Status of Lattice Flavor Physics

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Lattice 2004, Fermilab

Status

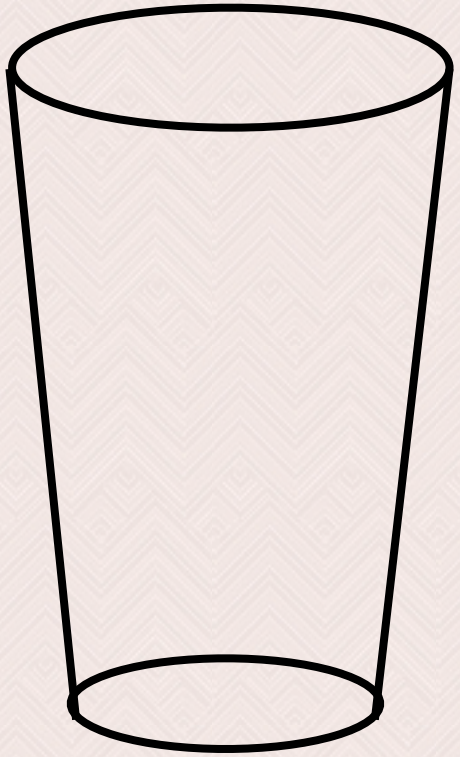
Not

Summary

Outline

- Comment on unquenching and (un)systematic uncertainties
- Focus of this talk
- Heavy-light decay constants
- Neutral B mixing
- Heavy-light semileptonic decay form factors
- Neutral K mixing - B_K

The Holy Grail



- Theoretically sound algorithm
- Good chiral properties - Ginsparg-Wilson-Luescher symmetry
- Simulate on large volumes, small lattice spacings, physical sea quark masses (or close enough for chi-PT)

The Grail of Purity

- Theoretically sound algorithm
- Good chiral properties - Ginsparg-Wilson-Luescher symmetry
- Simulate on large volumes, small lattice spacings, physical sea quark masses (or close enough for chi-PT)

Sea quarks and states of Sin

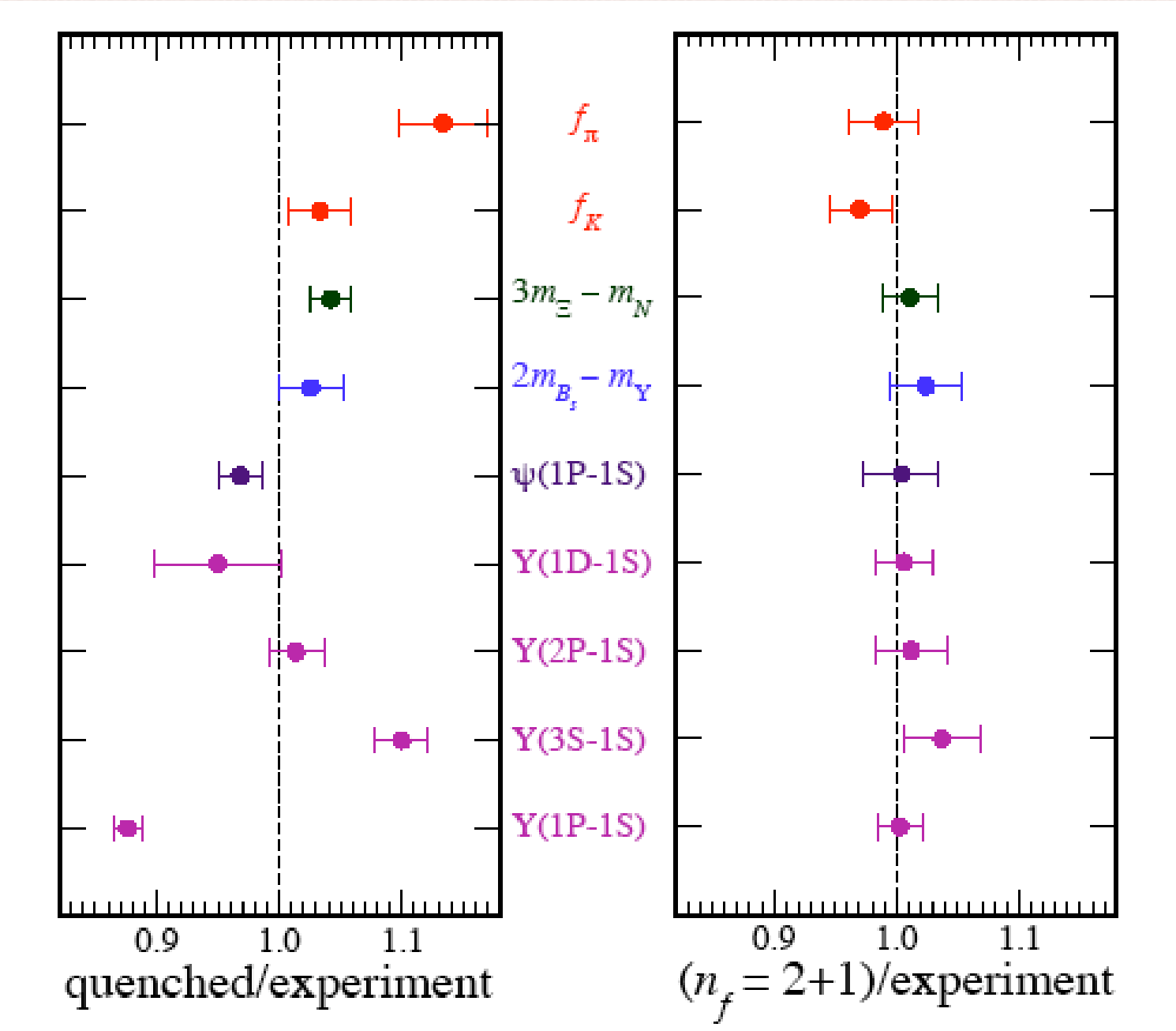
- Quenched

  Theoretically wrong. 10-20% disagreement with experiment.

- Lighter staggered

  Theoretically uncertain. Agreement with experiment within quoted uncertainties. Permits simulation inside chiral regime

Quenched vs. Light Improved Staggered



C. Davies, *et al.* PRL **92**, 022001 (2004)

Sea quarks and states of Sin

- Quenched

  Theoretically wrong. 10-20% disagreement with experiment.

- Lighter staggered

  Theoretically uncertain. Agreement with experiment within quoted uncertainties. Permits simulation inside chiral regime

- Heavier Wilson, twisted-mass, domain wall, overlap, fixed point

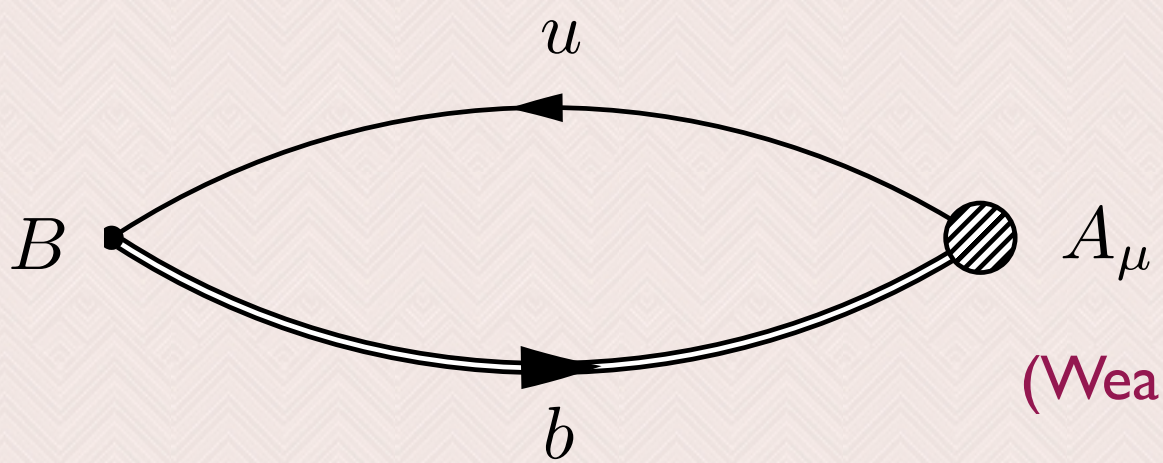
  Theoretically sound. More costly, so heavier mass required.
  Extrapolation to physical sea quark masses: inside chiral regime???

Focus of this Talk

- Application of Lattice QCD results to phenomenology
- Unquenched calculations of "golden" or cleanly computed quantities
- Quenched calculations show how to further reduce uncertainties - Result-oriented. Nice theoretic work going on, but outside the scope of this talk
- *K* to 2 *pi* is an important topic with lots of activity, but not enough time to discuss properly

Heavy light decay constants

Anatomy of decay constant calculation



(Weak axial current)

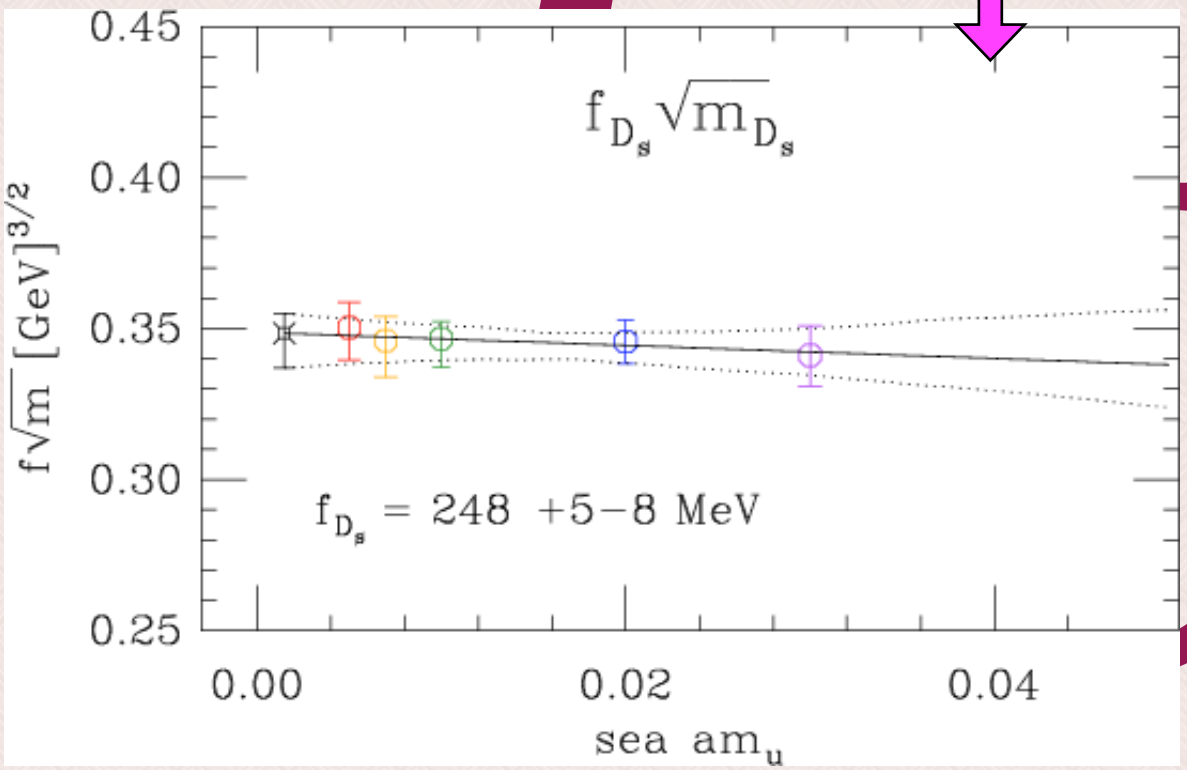
- Axial vector matrix element $\langle 0|A_\mu|H(p)\rangle \;=\; f_H p_\mu$
- Renormalization/matching
- Quark mass dependence -- directly at *s* quark, extrapolate to *u*/*d*

$$f_{D_s}$$

f_{D_s} by FNAL/MILC **PRELIMINARY**

Linear extrapolation in sea quark

physical m_s



$$f_{D_s}\sqrt{m_{D_s}} = 248 \begin{matrix} +5 \\ -8 \end{matrix} \pm 29 \text{ MeV}$$

- n_f= 2+1 impr. staggered

  $m_{ud}^{\text{sea}} \geq m_s/8$

- coarser MILC lattices

  $a \approx 0.12$ fm

- Volume $\approx (2.4 \text{ fm})^3$

- Mesons: FNAL-heavy impr. staggered light (AsqTad)

