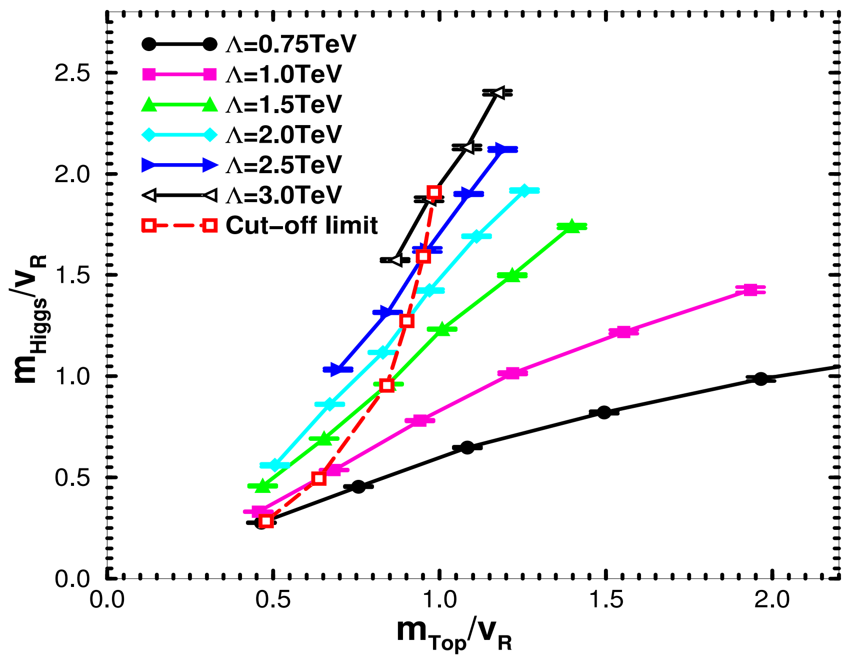
$V_{\rm eff}^{\rm c}$ not convex absolute minimum for finite Ω can measure $V_{\rm eff}^{\rm c}$ on lattice

Figures: Higgs–Yukawa model single ϕ $N_{\rm F} = 8$ $dV_{\rm eff}^{\rm c}/d\Phi$ from lattice

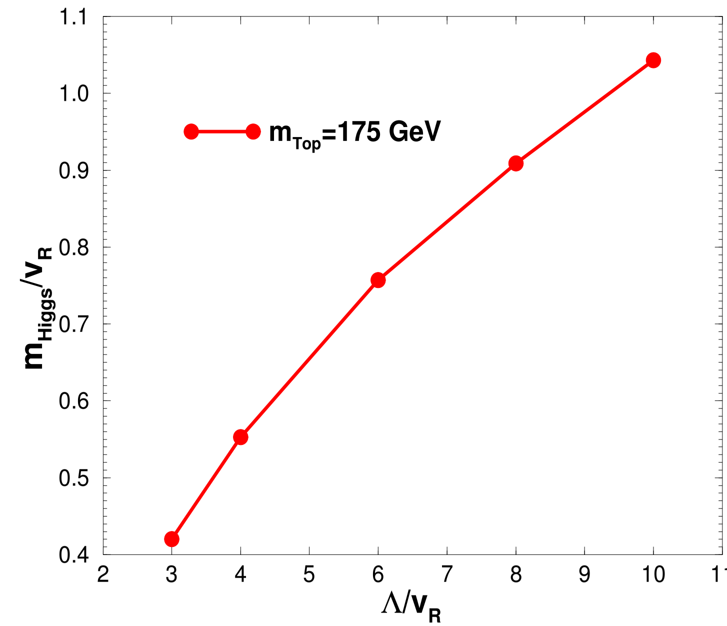
Higgs phase $\Rightarrow V_{\rm eff}^{\rm c}$ minimum for $\Phi \neq 0$ $V_{\rm eff}^{\rm c}$ not unstable at large Φ

Perturbation theory fails when cut-off effects ignored vacuum instability fake

Triviality and true m_{Higgs} lower bound



Higgs-Yukawa model with $N_{\text{F}} = 8$



summary of lattice simulations

1. For fixed m_{Top} and Λ/v_{R}, m_{Higgs} smallest when $\lambda = 0$ (recall upper bound saturated at $\lambda = \infty$)

2. Cut-off limit: $\xi/a = 1/m_{\text{phy}}a \geq 2 \Rightarrow$ cut-off effects (violation of Euclidean invariance) acceptably small — left of dashed line

m_{Higgs} lower bound for fixed m_{Top} $\qquad$ m_{Top} UPPER bound for fixed m_{Higgs}

