On modifying gravity as an alternative to dark matter

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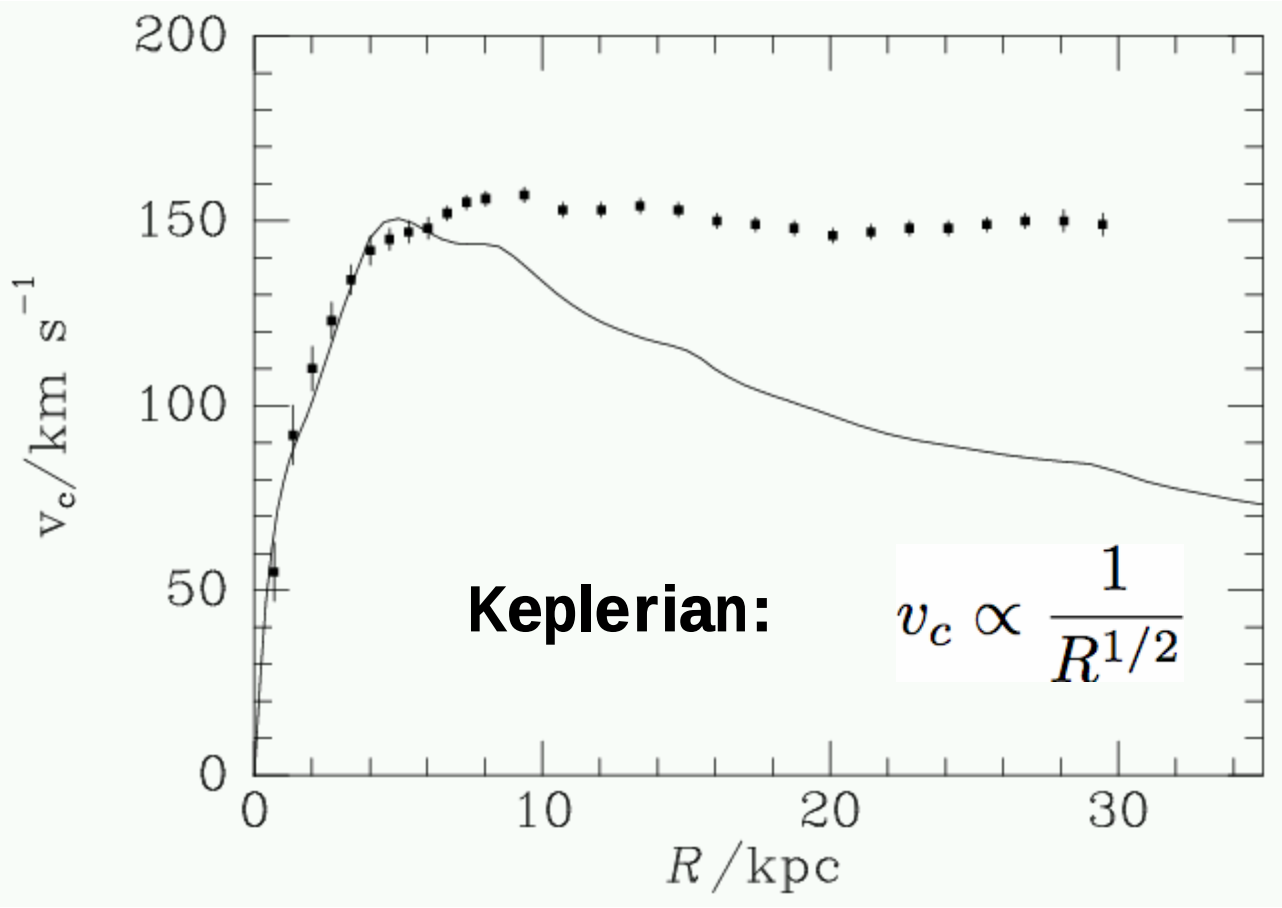
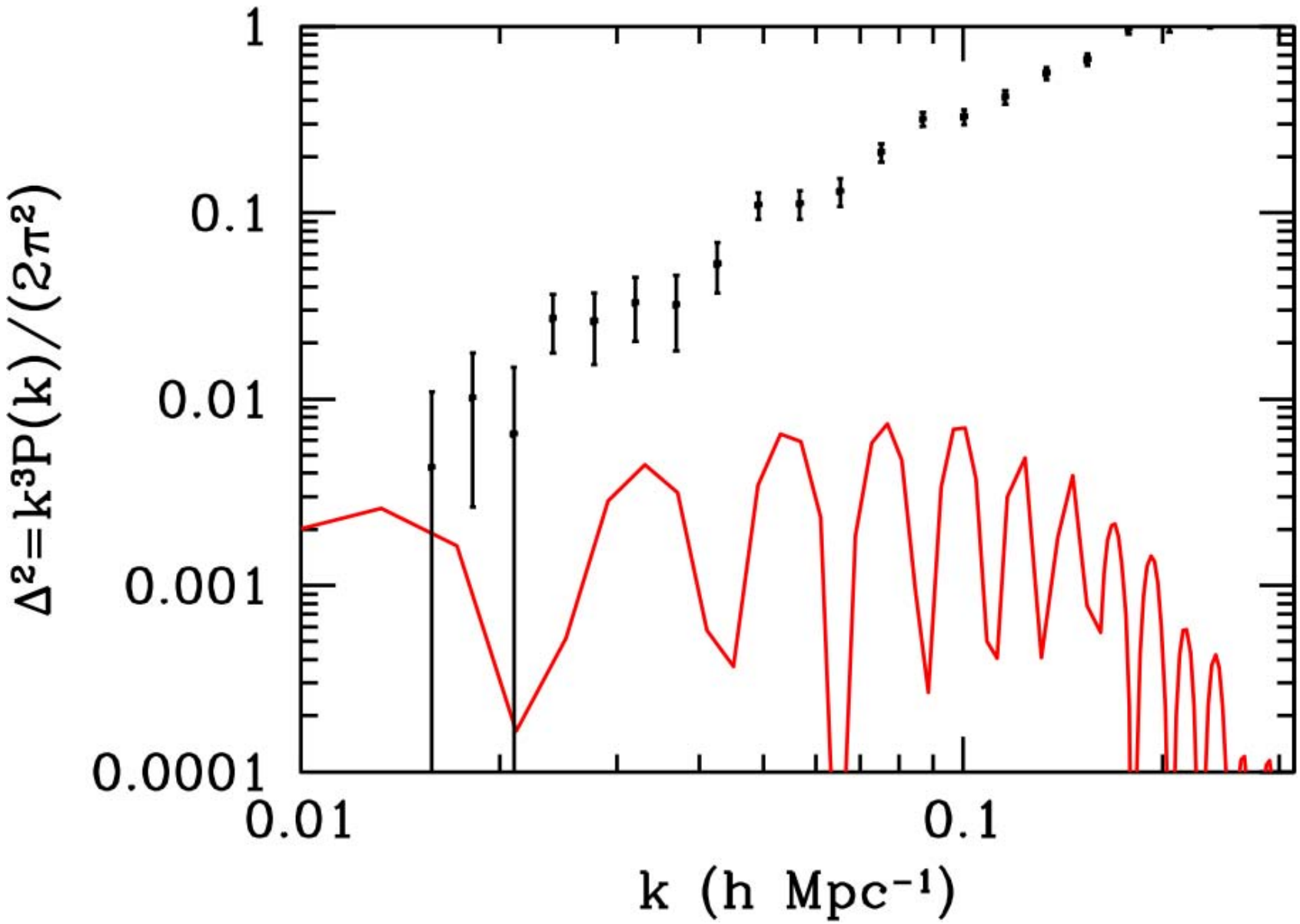
Roadmap