J. Simone, Lattice 2004

f_{D_s} by FNAL/MILC

Error budget

PRELIMINARY

source	uncertainty as percent of: $f_{D_s}\sqrt{m_{D_s}}$
stat.+extrapolation	3.3
higher order SχPt	?
HQ matching to QCD	10
LQ discr. effects	4
m_{charm} determination	4
valence m_s, m_d det.	1
lat. spac. & sea quark	2

f_{D_s}

J. Simone, Lattice 2004

Reduction of perturbative uncertainty

- Factor out part calculable nonperturbatively

  A. El Khadra, *et al.*, PR **D58** (1998)

- Perturbative calculation for this action, still preliminary

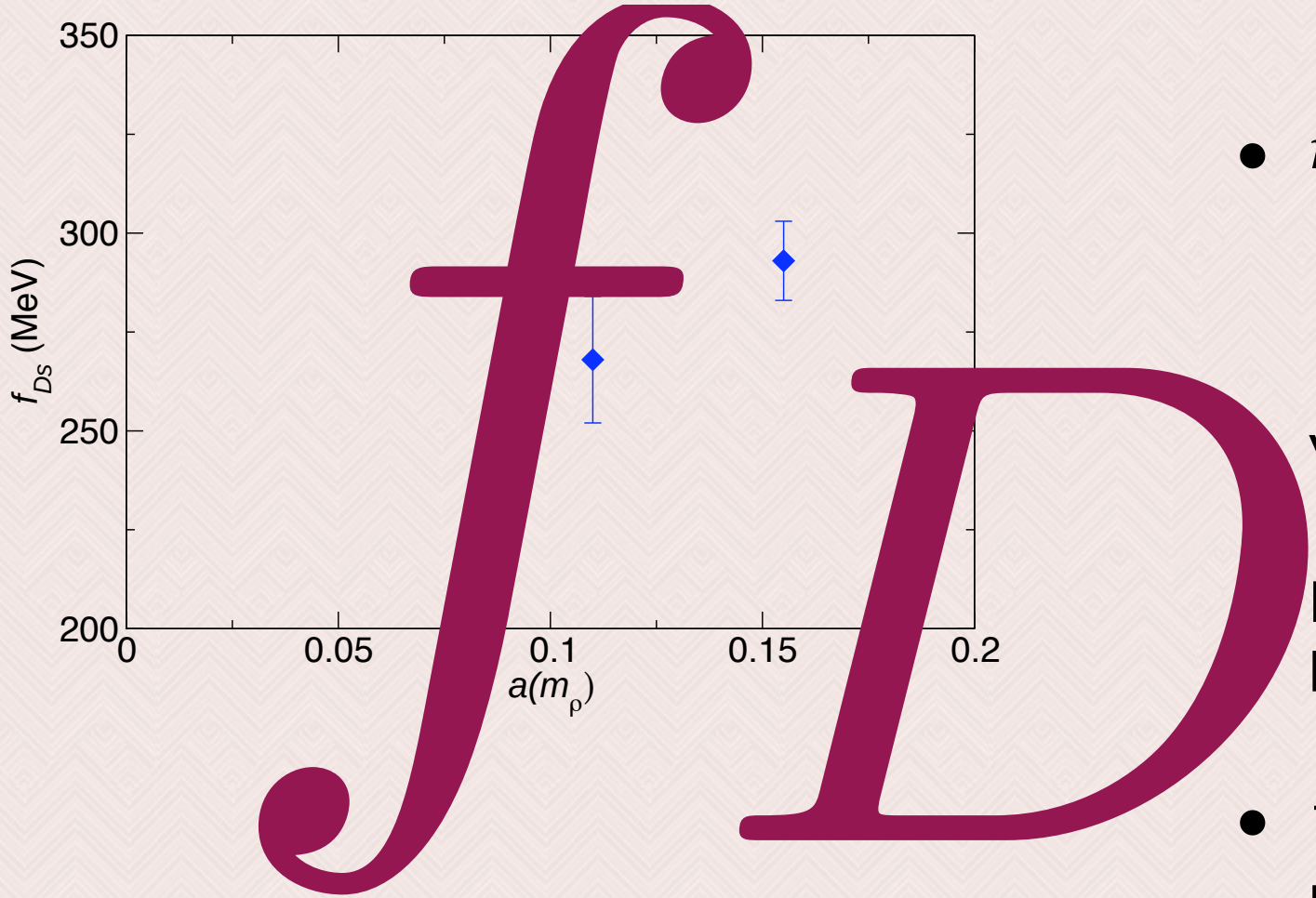
  A. El Khadra, M. Nobes, H. Trottier

- Perhaps two-loop corrections will be negligible

$$f_{D_s}$$

f_{D_s} by CP-PACS

PRELIMINARY



- n_f = 2 clover

  $m_{ud}^{\rm sea} \ \geq \ m_s/2$

Volume $\approx \ (2.5 \ \mathrm{fm})^3$

Mesons: Tsukuba-heavy /clover-light

- 1 value of sea quark mass presently

Y. Kayaba, Lattice 2004

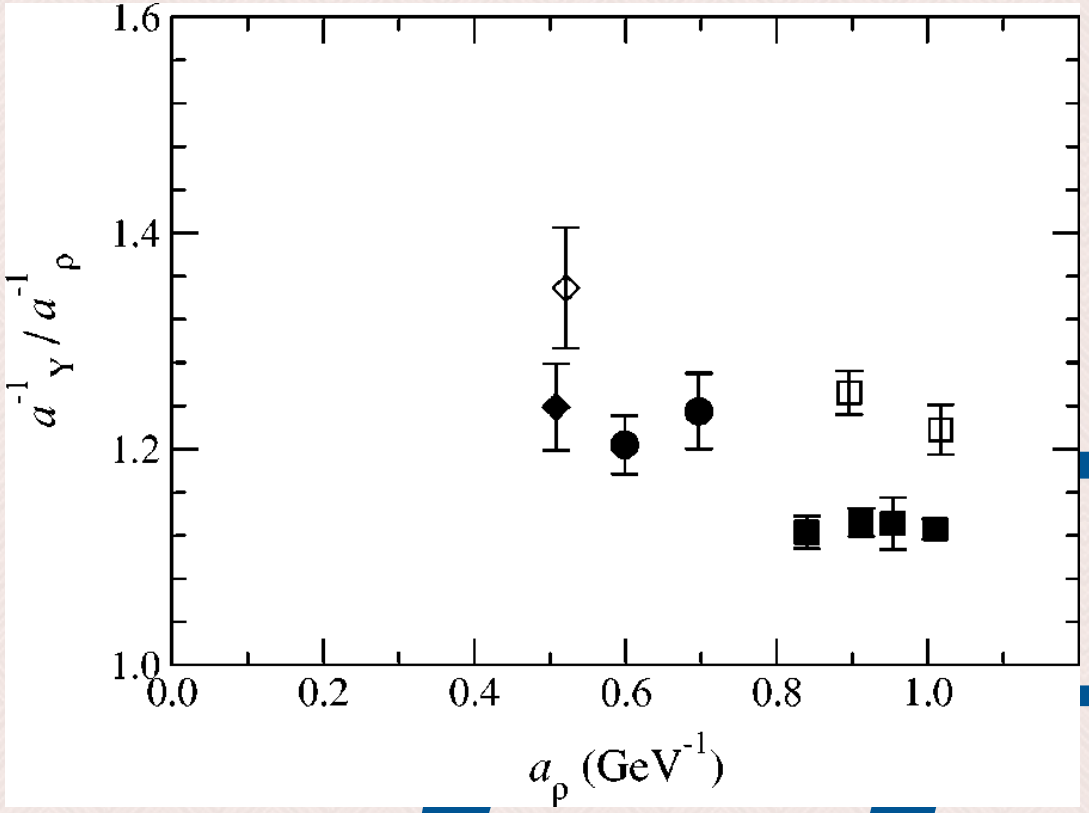
$$f_{B_s}$$

f_{B_s} with NRQCD

Ref	n_f	Configs	result (MeV)	
1	2	CP-PACS $m_{ud}^{\rm sea} \geq m_s/2$	$242 \pm 9 \pm 34$	
2	2	JLQCD $m_{ud}^{\rm sea} \geq m_s/2$	$215 \pm 9 \pm 13$	
3	2+1	MILC $m_{ud}^{\rm sea} \geq m_s/4$	$260 \pm 7 \pm 28$	

1. A. Ali Khan, *et al.* PRD 64 (2001)
2. S. Aoki, *et al.* PRL 91 (2003)
3. M.W., *et al.* PRL 92 (2004)

Lattice spacing ambiguity



(open points = quenched)

Solid points = 2 flavor

- CP-PACS - 2 flavors of clover (tadpole coeff)
- Upsilon 1P-1S splitting vs. rho mass to set lattice spacing
- JLQCD simulation sees scale agreement using m_ρ, f_K, r_0

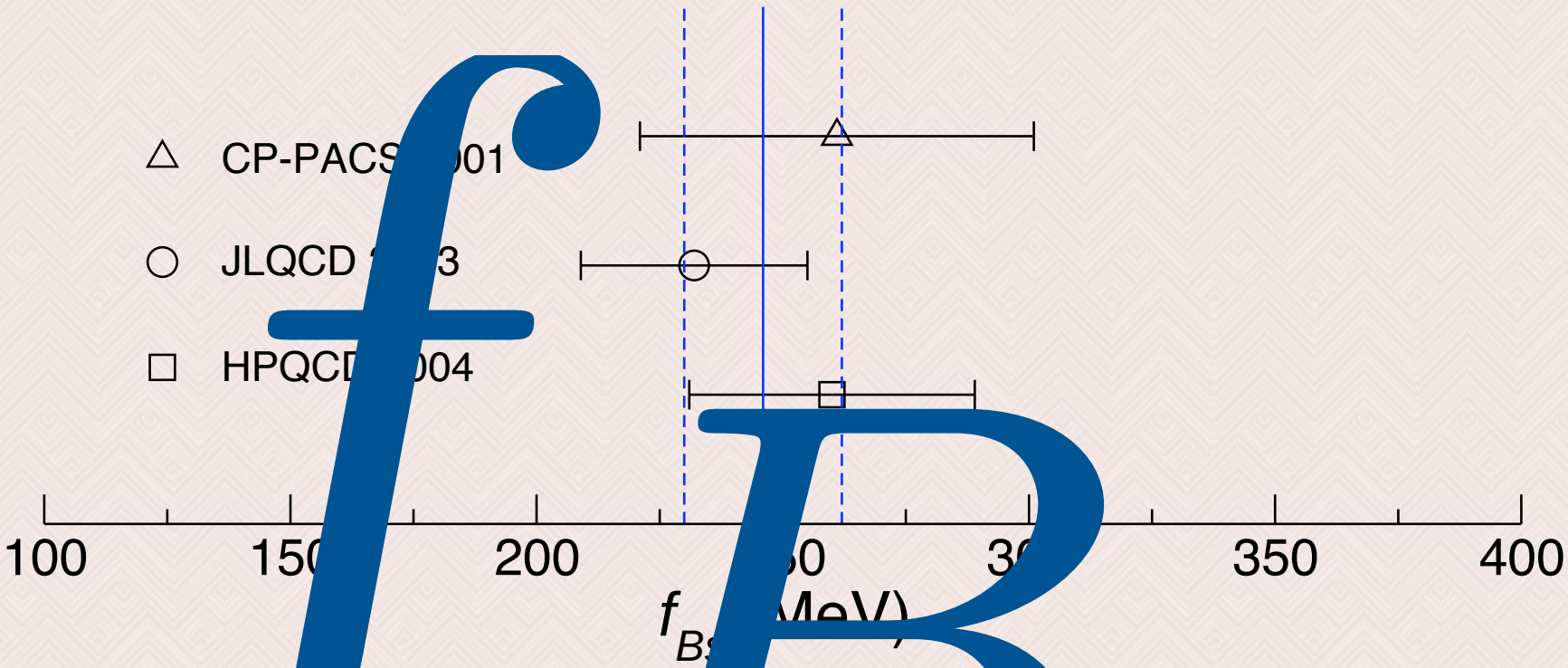
A. Ali Khan, et al. PRD 64 (2001)

f_{B_s} with NRQCD

Ref	n_f	Config	result (MeV)	scale ambiguity
1	2	CP-PACS $m_{ud}^{\rm sea} \geq m_s/2$	$242 \pm 9 \pm 34$	$^{+38}_{-0}$
2	2	JLQCD $m_{ud}^{\rm sea} \geq m_s/2$	$215 \pm 9 \pm 1$	$^{+34}_{-0}$
3	2+1	MILC $m_{ud}^{\rm sea} \geq m_s/4$	$260 \pm 7 \pm 3$	$< 4\%$

1. CP-PACS: A. Ali Khan, *et al.* PRD 64 (2001)
2. JLQCD: S. Aoki, *et al.* PRL 91 (2003)
3. HPQCD: M.W., *et al.* PRL 92 (2004)

Weighted Average



$$f_{B_s} = 246 \pm 16 \text{ MeV}$$

Experimental lower bound on f_{B_s}

- $\Delta m_{B_s} = \frac{G_F^2}{6\pi} m_W^2 \eta_B S(x_t) m_{B_s} |V_{ts} V_{tb}^*|^2 f_{B_s}^2 B_{B_s}$
- Factors in front $= 1.110 \times 10^{-7}$ GeV^{-1}
- Factors in blue, 90% confidence range (PDG):

  $|V_{tb}| = 0.9990 - 0.9994$ $|V_{ts}| = 0.037 - 0.044$
- $\Delta m_{B_s} > 15$ ps^{-1} (0.985×10^{-11} GeV) 95% C.L.
- Implies lower bound $f_{B_s} \sqrt{B_{B_s}^{\overline{MS}}(m_b)} > 214$ MeV