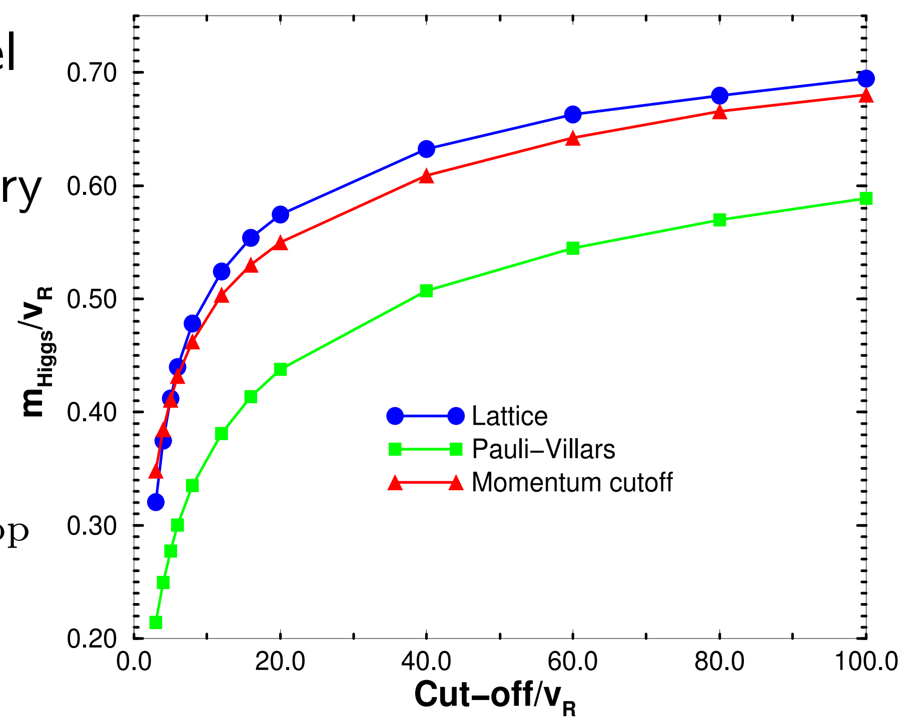
Bounds are regulator-dependent

Large $N_{\rm F}$ limit of Higgs–Yukawa model

Choose regulator for perturbation theory

1. Lattice **2.** Pauli-Villars
3. Hard momentum cut-off

Figure: $m_{\rm Higgs}$ lower bound, fixed $m_{\rm Top}$



Finite cut-off $\Lambda \Rightarrow$ loss of Universality

Lower bound varies with regulator even for $\Lambda/v_{\rm R}$ large
e.g. if $m_{\rm Higgs}/v_{\rm R} = 0.5 \Rightarrow \Lambda/v_{\rm R} \sim 10 - 40$

Pure Higgs $\quad m_{\rm Higgs}/\Lambda = C_\Lambda(\beta_1\lambda_{\rm R})^{-\beta_1/\beta_0^2}\exp(-1/\beta_0\lambda_{\rm R}) \qquad C_\Lambda$ not universal

Cannot make $m_{\rm Higgs}$ bounds arbitrarily precise AND regulator-independent
Inherently fuzzy — unavoidable aspect of Triviality

Conclusions

- Higgs-Yukawa model: Large N_{F} limit, lattice simulations
- RG flow of $\lambda_{\mathrm{R}}, y_{\mathrm{R}}$ non-perturbative measurement of constraint $V_{\mathrm{eff}}^{\mathrm{c}}$
- Vacuum instability is fake — comes from ignoring cut-off effects
- Determine m_{Higgs} lower bound by exploring phase diagram non-perturbatively
- Lower bound attained when $\lambda = 0$ (upper bound for $\lambda = \infty$)
- Bounds have a blur — regulator-dependent
- Proposal: lattice study of gluon–Higgs–Top system
  - most important pieces: $O(4)$ Higgs, single fermion flavor, α_{S}
  - remainder of SM has small effect, might be included perturbatively
- Where does SM break down? We don't know — Lattice can decide

Constraint effective potential $V_{\rm eff}^{\rm c}$

Renormalized potential with counter-terms and cut-off Λ

$$\begin{aligned}
V &= m^2\phi^2/2 + \lambda\phi^4/24 \\
V_{\rm eff}^{\rm c} &= V + 1/2\int_{k\neq 0}^{\Lambda} \ln[1 + V''/k^2] - 2N_{\rm F}\int_{k\neq 0}^{\Lambda} \ln[1 + y^2\phi^2/k^2] \\
&\quad -1/2\int_{k\neq 0}^{\Lambda} V''/k^2 + 1/4\int_{k\neq 0}^{\Lambda} (V'')^2/[k^2+\mu^2]^2 \\
&\quad +2N_{\rm F}\int_{k\neq 0}^{\Lambda} y^2\phi^2/k^2 - N_{\rm F}\int_{k\neq 0}^{\Lambda} y^4\phi^4/[k^2+\mu^2]^2
\end{aligned}$$

Naïve and wrong calculation $\Lambda \to \infty$

$$V_{\rm eff}^{\rm c} = V + \{\ln[\phi^2/\mu^2]/64\pi^2\}\{(V'')^2 - 4N_{\rm F}y^4\phi^4\}$$

Ignores Triviality: $\lambda, y \to 0$ as $\Lambda \to \infty$