- The problem with gravity.
- Preaching to the converted: Dark Matter.
- An alternative approach: Modified Gravity.
- How to build a theory.
- Isn't this dark matter?
- Conclusions.

NGC 3198



Begeman 1987


1
0.1
0.01
0.001
0.0001
0.01
0.1
$\Delta^2 = k^3 P(k)/(2\pi^2)$
k (h Mpc^{-1})

Density Perturbations

$$\delta_B = \frac{\rho_B - \bar{\rho}_B}{\bar{\rho}_B} \qquad \delta_\gamma = \frac{\rho_\gamma - \bar{\rho}_\gamma}{\bar{\rho}_\gamma}$$

Three Regimes

- Tight coupling

$$\ddot{\delta}_B + 2\frac{\dot{a}}{a}\dot{\delta}_B - c_S^2 k^2 \delta_B = \mathcal{S}(\dot{\Phi}, \ddot{\Phi})$$

- Recombination

$$\delta_B \propto e^{-(k_S t)}$$

- Free Streaming

$$\delta_B \propto a$$

$$\boxed{\nabla^2 \Phi = 4\pi G(\rho_B + \rho_\gamma)}$$

Preaching to the converted: Dark Matter

$$\boxed{\nabla^2\Phi = 8\pi G(\rho + \rho_C)}$$

If $\rho_C \propto 1/r^2$

then $$v^2 = \frac{GM(R)}{R} \propto \frac{R}{R} \rightarrow \text{constant}$$

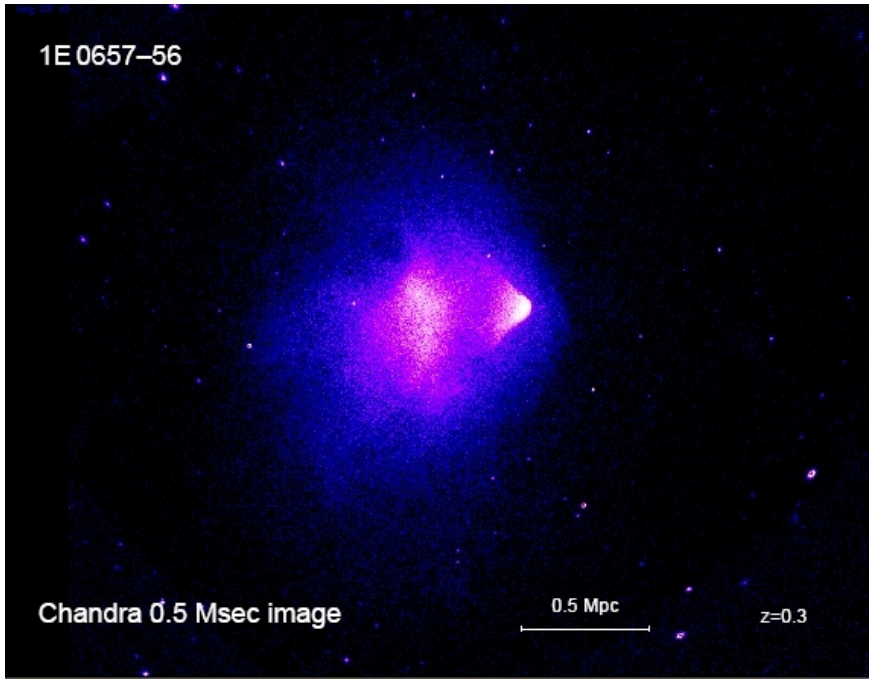
and $$\ddot{\delta}_B + 2\frac{\dot{a}}{a}\dot{\delta}_B - c_S^2 k^2 \delta_B \neq 0$$

through recombination

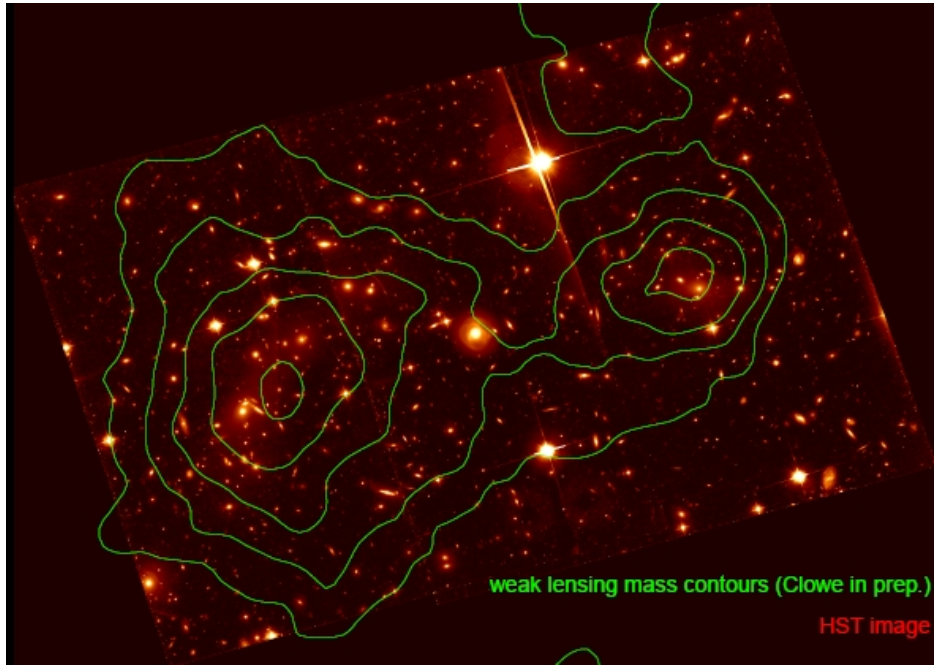
Simple and “predictive”

- Cross sections of order the weak scale, masses between Gev and Tev.
- Gravity may not be aligned with light.
- Globular clusters look different from Dwarf Galaxies.