  Or if $B_{B_s}^{\overline{MS}}(m_b) = 0.85 \Rightarrow f_{B_s} > 232$ MeV

f_{B_s}

Chiral extrapolations

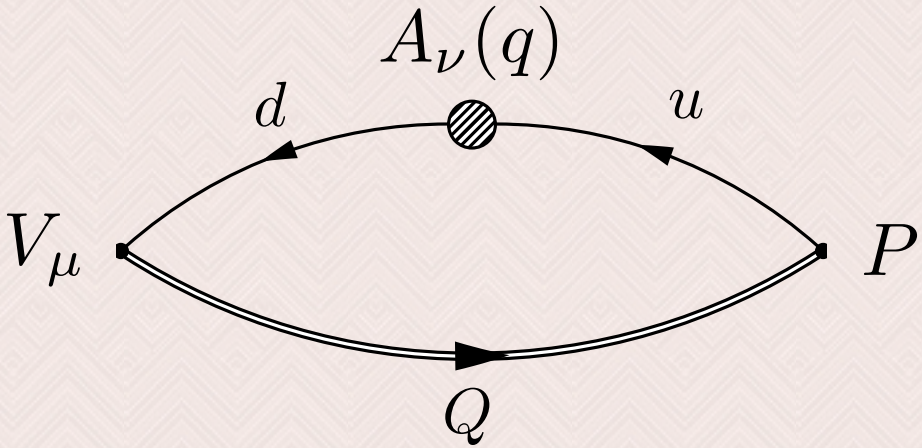
$$\xi_\Phi = \frac{f_{H_s}\sqrt{m_{H_s}}}{f_H\sqrt{m_H}} = 1 + \frac{8K(\Lambda_\chi)}{f^2}\left(m_K^2 - m_\pi^2\right)$$

$$+ \frac{1+3g^2}{4(4\pi f)^2} \times \left[5I_1(m_\pi^2) - 2I_1(m_K^2) - 3I_1(m_\eta^2)\right]$$

where $I_1(m^2) \equiv m^2 \ln\left(\frac{m^2}{\Lambda_\chi^2}\right)$

- Low energy constants, *independent* of chiral scale: f_π, g^2
- Extract from experiment or compute on lattice

Calculation of H^* - H -π coupling



$$g_{H^*H\pi} = \frac{2\sqrt{m_{H^*}m_H}}{f_\pi}\, g$$

Heavy-light chiral p.th.

- CLEO, PRL **87** (2001)

$$g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9 \;\Rightarrow\; g_c^{\rm exp} = 0.61(1)(7)$$

- Quenched calculation

$$g_c^{\rm quench} = 0.66(8)(5) \quad \& \quad g_\infty^{\rm quench} = 0.48(3)(11)$$

- Interpolate to B $\quad g_b^{\rm quench} = 0.58(6)(10)$

A. Abada, *et al.* PR **D66** (2002), JHEP (2004)

Chiral extrapolations

SχPT overview

C. Aubin talk for details.

Leading order in $1/m_Q$ expansion and χPT,

$$f_{Qq}\sqrt{m_{Qq}} = \alpha\left[1 + \frac{1}{16\pi^2 f^2}\Delta f_q + \ldots\right]$$

where ... are powers terms in sea and light valence quark masses; up to 3rd order in fit.

Chiral log terms

$$\Delta f_q = -\frac{1+3g^2}{2}\left(h^{av} + h^I + a^2\delta'_A h^A + a^2\delta'_V h^V\right)$$

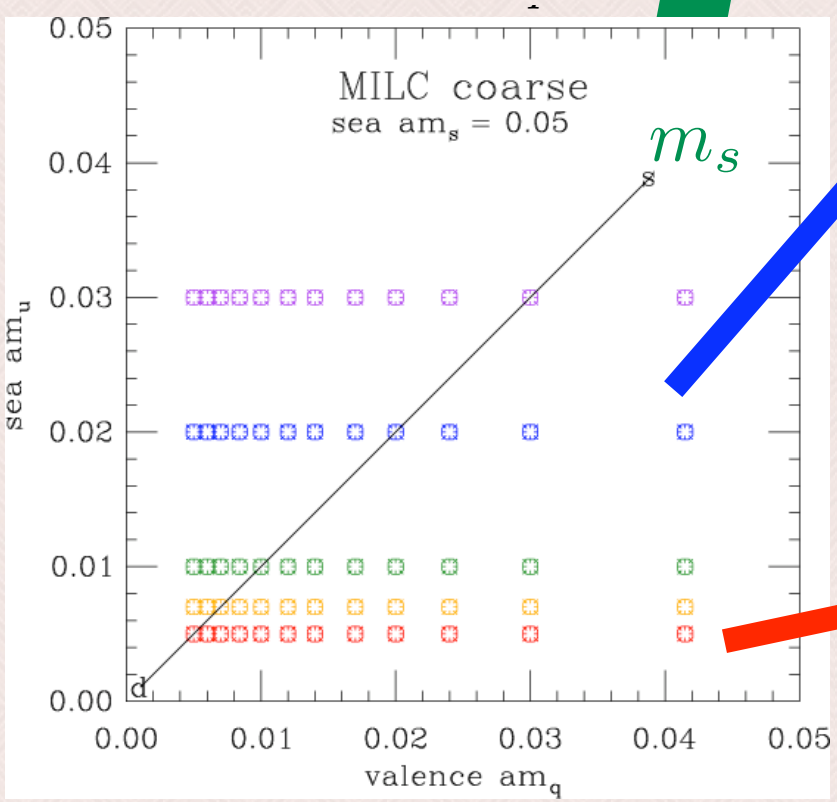
Discretization effects from taste violations in pion masses and h^A and h^V terms.

Slide stolen from J. Simone's talk.

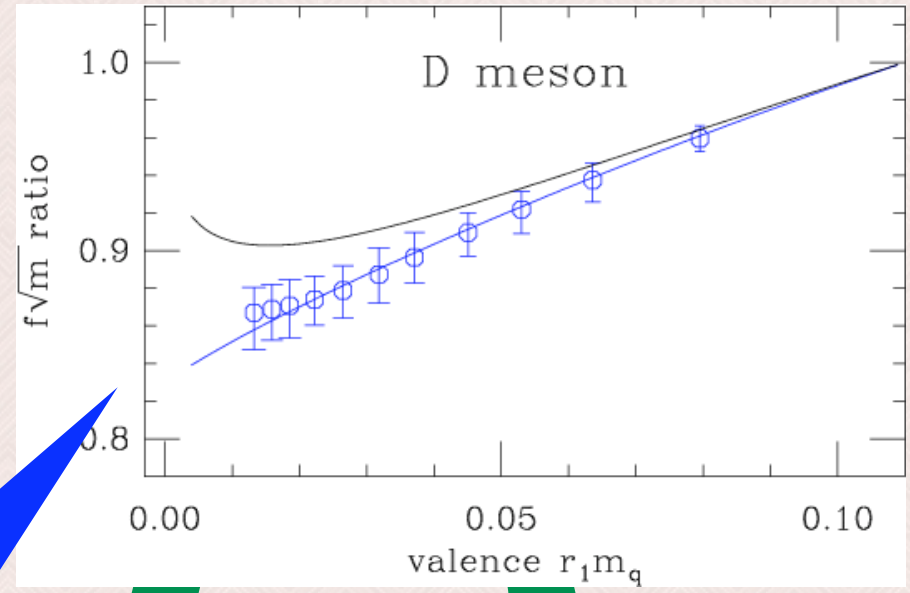
$$\int D$$

Light quark mass dependence

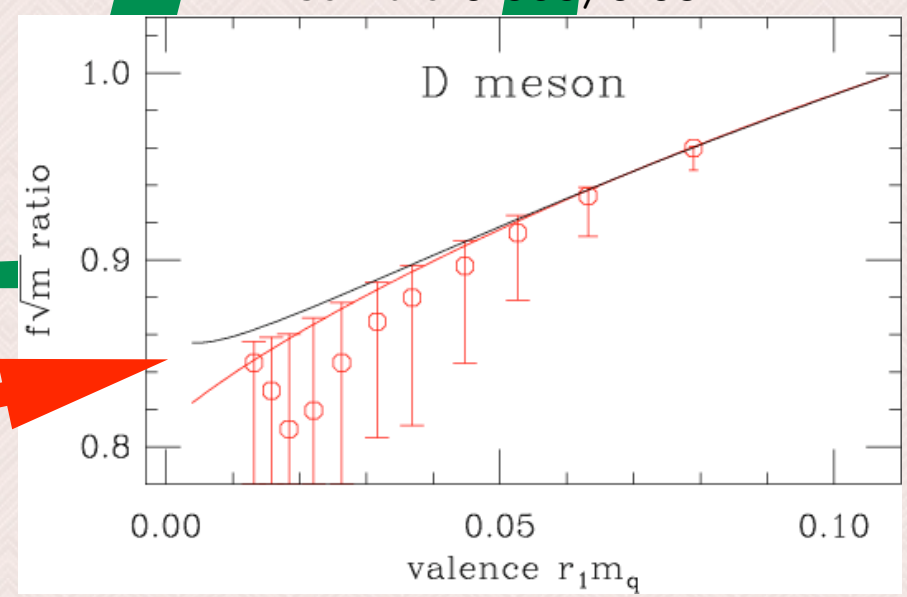
$$\frac{1}{\xi_\Phi} = \frac{f_D\sqrt{m_D}}{f_{D_s}\sqrt{m_{D_s}}}$$



Ensemble 0.020/0.05

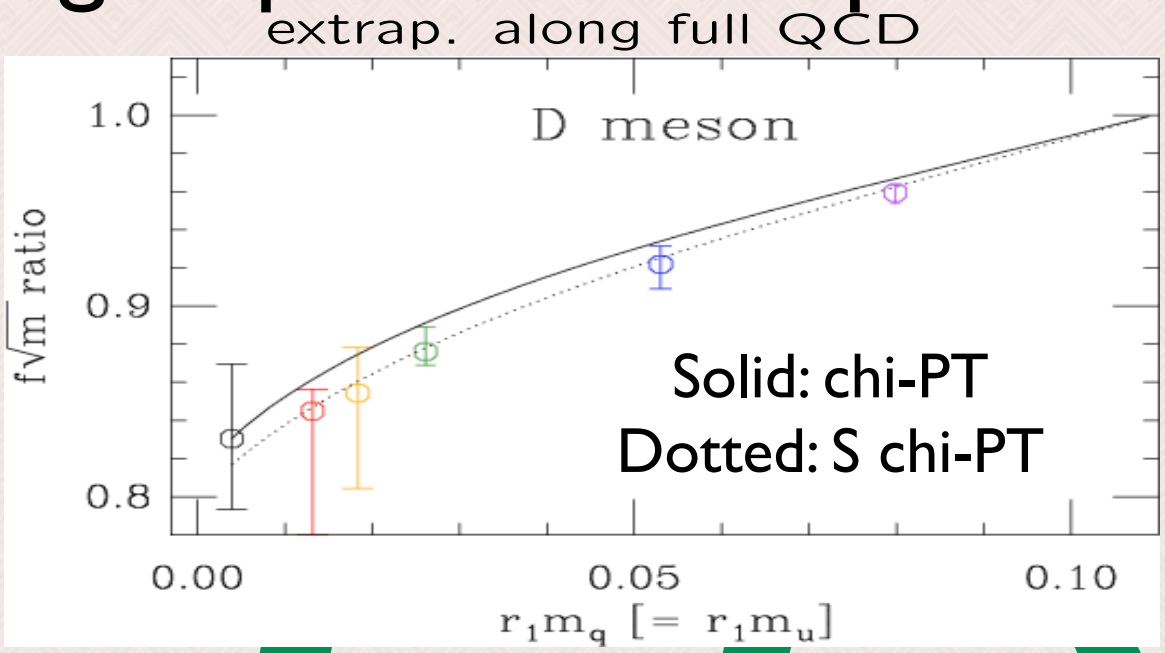


Ensemble 0.005/0.05



J. Simone, Lattice 2004

Light quark mass dependence



$$\xi_\Phi = \frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} = 1.20 \pm 0.06_{\rm stat} \pm 0.06_{\rm sys}$$

$$f_{D_s} = 263 \begin{array}{l} +5 \\ -9 \end{array} \pm 24 \text{ MeV}$$

$$f_D = 224 \begin{array}{l} +10 \\ -14 \end{array} \pm 22 \text{ MeV}$$

PRELIMINARY

J. Simone, Lattice 2004

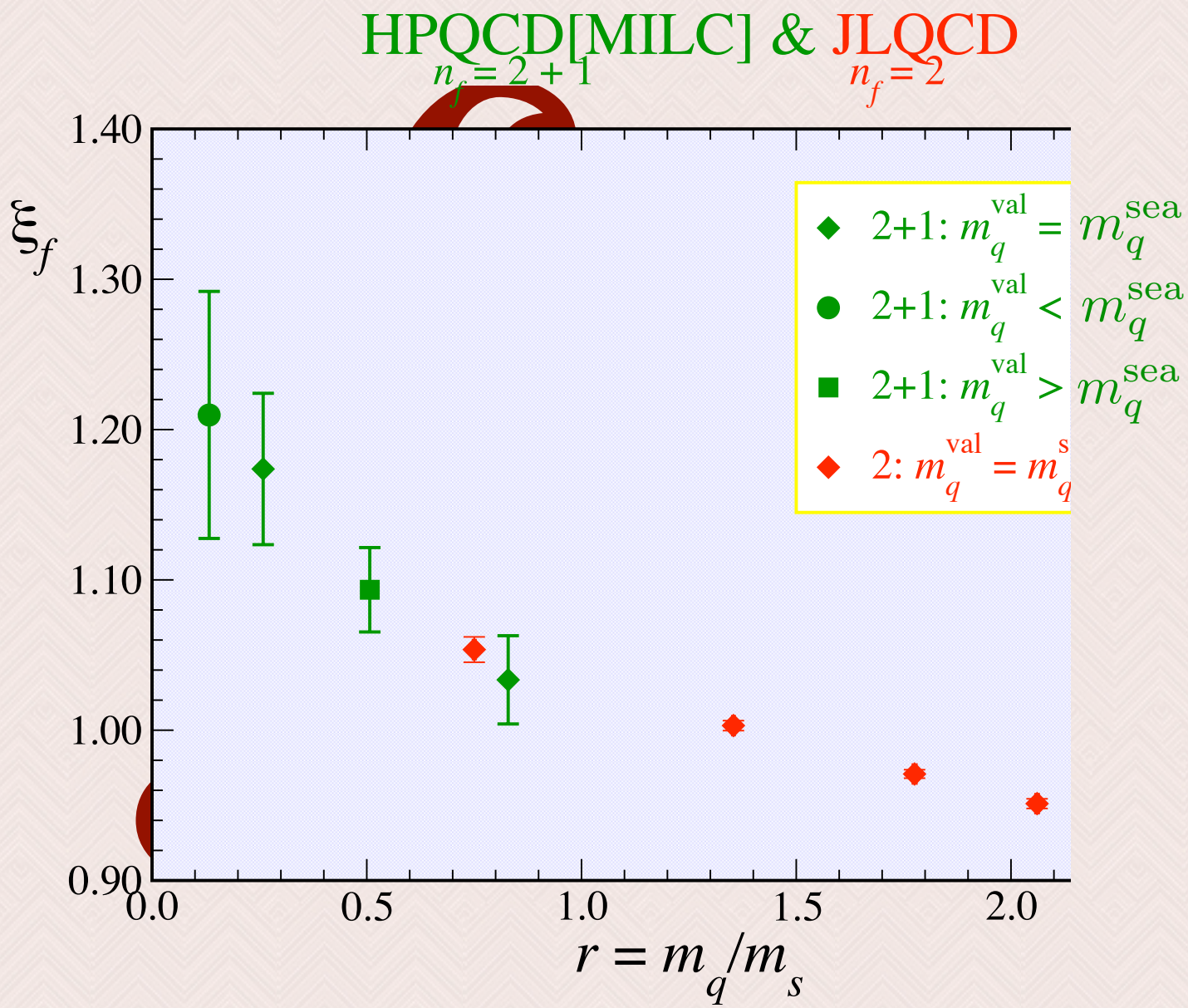
Error budget

PRELIMINARY

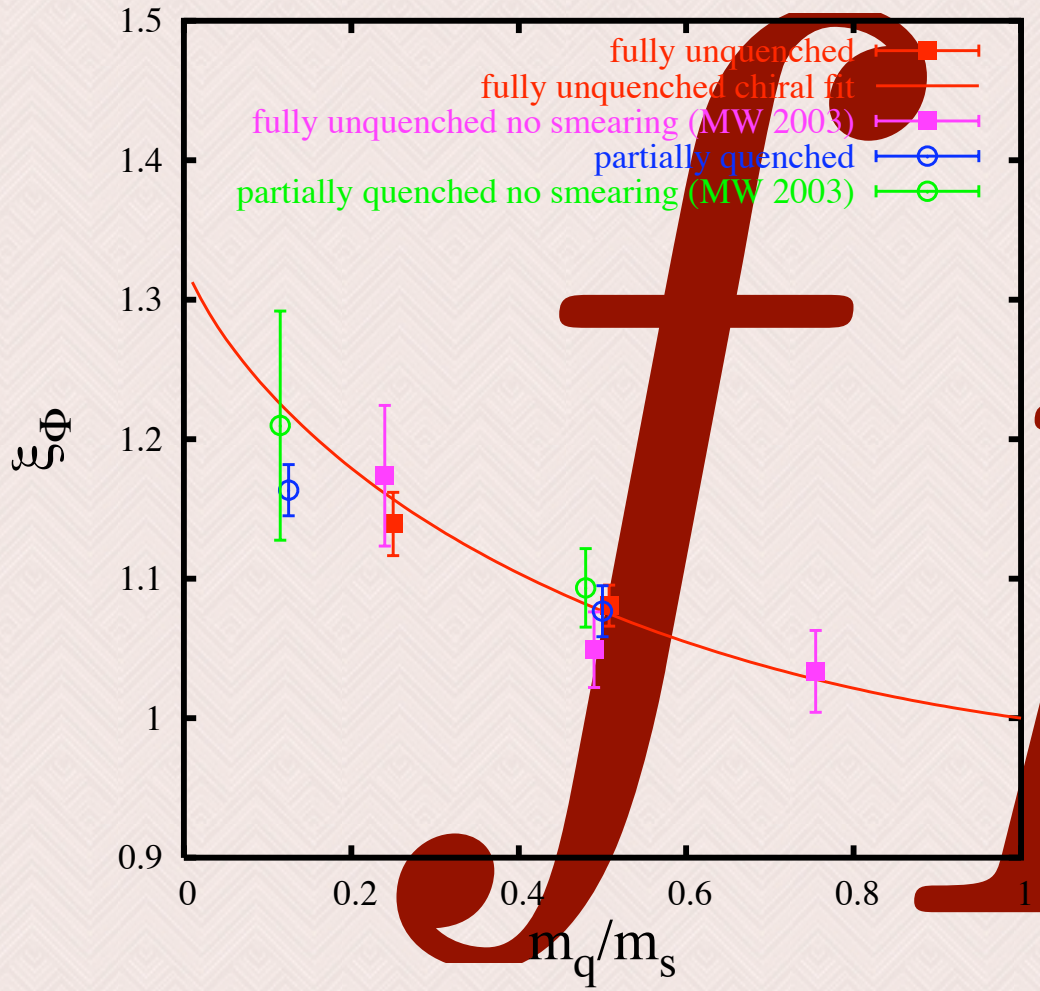
source	uncertainty as percent of:		
	d/s ratio	$f_{D_s}\sqrt{m_{D_s}}$	$f_D\sqrt{m_D}$
stat.+extrapolation	4.7	6.3	6.2
higher order SχPt	?	?	?
HQ matching to QCD	<1	10	10
LQ discret. effects		4	4
m_{charm} determination	1	4	4
valence m_s, m_d det.	2	1	2.2
lat. spac. & sea quark	<1	2	2

$$\int B$$

From A. Kronfeld, Lattice 2003



$$\xi_\Phi \equiv \frac{f_{B_s}\sqrt{m_{B_s}}}{f_B\sqrt{m_B}}$$

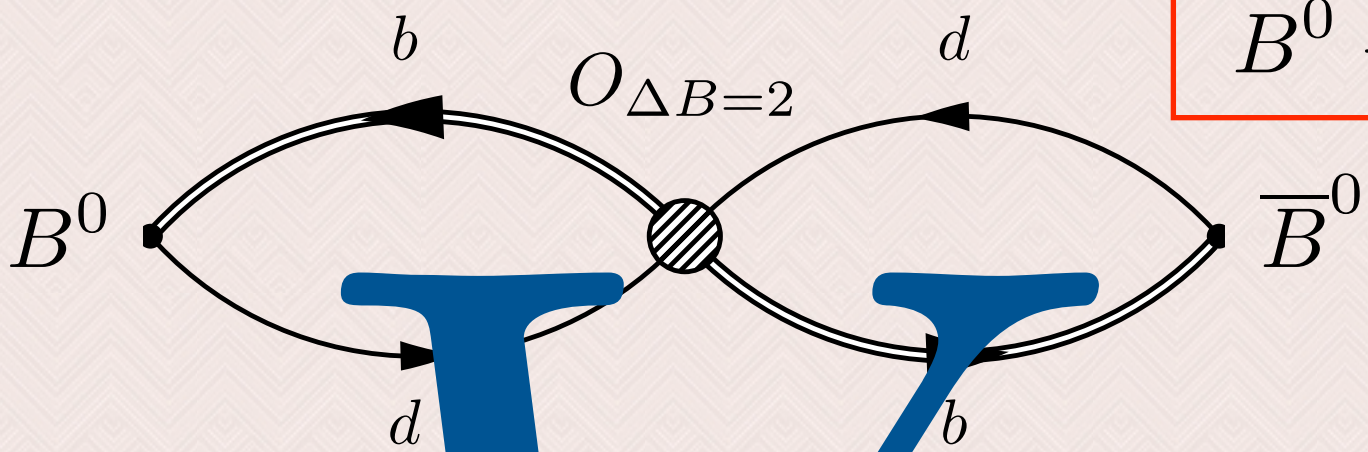


- MILC configurations
- NRQCD + KS
- Smeared sources/sinks improve statistics

A. Gray, Lattice 2004

$$V_{td}$$

$B^0 - \overline{B}^0$ mixing



$$\langle \overline{B}^0_s | (\bar{b}s)_{V-A} (\bar{b}s)_{V-A} | B^0_s \rangle = \frac{8}{3} f^2_{B_s} m^2_{B_s} B_{B_s}$$

$$\langle \overline{B}^0_s | (\bar{b}s)_{S-P} (\bar{b}s)_{S-P} | B^0_s \rangle = -\frac{5}{3} \left(\frac{f_{B_s} m_{B_s}}{m_b + m_s} \right)^2 B_{S_s}$$

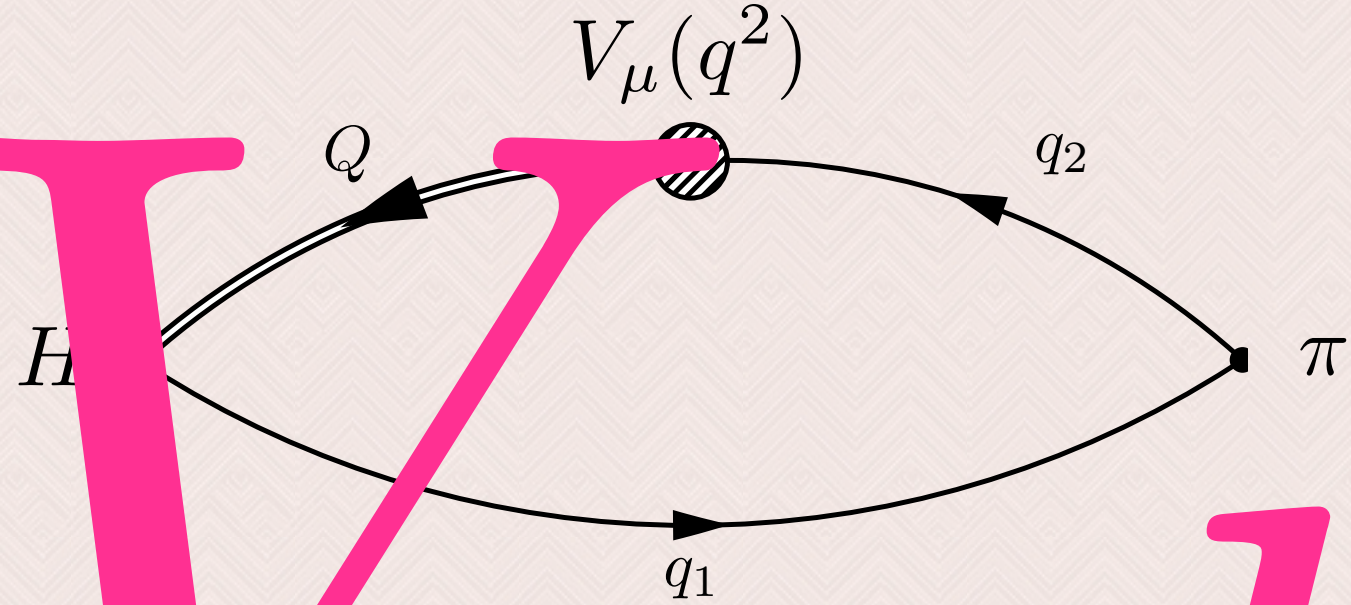
- JLQCD, PRL (2003); N. Yamada, Lattice 2001
- $B_{S_s}(m_b) = 0.86(3)(7)$
- A. Gray (HPQCD), Lattice 2004
  Calculation underway
- Chiral symmetry reduces mixings
  NRQCD+KS or Tsukuba+DWF (N. Yamada, Lattice 2004)

$$B_{B_d}(m_b) = 0.86(27)(^{+0}_{-27})(56),$$
$$B_{B_s}(m_b) = 0.90(22)(^{+18}_{-0})(57)(^{+5}_{-0}),$$
$$\frac{B_{B_s}}{B_{B_d}} = 1.04(16)(^{+53}_{-0})(17)(^{+6}_{-0}).$$