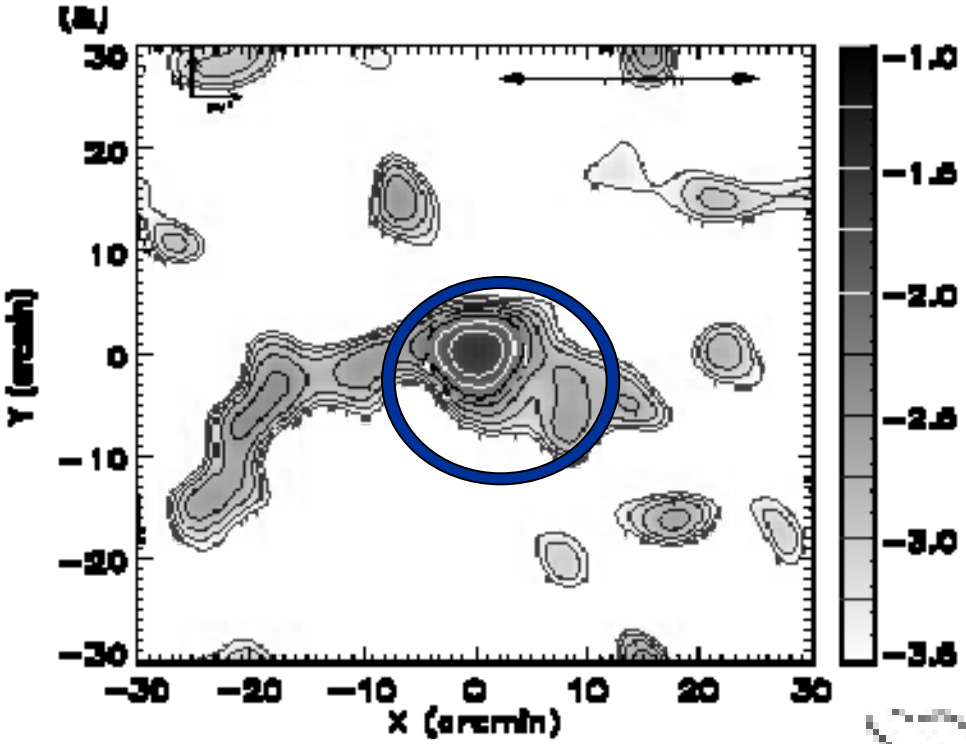
The Bullet Cluster



Displaced potential wells



Clowe et al, 2006

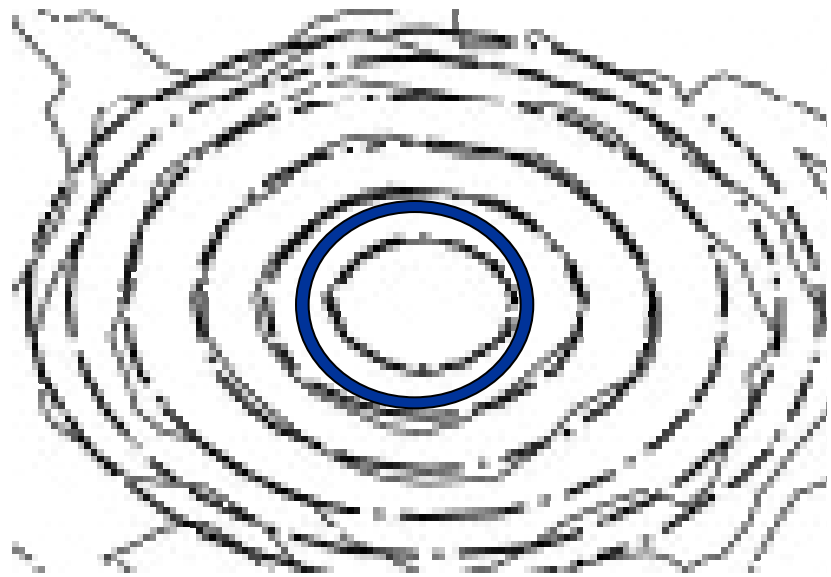


Zhao

$N = 10^5$ **Stars**