V_{td}

$$V_{cd}$$

Semileptonic 3 point function



$$\langle \pi | V_\mu | H \rangle \equiv f_+(q^2) \left(p_\pi + p_H - \frac{m_H^2 - m_\pi^2}{q^2} q \right)_\mu$$

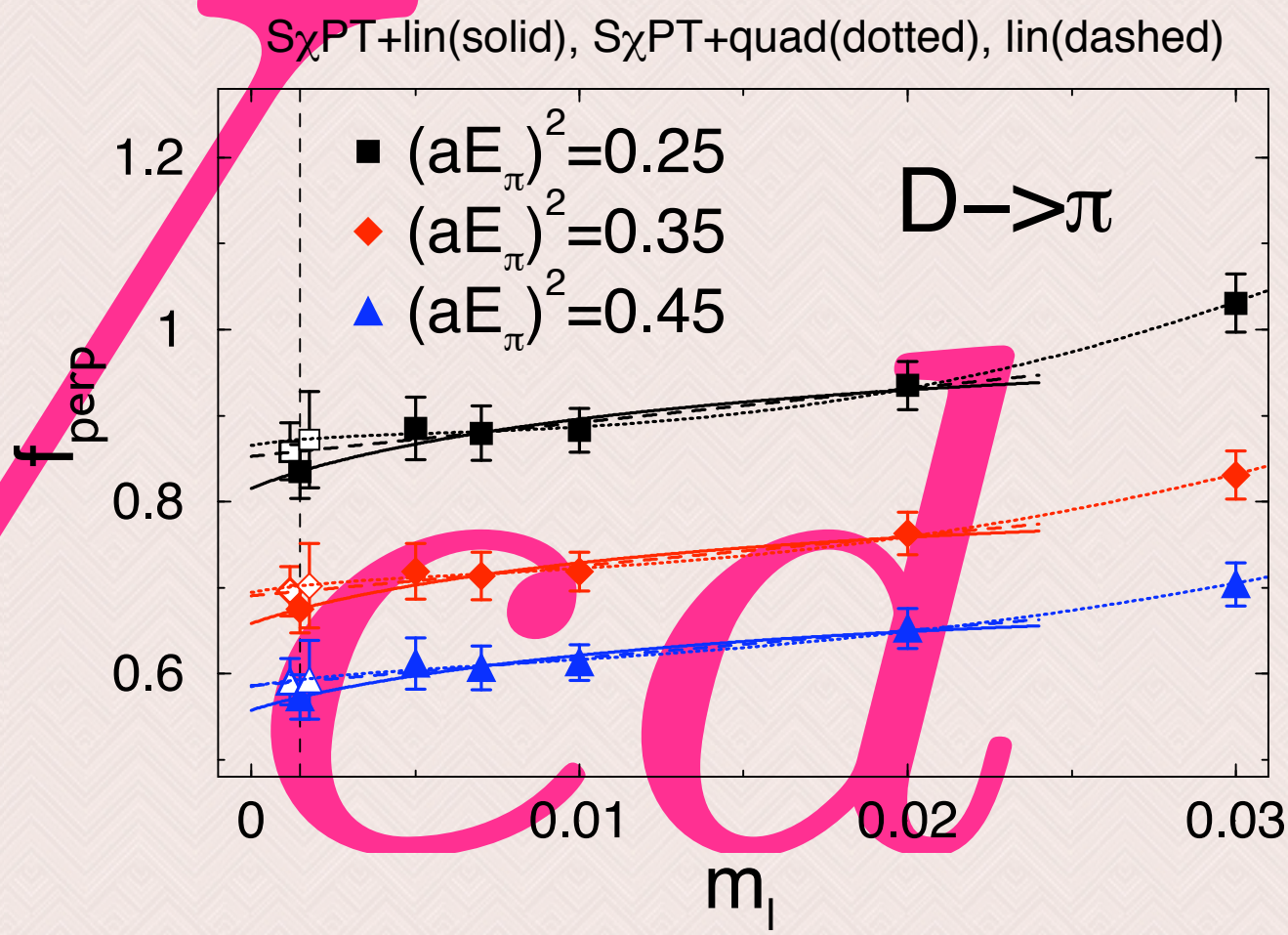
$$+ f_0(q^2) \frac{m_H^2 - m_\pi^2}{q^2} q$$

$$\equiv \sqrt{2m_H} \left(f_{||}(E_\pi) v_\mu + f_\perp(E_\pi) p_{\perp,\mu} \right)$$

V_{cd}

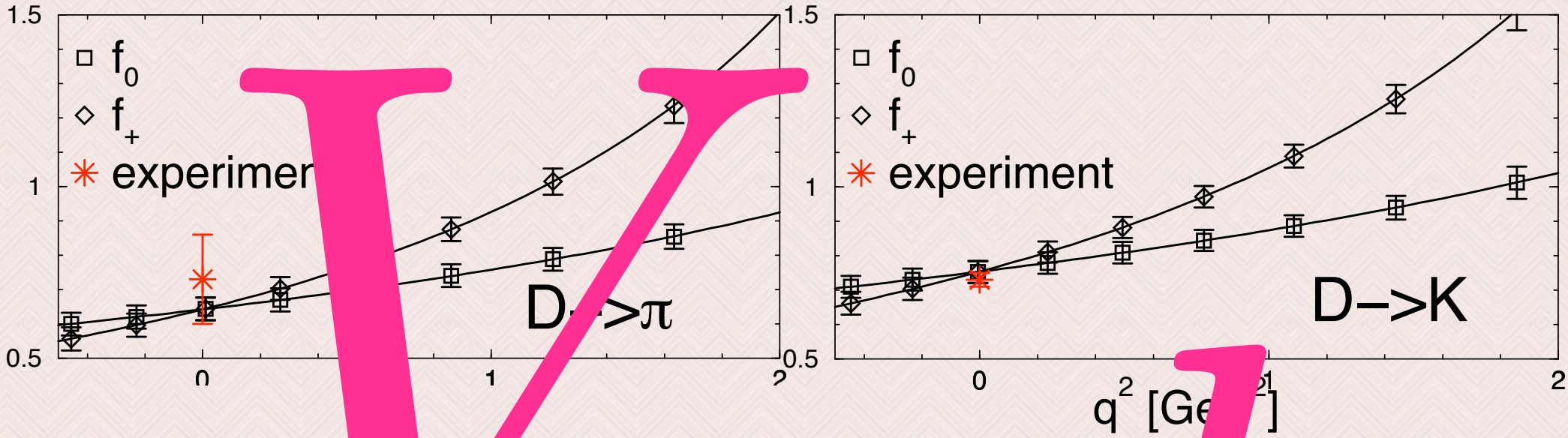
Physical up/down mass limit

- FNAL heavy + impr. stag. light (MILC configs)
- fully unquenched
- Aubin-Bernard S-chi-PT



M. Okamoto, Lattice 2004

q^2 dependence



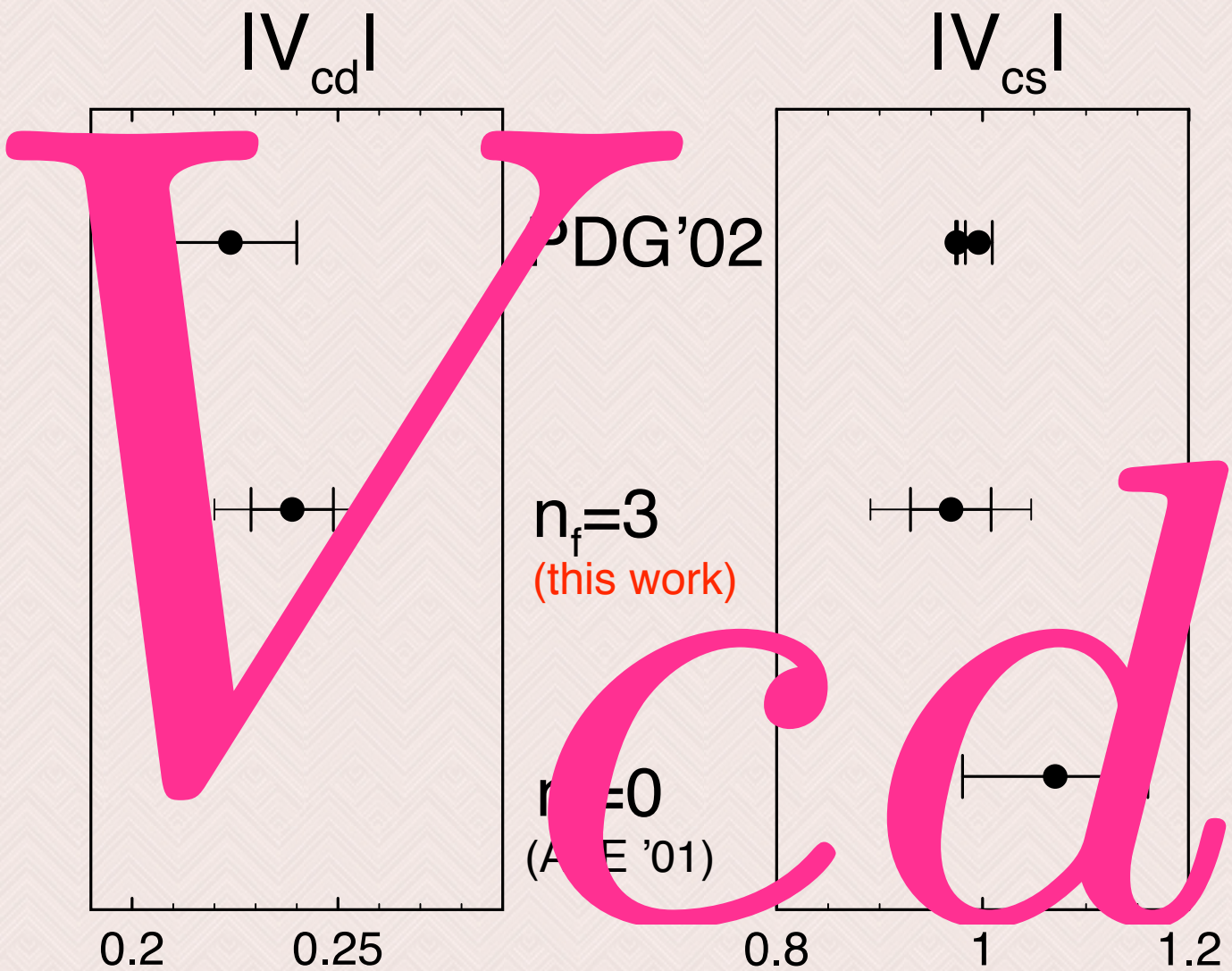
- Fit to Becirevic-Kaidalov ansatz
- Nearly final results:

$$f_+^{D\to\pi}(0) = 0.64(3)(6), \quad f_+^{D\to K} = 0.73(3)(6)$$

- Largest uncertainty due to heavy quark discretization: 7%

M. Okamoto, Lattice 2004

Combining f.f. with experiment

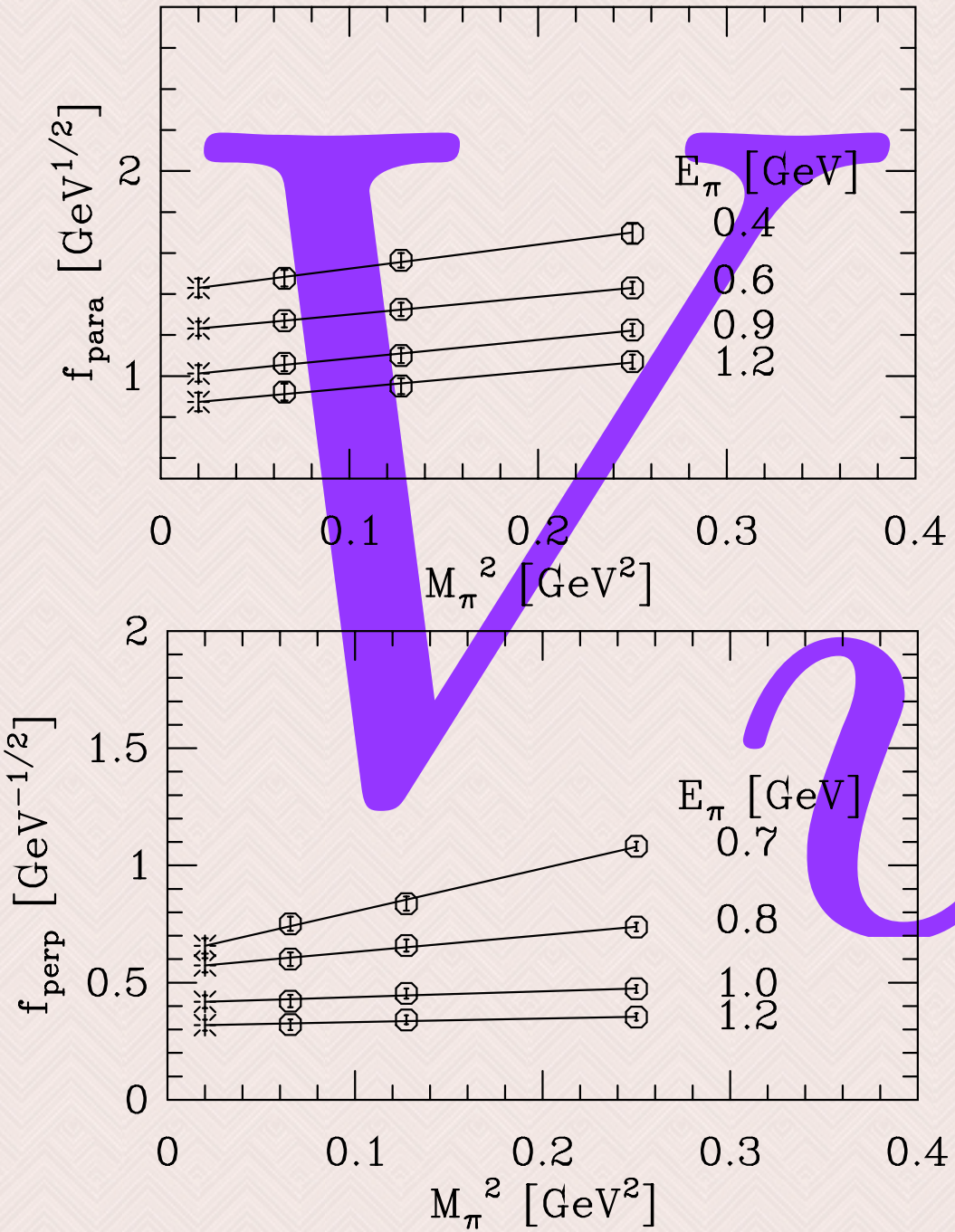


$$|V_{cd}| = 0.239(10)(19)(20), \quad |V_{cs}| = 0.969(39)(78)(24)$$

M. Okamoto, Lattice 2004

$$V_{ub}$$

Physical quark mass extrapolations



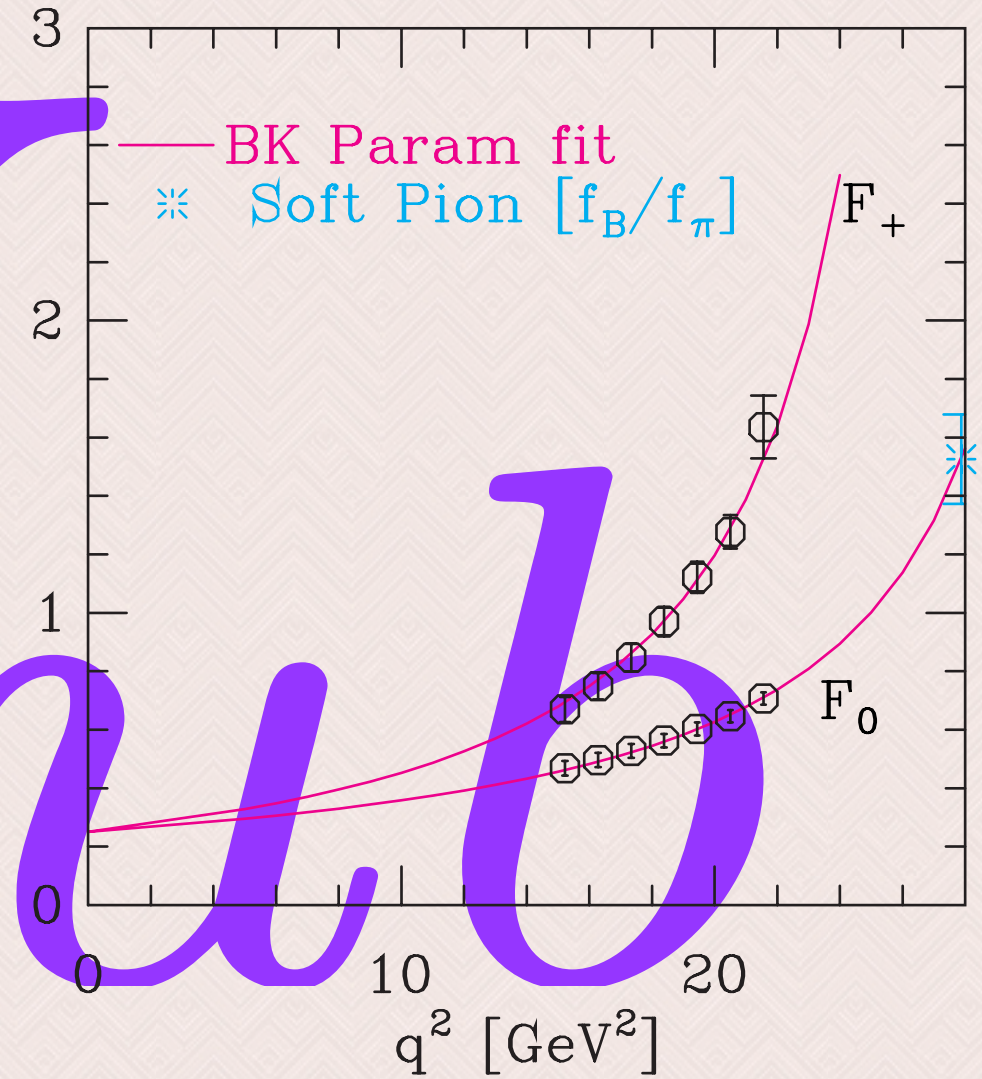
- NRQCD heavy + KS light
- coarser MILC configs
- partially quenched
- $m_{ud}^{\rm sea} = m_s/4$
- linear chiral extrapolation

J. Shigemitsu, Lattice 2004

q^2 dependence

- Fit to Becirevic-Kaidalov ansatz (B* pole plus effective pole)
- Result:

$$f_0(0) = f_+(0) = 0.25(2)$$



J. Shigemitsu, E. Gulez, Lattice 2004

Estimating $|V_{ub}|$

CLEO (hep-ex/0304019) has published the branching fractions,

$$\mathcal{B}(B^0 \to \pi^-, l^+\nu) = (1.33 \pm 0.18 \pm 0.11 \pm 0.01 \pm 0.07) \times 10^{-4}$$

for full range $0 \leq q^2 \leq q^2_{max}$ and

$$\mathcal{B}(q^2 \geq 16GeV^2) = (0.25 \pm 0.09 \pm 0.04 \pm 0.01 \pm 0.03) \times 10^{-4}$$

Using lattice determination of $f_+(q^2)$ one can integrate

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 |f_+(q^2)|^2$$

to get $\frac{\Gamma}{|V_{ub}|^2} \Longrightarrow \boxed{|V_{ub}|}$

Estimating $|V_{ub}|$ (cont'd)

Our preliminary results are (E.Gulez)

$$\frac{\Gamma}{|V_{ub}|^2} = \begin{cases} 5.80(93)\ ps^{-1} & 0 \leq q^2 \leq q^2_{max} \\ 1.31(16)\ ps^{-1} & 16GeV^2 \leq q^2 \end{cases}$$

hence,

$$|V_{ub}| = \begin{cases} 3.86(35)(62) \times 10^{-3} & 0 \leq q^2 \leq q^2_{max} \\ 3.52(70)(42) \times 10^{-3} & 16GeV^2 \leq q^2 \end{cases}$$

The errors (exper.)(lattice) are still tentative.

A recent review (Ali 2003) quotes an average of CLEO, BELLE and BABAR inclusive results

$$|V_{ub}|_{(inclusive)} = 4.32(57) \times 10^{-3}$$

slides from J. Shigemitsu, Lattice 2004

$$V_{cb}$$

$B \to Dl\nu$ decay

$$\langle D|V^\mu|B\rangle = \sqrt{m_B m_D} \times [h_+(w)(v_B + v_D)^\mu + h_-(w)(v_B - v_D)^\mu],$$

where $w = v_B \cdot v_D$.

$$\frac{d\Gamma(B \to Dl\nu)}{dw} \propto |\mathcal{F}_{B\to D}(w)|^2 |\mathbf{V_{cb}}|^2$$

$$\mathcal{F}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w).$$

We focus on the zero recoil limit ($w = 1$).

Ratio method (S. Hashimoto *et.al.* '99)

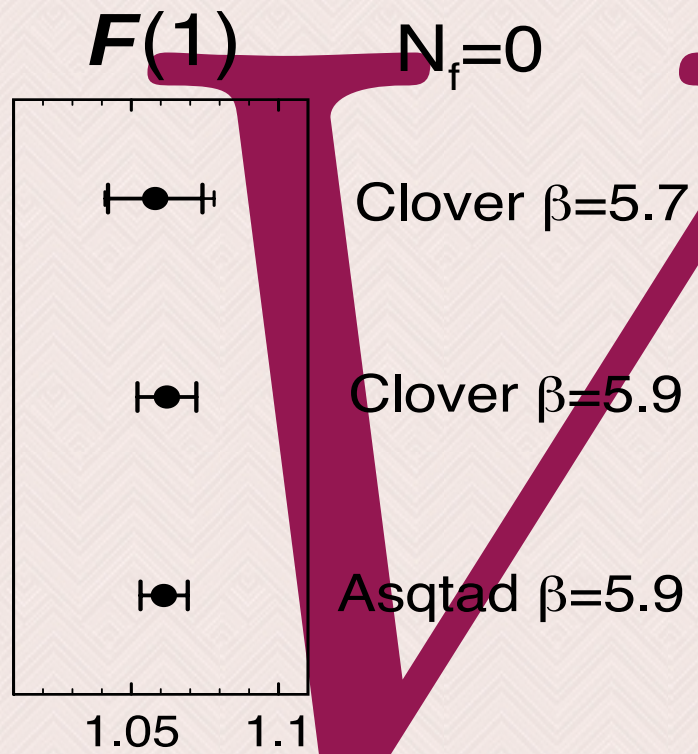
$$\frac{C^{DV_0B}(t)C^{BV_0D}(t)}{C^{DV_0D}(t)C^{BV_0B}(t)} \to \frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle} = |h_+^{B\to D}(1)|^2$$

$\mathcal{F}(1) = 1$ in $B = D$ limit.

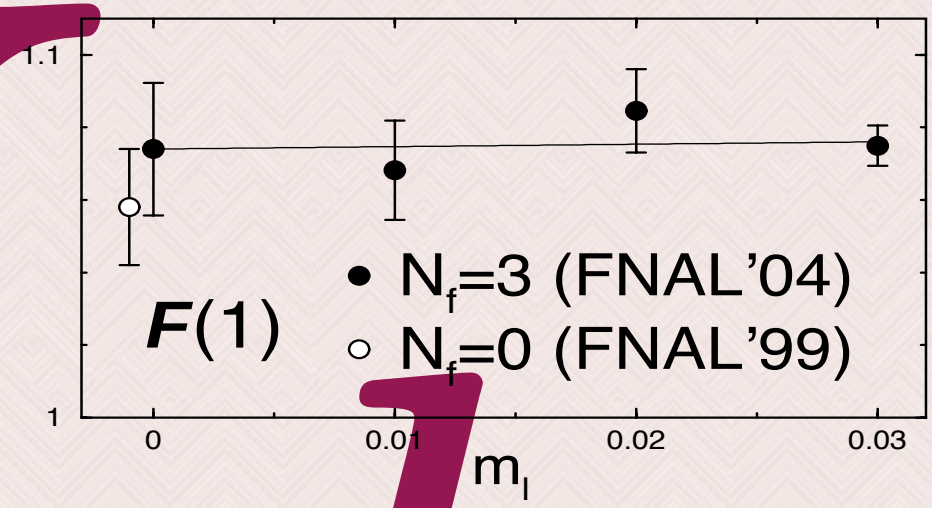
slide from M. Okamoto, Lattice 2004

$B \to D$ results (preliminary)

lattice spacing effect



dynamical quark effect



$$\mathcal{F}^{n_f=3}_{B\to D}(1) = 1.074(18)(15)$$

Using Belle's branching ratio

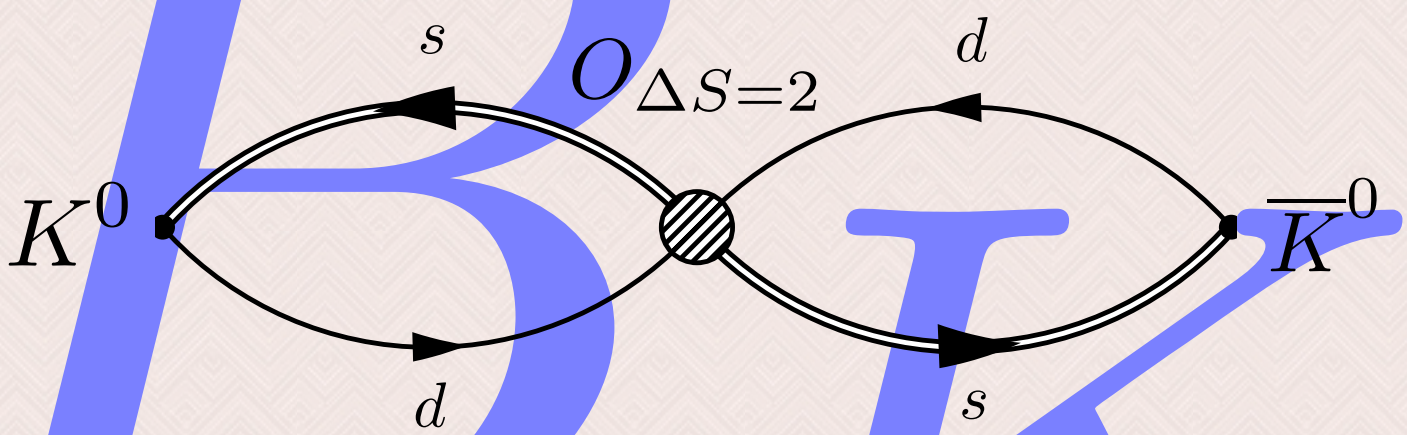
$$|V_{cb}| \times 10^2 = 3.83(07)(06)(64)$$

Unitarity check: $(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(4)(8)(2)$

slide from M. Okamoto, Lattice 2004

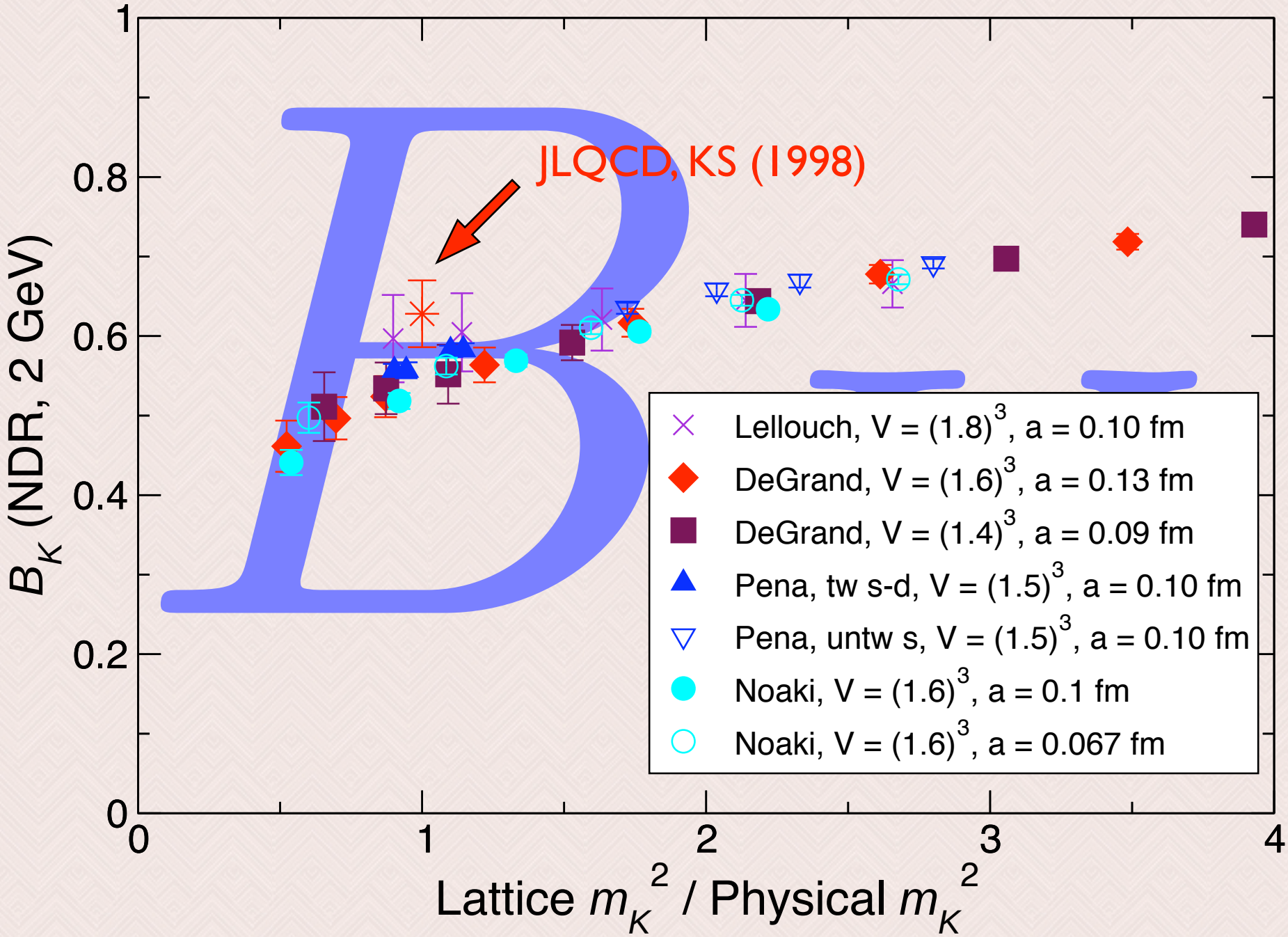
B_K

B_K

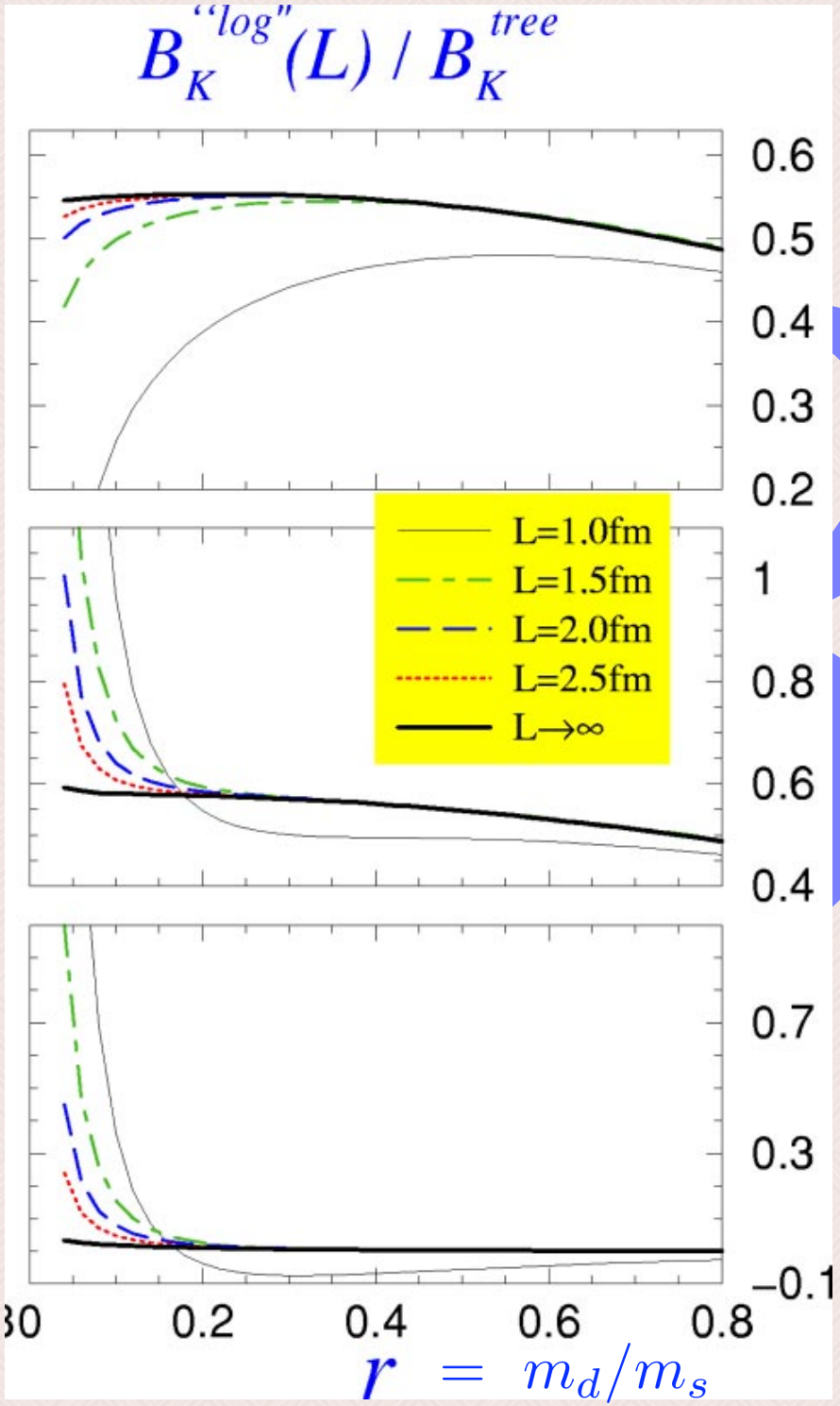


$$B_K(\mu) = \frac{\langle \overline{K^0} | O_{\Delta S=2} | K \rangle}{\frac{8}{3} f_K^2 m_K^2}$$

Quenched


JLQCD, KS (1998)
Lellouch, V = (1.8)^3, a = 0.10 fm
DeGrand, V = (1.6)^3, a = 0.13 fm
DeGrand, V = (1.4)^3, a = 0.09 fm
Pena, tw s-d, V = (1.5)^3, a = 0.10 fm
Pena, untw s, V = (1.5)^3, a = 0.10 fm
Noaki, V = (1.6)^3, a = 0.1 fm
Noaki, V = (1.6)^3, a = 0.067 fm
B_K (NDR, 2 GeV)
Lattice m_K^2 / Physical m_K^2
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4

Finite Volume and Partial Quenching



- Unquenched chi-PT
-
-
- Partially quenched chi-PT

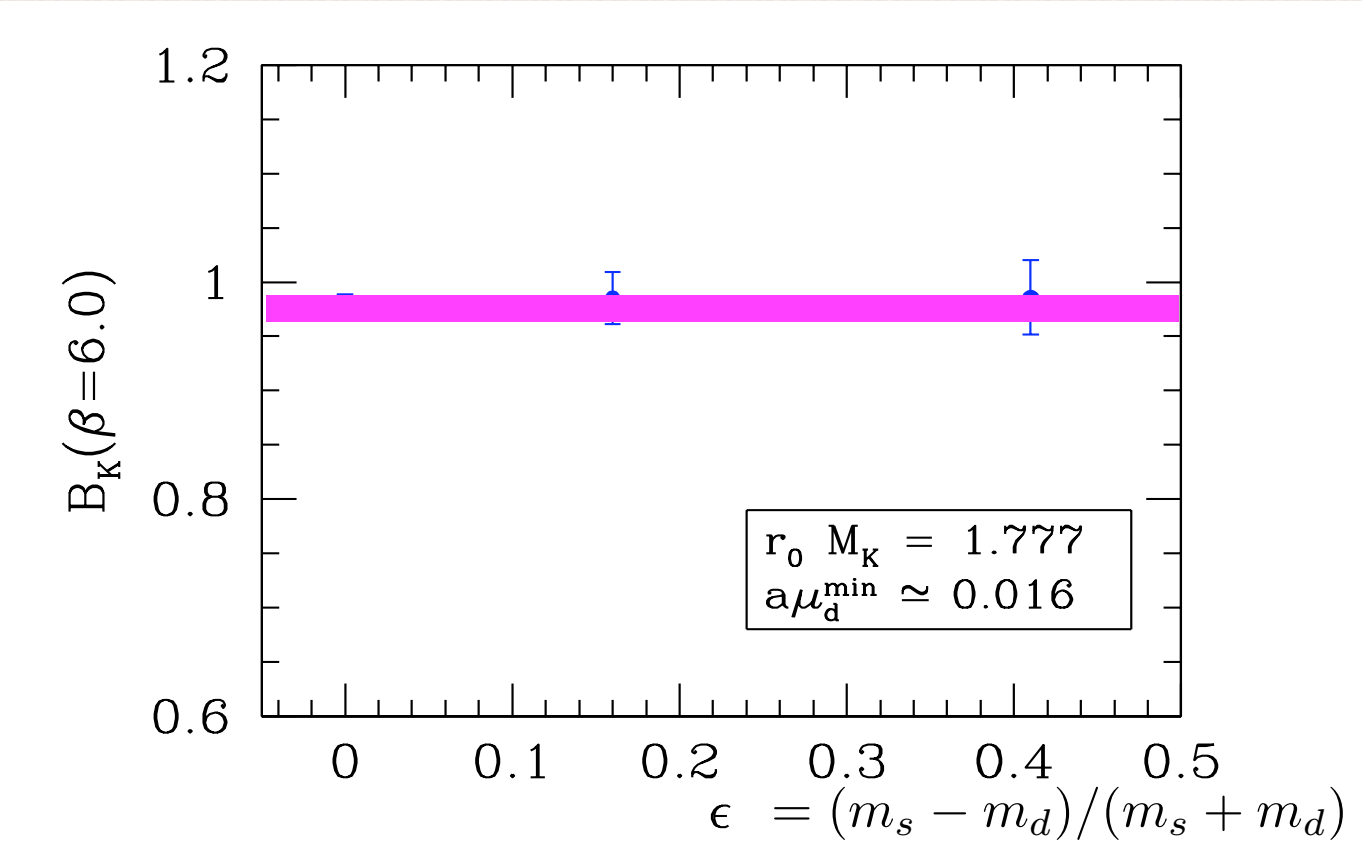
$$m_u^{\text{sea}} = m_{\text{strange}}/2$$