Globular Cluster

Dwarf Galaxy



(same external gravity)

Birkhoff's theorem: solution unique

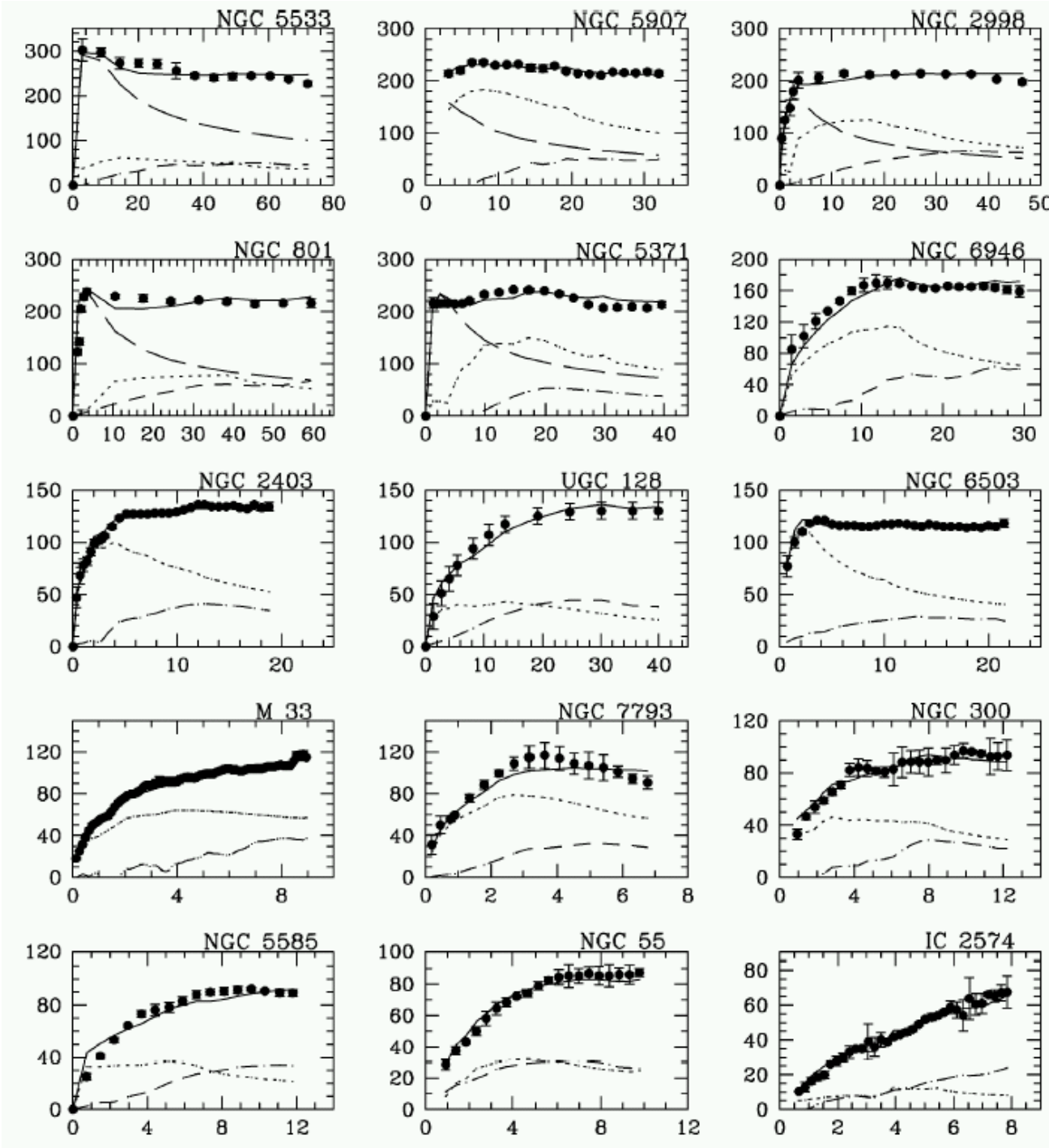
Alternative solution: modified law of inertia …