-
- Quenched chi-PT

D. Becirevic, G. Villadoro, PRD **69** (2004)

R. van de Water working on S-chi-PT

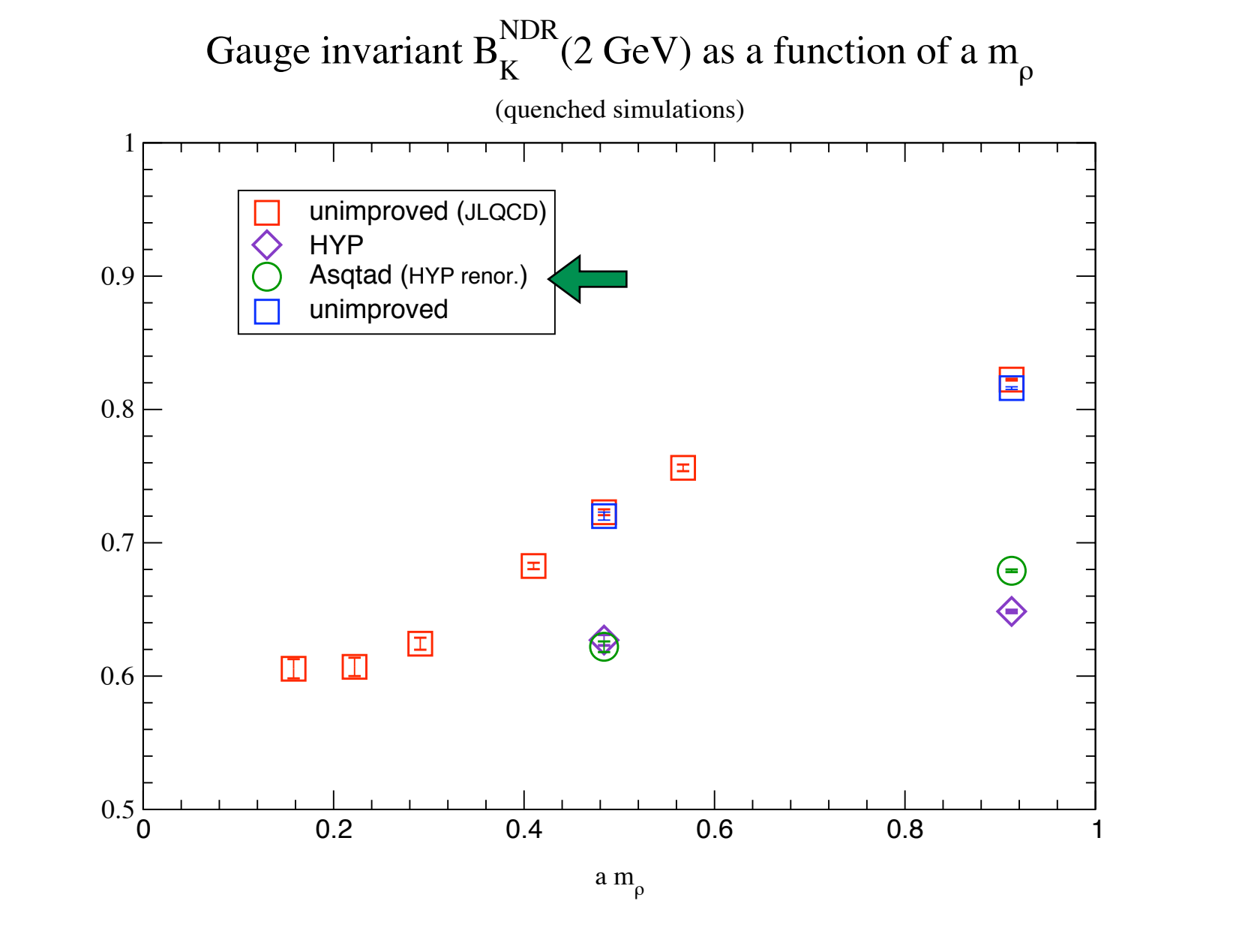
SU(3) breaking (quenched)



- tmQCD

C. Pena, Lattice 2004

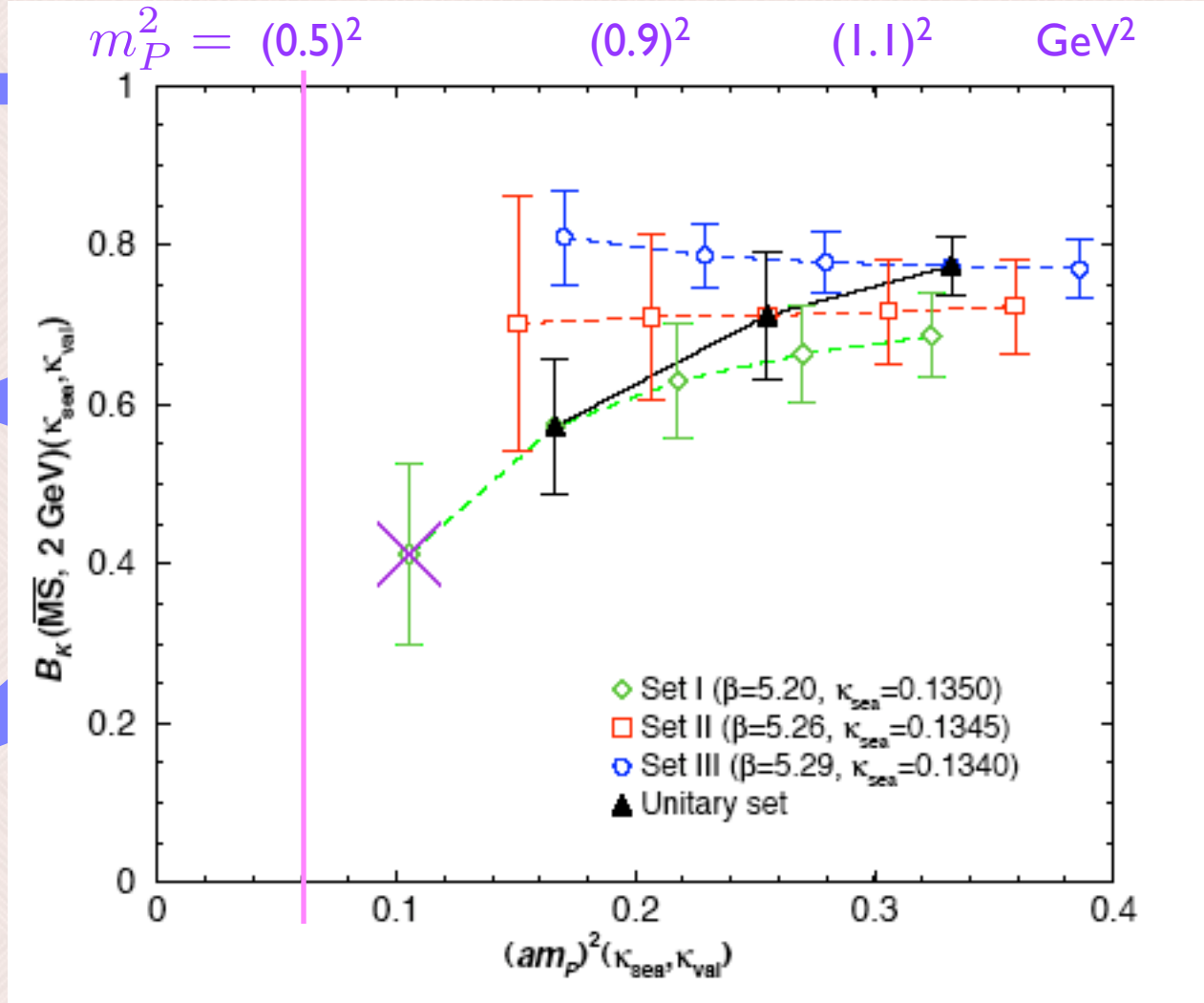
Quenched -- improved staggered



E. Gamiz, S. Collins (UKQCD), Lattice 2004

B_K on UKQCD lattices

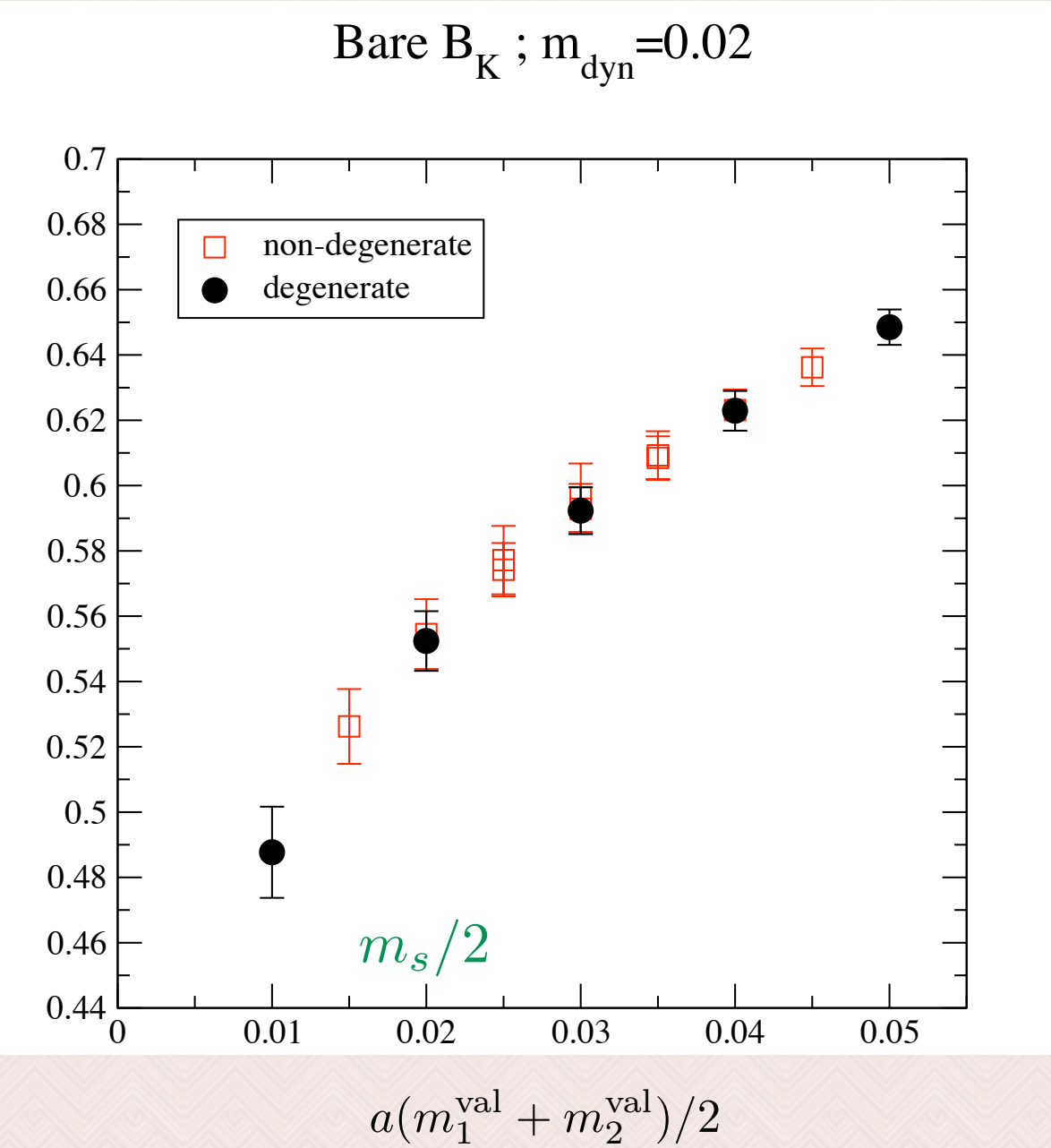
- 2 flavor sea quarks
- Clover w/ NP c_{SW}
- a = 0.1 fm
- $V = (1.6\ \text{fm})^3$



J. Flynn, F. Mescia, A.S. Bin Tariq, hep-lat/0406013

Unquenched DWF

- Downward trend with decreasing $m_{ud}^{\rm sea}$
- $V = (1.9\,{\rm fm})^3, a = 0.12$ fm
- also unquenched on MILC configs with impr. staggered (E. Gamiz, Lat2004) too preliminary



C. Dawson, Lattice 2004

Recapitulation

- Heavy-light decay constants
  - Impressive progress
  - Staggered chi-PT
  - Reductions of perturbative errors
- Neutral B mixing
  - JLQCD PRL (2003); HPQCD now working on MILC lattices
- Semileptonic form factors for heavy-light mesons
  - Impressive progress
  - Further explore q^2 dependence & related uncertainties
- Neutral K mixing
  - Need unquenched, larger volumes, smaller sea quark masses

Q&A