$$\mu\left(\frac{a}{a_0}\right)a = \frac{GM}{r^2} \qquad \mu(x) = \begin{cases} 1 & x \gg 1 \\ x & x \ll 1 \end{cases}$$

$$\frac{v^4}{a_0}\frac{1}{r^2} = \frac{GM}{r^2}$$

Milgrom, 83

$$v^4 = a_0 GM \rightarrow$$ **constant**



Sanders et al

… is a modified theory of gravity

$$\mu\vec{a} = -\vec{\nabla}\Phi_N \qquad \nabla^2\Phi_N = 4\pi G\rho$$

Rewrite as:

$$\vec{a} = -\vec{\nabla}\Phi \qquad \vec{\nabla}\cdot[f(\frac{\nabla\Phi}{a_0})\vec{\nabla}\Phi] = 4\pi G\rho$$

We need a relativistic theory (a generally covariant action) if we want true predictions.

Hints of relativity

$$a_0 \sim cH_0 \sim 10^{-8}\mathrm{cm/s}^2$$

All in the metric: $g_{00} \sim (1 + 2\Phi)$

Simplest approach: modify the Einstein-Hilbert action

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R$$

$$\Downarrow$$

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}F(R, R_{\mu\nu}, R_{\mu\nu\alpha\beta}, C_{\mu,\nu\alpha\beta}, ...)$$

- **Higher derivatives in the metric.**
- **Instabilities, ghosts (although not inevitable)**
- **New degrees of freedom (frozen modes become dynamical)- new fields.**

Keep Einstein Hilbert action

Dynamics
(geodesic equation)

$$a^\mu + \Gamma^\mu_{\alpha\beta} v^\alpha v^\beta = 0$$

Gravity
(Einstein equations)

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$g_{\mu\nu}$$

Common metric

“Bimetric” theories

Use two different metrics

“Physical” in Geodesic equations

“Geometric” metric in Einstein equations

$$g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu$$

Bekenstein, 04

$$\tilde{g}_{\alpha\beta} A^\alpha A^\beta = -1$$

Time-like (“aether”)

Again! New degrees of freedom (new fields)

A theory (TeVeS)

$$S = S_T + S_V + S_S + S_{matt}$$

$$S_T = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}R(\tilde{g}) \qquad S_{matt} = \int d^4x\sqrt{-g}\mathcal{L}(g, matter\ fields)$$

constants

$$S_V = -\frac{1}{32\pi G}\int d^4x\sqrt{-\tilde{g}}[K_B F^{\alpha\beta}F_{\alpha\beta} - 2\lambda(A^\mu A_\nu + 1)]$$

Lagrange multipliers

$$S_S = -\frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\{\mu[(\tilde{g}^{\mu\nu} - A^\mu A^\nu)\phi_{,\mu}\phi_{,\nu}] + V(\mu;\mu_0,\ell_B)\}$$

Einstein-Hilbert, one metric and Aether field

- Rewrite action solely in terms of physical metric and vector field

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} + \mathcal{L}(g, A) \right] + S_M$$

Extra degrees of freedom (the Aether)

Zlosnik et al

How do we modify gravity?

$$G_{\alpha\beta} = 8\pi G T^{matt}_{\alpha\beta} + 8\pi G T^{A}_{\alpha\beta}$$

Take non-relativistic, weak field limit:

$$\left.\begin{aligned} g_{00} &= -(1+2\Phi) \\ A &= (1+\alpha_0, \vec{\alpha}) \end{aligned}\right\} \Rightarrow g_{\mu\nu}A^{\mu}A^{\nu} = -1 \rightarrow \Phi = -\alpha_0$$

$$G^{mod}_{\alpha\beta}(\Phi) \equiv G_{\alpha\beta}(\Phi) - 8\pi G T^{A}_{\alpha\beta}(\Phi) = 8\pi G T^{matt}_{\alpha\beta} + 8\pi G T^{A}_{\alpha\beta}(\vec{\alpha})$$

Modified gravity

Extra d.o.f.

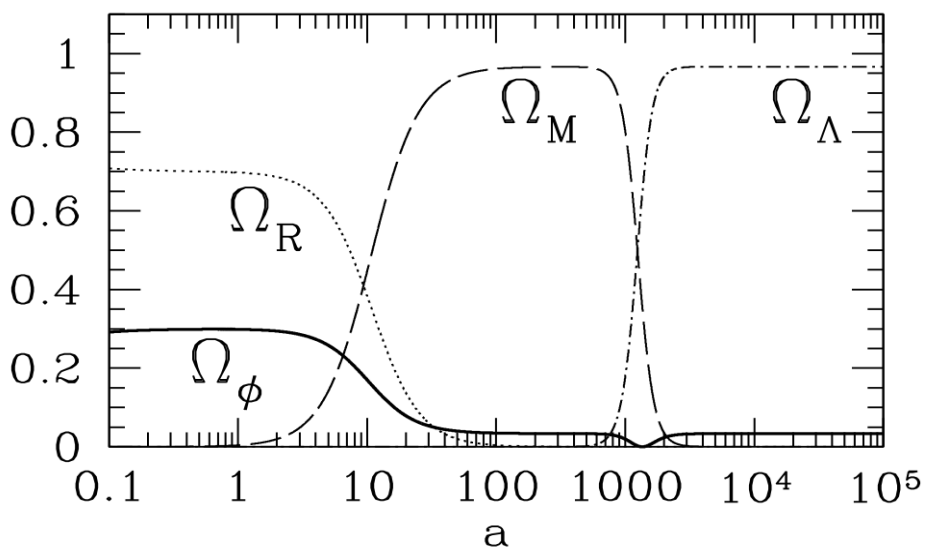
Extra degrees of freedom …

Is modified gravity simply a more contrived form of Dark Matter?

Cosmology of Bekenstein's theory

Skordis et al, 2006

$$H^2 = \frac{8\pi G_{eff}}{3}(\rho + \rho_\phi)$$



BBN:

$$\Omega_\phi < 10^{-2}$$

Not Dark Matter

Perturbations: definitions

Metric

$$g_{00} = -a^2(1+2\Phi) \quad g_{ij} = a^2(1-2\Psi)\delta_{ij}$$

Scalar Field

$$\phi = \bar{\phi} + \varphi$$

Vector Field

$$A = (1+\alpha_0, \vec{\alpha})$$

$$\Phi = -\alpha_0$$

Extra d.o.f

Perturbations

Approximation: $$\ddot{\alpha} + \frac{4}{\tau}\dot{\alpha} + \frac{2(1+\epsilon)}{\tau^2}\alpha = \mathcal{S}(\Phi, \Psi)$$

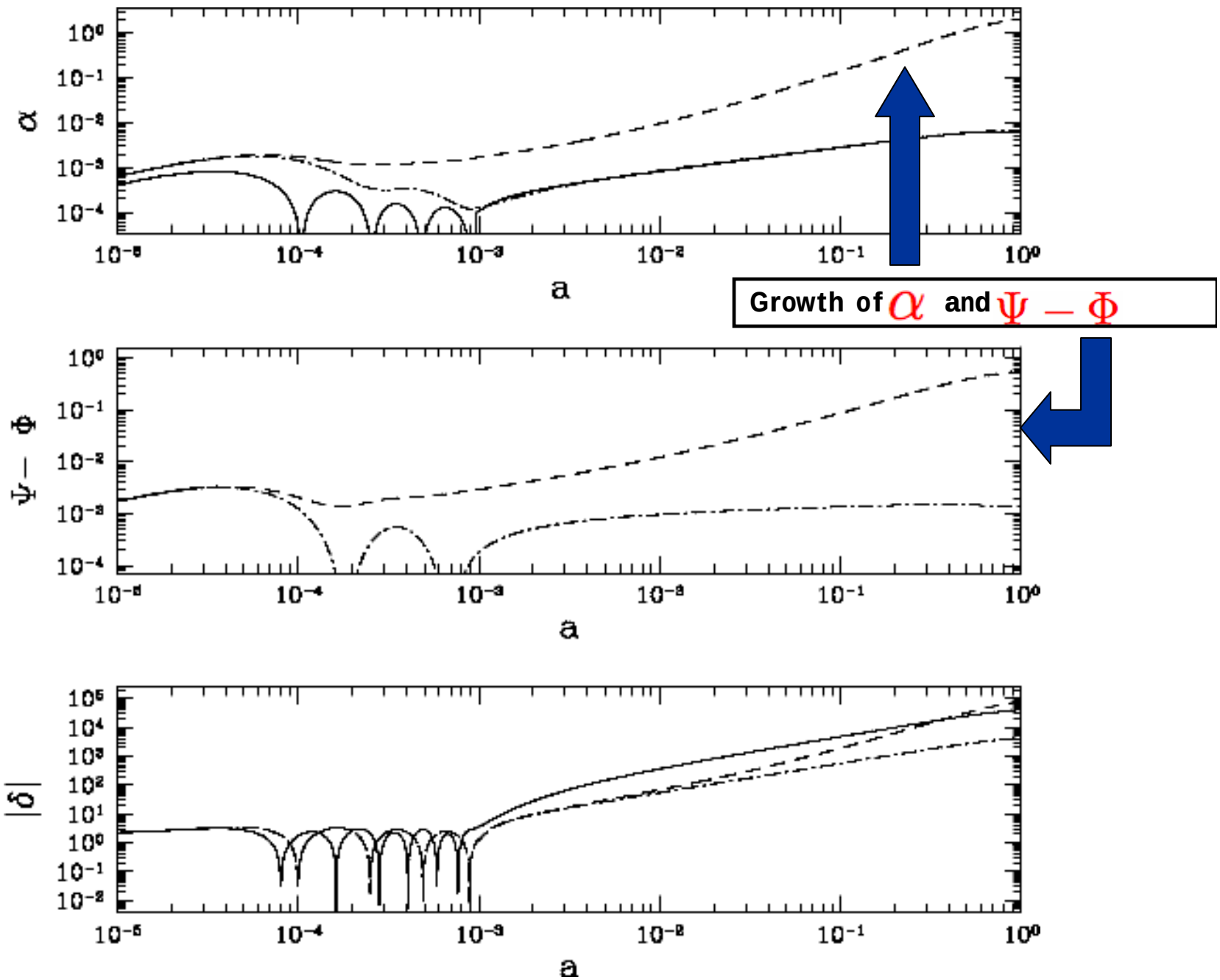
$$\alpha \propto \tau^p$$ **with** $$p_\pm = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9 - 8(1+\epsilon)}$$

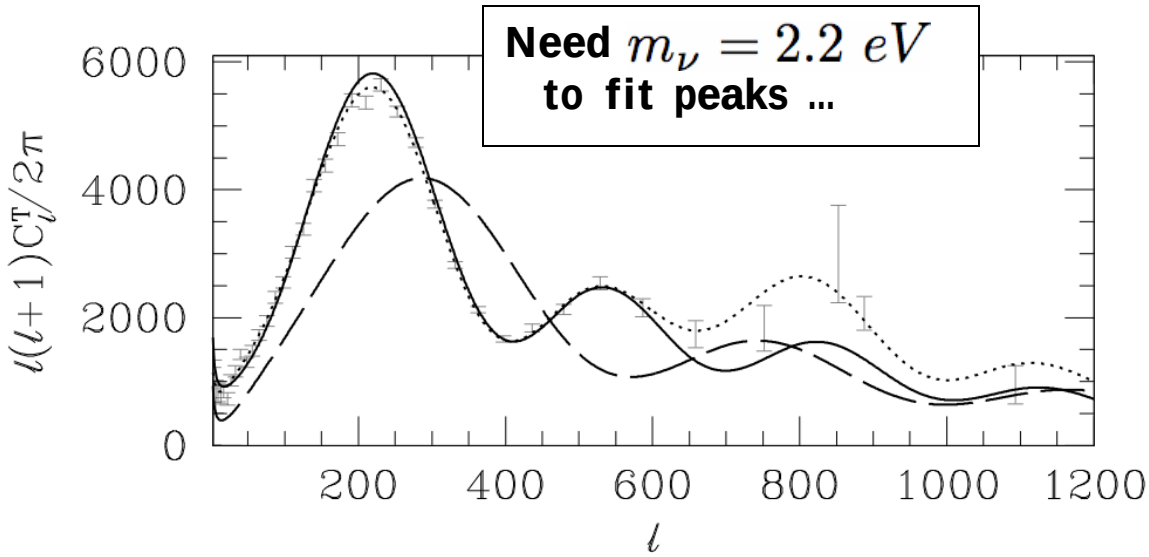
where

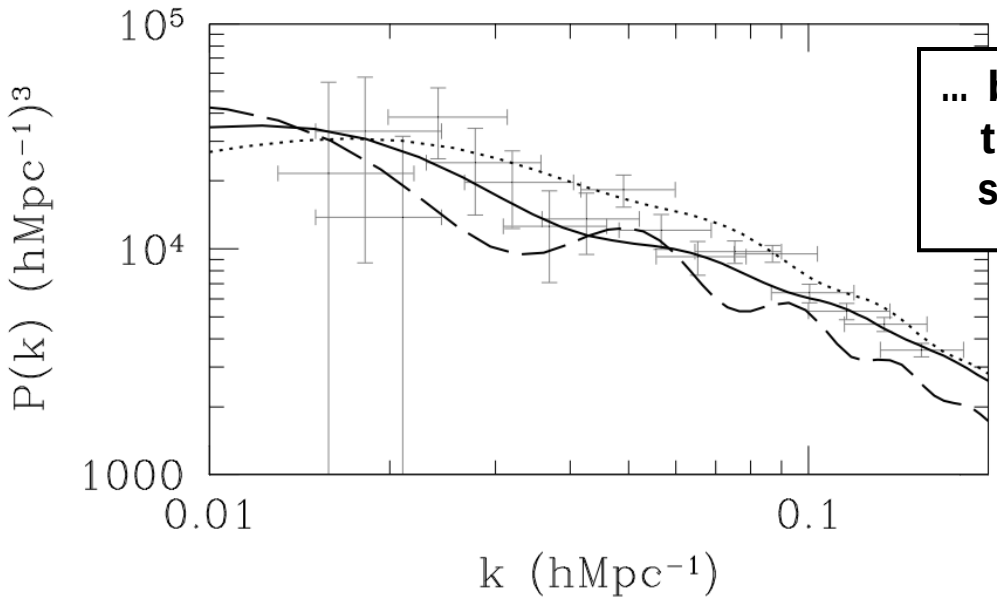
$$\epsilon \simeq -\frac{12\ln(a/5\times 10^{-5})}{\mu_0 K}$$ **Dodelson and Liguori, 2006**

$$\nabla^2\Phi = 4\pi G\delta\rho_B + 4\pi G\delta\rho_\alpha$$

$$\nabla^2(\Phi - \Psi) = 8\pi G\sigma_\alpha$$


α
a
Ψ − Φ
a
|δ|
a
Growth of α and Ψ − Φ





Skordis et al

$$\Omega_\phi \simeq 10^{-3}$$

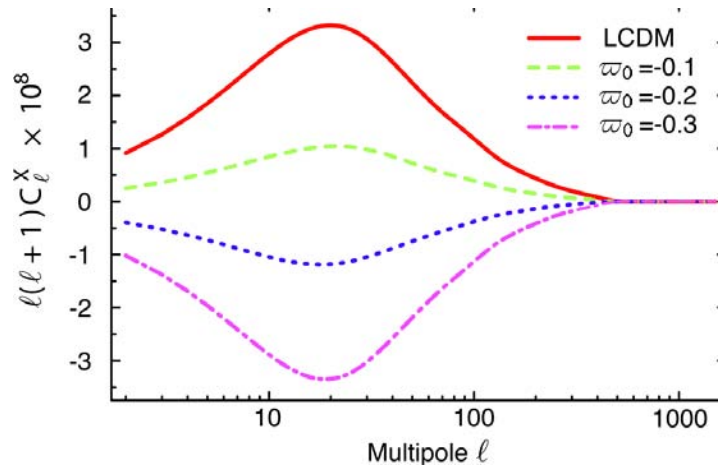
A smoking gun for modified gravity ?

Bertschinger

Caldwell et al

$$\Phi - \Psi \sim \omega \rho_{DE}$$

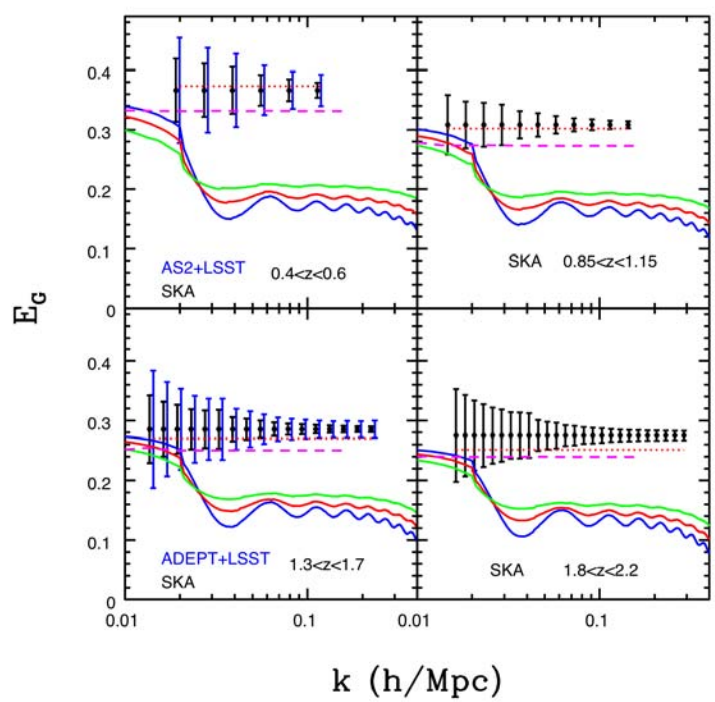
Cross correlate ISW and galaxies



$$\Phi - \Psi \neq 0$$

Cross correlate weak lensing and galaxies

Zhang et al



Main point:

Extra degrees of freedom in modified theories of gravity can play a significant role. This does <u>not</u> mean that they make a significant contribution to the overall energy density…

… and therefore:

Specific configurations (such as the "bullet cluster" or the discrepancy between globular clusters and dwarf galaxies) must be looked at <u>in detail</u> to see if extra degrees of freedom play a role.

PS: There may be a smoking gun …